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Session 16: Intersection of a Line and a Plane

2025.1.9.

$$Q_0 (-1, 2, 2) \quad Q_1 (1, 3, -1) \quad \text{plane: } x + 2y + 4z = 7$$

$$\text{Line } Q(t) \Rightarrow x(t) + 2y(t) + 4z(t)$$

$$= (-1 + 2t) + 2(2 + t) + 4(2 - 3t)$$

$$\textcircled{1} = -8t + 11 \quad \text{as } -8t + 11 = 7, \quad t = \frac{1}{2}$$

$$Q\left(\frac{1}{2}\right) = \left(0, \frac{5}{2}, \frac{1}{2}\right) \text{ in the plane}$$

if $\textcircled{1} = 7$, $\textcircled{1} = 7$, all values satisfy,
the line is contained in the plane

else if $\textcircled{1}$ can't = 7, the line is parallel to
the plane, there are no points of intersection

Problems:

$$\text{line: } (1, 3, 0), (1, 2, 4)$$

$$\text{plane: } (0, 0, 0), (1, 1, 0), (0, 1, 1)$$

Line:

$$\vec{v} = (0, -1, 4) \quad \text{and} \quad \begin{cases} x = 1 \\ y = 3 - t \\ z = 4t \end{cases}$$



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planes

$$\vec{M}_1 = (1, 1, 0), \quad \vec{M}_2 = (0, 1, 1)$$

$$\vec{N} = \vec{M}_1 \times \vec{M}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \vec{i} - \vec{j} + \vec{k} = \langle 1, -1, 1 \rangle$$

plane:

$$\therefore (x-1) - (y-1) + z = 0$$

substituting gives

$$-(3-t-1) + 4t = 0$$

$$-3 + t + 1 + 4t = 0$$

$$5t = 2, \quad t = \frac{2}{5}$$

$$\text{so point: } (1, 3 - \frac{2}{5}, \frac{8}{5}) = (1, \frac{13}{5}, \frac{8}{5})$$