Limited-angle Multi-energy CT using Joint Clustering Prior and Sparsity Regularization

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Outline

- Introduction
- Methodology
- Experiments
- Summary

Introduction
Motivation

- **Spectral (Multi-energy CT)**
  - ▷ differentiates materials
  - ▷ wide applications: extracting veins and kedney stones, detecting chemical elements (iodine, barium)
- **Problems**
  - ▷ The data required for reconstruction is multiplied
  - ▷ longer scan time, more cost, more dose
- **Goal**
  - ▷ Design an easy-to-implement scanning strategy
  - ▷ Lower does, cost and acquisition/reconstruction time (less angular views)
  - ▷ mitigate limited-angle artifacts

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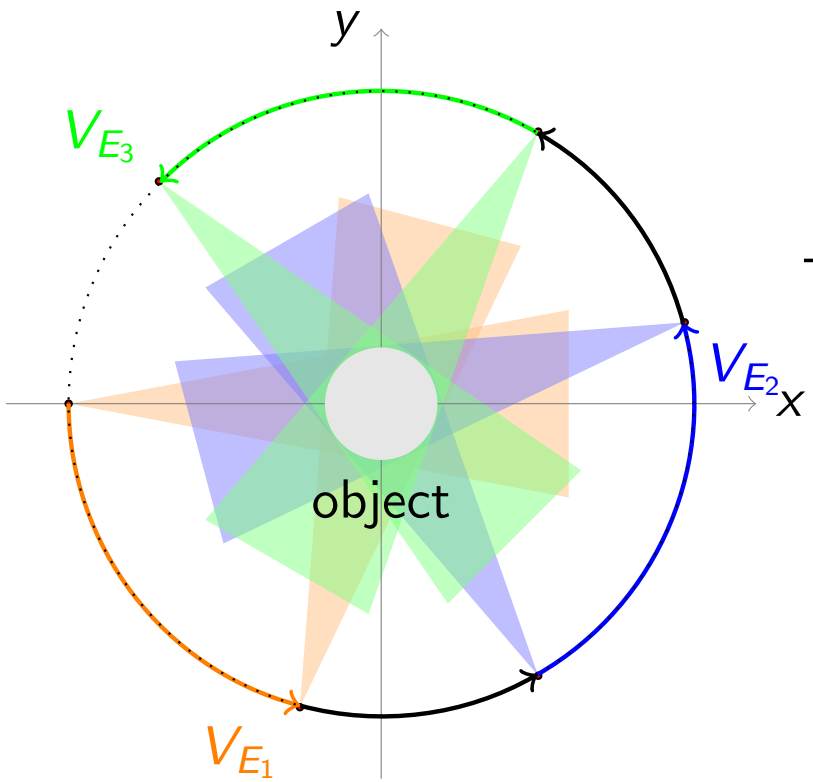
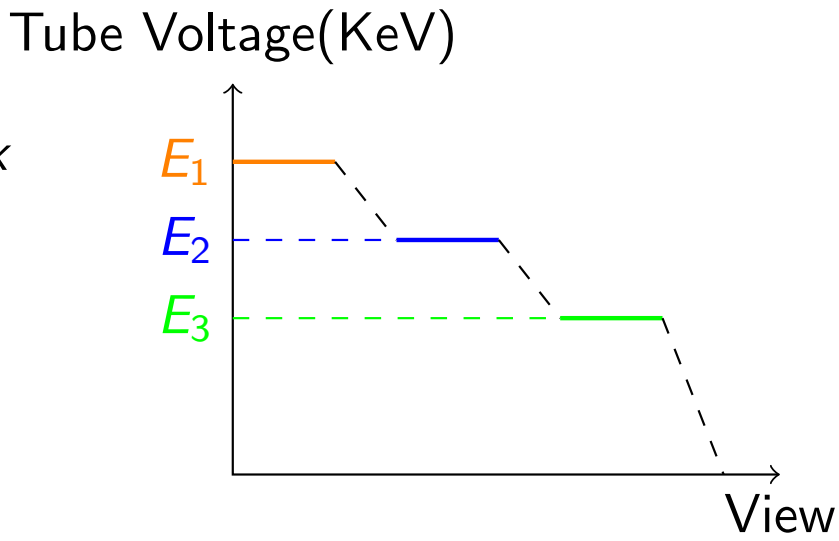
Methodology

Proposed data acquisition strategy: Multi-arc scan



(a) Scanning trajectory



(b) Voltage switching

Requirement: The angular coverage of all X-ray beams is no less than 180° plus fan beam angle.

