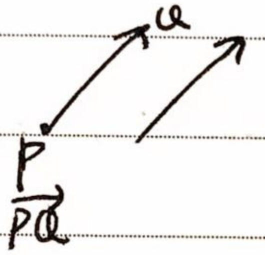
Session 1   2025.1.6.

vector: geometrically and algebraically
Geometrically:

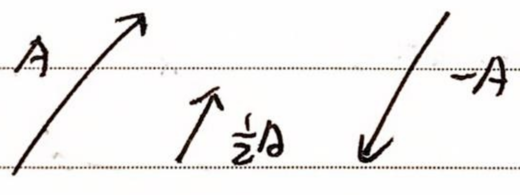
define: a magnitude and a direction

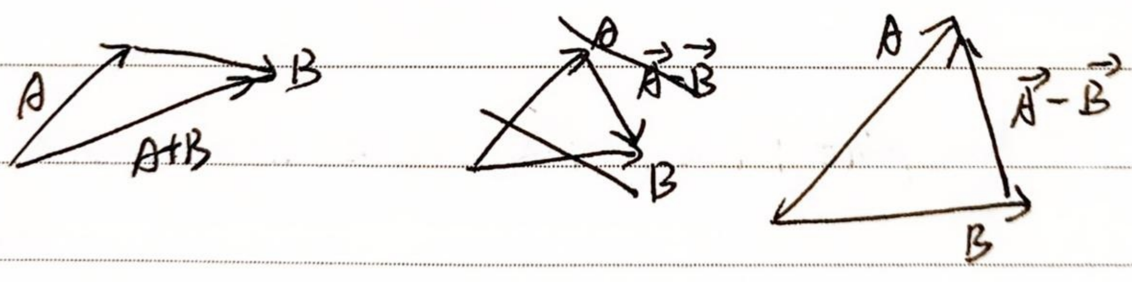
same vector:



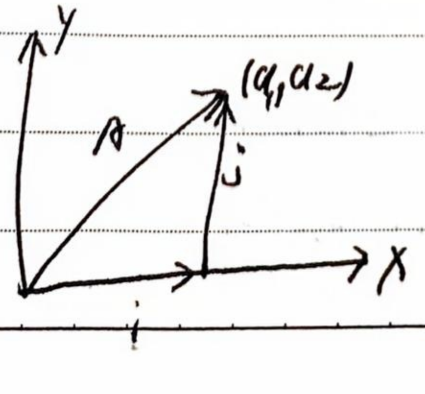
magnitude: $A = |A|$, length or norm

Scaling a vector (缩放 スケーリング)





coordinate:



$i=(1,0)$ $j=(0,1)$

$3,4\rangle$

$\frac{4}{5}\rangle$

$=\sqrt{14}$

Mo | Tu | We | Th | Fr | Sa | Su

Memo No.

Date / /

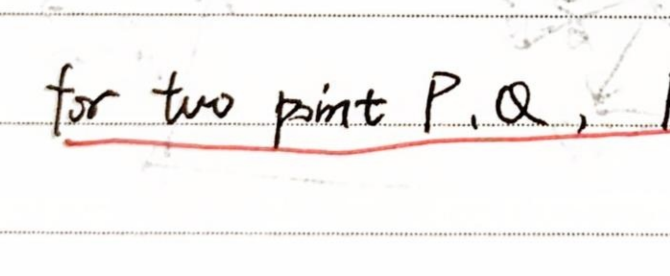
Notation and terminology

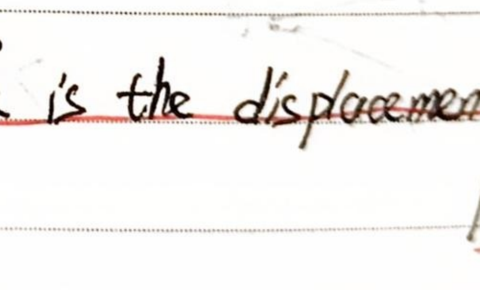
1. $\langle a_1, a_2 \rangle$ point at the plane
2. $\langle a_1, a_2 \rangle = a_1\vec{i} + a_2\vec{j}$
3. $\vec{A} = a_1\vec{i} + a_2\vec{j}$, a_1 / a_2 called components of A
5. $\vec{P} = \vec{OP}$ is the vector from the origin to P

