$$= \begin{pmatrix} 2 & -5 \\ -1 & 6 \end{pmatrix} \cdot \frac{1}{7}$$

Session 12: Equations of Planes 2025.1.8

recall $ax+by+cz=d$ defines a plane

$(\vec{n} = \vec{A} \times \vec{B} \quad (x,y,z)\cdot\vec{n} = 0)$

plane through $\vec{N} = \langle 1, 5, 10 \rangle$ $\qquad \vec{OP} \cdot \vec{N} = 0$

$P(x, y, z) \Rightarrow x + 5y + 10z = 0$

In equation $ax+by+cz=d$

$\langle a, b, c \rangle$ is normal vector $\vec{N}$. orthogonal to plane

d is

~~get $\vec{N}$ by cross product of 2 vectors in plane~~

~~Ex: $\vec{v} = \langle 1, 2, -1 \rangle$~~

~~Plane: $x+y+3z=5$ $\quad \vec{N} = \langle 1, 1, 3 \rangle$~~

$\cancel{\vec{v}\cdot\vec{N} = 1+2-3=0}$, ~~so~~ $\cancel{\vec{v} \parallel}$ ~~plane~~

$\cancel{\vec{v}: x+y+}$

Readings:

$\vec{N}$ is the normal (法线) to the plane

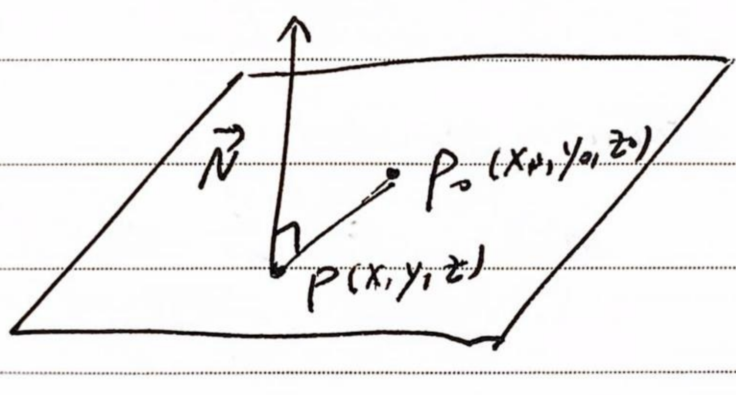
↳ orthogonal to the plane

the point-normal form for the plane:

$\vec{N} = \langle a, b, c\rangle \quad P_0 = \langle x_0, y_0, z_0)$

$\therefore \langle a, b, c\rangle \cdot \langle x-x_0, y-y_0, z-z_0\rangle = 0$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

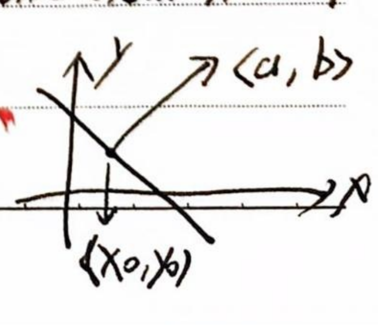


Lines in the plane

① $y = mx + b$

② point-normal form:

$P(x_0, y_0)$, vector $\langle a, b\rangle$ is the ~~nom~~ normal to the line : $a(x-x_0) + b(y-y_0) = 0$



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Memo No. ______
Date / /

Problems 1 Equation of a plane

1. $P_1 = (1,0,1)$ $P_2 = (0,1,1)$, $P_3 = (1,1,0)$

$\overrightarrow{P_1P_2} = (-1, 1, 0)$

$\overrightarrow{P_1P_3} = (0\ 1, -1)$

$$\vec{N} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\vec{i} - \vec{j} - 1\vec{k}$$