Methodology
Main Difficulty: Severe limited-angle Problem

- Limited-angle problem:
  - ▷ violates data sufficient condition: 180° plus fan beam angle coverage
  - ▷ severe artifacts
  - ▷ hard to eliminate the artifacts using compressed sensing
- Solutions:
  - ▷ Independent reconstruction will unavoidablely encounter limited-angle artifacts.
  - ▷ Leverage the structural coherence of images at all energies
  - ▷ jointly reconstruct images at all energy channels

Methodology
Pre-reconstruction

Combine the projection data from all energies to pre-reconstruct a prior image.

$$\boldsymbol{\mu}_p = \arg\min_{\boldsymbol{\mu}} \|\mathbf{H}\boldsymbol{\mu} - \mathbf{p}\|_2^2$$

where

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_{N_E} \end{pmatrix} \qquad \mathbf{p} = \begin{pmatrix} \mathbf{p}_1/\|\mathbf{p}_1\|_1 \\ \mathbf{p}_2/\|\mathbf{p}_2\|_1 \\ \vdots \\ \mathbf{p}_{N_E}/\|\mathbf{p}_{N_E}\|_1 \end{pmatrix}$$

Each indpendent $\mathbf{H}_i$ is ill posed, but the combined $\mathbf{H}$ not.

Figure : Prior image

L. Shen and Y. Xing, Medical physics, 42(1): 282, 2015

Methodology
Clustering

- Assumption:
  1. The number of tissues within the object is limited;
  2. Each tissue is spatially continuous;
  3. The pixels within one tissues share an identical value.
- Clustering:
  1. $k-$means clustering on the prior image;
  2. choosing features $(x, y, \mu(x, y))$. (x, y): coordinates, $\mu(x, y)$: pixel values;
  3. The image is divided into k patches;
  4. These k patches keep some structural details.

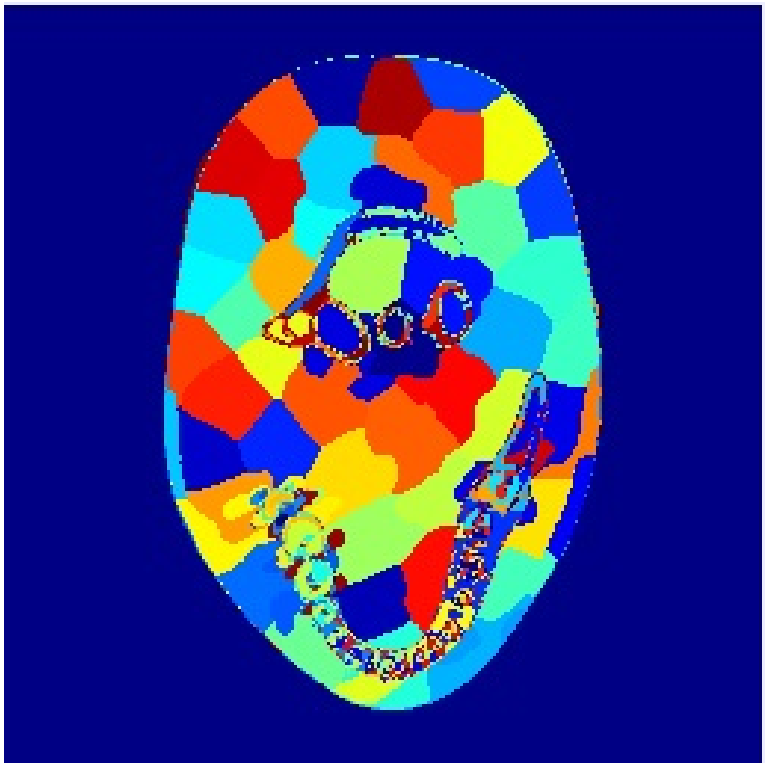


Figure : Clustering

Methodology

Structural information and constraint

Construct a constraint from the clustering

- k patches: $\Omega_c = \{(x, y) \mid \text{labeled with } c\}, c = 1, 2, \ldots, k.$
- construct a dictionary

