OTE: Ohjelmointitekniikka
Programming Techniques

course homepage: `http://www.cs.uku.fi/~mnykanen/OTE/`

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Solution sketch 1. This is by Gries (1981, Exercise 7.3 and its solution):

The assumption $\psi \Longrightarrow \psi'$ is logically equivalent to $\psi \Longleftrightarrow (\psi \wedge \psi')$.

(The intuition behind this can be described as follows: Let ψ be "it rains" and ψ' be "you get wet". Then the assumption is "if it rains, then you get wet". Similarly, the forward part $\psi \Longrightarrow (\psi \wedge \psi')$ of the equivalence is "if it rains, then it rains and you get wet" which makes sense given the assumption while the opposite part $\psi \Longleftarrow (\psi \wedge \psi')$ is "if it rains and you get wet, then it rains" which makes sense in general.)

This lets us calculate as follows:

$$\begin{aligned}
\mathrm{wp}(S,\psi) &= \mathrm{wp}(S,\psi \wedge \psi') && \text{(by their equivalence explained above)}\\
&\Leftrightarrow \mathrm{wp}(S,\psi) \wedge \mathrm{wp}(S,\psi') && \text{(by Theorem 7)}\\
&\Rightarrow \mathrm{wp}(S,\psi') && \text{(by logic, like in the opposite part above).}
\end{aligned}$$

This calculation shows the desired

$$\mathrm{wp}(S,\psi) \Rightarrow \mathrm{wp}(S,\psi')$$

when we look at its starting and ending points and the steps between. The green steps preserve equality, while the red step introduces the one-directional implication. Its use is okay, since it occurrs on the top level of the formula, which is inside an even number of negations, namely zero.

Solution sketch 2. Applying wp monotonicity (Theorem 8) to the clearly true formula

$$\phi \Longrightarrow (\phi \vee \phi')$$

gives

$$\mathrm{wp}(S,\phi) \Longrightarrow \mathrm{wp}(S,\phi \vee \phi'). \tag{1}$$

Similarly we can also get

$$\mathrm{wp}(S,\phi') \Longrightarrow \mathrm{wp}(S,\phi \vee \phi'). \tag{2}$$

Since formulae (1) and (2) have the same consequent (right side), propositional logic permits joining their antecedents (left sides) with '$\vee$' to get the desired

$$(\mathrm{wp}(S,\phi) \vee \mathrm{wp}(S,\phi')) \Longrightarrow \mathrm{wp}(S,\phi \vee \phi').$$

(The intuition behind this joining is: "If it rains, then you get wet. If it snows, then you get wet. So if it rains or snows, then you get wet.")

Solution sketch 3. Recall first that $f \in \mathrm{wp}(S, \psi)$ means in natural language that "if S is started in state f then it is *guaranteed* to terminate in some state $f' \in \mathrm{Sts}(\psi)$".

Let $f \in \mathrm{Sts}(\mathrm{wp}(S, \phi \vee \phi'))$ be arbitrary. Since S is deterministic, the terminating state f' corresponding to f is unique. If $f' \in \mathrm{Sts}(\phi)$ then we are therefore *guaranteed* to terminate in a state where ϕ is true. (Note that this is the part which cannot be assured for nondeterministic code S': some executions terminate with ϕ true, others with ϕ' true, but in general we *cannot guarantee* which one we will get when we start S'.) Hence we have $f \in \mathrm{Sts}(\mathrm{wp}(S, \phi))$ by the natural language meaning above.

Similarly we get also that if $f' \in \mathrm{Sts}(\phi')$ then $f \in \mathrm{Sts}(\mathrm{wp}(S, \phi'))$.

Hence $f \in \mathrm{Sts}(\mathrm{wp}(S, \phi)) \cup \mathrm{Sts}(\mathrm{wp}(S, \phi'))$. By set theory, we have therefore shown that

$$\mathrm{Sts}(\mathrm{wp}(S, \phi \vee \phi')) \subseteq \mathrm{Sts}(\mathrm{wp}(S, \phi)) \cup \mathrm{Sts}(\mathrm{wp}(S, \phi')).$$

Translating this from set theory into logic gives the desired

$$\mathrm{wp}(S, \phi \vee \phi') \Longrightarrow \mathrm{wp}(S, \phi) \vee \mathrm{wp}(S, \phi').$$

See Figure 2 about this translation.

Solution sketch 4. The first assumption is

$$\psi \Longrightarrow \phi_{\mathrm{pre}}$$

and the second assumption is

$$\phi_{\mathrm{pre}} \Longrightarrow \mathrm{wp}(S, \phi_{\mathrm{post}})$$

by View (4), so logically we can "follow the arrows" to get

$$\psi \Longrightarrow \mathrm{wp}(S, \phi_{\mathrm{post}})$$

which is the claim by View (4).

(The intuition behind this following the arrows is: "If it rains, then you get wet. If you get wet, then you get the flu. So if it rains, then you get the flu.")

Solution sketch 5. The first assumption is $\phi_{\mathrm{pre}} \Longrightarrow \mathrm{wp}(S, \phi_{\mathrm{post}})$ by View (4). Applying wp monotonicity (Theorem 8) to the second assumption gives $\mathrm{wp}(S, \phi_{\mathrm{post}}) \Longrightarrow \mathrm{wp}(S, \psi)$. Logically then (by following the arrows as in sketch 4) $\phi_{\mathrm{pre}} \Longrightarrow \mathrm{wp}(S, \psi)$ which is the claim by View (4).

Solution sketch 6. The first assumption is $\phi \Longrightarrow \mathrm{wp}(S_1, \varphi)$ by View (4). The second assumption is in turn $\varphi \Longrightarrow \mathrm{wp}(S_2, \psi)$ by View (4). Applying wp monotonicity (Theorem 8) to it gives $\mathrm{wp}(S_1, \varphi) \Longrightarrow \mathrm{wp}(S_1, \mathrm{wp}(S_2, \psi))$. Its consequent is $\mathrm{wp}(S_1; S_2, \psi)$ by Definition (7). Logically then (again by following the arrows as in sketch 4) $\phi \Longrightarrow \mathrm{wp}(S_1; S_2, \psi)$ which is the claim by View (4).

References

David Gries. *The Science of Programming.* Springer-Verlag, 1981.