$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \cdots 4xye^{\cdots} + 1$

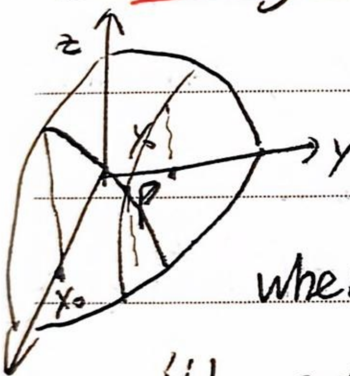
$$\text{so } \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) \Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

2)

$$\frac{\partial f}{\partial x}(1,3) = 2 \cdot e^{1+9} + 27 + 3 = 2e^{10} + 5$$

Session 27: Approximation Function 2025.1.11

For a function $w = f(x, y)$, the nature analogue is the tagent plane to the graph at point P



the plane: (i) must through $P(x_0, y_0, w_0)$ where $w_0 = f(x_0, y_0)$

(ii) cotain two tangent lines $\Rightarrow$ has the slopes in $\vec{i}$ ang $\vec{j}$ directions as the surface does the plane: $A(x-x_0)+B(y-y_0)+C(w-w_0)=0$

if plane is not vertical $\Rightarrow$ $C\neq 0$

(3) so $(w-w_0)=a(x-x_0)+b(y-y_0)$, $a=A/C$, $b=B/C$

what a?

putting $y=y_0$, $w-w_0=a\cdot(x-x_0)$ $\leftarrow$ x-w //

so a is the slope of graph in the $\vec{i}$-direction

so $\boxed{a=\left(\frac{\partial w}{\partial x}\right)_0}$

similarly: $b=\left(\frac{\partial w}{\partial y}\right)_0$

Therefore the equation of the tagent ~~line~~ plane:

$w(x_0,y_0) \rightarrow$ (4) $\boxed{w-w_0=\left(\frac{\partial w}{\partial x}\right)_0(x-x_0)+\left(\frac{\partial w}{\partial y}\right)_0(y-y_0)}$

The approximation formula

the intutive idea is the graph of the tagent plane will be a good approximation to the graph of the function $w=f(x,y)$

$\Rightarrow$ $\boxed{(5)\ f(x,y)\approx w_0+\left(\frac{\partial w}{\partial x}\right)_0(x-x_0)+\left(\frac{\partial w}{\partial y}\right)_0(y-y_0)}$

height of graph $\approx$ height of tagent plane

is often called the linerization

if we put Δx, Δy, Δw

(5) => (6) ~~$f(x,y) \approx$~~ $\boxed{\Delta w \approx \left(\frac{\partial w}{\partial x}\right)_0 \Delta x + \left(\frac{\partial w}{\partial y}\right)_0 \Delta y}$

more variables:

(7) $\Delta w \approx \left(\frac{\partial w}{\partial x}\right)_0 \Delta x + \left(\frac{\partial w}{\partial y}\right)_0 \Delta y + \left(\frac{\partial w}{\partial z}\right)_0 \Delta z + \cdots$

Examples:

1 give a ~~resenabe~~ reasonable square, centered at (1,1), over which the value of $w = x^3 y^4$ will not vary by more than ± 1.

at (1,1)

$w_x = 3x^2 y^4$ $\quad w_y = 4x^3 y^3$, $\Delta w \approx 3\Delta x + 4\Delta y$

because a square, $\Delta x = \Delta y$

$7\Delta x = \Delta w$, $|\Delta w| \le .1 \Rightarrow |\Delta x| \le \frac{.1}{7}$,

$|\Delta y| \le \frac{.1}{7} = 0.7$, take ~~$\Delta x = \Delta y =$~~ $|\Delta x| \le 0.01$

$|\Delta y| \le 0.01 \Rightarrow |\Delta w| \le 0.07$ meet the quest

so the square's length is 0.02

$|x-1| \le 0.1$, $|y-1| \le 0.1$

also can: $|\Delta x| = |\Delta y| \le 0.014$

$|x-1| \le 0.014$, $|y-1| \le 0.014$

Ex 2. a rectangular box length a, b, c to be 1, 2, 3, which of these measurements is the volume V most sensitive?

$V = abc$

so $\Delta V \approx \cancel{bc \cdot \left(\frac{\partial V}{\partial a}\right)} \ \Delta a \cdot \left(\frac{\partial V}{\partial a}\right)_0 \ \cdots$

$\approx \Delta a \cdot bc + \Delta b \cdot ac + \Delta c \cdot ab$

$= 6\Delta a + 3\Delta b + 2\Delta c$

we can see the Δa is most sensitive to the volume

The Reading 2:

$\boxed{\text{Smoothness hypothesis}}$:

We say $f(x,y)$ is smooth at (x_0, y_0) if

(8) f_x and f_y are continuous ~~in the~~ some rectangle centered at (x_0, y_0)

in general the normal way a function fails to be smooth is that one or both partial derivatives fail to exist at (x_0, y_0)

Ex 3. $w = \sqrt{x^2+y^2}$

$$w_x = \frac{x}{\sqrt{x^2+y^2}}, \quad \frac{\partial w}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$$

These are continuous at all points except $(0,0)$, where they are undefined. So the function is smooth except at the origin, we can use approximation formula (6) everywhere ~~expect~~ except at the origin

Examples:

a) Find the tangent plane $z = x^2+y^2$ at point $(2,1,5)$

$$\frac{\partial w}{\partial x} = 2x, \quad \frac{\partial w}{\partial y} = 2y$$

so plane: $2x \cdot (x-2) + 2y(y-1) = z-5 \Rightarrow 4(x-2) + 2(y-1) = (z-5)$

$\cancel{2x^2 + 4x + 2y^2 - 2y = z - 5}$

b) give the tagent approximation for z near the point $(x_0, y_0) = (2,1)$.

$$w - \overset{(5)}{w_0} = 4 \cdot (x-2) + 2(y-2)$$

$$\Delta w \approx 4\Delta x + 2\Delta y \Rightarrow \Delta z \approx 4\Delta x + 2\Delta y$$

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Problems:

1. find tagent plane:

$z=xy^2$ at $(1,1,1)$

$\frac{\partial z}{\partial x} = y^2, \ \frac{\partial z}{\partial y} = 2xy$

substitue with the point to partial derivative

$\Rightarrow z-1 = y^2\cdot(x-1) + 2xy(y-1) = (x-1)+2(y-1)$

2. give ~~linea~~ linearization of $f(x,y)=e^x+x+y$ at $(0,0)$.

$\frac{\partial w}{\partial x} = e^x+1, \ \frac{\partial w}{\partial y} = 1 \quad w(0)=1$

so $w \approx 1+(e^0+1)\cdot(x-0)+1\cdot(y-0)$

$f(x,y) = w \approx 1+2x+y$