$$\boldsymbol{\mu} = \boldsymbol{\Phi}\mathbf{a} = \sum_{c=1}^{k} a_c \boldsymbol{\varphi}_c \tag{1}$$

where

$\boldsymbol{\Phi} = (\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_k) \in \mathcal{R}^{N \times k}$ dictionary matrix

$\boldsymbol{\varphi}_i$ basis vector (element)

$$\varphi_{ij} = \mathbf{I}_{i \in \Omega_j} = \begin{cases} 1 & i \in \Omega_j \\ 0 & i \notin \Omega_j \end{cases}$$

Methodology
Reconstruction Model

Joint Clustering Prior and Sparsity Regularization (CPSR) model

$$\arg\min_{\boldsymbol{\mu}} \frac{1}{2}\|\mathbf{H}\boldsymbol{\mu}-\mathbf{p}\|_2^2+\lambda\|W\boldsymbol{\mu}\|_1 \quad s.t. \quad \boldsymbol{\mu}=\boldsymbol{\Phi}\mathbf{a} \tag{2}$$

Incorporates the structural constraint into general compressed sensing frame.

- $\|\mathbf{H}\boldsymbol{\mu}-\mathbf{p}\|_2^2$ linear projection model
- $\|W\boldsymbol{\mu}\|_1$ sparse constraint. W denotes wavelet transform.
- $\boldsymbol{\mu}=\boldsymbol{\Phi}\mathbf{a}$ structural constraint

Augmented Lagrangian Function:

$$L(\boldsymbol{\mu},\mathbf{a},\mathbf{z},\mathbf{y}_1,\mathbf{y}_2)=\underbrace{\frac{1}{2}\|\mathbf{H}\boldsymbol{\mu}-\mathbf{p}\|_2^2}_{\text{fedility}}+\underbrace{\frac{\rho_1}{2}\|\boldsymbol{\mu}-\boldsymbol{\Phi}\mathbf{a}+\mathbf{y}_1\|_2^2}_{\text{structural constraint}}+\underbrace{\lambda\|\mathbf{z}\|_1+\frac{\rho_2}{2}\|\mathbf{z}-\mathbf{W}\boldsymbol{\mu}+\mathbf{y}_2\|_2^2}_{\text{sparse constraint}}$$

Solution: Alternating direction method of multipliers (ADMM)

