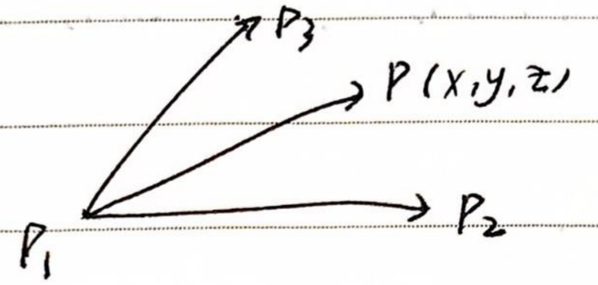
Mo | Tu | We | Th | Fr | Sa | Su

Memo No.

Date / /

Session 8, 2025.1.7 $\boxed{\text{Equations of Planes}}$



$$= (\overrightarrow{P_1P_3} \times \overrightarrow{P_1P_2}) \cdot \overrightarrow{P_1P}$$

$$\downarrow$$

$$\det(\overrightarrow{P_1P_3}, \overrightarrow{P_1P_2}, \overrightarrow{P_1P}) = 0$$

$$\Downarrow$$

telling us p is in the plane

$\vec{N} = \overrightarrow{P_1P_2} \times \overrightarrow{P_2P_3}$, if $\vec{N} \cdot \overrightarrow{P_1P} = 0$, $\vec{N} \perp \overrightarrow{P_1P}$

$\therefore$ so: $\overrightarrow{P_1P} \cdot (\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}) = 0$, triple product $= \det$

Examples:

Find a plane containing the three points

$P_1 = (1, 3, 1)$, $P_2 = (1, 2, 2)$, $P_3 = (2, 3, 3)$

$$\vec{N} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} i & j & k \\ 0 & -1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = \langle -2, 1, 1 \rangle$$

$\vec{N}$ is orthogonal to the plane, set p is any point

so $\vec{N} \cdot \overrightarrow{P_1P} = 0$, as $P(x, y, z)$

$$\langle -2, 1, 1 \rangle \cdot \langle x-1, y-3, z-1 \rangle = 0$$

$$\therefore -2(x-1) + (y-3) + (z-1) = 0$$

$$-2x + y + z = 2$$

Mo | Tu | We | Th | Fr | Sa | Su

Memo No.

Date / /

Problems, Find the plane

$P_1 = (1,0,1)$, $P_2 = (0,1,1)$, $P_3 = (1,1,0)$

$\overrightarrow{P_1P_2} = (-1,1,0)$

$\overrightarrow{P_1P_3} = (0,1,-1)$

$$\vec{N} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} i & j & k \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -\vec{i} - \vec{j} + (-1)\cdot\vec{k}$$

$$= -\vec{i} - \vec{j} - \vec{k}$$

set $P = (x, y, z)$

$$\overrightarrow{P_1P}\cdot\vec{N} = 0 = \langle x-1, y, z-1\rangle \cdot \langle -1, -1, -1\rangle$$

$$= -x+1-y-z+1 = 0$$

$$\therefore x+y+z=2$$