Limited-angle Multi-energy CT using Joint Clustering Prior and Sparsity Regularization

Huayu Zhang, Yuxiang Xing

Tsinghua University, University of Wisconsin-Madison

Outline

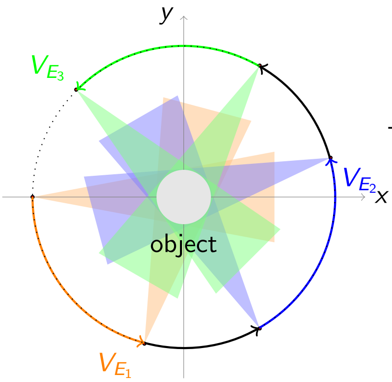
- Introduction
- Methdology
- Experiments
- Summary

- **Spectral (Multi-energy CT)**
  - differentiates materials
  - wide applications: extracting veins and kedney stones, detecting chemical elements (iodine, barium)
- **Problems**
  - The data required for reconstruction is multiplied
  - longer scan time, more cost, more dose
- **Goal**
  - Design an easy-to-implement scanning strategy
  - Lower does, cost and acquisition/reconstruction time (less angular views)
  - mitigate limited-angle artifacts

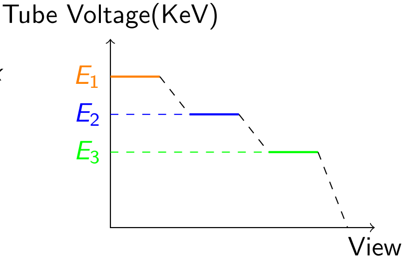
- Limited-angle problem:
  - violates data sufficient condition: 180° plus fan beam angle coverage
  - severe artifacts
  - hard to eliminate the artifacts using compressed sensing
- Solutions:
  - Independent reconstruction will unavoidably encounter limited-angle artifacts.
  - Leverage the structural coherence of images at all energies
  - jointly reconstruct images at all energy channels

Outline

- Introduction
- Methdology
- Experiments
- Summary



(a) Scanning trajectory



(b) Voltage switching

Requirement: The angular coverage of all X-ray beams is no less than 180° plus fan beam angle.

Combine the projection data from all energies to pre-reconstruct a prior image.

$$\boldsymbol{\mu}_p = \arg\min_{\boldsymbol{\mu}} \|\mathbf{H}\boldsymbol{\mu} - \mathbf{p}\|_2^2$$

where

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_{N_E} \end{pmatrix} \qquad \mathbf{p} = \begin{pmatrix} \mathbf{p}_1/\|\mathbf{p}_1\|_1 \\ \mathbf{p}_2/\|\mathbf{p}_2\|_1 \\ \vdots \\ \mathbf{p}_{N_E}/\|\mathbf{p}_{N_E}\|_1 \end{pmatrix}$$

Each indpendent $\mathbf{H}_i$ is ill posed, but the combined $\mathbf{H}$ not.

Figure : Prior image

- Assumption:
  1. The number of tissues within the ojbect is limited
  2. Each tissue is spatially continuous
  3. The pixels within one tissues share an identical value
- Clustering:
  1. $k-$means clustering on the prior image
  2. choosing features $(x, y, \mu(x, y))$. (x, y): coordinates, $\mu(x, y)$: pixel values
  3. The image is divided into k patches
  4. These k patches keep some structural details

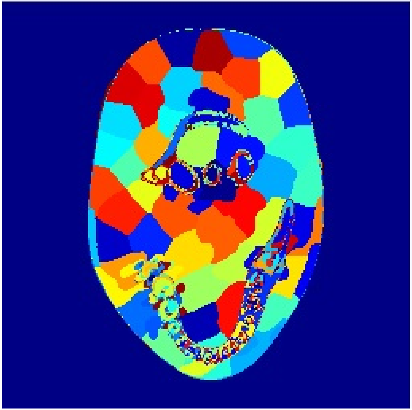


Figure : Clustering

Construct a constraint from the clustering

- k patches: $\Omega_c = \{(x, y) \mid \text{labeled with } c\}, c = 1, 2, \ldots, k$.
- construct a dictionary

$$\boldsymbol{\mu} = \mathbf{\Phi a} = \sum_{c=1}^{k} a_c \boldsymbol{\varphi}_c \tag{1}$$

where
$\mathbf{\Phi} = (\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_k) \in \mathcal{R}^{N \times k}$ dictionary matrix
$\boldsymbol{\varphi}_i$ basis vector (element)
$\varphi_{ij} = \mathbf{I}_{i \in \Omega_j} = \begin{cases} 1 & i \in \Omega_j \\ 0 & i \notin \Omega_j \end{cases}$

Joint Clustering Prior and Sparsity Regularization (CPSR) model

$$\arg\min_{\boldsymbol{\mu}} \frac{1}{2}\|\mathbf{H}\boldsymbol{\mu} - \mathbf{p}\|_2^2 + \lambda\|W\boldsymbol{\mu}\|_1 \quad s.t. \quad \boldsymbol{\mu} = \boldsymbol{\Phi}\mathbf{a} \tag{2}$$

Incorporates the structural constraint into general compressed sensing frame.

