Configuration space

In path planning, a complete description of the geometry of a robot $\mathcal{A}$ and of a workspace $\mathcal{W}$ is provided.

The workspace $\mathcal{W} = \mathbb{R}^3$ is a static environment populated with obstacles. The goal is to find a collision-free path for $\mathcal{A}$ to move from an initial position and orientation to a goal position and orientation.

A complete specification of the location of every point on the robot geometry, or a configuration $\mathbf{q}$ must be provided.

The configuration space or C-space $(q \in \mathcal{C})$ is the space of all possible configurations. C-space is a useful way to abstract planning problems in a unified way. The advantage of this is that a robot with a complex geometric shape is mapped to a single point in the C-space.

The number of DoF of a robot system is the dimension of the C-space, or a minimum number of parameters needed to specify a configuration.

Obstacle region

The closed set $\mathcal{O} \subset \mathcal{W}$ represent the obstacle region, usually expressed as a collection of polyhedra, 3-D triangles, or piecewise-algebraic surfaces.

The closed set $\mathcal{A}(\mathbf{q}) \subset \mathcal{W}$ denote the set of points occupied by the robot when at configuration $\mathbf{q} \in \mathcal{C}$. The C-space obstacle region is defined as:

$$\mathcal{C}_{\text{obs}} = \{\mathbf{q} \in \mathcal{C} \mid \mathcal{A}(\mathbf{q}) \cap \mathcal{O} \neq \emptyset\}$$

Free space

Since $\mathcal{O}$ and $\mathcal{A}(\mathbf{q})$ are closed sets in $\mathcal{W}$, the obstacle region is a closed set in $\mathcal{C}$

The set of configurations that avoid collision are:

$$\mathcal{C}_{\text{free}} = \mathcal{C} \setminus \mathcal{C}_{\text{obs}}$$