② $(\vec{r}_1(t) \times \vec{r}_2(t))' = \vec{r}_1'(t) \times \vec{r}_2(t) + \vec{r}_1(t) \times \vec{r}_2'(t)$

## Session 20: Velocity and Arc Length

2025.1.9

Speed

$$\text{speed} = |\vec{v}| = \left|\frac{d\vec{r}}{dt}\right|$$

Example:

radius 1  3 seconds $\pi$  $\Delta\theta = \frac{\pi}{3}/s$

What's avg velocity and average speed?

$$\text{speed} = \frac{2\pi}{3} \qquad \vec{v} = \vec{0}$$

Notations:

$\vec{r}(t)$ = position, plane: $x(t)\vec{i} + y(x)\vec{j} = \langle x, y \rangle$

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In space: $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

$\frac{d\vec{r}}{dt} = \vec{v}(t)$ = tagent vector = velocity

In plane: ~~$\vec{v} = x'(t)\vec{j} + y'(y)$~~

$\vec{v} = x'(t)\vec{i} + y'(t)\vec{j} = \langle x', y' \rangle$

In space: $\vec{v} = x'(t)\vec{i} + y'(t)\vec{j} + z'(t)\vec{k} = \langle x', y', z' \rangle$

$\vec{T} = \frac{\vec{v}}{|\vec{v}|}$ = unit tagent vector

$s$ = arclength, speed $= \frac{ds}{dt} = |\vec{v}|$

In plane: $\frac{ds}{dt} = \sqrt{(x')^2 + (y')^2}$

In space: $\frac{ds}{dt} = \sqrt{(x')^2 + (y')^2 + (z')^2}$

$\vec{v} = \frac{ds}{dt} \cdot \vec{T}$, $\vec{T} = \frac{\vec{v}}{ds/dt}$

$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$ = acceleration

$$\vec{v} = \frac{ds}{dt} \cdot \vec{T}$$

$$\uparrow |\vec{v}| \quad \uparrow \frac{\vec{v}}{|\vec{v}|}$$

$$\text{speed} = \frac{ds}{dt} = |\vec{v}|$$

$$\frac{ds}{dt} = \left|\frac{d\vec{r}}{dt}\right| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

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From Problems: Vector derivatives and arc length

1, let $\vec{r}(t) = t^2\vec{i} + t^3\vec{j}$

a)

velocity $= \frac{d\vec{r}(t)}{dt} = 2t\vec{i} + 3t^2\vec{j} = \langle 2t, 3t^2 \rangle$

unit tangent vector: $\vec{T}$

$|\vec{v}| = \sqrt{4t^2+9t^4} = \frac{ds}{dt}$

$\vec{T} = \left\langle \frac{2t}{\sqrt{4t^2+9t^4}}, \frac{3t^2}{\sqrt{4t^2+9t^4}} \right\rangle$

$a(t) = \frac{d\vec{v}}{dt} = 2\vec{i} + 3t\vec{j} = \langle 2, 6t \rangle$

b)

$s = \int_1^4 \sqrt{4t^2+9t^4}\, dt$

2, $x(t) = 3t+1$, $y(t) = 4t+3$

a) velocity $= \langle 3, 4 \rangle$

speed $= \sqrt{3^2+4^2} = 5$

$\vec{T} = \langle \frac{3}{5}, \frac{4}{5} \rangle$, $a(t) = \langle 0, 0 \rangle$

b) $\int_0^2 5\, dt = 5t\Big|_0^2 = 10$