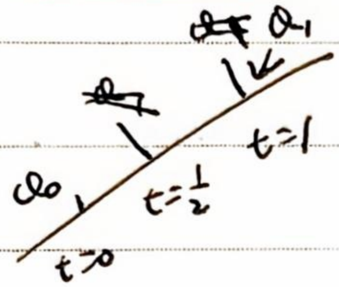
session 15. Equations of Lines

2025.1.9



$Q_0\,(-1, 2, 2)$

$Q_1\,(1, 3, -1)$

$Q(t)$: moving point

$\overrightarrow{Q_0Q(t)} = t\,\overrightarrow{Q_0Q_1}$ $\quad Q(t) = (x(t), y(t), z(t))$

$= t\langle 2, 1, -3\rangle$

$$\begin{cases} x(t)+1 = 2t \\ y(t)-2 = t \\ z(t)-2 = -3t \end{cases} \rightarrow \begin{cases} x(t) = 2t-1 \\ y(t) = t+2 \\ z(t) = -3t+2 \end{cases}$$

$$\vec{Q}(t) = Q_0 + t\cdot\overrightarrow{Q_0Q_1}$$

Reading:

$x(t), y(t), z(t)$: parametric equations

a point + a direction (vector)

$\| \; P_0 \qquad \| \; \vec{v}$

Example:

$P_0 = \langle 1, 2, 3\rangle$, $\vec{v} = \langle 1, 3, 5\rangle$

if $P(x, y, z)$ on the line, then

$\overrightarrow{P_0P} = \langle x-1, y-2, z-3\rangle$

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and $\vec{P_0P}$ is parallel to $\langle 1,3,5\rangle$, so $\vec{P_0P}$ is a scalar multiple of $\langle 1,3,5\rangle$, so

$$\cancel{\langle x,y,z\rangle} = \langle x-1, y-2, z-3\rangle = t\langle 1,3,5\rangle$$

$$\therefore \begin{cases} x-1 = t \\ y-2 = 3t \\ z-3 = 5t \end{cases} \Rightarrow \begin{cases} x = t+1 \\ y = 3t+2 \\ z = 5t+3 \end{cases}$$

in general — parametric equation of lines

$$\boxed{\begin{array}{l} P_0 = (x_0, y_0, z_0) \quad \vec{v} = \langle v_1, v_2, v_3\rangle \\ \langle x,y,z\rangle = \langle x_0 + tv_1, y_0 + tv_2, z_0 + tv_3\rangle \end{array}}$$

Problems:

1. $P\langle 1,1,2\rangle$ $\vec{v}\langle 2,-3,-1\rangle$

line: $\langle x,y,z\rangle = \langle 1+2t, 1-3t, 2-t\rangle$

$$\rightarrow \begin{cases} x = 1+2t \\ y = 1-3t \\ z = 2-t \end{cases}$$

2. intersection

$$\begin{cases} x+y+z=1 \\ x+2y+3z=2 \end{cases}$$

$$\langle 1,1,1\rangle \times \langle 1,2,3\rangle = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$= \vec{i} - 2\vec{j} + \vec{k}$ $\therefore \vec{v} = \langle 1,-2,1\rangle$

$P(0,1,0)$ $\therefore$ $\begin{cases} x = t \\ y = 1-2t \\ z = t \end{cases}$

$x+2y+\cancel{3-x-z}\ 3-3x-3y = 2$

$1 = 2x+y$, $x=0, y=1, z=0$