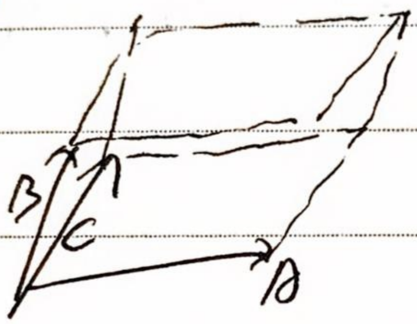
Session 6: Volumes and Determinants in Space

chalkboard: (Laplace expansion)

$\vec{A}, \vec{B}, \vec{C}$

$$\det(\vec{A},\vec{B},\vec{C}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$\boxed{(-1)^{i+j}}$



$V = \pm \det(\vec{A},\vec{B},\vec{C})$

Reading:

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$

D-1. $|A|$ is multiplied by -1 if change two rows or two columns

D-2. $|A|$ is zero, ~~is all~~ if one row or column is all zero, or if two rows or two columns are the same

D-3. $|A|$ is multiplied by C, if every element of row or column is multiplied by C.

D-4. The value of $|A|$ is unchanged if we add to one row (or column) a constant multiple of another ~~ro~~ row (column) notation $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$|A_{ij}|$ is the determinant that's left after deleting form $|A|$ the row and column containing a_{ij}

called ij-minor a_{ij} called ij-entry

Computing determinant   ij-cofactor called A_{ij}

$$A_{ij} = \cancel{(-1)^{ij}}\ (-1)^{i+j}\ |A_{ij}|$$

Example 2:

$$|A| = \begin{vmatrix} 1 & 0 & 3 \\ 1 & 2 & -1 \\ 2 & 1 & -1 \end{vmatrix}$$

-1-6

$|A_{12}| = \cancel{(-1)^3} \cdot \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = \cancel{0}\ 1$, $|A_{22}| = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -7$

$A_{12} = (-1)^3 \cdot |A_{12}| = -1$, $A_{22} = (-1)^{2+2} \cdot |A_{22}| = -7$

Laplace expansion by cofactors

(Select any row (or column) of the determiant. Multiply each a_{ij} in that (or column) row by its cofactor A_{ij}, and add the resulting; you get the value of the determinant)

in 3x3: $|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$|A| = \cancel{a_{ij}}\ a_{1j}A_{1j} + a_{2j}A_{2j} + a_{3j}A_{3j}$

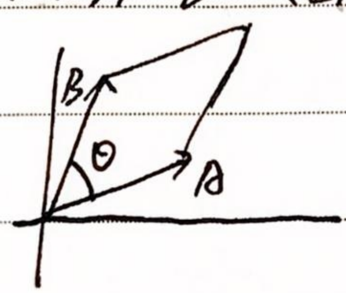
Example : evaluate $\begin{vmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 1 & 4 \\ -1 & 4 & 1 & 0 \\ 0 & 4 & 2 & -1 \end{vmatrix}$ by

$$= 1 \cdot \begin{vmatrix} -1 & 1 & 4 \\ 4 & 1 & 0 \\ 4 & 2 & -1 \end{vmatrix} - 0 \cdot A_2 + 2 \cdot \begin{vmatrix} 2 & -1 & 4 \\ -1 & 4 & 0 \\ 0 & 2 & -1 \end{vmatrix} - 3 \cdot \begin{vmatrix} 2 & -1 & 1 \\ -1 & 4 & 1 \\ 0 & 4 & 2 \end{vmatrix}$$

$$= 1 \cdot 21 + 2 \cdot (-23) - 3 \cdot 2 = -31$$

Determinants 2. Area and Volume

(1) $\pm \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$ = area of parallelogram with edges $\vec{A} = (a_1, a_2)$, $\vec{B} = (b_1, a_2)$



(2) $\pm \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ = volume of parallelepiped with edges row-vectors $\vec{A}, \vec{B}, \vec{C}$