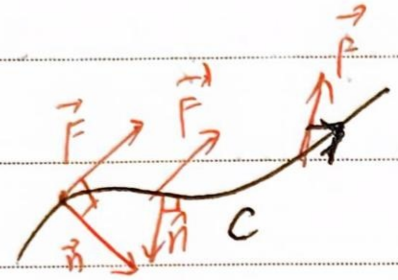
2025.2.20. Flux LEC 23

C plane $\vec{F}$ vctor Field

Flux of $\vec{F}$ acceross C is $\boxed{\int_C \vec{F}\cdot\hat{n}\,ds}$

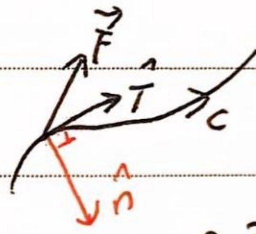


$\hat{n}$: unit normal vector to C, 90° degree closewise

If break C into small pieces Δs:

$$\text{Flux} = \lim_{\Delta s \to 0} \left(\sum \vec{F}\,\hat{n}\cdot\Delta s\right)$$

Work: $\int_C \vec{F}\cdot d\vec{r} = \underline{\int_C \vec{F}\cdot\hat{T}\,ds}$



summing tagential component of $\vec{F}$

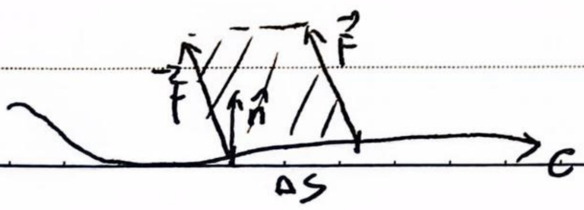
Flux: $\int_C \vec{F}\cdot\hat{n}\,ds$ — summing normal component of $\vec{F}$

what's interpertation?

for $\vec{F}$ a velocity field

flux measure how much fluid pass through C per unit time (通量)

↓ $\propto$ contents of a parallelogram



Area = base · height

$= \Delta s \cdot (\vec{F}\cdot\hat{n})$

Mo | Tu | We | Th | Fr | Sa | Su

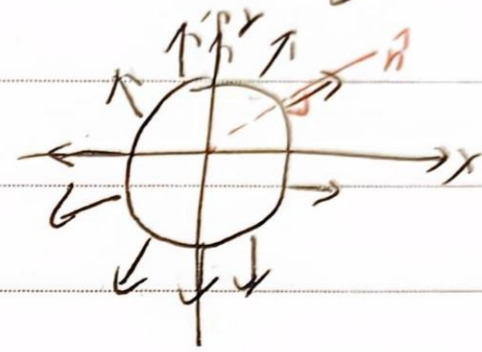
Memo No. ______

Date / /

What flows across C left to right is counted positively right-to-left negatively

Examples: C is a circle radius a at origin

counterclockwise



$\vec{F} = x\hat{i} + y\hat{j}$

along C, $\vec{F} \parallel \vec{n}$, $\vec{F} \cdot \hat{n} = |\vec{F}|$

$= a$

$$\int_C \vec{F} \cdot \hat{n}\, ds = \int_C a\, ds = a \cdot \text{length } C = 2\pi a^2$$

2. same C, $\vec{F} = \langle -y, x \rangle \Rightarrow \vec{F}$ tangent to C, so

$\vec{F} \cdot \hat{n} = 0$, Flux $= 0$

* Calculation using components.

before: $d\vec{r} = \vec{T}\, ds = \langle dx, dy \rangle$

remember this

flux: $\hat{n}$ is $\vec{T}$ rotated $90°$ clockwise

so $\hat{n}\, ds = \langle dy, -dx \rangle$

$\int_C M\,dy + N(-dx)$

$\parallel$

$\int_C M\,dy - N\,dx$

$\cancel{\int_C -N\,dx + M\,dy}$

$\vec{T} \cdot \Delta s = \Delta\vec{r} = \langle \Delta x, \Delta y \rangle$

$\hat{n} \cdot \Delta s = \langle \Delta y, -\Delta x \rangle$

So, if $\vec{F} = \langle P, Q \rangle$ then $\int_C \vec{F} \cdot \hat{n}\, ds = \int_C \langle P, Q \rangle \cdot \langle dy, -dx \rangle$

$$\boxed{= \int_C -Q\,dx + P\,dy}$$

Mo Tu We Th Fr Sa Su

Memo No. ______

Date / /

Green's Theorem for flux:

if C enclosed a region R $\boxed{\text{counterclockwise}}$

and $\vec{F}$ defined $\langle P, Q\rangle$ defined in R

then $\oint_C \vec{F}\cdot\hat{n}\,ds = \iint_R \operatorname{div}(\vec{F})\cdot dA$

↓ divergence (散度) $M\,dy$ $-N\,dx$

$\operatorname{div}\langle P, Q\rangle = P_x + Q_y$ (curl: $N_x - M_y$, div: $M_x + N_y$)

↳ green's theorem in normal form

(curl, in tangential form)

Proof:

$$\oint_C \underbrace{-Q}_{M}\,dx + \underbrace{P}_{N}\,dy = \iint_R (P_x + Q_y)\,dA$$

$N_x - M_y$

set $\Rightarrow M = -Q,\ N = P$

$$\oint_C M\,dx + N\,dy = \iint_R (N_x - M_y)\,dA$$

$$\boxed{\text{if } \vec{F} = \langle M, N\rangle:\ \oint_C M\,dy - N\,dx = \iint_R (M_x + N_y)\,dx\,dy}$$

Example: $\vec{F} = x\hat{i} + y\hat{j}$, C: circle of radius a

$$\operatorname{div}\vec{F} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} = 2$$

$$\text{flux} = \oint_C x\cdot dy - y\cdot dx = \oint_C \vec{F}\cdot\hat{n}\,ds = \iint_R 2\,dA = 2\cdot\pi a^2$$

Mo | Tu | We | Th | Fr | Sa | Su

Memo No.

Date / /

what div($\vec{F}$) measure? diverage

interpetation of div $\vec{F}$

① measure how much the flow is "expending"

② the "Source rate" = amount of ~~fluere~~ fluid added to the system per unit time & area