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UNIVERSITI TEKNOLOGI MALAYSIA

FACULTY OF COMPUTING

SEMESTER 1 2023/2024

SECI1013 – DISCRETE STRUCTURE

SECTION 3

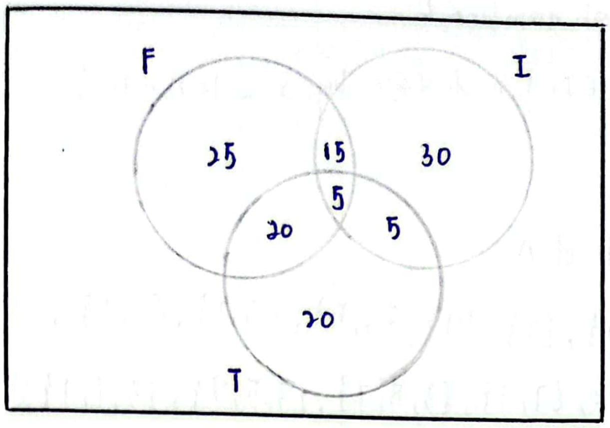
ASSIGNMENT 1 – CHAPTER 1

LECTURER: DR. NOR HAIZAN BT MOHAMED RADZI

STUDENT NAME	MATRIC NO
LAU YEE WEN	A23CS0099
GUI KAH SIN	A23CS0080

Assignment

1. a) i)



F = students have Facebook account

I = students have Instagram account

T = students have Twitter account

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Total students = 150
Total Facebook users = 65
Total Instagram users = 55
Total Twitter users = 50

Facebook only = 25
Instagram only = 30
Twitter only = 20

$F \cap I = 15$

$F \cap I \cap T = 5$

$I \cap T = 55 - 30 - 15 - 5$
$= 5$

$F \cap T = 65 - 25 - 15 - 5$
$= 20$

ii) students do not have an account in any three social networks

$= (F \cup I \cup T)'$

$= 150 - (25 + 20 + 15 + 5 + 5 + 30 + 20)$

$= 30$

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iii) students have exactly two social networks

$= (F \cap I) + (I \cap T) + (T \cap F)$

$= 15 + 5 + 20$

$= 40$

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iv) students have social media account other than Facebook

$= F'$

$= 30 + 5 + 20$

$= 55$

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b) $A = \{n \in N \mid n \text{ odd}, 1 < n < 10\}$, $N = \{\text{natural number}\}$

$B = \{n \in N \mid n \text{ is prime}, 1 < n < 10\}$, $C = \{n \in N \mid n \text{ divisible by } 3, 1 < n < 10\}$

i) $A = \{3,5,7,9\}$

$B = \{2,3,5,7\}$

$C = \{3,6,9\}$

$|A| = 4$

$|B| = 4$

$|C| = 3$

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ii) proper subsets of A

$\emptyset, \{3\}, \{5\}, \{7\}, \{9\}, \{3,5\}, \{3,7\}, \{3,9\},$
$\{5,7\}, \{5,9\}, \{7,9\}, \{3,5,7\}, \{3,5,9\}, \{3,7,9\}, \{5,7,9\}$

number of proper subsets of A $= 2^4 - 1$
$= 16 - 1$
$= 15$

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iii) $C = \{3,6,9\}$

$B = \{2,3,5,7\}$

$C \times B = \{(3,2), (3,3), (3,5), (3,7), (6,2), (6,3), (6,5), (6,7), (9,2), (9,3),$
$(9,5), (9,7)\}$

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2. a) $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

Truth table:

p	q	$\sim(p \vee q)$	$(\sim p \wedge q)$	$\sim(p \vee q) \vee (\sim p \wedge q)$	$\sim p$
T	T	F	F	F	F
T	F	F	F	F	F
F	T	F	T	T	T
F	F	T	F	T	T

