$5t = 2$, $t = \frac{2}{5}$

so point : $(1, 3 - \frac{2}{5}, \frac{8}{5}) = (1, \frac{13}{5}, \frac{8}{5})$

Session 17:

(摆线)

General Parametric Equations; the Cycloid

position vector:

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k} = \langle x(t), y(t), z(t) \rangle$$

the vector from origin to point

The most important are circles and lines.

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Ex parametric form

circle: $x = a\cos t,\ y = a\sin t$

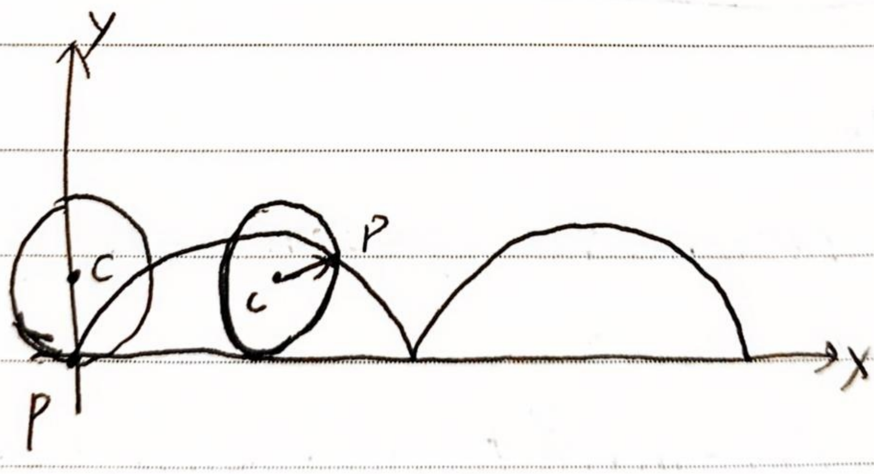
symmetric form: $x^2 + y^2 = a^2$

ellipse: $x(t) = a\cos t \quad y(t) = b\sin t$

Lines: $r(t) = \langle x, y\rangle = \langle x_0 + tb_1,\ y_0 + tb_2\rangle$

The cycloid:

no symmetric form, only can work with it in its parametric form



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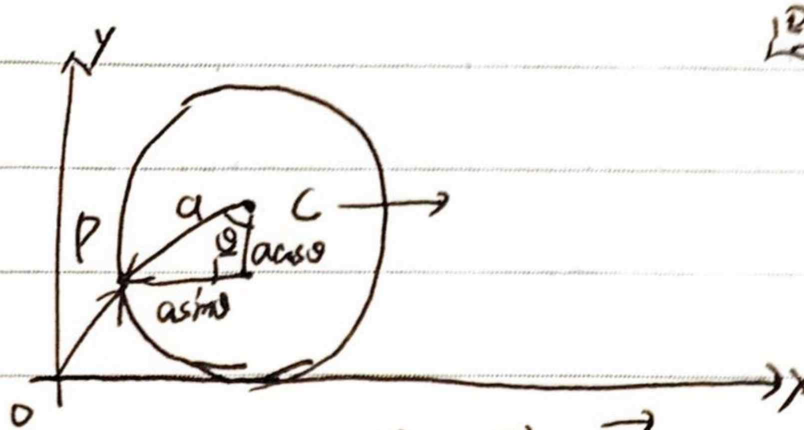
思路: $\vec{v}$ θ

get $\vec{OC}$, get $\vec{CP}$



$$\vec{OP} = \vec{OC} + \vec{CP}$$

now, $C(a\theta, a)$, $\vec{OC} = \langle a\theta, a \rangle$ $\vec{CP}$ turn to the (0,0)

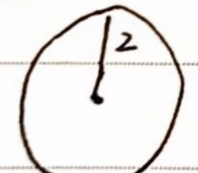
$$\boxed{\vec{CP} = \langle -a\sin\theta, -a\cos\theta \rangle}$$ ← from figure

$$\therefore \vec{OP} = \langle a\theta - a\sin\theta, \theta - a\cos\theta \rangle$$

$$\boxed{\begin{cases} x(\theta) = a\theta - a\sin\theta \\ y(\theta) = a - a\cdot\cos\theta \end{cases}}$$

the paramtric equation of cycloid

Problems:



$12\sqrt{2}$ cm/sec $\vec{v} = \langle 1, 1 \rangle$

3 revolution per second, counterclockwise

$t=0$, $(0,0)$ $(2,0)$

$\theta = 6\pi t$

$$\vec{OC} = 12\sqrt{2}\, t \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = (12t, 12t)$$

$$\vec{CP} = \langle 2\cos(6\pi t), 2\sin(6\pi t) \rangle$$

$$\vec{OP} = \vec{OC} + \vec{CP} = \langle 12t + 2\cos(6\pi t), 12t + 2\sin(6\pi t) \rangle$$