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Unit 2 Partial Derivatives 2025.6.10

Session 24: Functions of two Variables: Graphs

~~Fun~~

Readings:

dependent and ~~indepent~~ independent variables

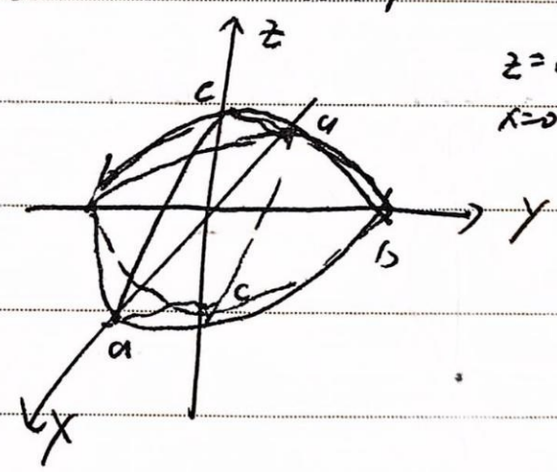
$z = f(x, y)$

x, y can be any

How to graph: take a $x = a$ constant, draw y-z plane and then take $y = a$ constant, draw x-z plane then take $z = a$ constant, ...

Example

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

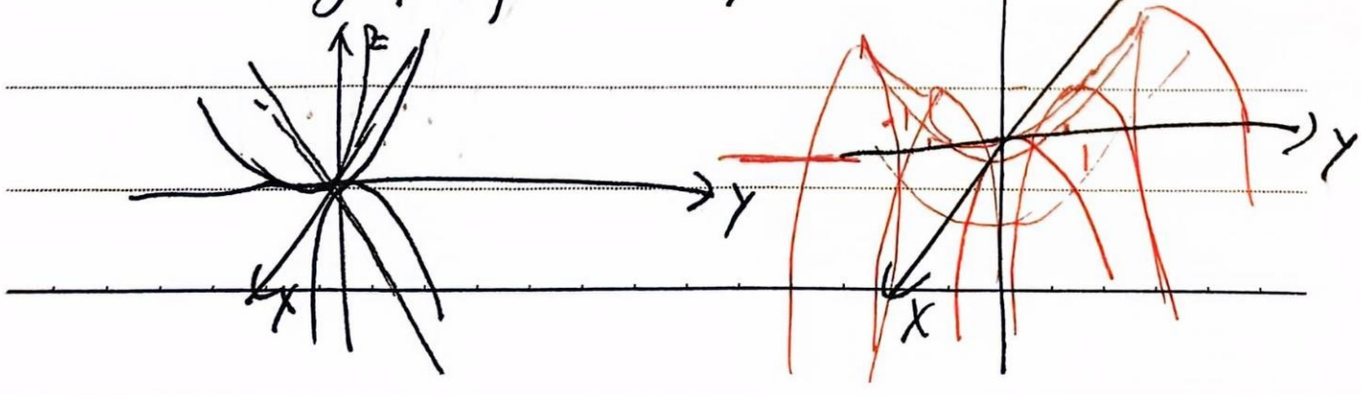


$z = 0$ ...

$x = 0$ ...

Problems:

1. Draw a graph of $z = y^2 - x^2$



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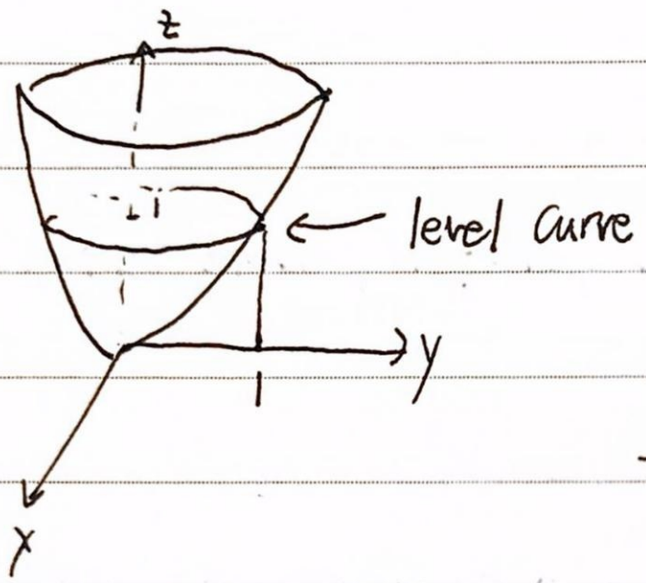
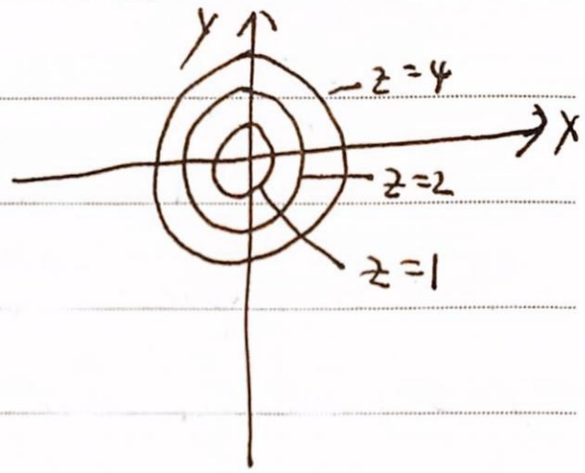
(水平曲线) (等高曲线)

Session 25. Level Curves and Contour Plots

Level Curves: z = constant

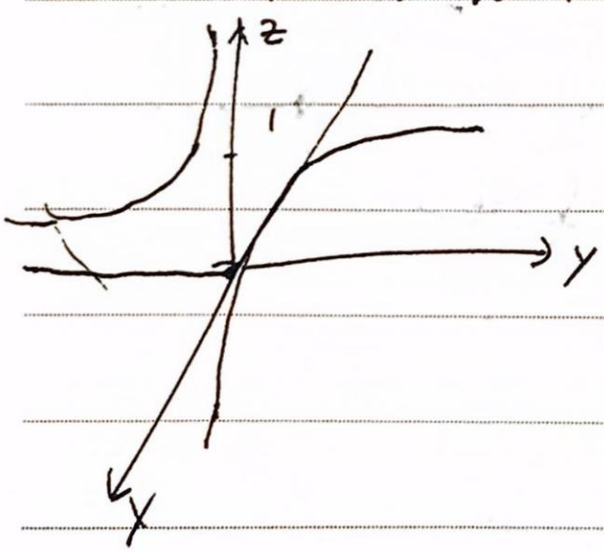


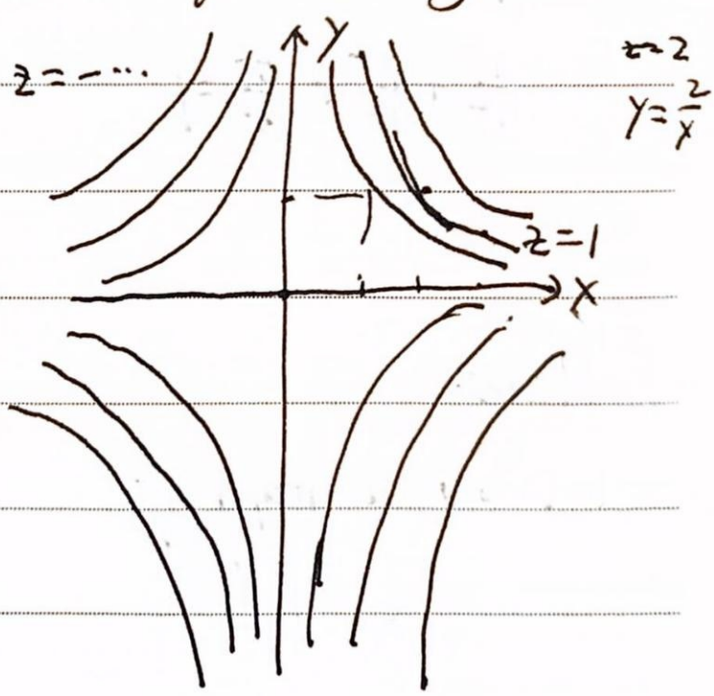
Contour Plot: draw in xy-plane and label their z-value



Problems:

Sketch the level curves for $z = xy$.