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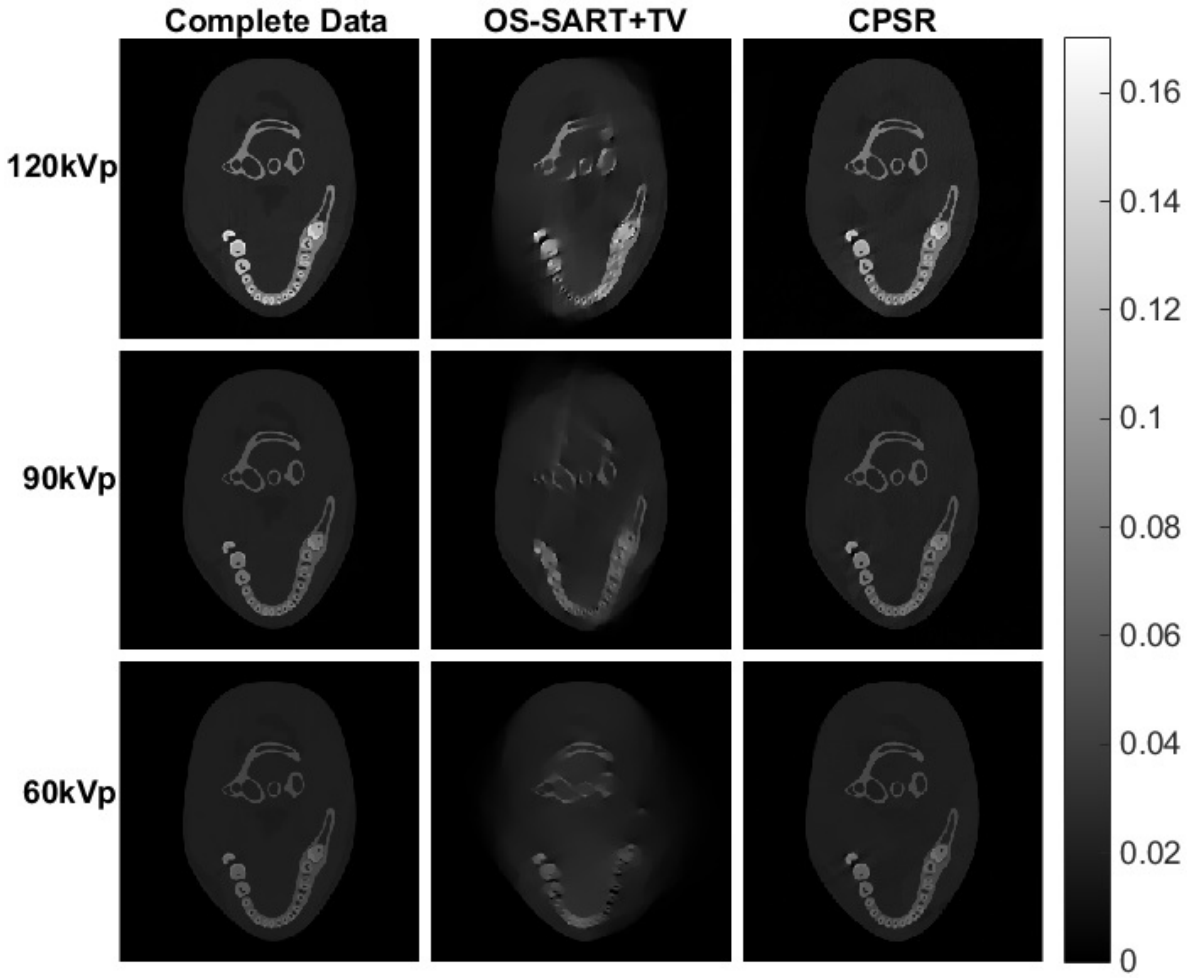
Numerical Experiments
Settings

- Scan
  - modality: Fan beam
  - X-ray energies: 120kVp, 90kVp, 60kVp
  - Projection data for each energy: 75 (views) x 320 (detectors)
- Reconstruction
  - Image size: 256×256
  - Clustering number: $k = 100$
  - Reconstruction algorithm: OS-SART
  - Sparse constraint: Total variation in wavelet space

