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Lec15. Topics for Exam 2 2025.1.15

- Functions of ~~severall~~ several variables

contour plots

- Partial derivatives $\boxed{f_x = \frac{\partial f}{\partial x}}$

- Gradient : $\nabla f = \langle f_x, f_y, f_z \rangle$

Approximation : $\boxed{\begin{aligned} \Delta f &\approx f_x \Delta x + f_y \Delta y + f_z \Delta z \\ &= \nabla f \cdot \Delta \vec{r} \end{aligned}}$

↑

tangent plane → $\boxed{\text{normal vector} = \nabla f}$

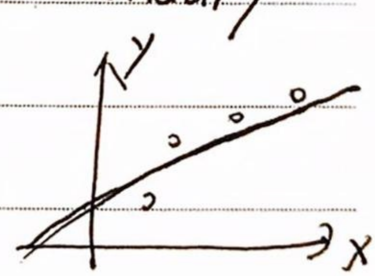
Partial differential equations.

- minimum/maximum : ~~critical problems~~

critical points all partial derivative $=0$

second derivative test +/ boundary point

- least squares approximation



$\boxed{D = \sum (y_i - (ax_i + b))^2}$

- Differentials.

$\boxed{df = f_x dx + f_y dy + f_z dz}$

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↓ chain rule, if $x = x(u,v)$ $y = y(u,v)$, $z = z(u,v)$

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial u}$$ changes variables

- Non-independent variable $\underbrace{g(x,y,z) = c}_{\text{constraint}}$

① substitude

→② min/max problems: lagrange multipliers

→ $\nabla f = \lambda \nabla g$ $\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ f_z = \lambda g_z \end{cases}$ $+ \ g = c$

→ constrained partial derivatives:

$f(x,y,z)$ where $g(x,y,z) = c$

To find $\left(\frac{\partial f}{\partial z}\right)_y$ ← y is constant, z varies, $x = x(y,z)$.

Rate of change of f with respect to z?

1) use differentials:

$df = f_x dx + \cancel{f_y} \overset{0}{dy} + f_z dz$; $dy = 0$, $dx = (\ )dz$

$dg = g_x dx + \cancel{g_x dy} + g_z dz = 0 \Rightarrow$ $dx = -\frac{g_z}{g_x} dz$

plug in to $df \Rightarrow df = f_x \cdot \left(\frac{-g_z}{g_x}\right) dz + f_z dz$

$\Rightarrow \left(\frac{\partial f}{\partial z}\right)_y =$ ∽

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$\boxed{\text{2) using chain rule:}}$

$$\left(\frac{\partial f}{\partial z}\right)_y = \frac{\partial f}{\partial x}\left(\frac{\partial x}{\partial z}\right)_y + \cancel{\left(\frac{\partial y}{\partial z}\right)_y}^{\,0}\frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}\left(\frac{\partial z}{\partial z}\right)_y \rightarrow 1$$

$$0 = \left(\frac{\partial g}{\partial z}\right)_y = \frac{\partial g}{\partial x}\left(\frac{\partial x}{\partial z}\right)_y + \frac{\partial g}{\partial y}\cancel{\left(\frac{\partial y}{\partial z}\right)_y}^{\,0} + \frac{\partial g}{\partial z}\left(\frac{\partial z}{\partial z}\right)_y$$

$$\Rightarrow \quad 0 = \frac{\partial g}{\partial x}\left(\frac{\partial x}{\partial z}\right)_y + \frac{\partial g}{\partial z}\left(\frac{\partial z}{\partial z}\right)_y$$

$$\Rightarrow \left(\frac{\partial x}{\partial z}\right)_y = -\frac{g_z}{g_x}$$

– direction derivative $\boxed{\frac{df}{ds}\Big|_{\hat{u}} = \nabla f \cdot \hat{u}}$