What is an IMU?

An IMU is an Inertial Measurement Unit, generally being either 6-DoF or 9-DoF.

An 6-DoF IMU is made up of 3 gyroscopes and 3 accelerometers (one in each of X, Y, Z axes).

A 9-DoF IMU also includes a 3-axis magnetometer.

Gyroscopes measure angular velocity across an axis. Gyroscopes are not affected by external forces and acceleration meaning they work well under dynamic conditions when rotational velocities are high, however they drift significantly with regard to time. Thus the simplest filtering operation on gyroscope data is a high pass filter to remove low frequency drift.

Accelerometers measure effective acceleration along an axis. Accelerometers are affected by vibration and other external forces and hence cannot be directly used for computing angles / attitude accurately. However, accelerometers work well in static conditions as opposed to gyroscopes. Hence, the simplest filtering operation performed on accelerometer data is a low pass filter to remove dynamic noise from vibrations and other external factors.

Magnetometers measure the Earth's magnetic field which can be used to obtain orientation information as well. For indoor operation with lots of metal structure, magnetometers are generally inaccurate and excluded from data fusion. Magnetometers are usually called Digital Compasses as they can be directly used to compute the North Pole direction.

Gyroscope mathematical model

The mathematical model for a gyroscope is:

$$\omega = \hat{\omega} + \mathbf{b}_g \ + \mathbf{n}_g$$

Where:

- ω is the measured angular velocity from the gyroscope
- $\hat{\omega}$ is the latent ideal angular velocity we wish to recover
- $\mathbf{b}_g$ is the gyroscope bias which changes with time and other factors such as temperature
- $\mathbf{n}_g$ is the white gaussian gyroscope noise

The gyroscope bias is modelled as:

$$\dot{\mathbf{b}}_g = \mathbf{b}_{bg}(t) \sim \mathcal{N}(0, Q_g)$$

Where

- $\mathbf{b}_{bg}(t)$ is the bias random walk. i.e. the bias is modelled as a random walk process, evolving randomly over time
- Q_g is the covariance matrix which models gyroscope noise [Multivariate normal distribution > Covariance matrix]

Q_g quantifies the magnitude and relationships of the noise in the gyroscope bias

$$Q_g = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 \end{bmatrix}$$

The diagonal entries represent the variance of the noise in each axis and the off-diagonal entries represent the covariance between axis (how noise in one axis is related to noise in another)

Thus, the bias model suggests that at any given time, the gyroscope bias is a random value that evolves continuously over time. This evolution is unpredictable but statistically described by a multivariate Gaussian distribution with a mean of 0 (the bias changes are centered around zero, so on average, there's no systematic trend in any specific direction) and a covariance of Q_g (determines how much the bias can vary and how correlated the changes in different axes are)

$\mathbf{n}_g$ is a random noise term added to the gyroscope's output, representing measurement inaccuracies due to imperfections in the sensor. The term "white" refers to the fact that the value of this noise is independent of its value at any other time. There's no predictable trend or relationship in the noise over time.

But we still model this using a Gaussian distribution:

$$\mathbf{n}_g \sim \mathcal{N}(0, R_g)$$

Where:

- R_g specifies the variance of the noise along each axis and the correlation (if any) between the noise on different axes.

The white Gaussian noise in gyroscopes has the following characteristics:

- Zero mean: $\mathbb{E}[\mathbf{n}_g] = 0$, meaning the noise doesn't introduce a systematic offset
- Constant variance over time

Accelerometer mathematical model

The mathematical model for an accelerometer is:

$$\mathbf{a} = R^T(\hat{\mathbf{a}} - \mathbf{g}) + \mathbf{b}_a + \mathbf{n}_a$$

Where:

- $\mathbf{a}$ is the measured acceleration from the accelerometer
- $\hat{\mathbf{a}}$ is the latent ideal acceleration we wish to recover
- R is the orientation of the sensor in the world frame, i.e. the rotation matrix that describes the sensor orientation relative to the world frame
- R^T is the rotation matrix that converts a vector from the world frame into the sensor's frame
- $\mathbf{g}$ is the acceleration due to gravity in the world frame
- $\mathbf{b}_a$ is the accelerometer bias which changes with time and other factors such as temperature
- $\mathbf{n}_a$ is the white gaussian accelerometer noise

Thus the model is the latent acceleration is subtracted from gravity and then transformed into the sensor's frame and bias and noise are added.

The accelerometer bias is modelled as:

$$\dot{\mathbf{b}}_a = \mathbf{b}_{ba}(t) \sim \mathcal{N}(0, Q_a)$$

Where:

- $\mathbf{b}_{ba}(t)$ is the bias random walk. i.e. the bias is modelled as a random walk process, evolving randomly over time
- Q_a is the covariance matrix which models accelerometer noise

Here the orientation of the sensor is either known from external sources such as estimated by sensor fusion.