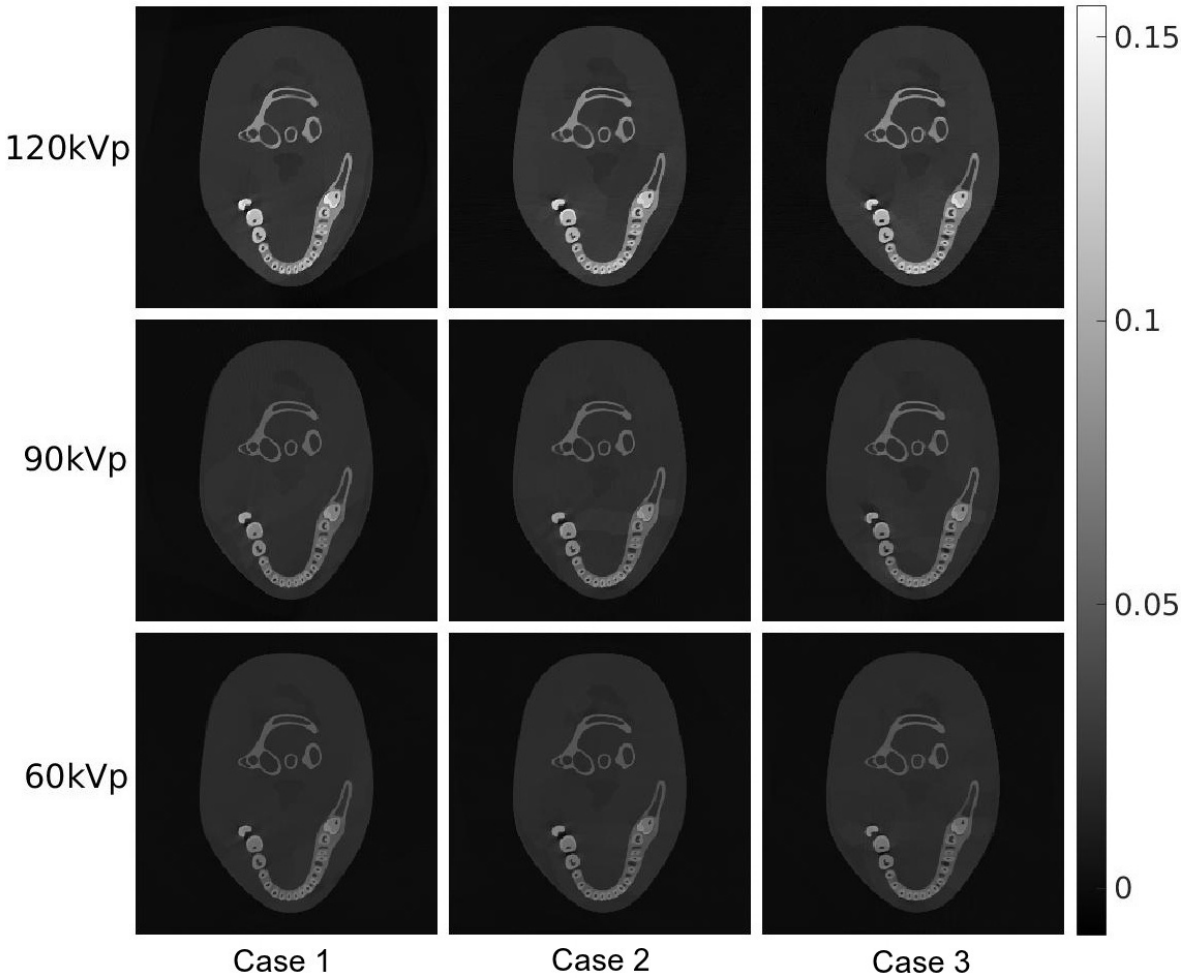
Numerical experiments
Reconstruction results



- Complete Data: Independent reconstruction from 180° plus fan beam angle projection data
- OS-SART: Independent reconstruction using OS-SART + TV constraint from 75° angular coverage projection data.
- View 3: CPSR method from 75° angular coverage projection data.

Numerical experiments

Impact of view selection on reconstruction



Compare results from different view configurations:

