Mo | Tu | We | Th | Fr | Sa | Su

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Session 9: ~~Mart~~ Matrix Multiplication 2025.1.8

矩阵乘法

$$\begin{bmatrix} 2 & 3 & 3 \\ 2 & 4 & 5 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$u_1 = 2x_1 + 3x_2 + 3x_3$ $\qquad A \cdot X = U$

$u_2 = 2x_1 + 4x_2 + 5x_3$

$u_3 = x_1 + x_2 + 2x_3$

$$\begin{array}{ccc} & 3 \times 2 & 2 \times 3 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ \vdots & \vdots & \vdots & \vdots \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \vdots & 0 \\ \vdots & 3 \\ \vdots & 0 \\ \cdot & 2 \end{bmatrix} = \begin{bmatrix} \cdot & 14 \\ \vdots & \vdots \\ \cdot & \cdot \end{bmatrix}$$

$$\begin{array}{cc} A & B \end{array}$$

width = hight

Readings:

$m \times n$ matrix A, (a_{ij})

i-j entry, i row, j column $\quad a_{ij}$

row-vectors: $1 \times n$

column-vectors: $m \times 1$

four basic operations which produce new matrices from old.

1. Scalar Multiplication: $cA = (ca_{ij})$

2. Matrix addition : $A + B = (a_{ij} + b_{ij})$, A and B must have same wideth and hight

3: transposition : $m \times n$ $A \to n \times m$ A^T or A'

$A^T = (a_{ji})$

Example:

$$\vec{A} = \begin{pmatrix} 2 & -3 \\ 0 & 1 \\ -1 & 2 \end{pmatrix}, \quad \vec{B} = \begin{pmatrix} +1 & 5 \\ -2 & 3 \\ -1 & 0 \end{pmatrix}$$

$$\vec{A} + \vec{B} = \begin{pmatrix} 3 & 2 \\ -2 & 4 \\ -2 & 2 \end{pmatrix} \qquad \vec{A}^T = \begin{pmatrix} 2 & 0 & -1 \\ -3 & 1 & 2 \end{pmatrix}$$

4. Matrix multiplication: $\underset{m\times n}{\vec{A}} \cdot \underset{n\times p}{\vec{B}} = \underset{m\times p}{\vec{C}}$

$$\boxed{c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}}$$

$\boxed{\vec{A}\text{'s columns same as } \vec{B}\text{'s rows.}}$

$$c_{ij} = a_i \cdot b_j$$

Ex: $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ $\quad (2 \cdot 1 -1)\begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} = (-2+4-2) = (0)$

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}(4 \; 5) = \begin{pmatrix} 4 & 5 \\ 8 & 10 \\ -4 & -5 \end{pmatrix}$$

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Laws and properties

M-1. $A(B+C) = AB + AC$ $(A+B)C = AC + BC$

M-2. $(AB)C = A(BC)$ $(cA)B = c(AB)$

M-3. identity martrix $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\vec{A} \cdot \vec{I} = \vec{A}$, $\vec{I} \cdot \vec{A} = \vec{A}$ (3x3)

M-4. for two square n×n matrix $AB \neq BA$

M-5. $|AB| = |A||B|$ $\det(AB) = \det(A)\det(B)$

M-6. martrix can be used to pick out a row or column of a given martrix : multiply by a simple row or column vector to do this $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

↓

Ex: $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}$, the second column

$(1\ 0\ 0) \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = (1\ 2\ 3)$