Mo Tu We Th Fr Sa Su

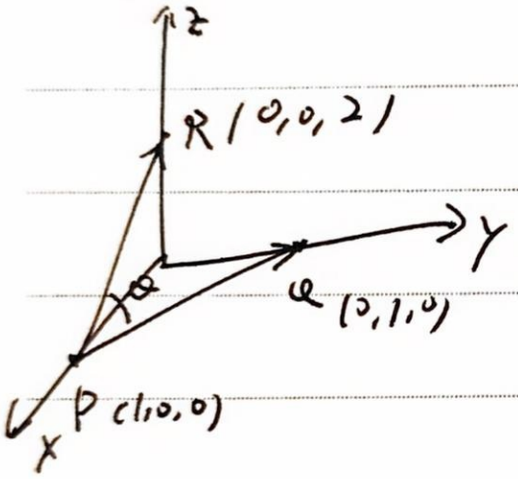
Memo No. _____

Date / /

session 3: Use of the Dot Product: Lengths and Angles 2025.1.6.

chalkboard:

Ex:



$\vec{PQ} \cdot \vec{PR} = |\vec{PQ}| \cdot |\vec{PR}| \cdot \cos\theta$

$\cos\theta = \dfrac{\vec{PQ} \cdot \vec{PR}}{|\vec{PQ}| \cdot |\vec{PR}|}$

$= \dfrac{\langle -1, 1, 0 \rangle \cdot \langle -1, 0, 2 \rangle}{\sqrt{1+1} \cdot \sqrt{1+4}}$

$= \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{10}}{10}$, $\theta \approx 71.5°$

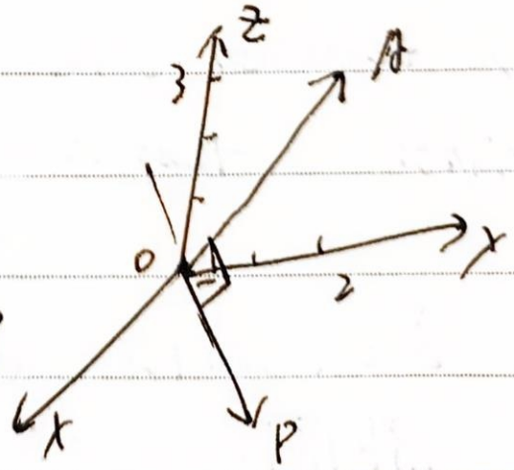
Ex2.

$x + 2y + 3z = 0$

as $\vec{OP} = \langle x, y, z \rangle$

$\vec{OA} = \langle 0, 2, 3 \rangle$

$\vec{OP} \cdot \vec{OA} = 0$, $\therefore \vec{OP} \perp \vec{OA}$



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Examples:

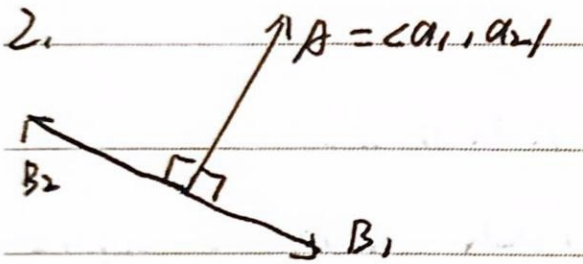
1. Find angle between $\vec{i}+\vec{j}+2\vec{k}$ and $2\vec{i}-\vec{j}+\vec{k}$

$$\langle \vec{i}+\vec{j}+2\vec{k} \rangle \cdot \langle 2\vec{i}-\vec{j}+\vec{k} \rangle = 2-1+2 = 3$$

$$= |\sqrt{6}| \cdot |\sqrt{6}| \cdot \cos\theta = 6\cdot\cos\theta$$

$$\therefore \cos\theta = \frac{1}{2}, \quad \theta = \frac{\pi}{3}$$

2.



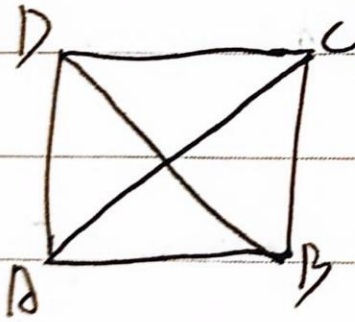
length

they all equal, what B_1 and B_2's coordinates?

$\vec{B_1}$: $\langle a_1, -a_2 \rangle$, $\vec{A} \cdot \vec{B_1} = 0$

$\vec{B_2}$: $\langle -a_1, a_2 \rangle$, $\vec{A} \cdot \vec{B_2} = 0$

3.



show the diagonals of a parallelogram have equal lengths if and only if it's a reetangle

$$\vec{AC} = \vec{AB} + \vec{BC}$$

$$\vec{BD} = \vec{BC} + \vec{CD} = \vec{BC} + \vec{BA} = \vec{BC} - \vec{AB}$$

$$|\vec{AC}|^2 = \vec{AC} \cdot \vec{AC} = (\vec{AB} + \vec{BC})^2 = |\vec{AB}|^2 + |\vec{BC}|^2 + 2\vec{AB} \cdot \vec{BC}$$

$$|\vec{BD}|^2 = (\vec{BC} - \vec{AB})^2 = |\vec{AB}|^2 + |\vec{BC}|^2 - 2\vec{AB} \cdot \vec{BC}$$

$\therefore$ only $\vec{AB} \perp \vec{BC}$ can make $AC = BD$, if $AC = BD$, it's a rectangle