$P(x, y, z)$ $\therefore$ $-(x-1) - (y-1) - z = 0$

$-x + 1 - y + 1 - z = 0$

$x + z + y = 2$

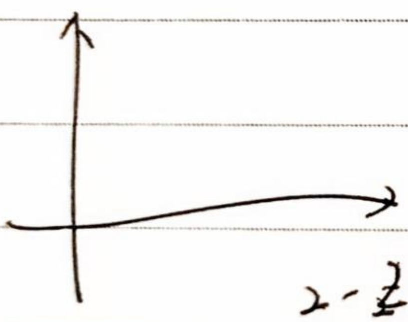
2. line through $(1,2)$ and $(3,1)$

in point-normal form

$m = \frac{2-1}{1-3} = -\frac{1}{2}$ $\therefore$ $m^{-1} = +2$

set $\vec{N} = \langle 1, +2 \rangle$

$\therefore$ $(x-1) + 2(y-2) = 0$



$2 - 2(-1)$

$\therefore (x-1) + 2(y-2) = 0$

$2 - 2(-1)$

vector : $\langle 2, -1 \rangle$, so $\vec{N} = \langle 1, 2 \rangle$

↑one of normals

Mo | Tu | We | Th | Fr | Sa | Su

Memo No. ________

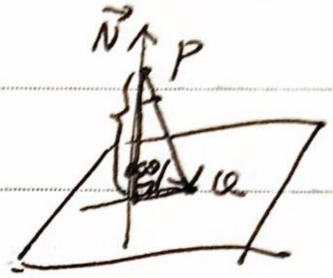
Date / /

Reading 2. distance to planes and lines

1. Distance : Point to plane:

i) a Point P ii) a plane with normal $\vec{N}$ and containing a point Q.

$$\boxed{d = |\vec{PQ}| \cos\theta = \left|\vec{PQ} \cdot \frac{\vec{N}}{|\vec{N}|}\right|}$$



distance $= |\vec{PQ}| \cdot \cos\theta$

$\vec{PQ} \cdot \vec{N} = |\vec{PQ}| \cdot |\vec{N}| \cdot \cos\theta$

$\therefore$ distance $= \left|\frac{\vec{PQ} \cdot \vec{N}}{|\vec{N}|}\right|$

Ex1. let $P = (1, 3, 2)$

Find P to plane $x + 2y = 3$'s distance

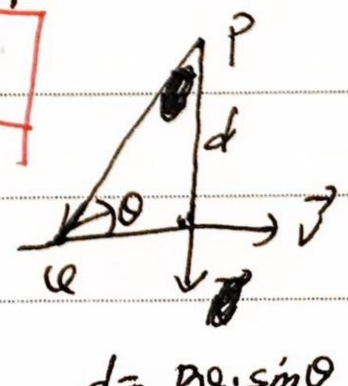
get $Q(3, 0, 0)$ $\vec{N} = \langle 1, 2, 0 \rangle$, $\vec{PQ} = \cancel{(1,3,1)}$ $(2, -3, -2)$

$$d = |\vec{PQ}| \cdot \cos\theta = \left|\vec{PQ} \cdot \frac{\vec{N}}{|\vec{N}|}\right| = \left|\frac{2-6}{\sqrt{5}}\right| = \frac{-4}{\sqrt{3}} = -\frac{4}{\sqrt{5}}$$

2. Distance : point to line

i) point P ii) vector $\vec{v}$ and Point Q

$$\boxed{d = |\vec{QP}| \cdot \sin\theta = \left|\vec{QP} \times \frac{\vec{v}}{|\vec{v}|}\right|}$$



$d = PQ \cdot \sin\theta$

3. Distance between parallel planes:

trick: reduce it to the distance from a point to a plane.

get two point at two planes

Mo | Tu | We | Th | Fr | Sa | Su

Memo No. ____________

Date / /

(异面直线)

4. Distance between skew lines:

replace the lines in parallel plane

~~$\vec{N} = \vec{v_1} +$~~ $\vec{N} = \vec{v_1} \times \vec{v_2}$

Problems:

1. from $(1,0,0)$ to the plane $2x+y-2z=0$

$\vec{N} = \langle 2, 1, -2 \rangle$, set $Q = (0,0,0)$

$d = \left| \vec{PQ} \cdot \frac{\vec{N}}{|\vec{N}|} \right| = \left| \frac{-2}{|\vec{N}|} \right| = \frac{2}{\sqrt{4+4+1}} = \frac{2}{3}$

$\vec{PQ} = \langle -1, 0, 0 \rangle$

$\therefore d = \frac{2}{3}$

2. d from point $(0,0)$ to the line $2x+y=2$

① $O(0,0)$ $\vec{v} = \langle 1, -2 \rangle$ $Q(0,2)$, $(1,0)$

$d = \left| \vec{OQ} \times \frac{\vec{v}}{|\vec{v}|} \right|$ $\vec{OQ} = (0,2)$

$\vec{OQ} \times \vec{v} = \begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ 1 & -2 & 0 \end{vmatrix} = -2\vec{k}$ $|\vec{v}| = \sqrt{5}$

$\therefore d = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

② $\vec{N} = \langle 2, 1 \rangle$ $Q = (1,0)$

$d = \left| \vec{PQ} \cdot \frac{\vec{N}}{|\vec{N}|} \right| = \frac{2}{\sqrt{5}}$