$\sec^2\theta = \tan^2\theta + 1$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

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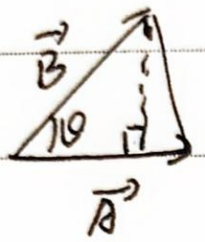
session 5 Area and Determinants in 2D

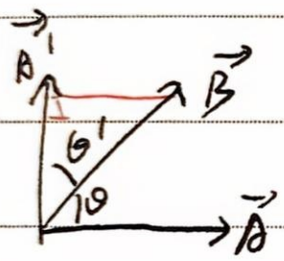
chalkboard: (行列式)

area of a triangle



$\text{Area} = \frac{1}{2}|\vec{A}|\cdot|\vec{B}|\cdot\sin\theta$

we could find $\cos\theta$, the $\sin^2\theta + \cos^2\theta = 1$

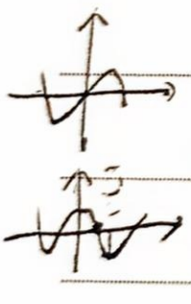


$\vec{A}' = \vec{A}$ rotated $\frac{\pi}{2}$, $\begin{cases} A = \langle a_1, a_2 \rangle \\ A' = \langle -a_2, a_1 \rangle \end{cases}$

$\theta' = \frac{\pi}{2} - \theta$, $A \perp A'$

~~$\frac{1}{2}|\vec{A}||\vec{B}|\cos\theta = \frac{1}{2}$~~

$\cos(\frac{\pi}{2} - \theta)$

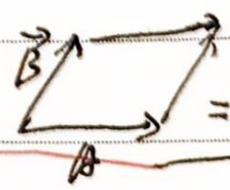


$\cos(\theta') = \sin\theta$

$\frac{1}{2}\cdot|\vec{A}|\cdot|\vec{B}|\cdot\sin\theta$

$= \frac{1}{2}|\vec{A}'||\vec{B}|\cdot\cos\theta' = \vec{A}'\cdot\vec{B}' = a_1b_2 - a_2b_1$

$\uparrow |\vec{A}'| = |\vec{A}|$

$\det(\vec{A}, \vec{B}) = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = \pm$ area of  $= a_1b_2 - a$

$\pm\text{area}(\diamond) = |\vec{A}|\cdot|\vec{B}|\cdot\sin\theta = \det(\vec{A}, \vec{B})$

$\pm\text{area}(\triangle) = \frac{1}{2}|\vec{A}|\cdot|\vec{B}|\cdot\sin\theta = \frac{1}{2}\det(\vec{A}, \vec{B})$

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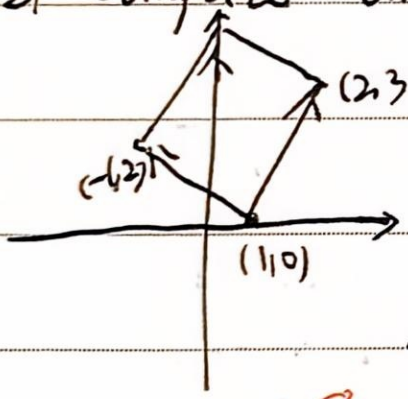
Examples:

1. $\begin{vmatrix} 6 & 5 \\ 1 & 2 \end{vmatrix} = 2\cdot 6 - 5\cdot 1 = 12-5 = 7$

2. Compute the area of the parallelogram shown



$\vec{A} = \langle 1,3\rangle$, $\vec{B} = \langle -2,2\rangle$

Area $= |\det(\vec{A},\vec{B})| = \left|\det\begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix}\right| = 2+6 = 8$

$= |\vec{A}||\vec{B}|\sin\theta = |\vec{A}'||\vec{B}|\cos\theta = \vec{A}'\cdot\vec{B} = 6+2 = 8$

$\vec{A}' = \langle -3,1\rangle$, $\langle 3,-1\rangle$ $= |-6-2| = |-8| = 8$

Problems:

1) a. Compute $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4-6 = -2$

b) $\begin{vmatrix} 1 & \vec{2} \\ 3 & 4 \end{vmatrix} = 4-(-6) = 10$

c) $\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 6-4 = 2$

2. Find the area

$|\det(\vec{CA},\vec{CB})|$

C (4,2)

Area $= \frac{1}{2}\det(\vec{A},\vec{B}) + \frac{1}{2}\det(\vec{AC},\vec{BC})$

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