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Session 26 : Partial Derivatives 2025.1.11

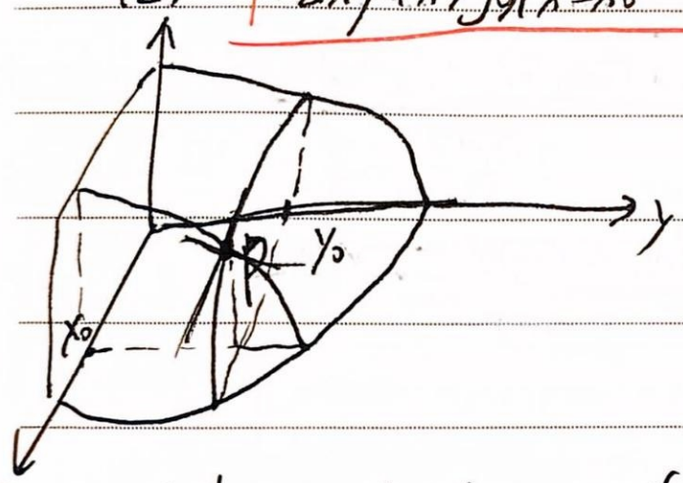
(2. 偏微分)

defination:

(1) $w = f(x, y)$, y fixed in y_0, x is vary

so $w = f(x, y_0)$ ⇒ the partial function for $y = y_0$ whose slope at the point P where $x = x_0$ is given by the derivate

(2) $\frac{d}{dx} f(x, y_0)\Big|_{x=x_0}$, or $\frac{\partial f}{\partial x}\Big|_{(x_0, y_0)}$ ← standard



↑ Partial derivative

Other notations: $f_x(x_0, y_0)$, $\frac{\partial w}{\partial x}\Big|_{(x_0, y_0)}$, $\left(\frac{df}{dx}\right)_0$, $\left(\frac{\partial w}{\partial x}\right)_0$

as the same way : x fixed x_0, y is vary

$\left(\frac{df}{dy}\right)_0$, $\frac{\partial f}{\partial y}\Big|_{(x_0, y_0)}$, ~~$f_y(x_0, y_0)$~~ $f_y(x_0, y_0)$

$\frac{\partial w}{\partial y}\Big|_{(x_0, y_0)}$, ~~$\left(\frac{df}{dy}\right)$~~ $\left(\frac{\partial w}{\partial y}\right)_0$

$\partial f/\partial x$ and $\partial f/\partial y$ depend on (x_0, y_0) and therefore functions of x and y. $\frac{\partial w}{\partial x}$ ⇒ how fast W increase by x increase when y is held constant

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for more variables, $w = f(x, y, z \cdots)$

as the same way, make the other variables constant and take the ordinary derivative with respect to $x \Rightarrow \frac{d}{dx} f(x, y_0, z_0 \cdots) = f_x(x, y_0, z_0, \cdots)$

$\left(\frac{\partial f}{\partial x}\right)_0$, $\left(\frac{\partial w}{\partial x}\right)_0$

chalk board:

$x_0 \rightsquigarrow f(x_0)$ $\quad f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

$\frac{\partial f}{\partial x}(x, y_0) \Rightarrow \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$

$\frac{\partial f}{\partial y}(x_0, y) \Rightarrow \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$

Example: $f(x, y) = x^3 y + y^2$

$\frac{\partial f}{\partial x} = 3x^2 y + 0$, $\frac{\partial f}{\partial y} = x^3 + 2y$

Problems:

Let $f(x, y) = e^{(x^2 + y^2)} + x^2 + y^2 + xy + 2y + 3$.

a) $\frac{\partial f}{\partial x} = e^{y^2} \cdot e^{x^2} \cdot 2x + 2x + y$

$= 2x e^{x^2 + y^2} + 2x + y$

$\frac{\partial f}{\partial y} = 2y \cdot e^{x^2 + y^2} + 2y + x + 2$

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b) show $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = 2y \cdot 2x\, e^{y^2} e^{x^2} + 1$$

$$= 4xy\, e^{x^2} e^{y^2} + 1$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = 4xy\, e^{x^2} e^{y^2} + 1$$

so $\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) \Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

2)

$$\frac{\partial f}{\partial x}(1,3) = 2 \cdot e^{1+9} + 2 + 3 = 2e^{10} + 5$$

Session 27: Approximation Function 25.1.11

For a function $w = f(x,y)$, the nature analogue is the tagent plane to the graph at point P