Mo | Tu | We | Th | Fr | Sa | Su

Memo No.

Date / /

Session 14 : solution to Square Systems

2025.1.8

Recall:

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A) \quad (\det(A) \neq 0)$$

e.g.
$$\begin{cases} x + z = 0 \\ x + y = 0 \\ x + 2y + 3z = 0 \end{cases}$$
solution $(0,0,0)$

general case:

$AX = B$, if $\det(A) \neq 0$, then the unique solution

$X = A^{-1}B$

if $\det(A) = 0$, then 0 or ∞ many solutions

(齐次) (非齐次)

trivial solution $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

Readings: Homogeneous and Inhomogeneous System

Definition: $Ax = b$ is called Homogeneous if $b = 0$ otherwise it is called inhomogeneous

Theorem 1. A be $n \times n$ matrix

(20) $|A| \neq 0$, $Ax = b \Rightarrow$ has the unique solution $X = A^{-1}b$

(21) $|A| \neq 0$, $AX = 0 \Rightarrow$ has only the trivial solution $X = 0$

~~Theg~~ Theorem 2. A be $n \times n$

(22) $|A| = 0$, $Ax = 0$, has non-trivial solutions

(23) $|A| = 0$, $\Rightarrow Ax = b$, usually has no solution, but some b have

inconsistent consistent

Memo No.
Date / /

~~2.3~~ Homogeneous systems: $Ax=0$ has non-trivial solutions $\Leftrightarrow |A|=0$

Inhomogeneous systems: $Ax=b$ has the unique solution $x=A^{-1}b$, if $|A|\neq 0$

proof for 22: (just 3×3) $\vec{A}\vec{x}=0$ (24)

$Ax=0$, $|A|=0$, has non-trivial

$\vec{A}=(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ ← three vector

$\therefore \vec{a}_1\cdot\vec{x}=0$, $\vec{a}_2\cdot\vec{x}=0$, $\vec{a}_3\vec{x}=0$

$|A|=0$, so the parallelepiped has zero volume, so it is a plane, so all non-zero vector x which is orthogonal to this plane will be orthogonal to $\vec{a}_1$, $\vec{a}_2$ and $\vec{a}_3$, and therefore will be a solution to (24), this prove (22) if $|A|=0$, the $|A|\cdot x=0$ has a nontrivial solution

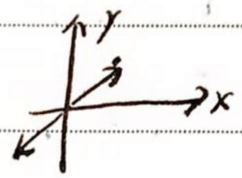
~~Sig~~ Singular Matrices (奇异矩阵)

$\boxed{\text{square } \cancel{A}\text{ matrix } A \text{ is singular if } |A| = 0}$ (系数矩阵平行,)

nonsingular or <u>invertible</u> if $|A| \neq 0$

because A^{-1} exist only (可逆) if $|A| \neq 0$

#0 $\begin{vmatrix} 1 & 1 \\ -1 & -1 \end{vmatrix} = 0$



homogeneous homogeneous homogeneous homogeneous homogeneous homogeneous

Problems:

1. $x + y + 2z = 0$
$2x + y + cz = 0$
$3x + y + 6z = 0$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & c \\ 3 & 1 & 6 \end{bmatrix}$$

(a) $c = 1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 3 & 1 & 6 \end{bmatrix} x = \vec{0}$$

$\langle 1, 1, 2\rangle$ $\langle 2, 1, 1\rangle$ $\langle 3, 1, 6\rangle$

$\begin{matrix} i & j & k \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{matrix} = -i + 3j - k$

$-3 + 3 - 6 = -6$

det($\vec{A}$)

~~det(A)=~~ $|\vec{A}| = 5 - 9 + (-2) = -6$ $\quad \therefore x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

also can, but det(A) better

(b) $c = 4$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 4 \\ 3 & 1 & 6 \end{bmatrix}$$

det($\vec{A}$)

$|\vec{A}| = +2 \cancel{平} - 2 = 0$ $\quad x = \begin{pmatrix} 2n \\ 0 \\ -n \end{pmatrix}$

$\langle 1, 1, 2\rangle \times \langle 2, 1, 4\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 2 & 1 & 4 \end{vmatrix} = 2\vec{i} - \vec{k}$