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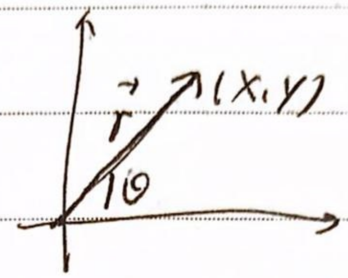
$\boxed{\text{LEC 17. in polar coords 2025.1.16}}$

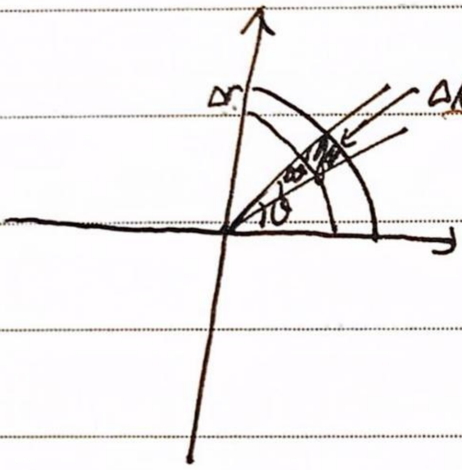
Example:

$$\iint 1-x^2-y^2 \, dA = ?$$

$x^2+y^2 \le 1$, $x, y \ge 0$



$x = r\cos\theta$, $y = r\sin\theta$



$\Delta A \approx \Delta r \cdot r\Delta\theta \Rightarrow dA = dr \cdot r\,d\theta$

$$\int_0^{\frac{\pi}{2}}\int_0^{2} (1-r^2)\, r \cdot dr\, d\theta$$

$f \cdot dA = (1-r^2)\, r\,dr\,d\theta$

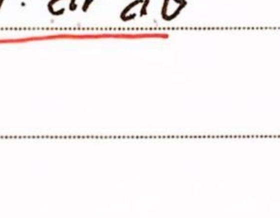
$$\left(f = 1-x^2-y^2 = 1-r^2 \right)$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \left[\frac{r^2}{2} - \frac{r^4}{4} \right]_0^1 d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{4}\, d\theta = \frac{\pi}{8}.$$

Applications:

i) find area of given R

$$\boxed{\text{Area}(R) = \iint_R 1 \cdot dA}$$



ii) Mass of a (flat) object with density

δ = mass unit $\Delta m = \delta \cdot \Delta A$

$\text{Mass} = \iint_R \delta \cdot dA$

2) find Average value of f in R

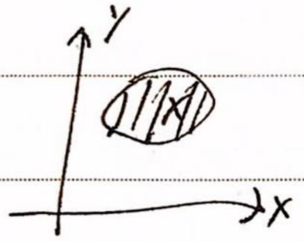
~~Are~~ Average of $f = \bar{f} = \frac{1}{\text{Area}(R)} \iint_R f \, dA$

~~with~~

weighted average of f : $\frac{1}{\text{mass}(R)} \iint_R f \delta \, dA$

identity δ

2a ~~3)~~. Center of mass of a object with density δ ?



$(\bar{x}, \bar{y})$

$\bar{x} = \frac{1}{\text{Mass}} \iint x \delta \, dA$

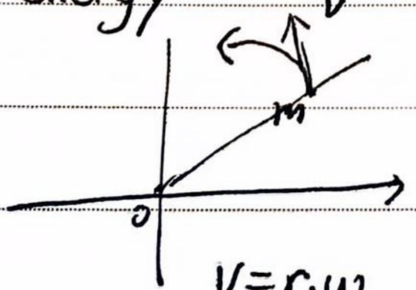
$\bar{y} = \frac{1}{\text{Mass}} \iint y \delta \, dA$

3) Moment of inertia: (惯性) (转动惯量)

mass = how hard it is to import translation motion

Idea

Kinetic energy $\frac{1}{2} m v^2$



for a mass m at distance r and angular ~~ro~~ velocity ω

$V = r \cdot \omega$

$\frac{1}{2} m v^2 = \frac{1}{2} \boxed{m r^2} \omega^2$ moment of inertia

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For a solid with density δ:

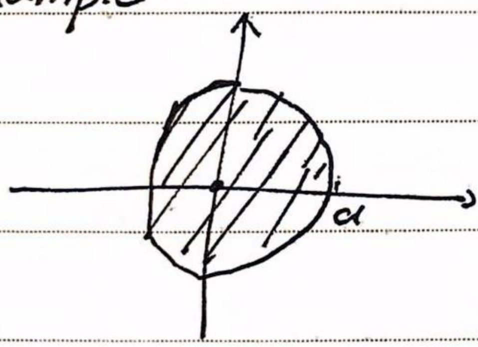
$\Delta m \approx \delta \Delta A$,

has moment of inertia $\Delta m \cdot r^2 = \delta \Delta A \cdot r^2$

Moment of inertia about the origin is $\iint_R r^2 \delta \, dA = I_0$

(Rotational kinetic energy is $\frac{1}{2} I_0 w^2$)

Example:



disk of radius a, $\delta = 1$

$$I_0 = \iint r^2 \cdot dA = \int_0^{2\pi}\int_0^a r^2 \, r \, dr \, d\theta = \frac{\pi}{2} a^4$$

~~About~~