Mo Tu We Th Fr Sa Su

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Lec 12. The grandient vector (梯度向量)

$$\frac{dw}{dt} = \cancel{f_x \cdot \frac{dx}{dt}} \; w_x \frac{dx}{dt} + w_y \frac{dy}{dt} + w_z \frac{dz}{dt}$$

$$= \nabla w \cdot \frac{d\vec{r}}{dt}$$

$\boxed{\Rightarrow \nabla w = \langle w_x, w_y, w_z \rangle}$ GRANDIENT of w

at some point (x, y, z)

$$\frac{d\vec{r}}{dt} = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$$

$\boxed{\text{Theorem: } \nabla w \perp \text{level surface}}$ point foward higher w values

(the tagent plane)

Ex 1:

$w = a_1 x + a_2 y + a_3 z$ set $w =$ a constant

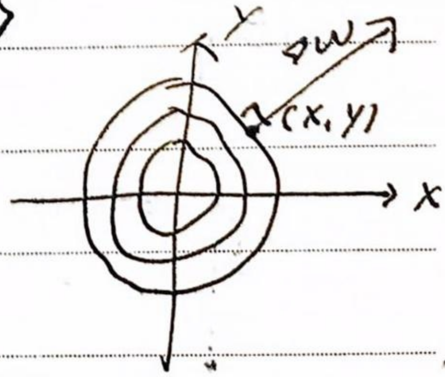
$\nabla w = \langle a_1, a_2, a_3 \rangle$ level surface: $a_1 x + a_2 y + a_3 z = c$

$\vec{N} = \langle a_1, a_2, a_3 \rangle$

Level curve

Ex: $w = x^2 + y^2$. $w = c$ is a circle, $x^2 + y^2 = c$

grandiant Vector $\nabla w = \langle 2x, 2y \rangle$



Proof:

Take curve $\vec{r} = \vec{r}(t)$ that

stays on the level $w = c$

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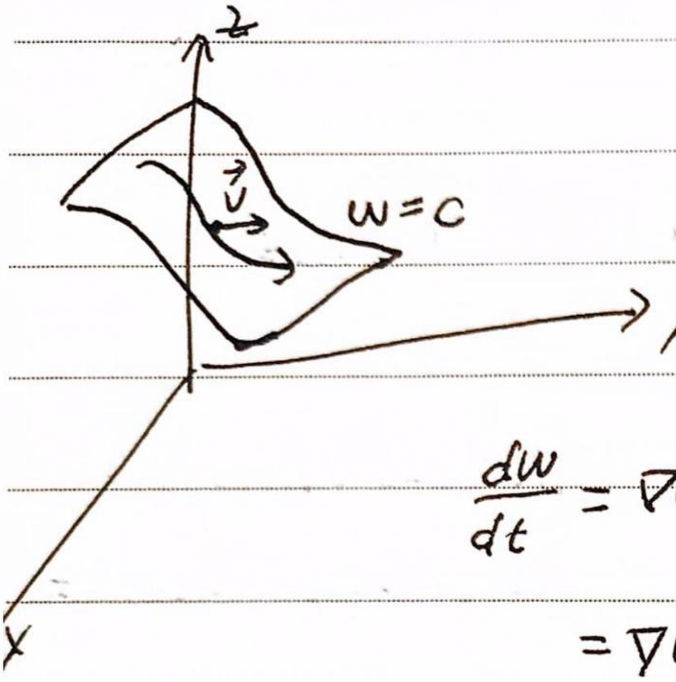


Velocity $\vec{v} = \frac{d\vec{r}}{dt}$ is tangent to the level $w=c$

$$\frac{dw}{dt} = \nabla w \cdot \frac{d\vec{r}}{dt}$$

$$= \nabla w \cdot \vec{v} = \underline{0}$$ because $w=c$, $\frac{dw}{dt}=0$

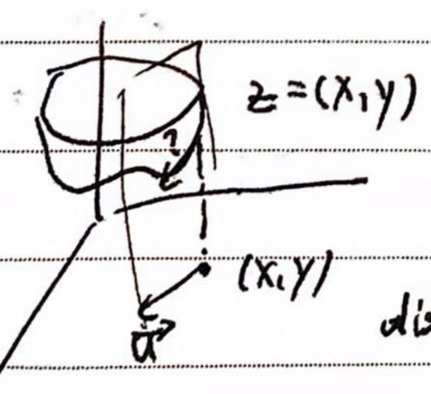
so ∇w is always perpendicular to $\vec{v}$

$\nabla w \perp \vec{v}$, This is true for any motion on $w=c$, $\vec{v}$ can be and vector tangent to $w=c$

Directional derivatives

$w = w(x,y) \rightsquigarrow$ know $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial y}$

what if we move in direction of $\hat{u}$ = unit vector?



$\vec{r}(s)$, $\underline{\frac{d\vec{r}}{ds} = \hat{u}}$

arclength distance along line

$\Rightarrow \frac{dw}{ds}$?

If $\hat{u} = \langle a, b \rangle$ $\quad \begin{cases} x(s) = x_0 + as \\ y(s) = y_0 + bs \end{cases}$ plug into w,

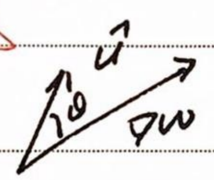
$\left.\frac{dw}{ds}\right|_{\hat{u}}$ = slope of slice of graph by a vertical plane

$$\boxed{\frac{dw}{ds} = \nabla w \cdot \frac{d\vec{r}}{ds} = \nabla w \cdot \hat{u}}$$

the component of ∇w in dir^n of $\hat{u}$

Ex: $\left.\frac{dw}{ds}\right|_{\hat{i}} = \nabla w \cdot \hat{i} = \frac{\partial w}{\partial x}$

$$\boxed{\left.\frac{dw}{ds}\right|_{\hat{u}} = \nabla w \cdot \hat{u} = |\nabla w| \cdot \cos\theta}$$



so, when $\cos\theta = 1$, $\theta = 0°$, that means $\hat{u} = \text{dir}(\nabla w)$

the direction of ∇w = the direction of fastest increase of w $\quad \boxed{|\nabla w| = \left.\frac{dw}{ds}\right|_{\hat{u}\,\text{dir}(\nabla w)}}$

→ min $\quad \theta = 180°$, $\cos\theta = -1$, $\hat{u}$ is in dir $\langle -\nabla w \rangle$

→ $\left.\frac{dw}{ds}\right|_{\hat{u}} = 0$ when $\cos\theta = 0$, $\theta = 90°$ $\hat{u} \perp \nabla w$.