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Mo Tu We Th Fr Sa Su

Session 18: Point (Cusp) on Cycloid 1.9

cusps: the point of the Cycloid where the graph touches the x-axis

take $a=1$, then

$$x(\theta)=\theta-\sin\theta, \quad y(\theta)=1-\cos\theta$$

take derivatives

$$\frac{dx}{d\theta}=1-\cos\theta \qquad \frac{dy}{d\theta}=\sin\theta$$

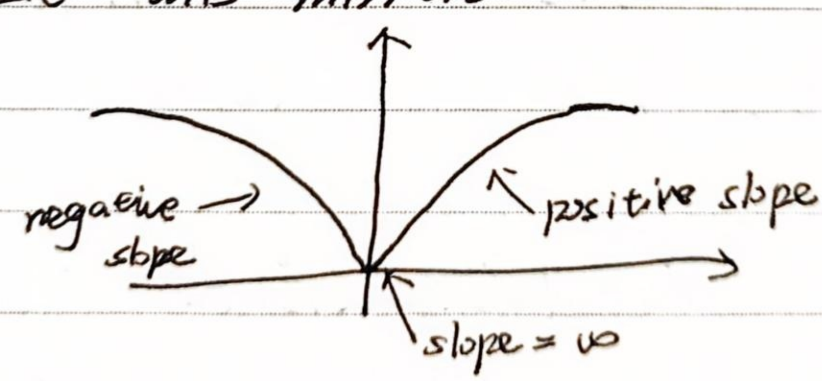
$$\frac{dy}{dx}=\frac{\sin\theta}{1-\cos\theta}$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{1-\cos\theta}=\lim_{\theta\to 0}\frac{\cos\theta}{\sin\theta}=\frac{1}{0}=\infty$$

$\theta\to 0^-$, limit $\to -\infty$

$\theta\to 0^+$, limit $\to +\infty$

~~where~~ this mirrors:



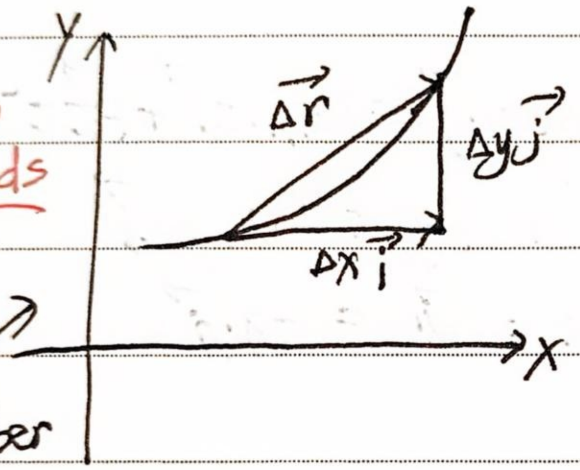
Session 19. Velocity and Acceleration

2025.1.9.

the benefits of the position vector.

$$\boxed{\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}}$$

Velocity:



$dt \to 0$
$dr \to ds$

remember

this

avg velocity: $\frac{\Delta r}{\Delta t}$

$$\Delta\vec{r} = \Delta x\vec{i} + \Delta y\vec{j}$$

$$\frac{\Delta r}{\Delta t} = \frac{\Delta x}{\Delta t}\vec{i} + \frac{\Delta y}{\Delta t}\vec{j}$$

$$\text{Velocity} = \frac{dr}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} = x'\vec{i} + y$$

$$= \langle x', y' \rangle$$

Tagent vector:

if Δr shrinks to 0, the vector $\frac{\Delta r}{\Delta t}$ becomes tangent to the curve.

$$\vec{r}'(t) = \text{velocity}$$

called the tagent ~~velocity~~ vector

Acceleration:

$$\text{acceleration} = \vec{a}(t) = \frac{d\vec{v}}{dt}$$

$$\frac{d\vec{v}}{dt} = \vec{r}''(t) = x''(t)\vec{i} + y''(t)\vec{j} = \langle x''(t), y''(t)\rangle$$

Examples

A rocket follows a trajectory

$\cancel{\vec{r}(x) =}$

$$\vec{r}(t) = x(t)\,\vec{i} + y(t)\cdot\vec{j} = v_{0,x}\cdot t\vec{i} + (-\frac{g}{2}t^2 + v_{0,y}t)\vec{j}$$

1. $\vec{v} = \frac{d\vec{r}}{dt} = v_{0,x}\vec{i} + (-gt + v_{0,y})\vec{j}$
2. acceleration: $\vec{a}(t) = \vec{v}' = -g\cdot\vec{j}$

Problems:

$\vec{r_1}(t)$ $\vec{r_2}(t)$

$$(\vec{r_1}(t) \times \vec{r_2}(t))' = \vec{r_1}'(t) \times \vec{r_2}(t) + \vec{r_1}(t) \times \vec{r_2}'(t)$$

$\vec{r_1}(t) = \langle x_1, y_1, z_1\rangle$ $\vec{r_2}(t) = \langle x_2, y_2, z_2\rangle$

$$\vec{r_1} \times \vec{r_2} = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1 z_2 - z_1 y_2)\vec{i} - \vec{j}(x_1 z_2 - x_2 z_1) + \vec{k}(x_1 y_2 - x_2 y_1)$$

$$= \langle y_1 z_2 - z_1 y_2,\ x_2 z_1 - x_1 z_2,\ x_1 y_2 - x_2 y_1 \rangle$$

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$$\frac{d(\vec{r_1}\times\vec{r_2})}{dt} = \text{---} = \langle x_1', y_1', z_1' \rangle \times \langle x_2, y_2, z_2 \rangle$$
$$+ \langle x_1, y_1, z_1 \rangle \times \langle x_2', y_2', z_2' \rangle$$

properties:

① $$\frac{d\,\vec{r_1}(t)\cdot\vec{r_2}(t)}{dt} = \vec{r_1}'(t)\cdot\vec{r_2}(t) + \vec{r_1}(t)\cdot\vec{r_2}'(t)$$

② $$(\vec{r_1}(t)\times\vec{r_2}(t))' = \vec{r_1}'(t)\times\vec{r_2}(t) + \vec{r_1}(t)\times\vec{r_2}'(t)$$

Session 20: Velocity and Arc Length

2025.1.9

Speed

$$\boxed{\text{speed} = |\vec{v}| = \left|\frac{d\vec{r}}{dt}\right|}$$

Example:

$- \ \ \frac{\pi}{}$ /