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Logic property law:

$$\begin{aligned} \sim(p \vee q) \vee (\sim p \wedge q) &= (\sim p \wedge \sim q) \vee (\sim p \wedge q) && \text{De Morgan's Laws} \\ &= \sim p \wedge (\sim q \vee q) && \text{Distributive Laws} \\ &= \sim p \wedge U \\ &= \sim p \end{aligned}$$

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$\therefore \sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

(b) (i) $(r \wedge q) \rightarrow p$

(ii) $\sim(r \vee q) \rightarrow \sim p$

$\equiv (\sim r \wedge \sim q) \rightarrow \sim p$

(iii) $\sim p \rightarrow \sim(r \vee q)$

$\equiv \sim p \rightarrow (\sim r \wedge \sim q)$

2

2

2

(c) Negation of $\forall x (x^2+2x-3=0) = \exists x (\sim(x^2+2x-3=0))$

The domain of discourse is integer $x \in Z$

If $x=2$, $x^2+2x-3 = (2)^2+2(2)-3$

$= 5 \ (\neq 0)$

(shown)

$\therefore$ The statement $\exists x (\sim(x^2+2x-3=0))$ is TRUE.

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(d) Let $P(x)$: x is student who can speak Rusian.

$Q(x)$: x is student who know C++.

The domain of discourse consist of all students at school.

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(i) $\exists x (P(x) \wedge \sim Q(x))$ 2

(ii) $\forall x (P(x) \vee Q(x))$ 2

(iii) $\sim(\exists x (P(x) \vee Q(x)))$ 1

3-(a) For all integer, if a^2-3b is even, then a is even and b is even.

Let $P(x)$: a^2-3b is even.

$Q(x)$: a is even and b is even

$\forall x\,(P(x) \rightarrow Q(x))$

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$P(x) \rightarrow Q(x) \equiv \sim Q(x) \rightarrow \sim P(x)$

Assume $\sim Q(x)$ is true, if a is odd or b is odd, then a^2-3b is odd.

- Case 1: a is even and b is odd
- Case 2: a is odd and b is even
- Case 3: a is odd and b is odd

Case 1: if a is even and b is odd, let $a=2k$, $b=2n+1$

$$\begin{aligned} a^2-3b &= (2k)^2 - 3(2n+1) \\ &= 4k^2 - 6n - 3 \\ &= 4k^2 - 6n - 4 + 1 \\ &= 2(2k^2 - 3n - 2) + 1 \end{aligned}$$

Let $m = 2k^2 - 3n - 2$

$a^2 - 3b = 2m + 1$ (odd)

$\therefore$ Since $2m+1$ is an odd integer, thus the statement is true.

$\sim Q(x)$ is true, $\sim P(x)$ is true, $\sim Q(x) \rightarrow \sim P(x)$ is true.

Case 2 : if a is odd and b is even, let $a = 2n+1$, $b = 2k$

$$a^2 - 3b = (2n+1)^2 - 3(2k)$$
$$= 4n^2 + 4n + 1 - 6k$$
$$= 2(2n^2 + 2n - 3k) + 1$$

Let $m = 2n^2 + 2n - 3k$

$$a^2 - 3b = 2m + 1 \quad \text{(odd)}$$

∴ Since a^2-3b is an odd integer, thus the statement is true.

$\sim Q(x)$ is true, $\sim P(x)$ is true, $\sim Q(x) \rightarrow \sim P(x)$ is true.

Case 3 : if a is odd and b is odd, let $a = 2k+1$, $b = 2m+1$

$$a^2 - 3b = (2k+1)^2 - 3(2m+1)$$
$$= 4k^2 + 4k + 1 - 6m - 3$$
$$= 4k^2 + 4k - 6m - 2$$
$$= 2(2k^2 + 2k - 3m - 1)$$

Let $m = 2k^2 + 2k - 3m - 1$

$$a^2 - 3b = 2m \quad \text{(even)}$$

∴ Since a^2-3b is an even integer, thus the statement is false.

$\sim Q(x)$ is true, $\sim P(x)$ is false, $\sim Q(x) \rightarrow \sim P(x)$ is false.

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∴ The theorem is FALSE.