- $\|\mathbf{H}\boldsymbol{\mu} - \mathbf{p}\|_2^2$ linear projection model
- $\|W\boldsymbol{\mu}\|_1$ sparse constraint, W denotes wavelet transform.
- $\boldsymbol{\mu} = \boldsymbol{\Phi}\mathbf{a}$ structural constraint

Augmented Lagrangian Function:

$$L(\boldsymbol{\mu}, \mathbf{a}, \mathbf{z}, \mathbf{y}_1, \mathbf{y}_2) = \underbrace{\frac{1}{2}\|\mathbf{H}\boldsymbol{\mu} - \mathbf{p}\|_2^2}_{\text{fedility}} + \underbrace{\frac{\rho_1}{2}\|\boldsymbol{\mu} - \boldsymbol{\Phi}\mathbf{a} + \mathbf{y}_1\|_2^2}_{\text{structural constraint}} + \underbrace{\lambda\|\mathbf{z}\|_1 + \frac{\rho_2}{2}\|\mathbf{z} - \mathbf{W}\boldsymbol{\mu} + \mathbf{y}_2\|_2^2}_{\text{sparse constraint}}$$

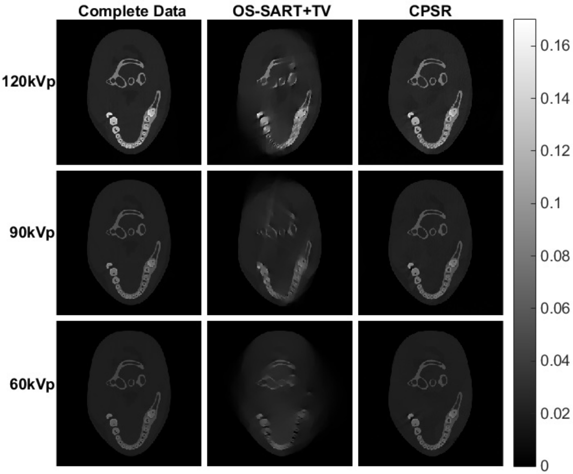
Solution: Alternating direction method of multipliers (ADMM)

Outline

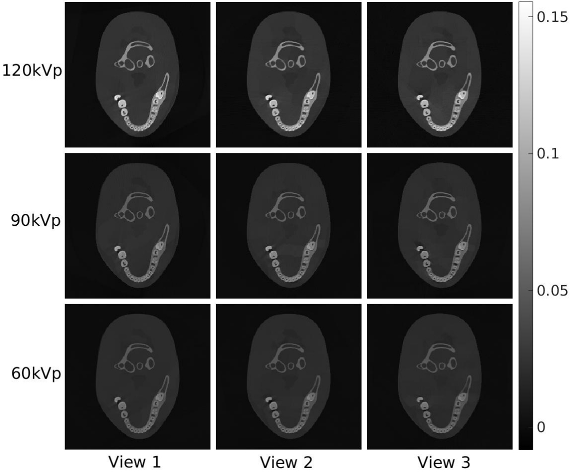
- Introduction
- Methdology
- Experiments
- Summary

- Scan
  - modality: Fan beam
  - X-ray energies: 120kVp, 90kVp, 60kVp
  - Projection data for each energy: 75 (views) x 320 (detectors)
- Reconstruction
  - Image size: 256×256
  - Clustering number: $k = 100$
  - Reconstruction algorithm: OS-SART
  - Sparse constraint: Total variation

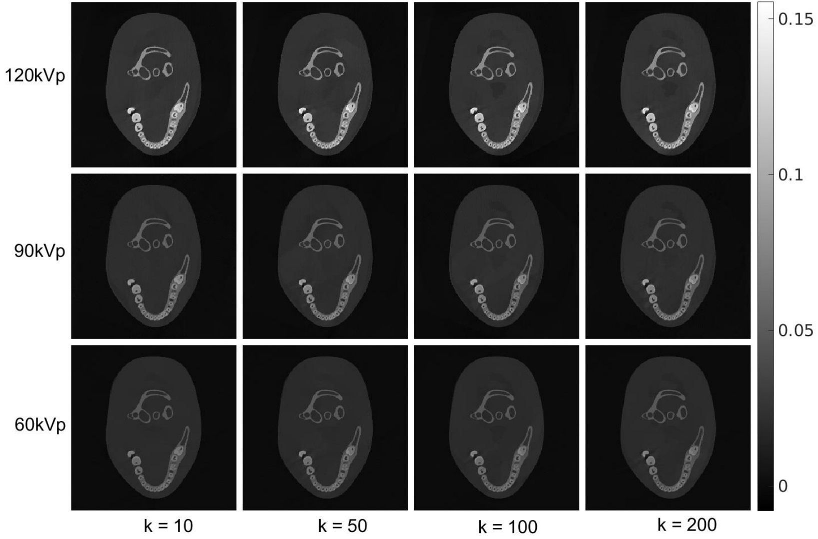
Numerical experiments
Reconstruction results