$|\vec{A}| = \sqrt{a_1^2 + a_2^2}$

$\vec{A}+\vec{B} = (a_1+b_1)\vec{i} + (a_2+b_2)\vec{j}$ $\quad \langle a_1+b_1, a_2+b_2 \rangle$ ~~<a1+a2>~~

$\vec{A}-\vec{B} = (a_1-b_1)\vec{i} + (a_2-b_2)\vec{j}$ $\quad \langle a_1-b_1, a_2-b_2 \rangle$

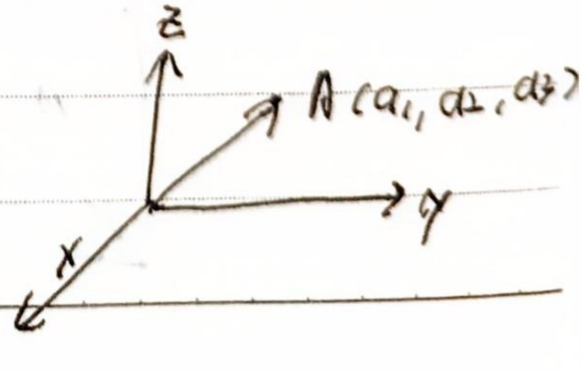




for two point P, Q, $\vec{PQ}$ is the displacement from P to Q

the three dimensions

$\vec{A} = \langle a_1, a_2, a_3 \rangle$



$i = \langle 1,0,0 \rangle$, $j = \langle 0,1,0 \rangle$, $k = \langle 0,0,1 \rangle$

$\langle a_1, a_2, a_3 \rangle = a_1 i + a_2 j + a_3 k$

$|\langle a_1, a_2, a_3 \rangle| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

unit vector: $\hat{\vec{a}}$

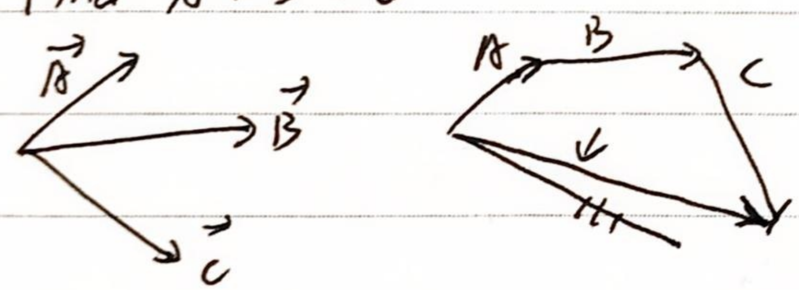
Ex: find a unit vector that is ~~paralel~~ parallel to $\langle 3,4 \rangle$

$|\langle 3,4 \rangle| = 5$, ∴ unit vector $= \frac{1}{5}\langle 3,4 \rangle = \langle \frac{3}{5}, \frac{4}{5} \rangle$

Examples:

① Vector addition

1. Find $\vec{A} + \vec{B} + \vec{C}$



2. $\vec{A} + \vec{B} + \vec{C}$, $A = \langle 1,2 \rangle$, $B = \langle 1,0 \rangle$, $C = \langle 2,-1 \rangle$

① $= \langle 1+1+2,\ 2+0-1 \rangle = \langle 4, 1 \rangle$

② $= \vec{i} + 2\vec{j} + \vec{i} + 2\vec{i} - \vec{j} = 4\vec{i} + \vec{j}$

② Vector Length

$\vec{A} = \langle 1,2 \rangle$, $\vec{B} = \langle 1,-1 \rangle$, $\vec{C} = \vec{i} + 2\vec{j} + 3\vec{k}$ $|\vec{C}| = \sqrt{1+4+9} = \sqrt{14}$