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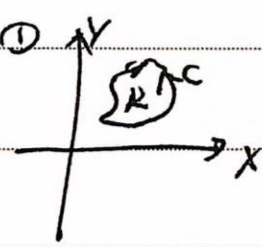
LEC 24: 2025.2.20

More about validity of Green's theorem:

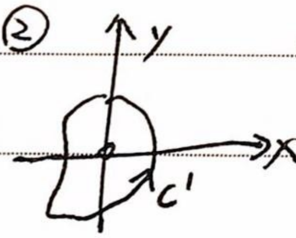
We have seen two forms $\begin{cases} \oint_C \vec{F}\cdot\hat{T}\,ds = \iint_R \mathrm{curl}(\vec{F})\,dA \\ \oint_C \vec{F}\cdot\hat{n}\,ds = \iint_R \mathrm{div}(\vec{F})\,dA \end{cases}$

only work if $\vec{F}$ (and antiderivative) defined everywhere in R

Example: $\vec{F} = \frac{-y\hat{i}+x\hat{j}}{x^2+y^2}$, $\vec{F}$ not defined at origin, curl $\vec{F} = 0$ everywhere else

① 

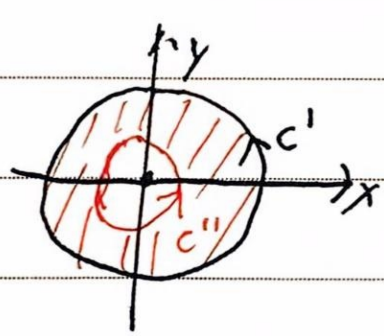
$\oint \vec{F}\cdot d\vec{r} = 0$

② 

can't use green thm

$\oint_{C'} \vec{F}\cdot d\vec{r} = ?$

② ⇒ extend to green theorem.



$\oint_{C'} \vec{F}\cdot d\vec{r} - \oint_{C''} \vec{F}\cdot d\vec{r} = \iint_R \mathrm{curl}(\vec{F})\,dA$

$= 0$ (in this case)

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Definition: a connected region in the plane is simply connected is ~~any closed curve in R~~ the interior of any closed curve in R is also containted in R

not simply connected

simply connected

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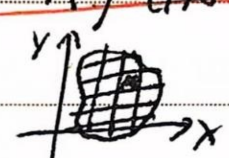
exam Review Unit2

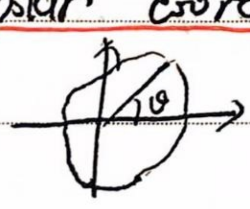
2 main objects: $\iint_R f\,dA$ / $\int_C \vec{F}\cdot \overset{\text{or}}{\hat{T}/\hat{n}}\,ds$



- set up $\iint_R$: draw picture of R & take slice

① exchange value ⇒ draw a picture $\iint_R dx\,dy \Rightarrow \iint_R dy$

② in polar coordinates: $x = r\cos\theta$, $y = r\sin\theta$, $dxdy = r\,dr$



③ Remember: area(R) $= \iint_R 1\,dA$, mass avg value $\bar{f}$

jacobin deteminator ($\bar{x}, \bar{y}$ center of mass) polar moment of inertia $I_0 = \iint (x^2+y^2)\,\delta\, dA$.

inertia $r^2 \cdot m$

- Evaluating $\int$:

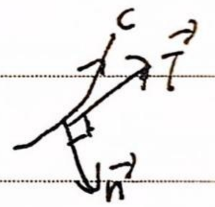
MUST KNOW: usual integral; substitution; integration by parts

- change of variable $u = u(x,y)$, $v = v(x,y)$

① find $\frac{\partial(u,v)}{\partial(x,y)}$ $\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$ $du\,dv = \left|\frac{\partial(u,v)}{\partial(x,y)}\right| dx\,dy$

② substitute x, y's in the integral absolute

③ Setting up boards:



2. Line Integrals: $\vec{F} = \langle M, N \rangle$

$$\int_C \vec{F}\cdot d\vec{r} = \int_C M\,dx + N\,dy \quad (\vec{F}\cdot d\vec{r})$$

$$\int_C \vec{F}\cdot \hat{n}\, ds = \int_C M\,dy - N\,dx \quad (\vec{F}\cdot \langle dy, -dx\rangle)$$

Evaluation: reducing to a single parameter

$x = x(t)$, $y = y(t)$

☆ if $\mathrm{curl}(\vec{F}) = N_x - M_y = 0$

(& domain simply-connected)

$\begin{cases} f_x = M \\ f_y = N \end{cases} \Leftrightarrow \vec{F} = \nabla f \Rightarrow$ find f

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— Green's theorem: $\oint_C \vec{F} \cdot d\vec{r} = \iint_R \underbrace{\text{curl}}_{N_x - M_y} \vec{F} \cdot dA$

$\oint_C \vec{F} \cdot \hat{n}\, ds = \iint_R \underbrace{\text{div}}_{M_x + N_y} \vec{F} \cdot dA$