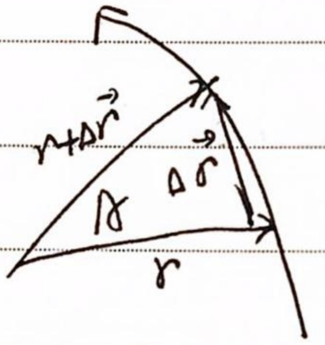
Memo No. ______

Date / /

Session 21: Kepler's Second Law

" A planet moves in a planet, and the radius vector (from sun to the planet sweep out equal areas in equal times) "

First Law " The planet's orbit in that plane is a ellispe , with the sun at one focus"



$$\Delta A \approx \frac{1}{2}|\vec{r}\times\Delta\vec{r}|$$

$$\frac{2\Delta A}{\Delta t} \approx \left|\vec{r}\times\frac{\Delta\vec{r}}{\Delta t}\right|$$

$\Delta t \to 0$ $$2\frac{dA}{dt} = \left|\vec{r}\times\frac{d\vec{r}}{dt}\right| = |\vec{r}\times\vec{V}|$$

$\uparrow$ constant so $|\vec{r}\times\vec{V}|$ is constant

so $\vec{r}\times\vec{V}$ direction $\perp$ perpendicular to the plane

so $\vec{r}\times\vec{V} = \vec{S}\,\vec{K}$ is a constant

$\Rightarrow \frac{d}{dt}(\vec{r}\times\vec{V}) = 0$

Mo | Tu | We | Th | Fr | Sa | Su

Memo No.
Date / /

$$\frac{d}{dt}(\vec{r}\times\vec{v}) = \vec{v}\times\vec{v} + \vec{r}\times\vec{a}$$

$$= \vec{r}\times\vec{a} \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

$$\therefore \vec{r}\times\vec{a} = 0$$

so ~~circle~~ acceleration vector $\vec{a}$ is parallel to $\vec{r}$, but in the opposite direction

Problems:

$\vec{r}(t)$ prove $\frac{d\vec{r}}{dt} = 0 \Rightarrow r(t) = \vec{k}$, $\vec{k}$ is a constant vector

$$\cancel{\frac{d\vec{r}}{dt} = \lim_{\Delta t\to 0}\frac{r(t+\Delta t)-r(t)}{\Delta t} = 0}$$

$$\cancel{\therefore r(t+\Delta t) - r(t) = 0}$$

$$\cancel{\therefore r(t) = \vec{k}}$$

$r'(t) = 0$, so $x'(t) = 0$, $y'(t) = 0$

$\Rightarrow x(t) = k_1$, $y(t) = k_2$

$\vec{r}(t) = \langle k_1, k_2 \rangle$