- Complete Data: Independent reconstruction from 180° plus fan beam angle projection data
- OS-SART: Independent reconstruction using OS-SART from 75° angular coverage projection data.
- View 3: CPSR method from 75° angular coverage projection data.

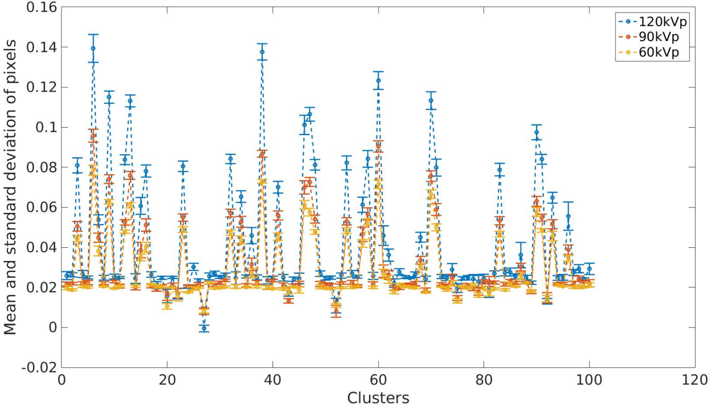


- View 1:
  $V_{120kVp} = [0^\circ, 75^\circ]$
  $V_{90kVp} = [120^\circ, 195^\circ]$
  $V_{60kVp} = [240^\circ, 315^\circ]$
- View 2:
  $V_{120kVp} = [30^\circ, 105^\circ]$
  $V_{90kVp} = [150^\circ, 225^\circ]$
  $V_{60kVp} = [270^\circ, 345^\circ]$
- View 3:
  $V_{120kVp} = [60^\circ, 135^\circ]$
  $V_{90kVp} = [180^\circ, 255^\circ]$
  $V_{60kVp} = [300^\circ, 375^\circ]$



120kVp
90kVp
60kVp
k = 10
k = 50
k = 100
k = 200
0.15
0.1
0.05
0

- Assumption of identical pixel value within one cluster may be too strong.
- Our method is flexible and tolerates some variation within one cluster by assigning a weight on the prior structural constraint term.



Outline

- Introduction
- Methdology
- Experiment
- Summary

- Multi-energy CT scan and reconstruction L. Shen *et al* [1]
- Compressed sensing
  1. sparsity, E.Y. Sidky *et al* [2, 3]
  2. low rank, H. Gao *et al* [4]
- Limited-angle CT, X. Jin *et al* [5]
- Sparse dictionary learning, M. Cao *et al* [6]

- Largely reduce the projection data required.
  From 180° plus fan beam angle to at least 75°.
- Design and implement a reconstruction approach.
- Solve the limited angle problem by leveraging the coherence among all data at different energies.

References

L. Shen and Y. Xing, "Multienergy ct acquisition and reconstruction with a stepped tube potential scan," *Medical physics*, vol. 42, no. 1, pp. 282–296, 2015.

E. Y. Sidky, C.-M. Kao, and X. Pan, "Accurate image reconstruction from few-views and limited-angle data in divergent-beam ct," *Journal of X-ray Science and Technology*, vol. 14, no. 2, pp. 119–139, 2006.

E. Y. Sidky and X. Pan, "Image reconstruction in circular cone-beam computed tomography by constrained, total-variation minimization," *Physics in Medicine and Biology*, vol. 53, no. 17, p. 4777, 2008.

H. Gao, H. Yu, S. Osher, and G. Wang, "Multi-energy ct based on a prior rank, intensity and sparsity model (prism)," *Inverse problems*, vol. 27, no. 11, p. 115012, 2011.

X. Jin, L. Li, Z. Chen, L. Zhang, and Y. Xing, "Anisotropic total variation minimization method for limited-angle ct reconstruction," in *SPIE Optical Engineering+ Applications*, pp. 85061C–85061C, International Society for Optics and Photonics, 2012.

M. Cao and Y. Xing, "Limited angle reconstruction with two dictionaries," in *Nuclear Science Symposium and Medical Imaging Conference (NSS/MIC), 2013 IEEE*, pp. 1–4, IEEE, 2013.