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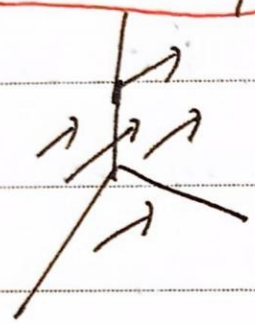
LEC 31 STokes' Theorem 225.2.25.

Recall: curl $\vec{F} = \nabla \times \vec{F}$

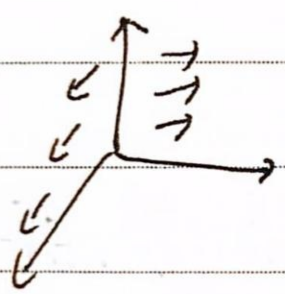
(measure rotation part of a velocity field:

dir = axis of rotation, magnitude = 2 · angular velocity)

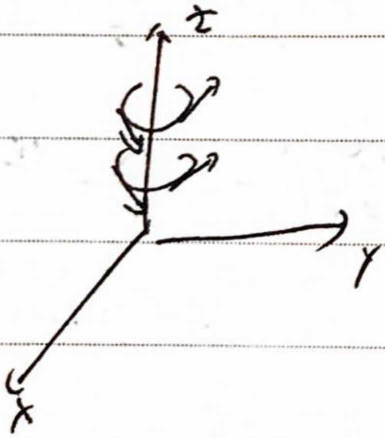
Ex:



$\vec{F} = \langle a, b, c \rangle$ curl $\vec{F} = 0$



$\vec{F} = \langle x, 0, 0 \rangle$, curl $\vec{F} = 0$, (but div $\vec{F} = 1$)



$\vec{F} = \langle -y, x, 0 \rangle$, curl $\vec{F} = 2\hat{k}$

STokes' THM (3D Green's Theorem)

$$\boxed{\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \hat{n} \, dS}$$

If C ~~at~~ closed curve
S = any surface bounded by C

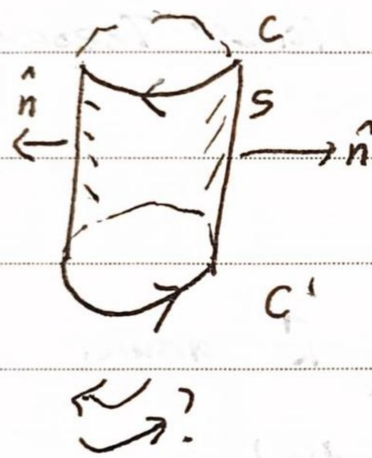
① or ② all ok

Orientation?

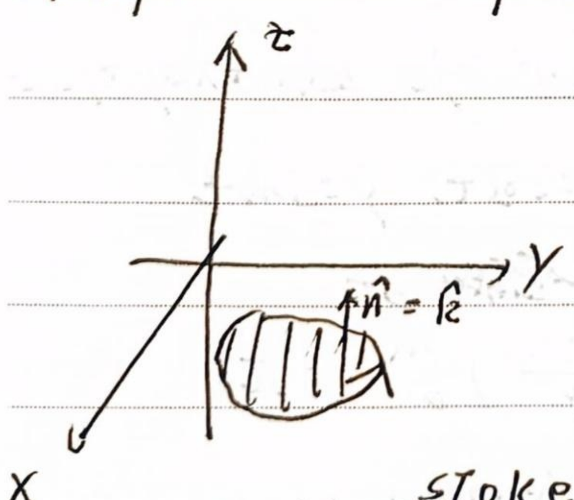
need orientation of C and S to be compatible:

- if I walk along C, S is left of me, then $\hat{n}$ is positive (up) or use the right-hand rule

{ thumb (拇) along C positive
index (食) tangent to S
middel (中) points // $\hat{n}$



Example: Comparing Stoken with Green.



S = portion of x-y plance bounded by a curve C (counterclockwise)

$\oint_C \vec{F} \cdot d\vec{r} = \oint_C P\,dx + Q\,dy$ $\quad (\vec{F} = \langle P, Q, R \rangle)$

z-component

Stokes $= \iint_S \text{curl}\,\vec{F} \cdot \hat{n}\, dS = \iint_S \boxed{(\nabla \times \vec{F}) \cdot \hat{k}}\, dS$

$(\nabla \times \vec{F}) \cdot \vec{k} = (Q_x - P_y)$

$= \iint_S (Q_x - P_y)\, dS$

is same as Green theorem $\longrightarrow = \iint_S (Q_x - P_y)\, dx\,dy$

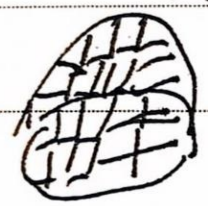
Green's theorem is special case of stokes in the space

· Why stokes is true?

☆ Know it for C, S on x-y plane

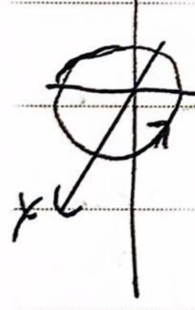
☆ also for C,S in any plane

⇒ give any S: decompose it in to tiny, almost flat pieces, use Stokes' Theorem



on each ~~piece~~ pieces

Example: work of $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$ around unit circle in x-y plane (counterclockwise)



① directly $\oint_C \vec{F} \cdot d\vec{r} = \oint_C z\,dx + x\,dy + y\,dz$

$\Rightarrow z = 0,\ x = \cos t,\ y = \sin t$

$= \int_0^{2\pi} \cos^2 t \ \cancel{\ \ }\ dt$

$= \left(\frac{t}{2} + \frac{\cos 2t}{4} \right)_0^{2\pi} = \pi$

② I can choose any surface bounded by C

use $z = 1 - x^2 - y^2$

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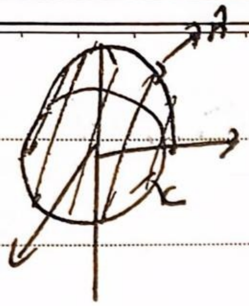

$$= \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$$

$$\text{curl}\, \vec{F} = \nabla \times \langle z, x, y \rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z & x & y \end{vmatrix} = 1\hat{i} + \hat{j} + \hat{k}$$

$$= \langle 1, 1, 1 \rangle$$

$$\hat{n}\, dS = \langle -f_x, -f_y, 1 \rangle\, dx\, dy$$ → from the LEC about divergence

$$= \langle 2x, 2y, 1 \rangle\, dx\, dy$$

$$\Rightarrow \iint_S (2x + 2y + 1)\, dx\, dy$$

$=$

① switch to polar $= \pi$

② by symmetry $= \pi$

($\int x$ and $\int y\, dx\, dy = 0$)

$$= \iint_S 1\, dx\, dy = \pi$$