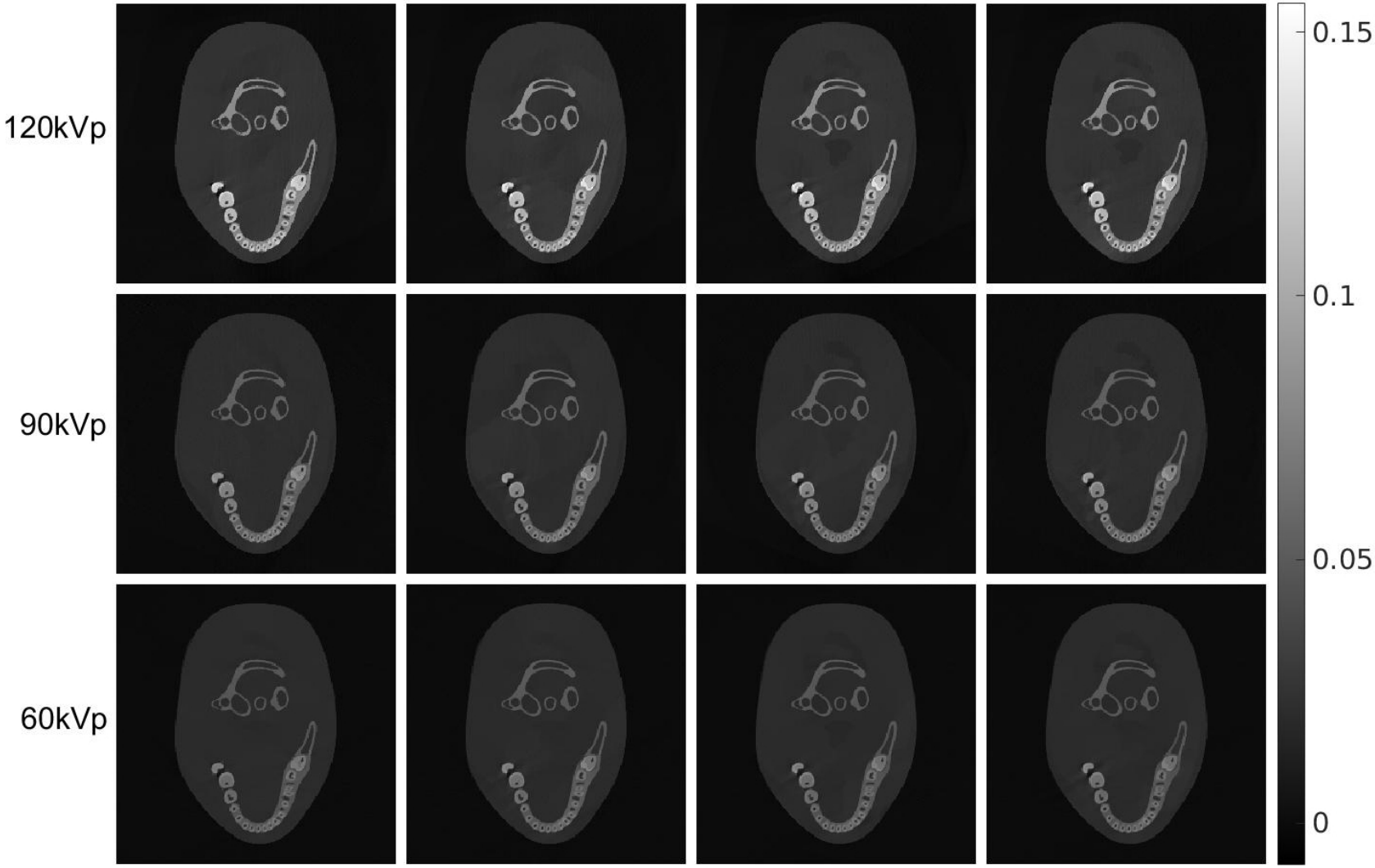
Case 1: $V_{120kVp} = [0°, 75°]$
$V_{90kVp} = [120°, 195°]$
$V_{60kVp} = [240°, 315°]$

Case 2: $V_{120kVp} = [30°, 105°]$
$V_{90kVp} = [150°, 225°]$
$V_{60kVp} = [270°, 345°]$

Case 3: $V_{120kVp} = [60°, 135°]$
$V_{90kVp} = [180°, 255°]$
$V_{60kVp} = [300°, 375°]$

Numerical experiments

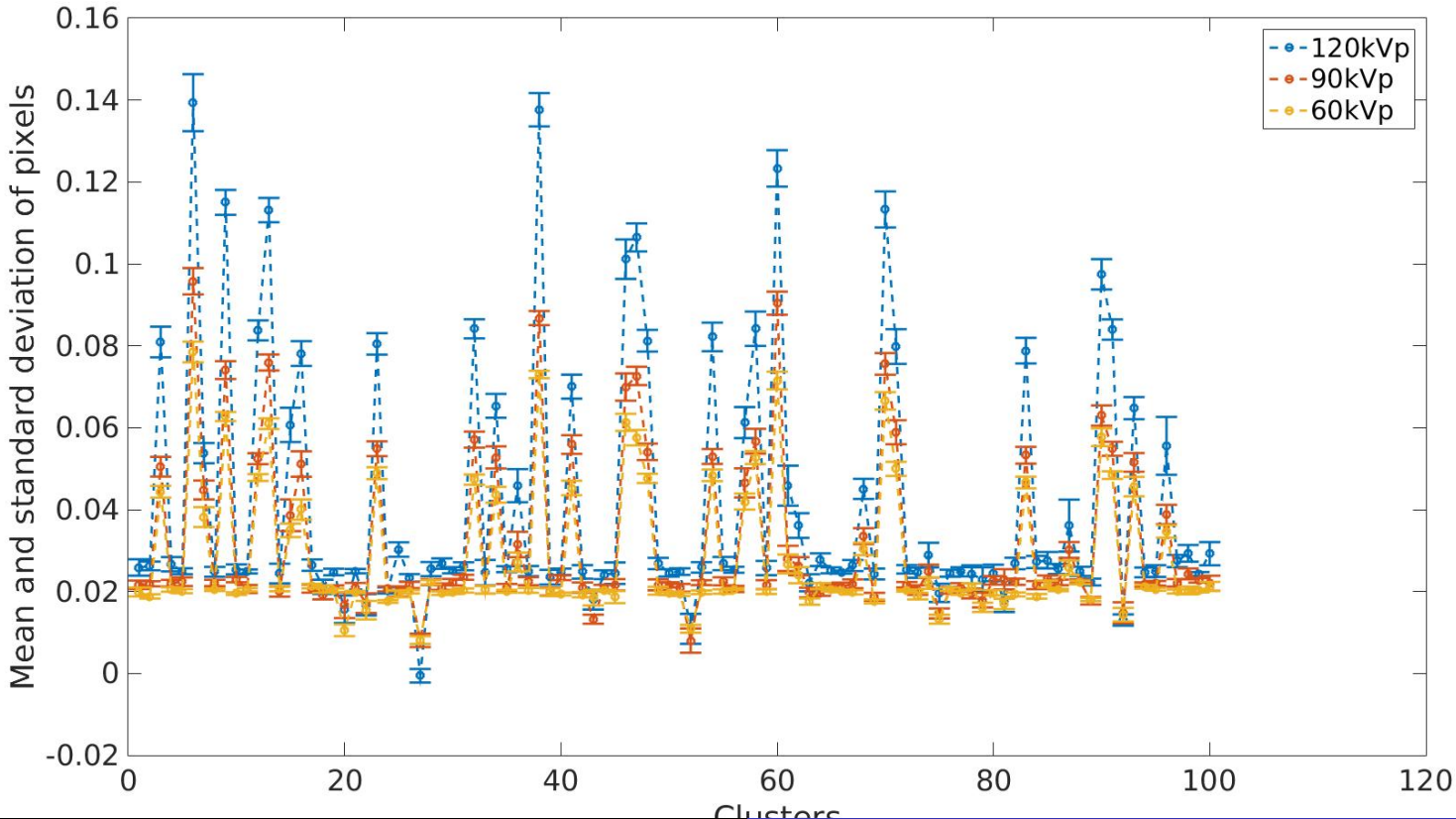
Impact of the clustering number on reconstruction



Numerical experiments

Impact of the clustering constraint on reconstruction

- Assumption of identical pixel value within one cluster may be too strong.
- Our method is flexible and tolerates some variation within each cluster by assigning a weight on the prior structural constraint term.



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Related works

- Multi-energy CT scan and reconstruction [Shen and Xing 2015]
- Compressed sensing
  1. sparsity[Sidky, Kao, and Pan 2006; Sidky and Pan 2008]
  2. low rank[Gao et al. 2011]
- Limited-angle CT[Jin et al. 2012]
- Sparse dictionary learning[Cao and Xing 2013]

Summary
Contributions

- Largely reduce the projection data required.

From 180° plus fan beam angle to at least 75°.

- Design and implement a reconstruction approach using joint clustering prior and sparsity in wavelet space.
- Solve the limited angle problem by leveraging the coherence among all data at different energies.
- Our method enable flexible angular configuration and broaden spectral CT system design.

Main References


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