just as the same reason: $\vec{MR} = \vec{JN}$

so we are done.

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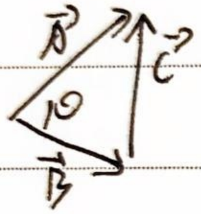
session 2: dot products 2025.1.6

defination $\vec{A}\cdot\vec{B} = \sum a_i b_i = a_1b_1 + a_2b_2 + a_3b_3$

this is a scalar

Geometrically $\vec{A}\cdot\vec{B} = |\vec{A}|\cdot|\vec{B}|\cdot\cos\theta$

1) $\vec{A}\cdot\vec{A} = |\vec{A}|^2\cdot\cos 0^\circ = |\vec{A}|^2 = a_1^2 + a_2^2 + a_3^2$



$\vec{C} = \vec{A} - \vec{B}$ $|\vec{C}|^2 = |\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cdot\cos\theta$

$$|\vec{C}|^2 = |\vec{C}|\cdot|\vec{C}|\cdot\cos 0^\circ = |\vec{A}-\vec{B}|\cdot|\vec{A}-\vec{B}|$$
$$= \vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{B} - 2\vec{A}\cdot\vec{B}$$
$$= |\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}|\cdot|\vec{B}|\cdot\cos\theta$$

defination $\vec{A}\cdot\vec{B} = a_1b_1 + a_2b_2$ example: $\langle 6,5\rangle\cdot\langle 1,2\rangle = 6+10=16$

$$|\vec{A}-\vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cdot\cos\theta$$
$$= (a_1^2+a_2^2) + (b_1^2+b_2^2) - \left((a_1-b_1)^2 + (a_2-b_2)^2\right) = 2|\vec{A}|\cdot|\vec{B}|\cdot\cos\theta$$

$$\therefore a_1b_1 + a_2\cdot b_2 = |\vec{A}|\cdot|\vec{B}|\cdot\cos\theta$$

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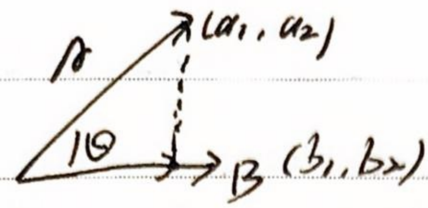
Memo No.

Date / /

$$\langle a_1, a_2\rangle \cdot \langle b_1, b_2\rangle = a_1 b_1 + a_2 \cdot b_2 = |\vec{A}|\cdot|\vec{B}|\cdot\cos\theta$$



$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

$$\Rightarrow \langle a_1, a_2\rangle \cdot \langle b_1+c_1, b_2+c_2\rangle$$

$$= a_1\cdot b_1 + a_1\cdot c_1 + a_2\cdot b_2 + a_2\cdot c_2$$

$$= a_1b_1 + a_2b_2 + a_1c_1 + a_2c_2$$

$$= \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

Ex:

i) $|\vec{A}| = 2$, $|\vec{B}| = 5$, $\theta = \frac{\pi}{4}$

$$\vec{A}\cdot\vec{B} = 2\cdot 5\cdot\cos\frac{\pi}{4} = 5\sqrt{2}$$

ii) $\vec{A} = \vec{i} + 2\vec{j}$ $\quad \vec{B} = 3\vec{i} + 4\vec{j}$

$$\vec{A}\cdot\vec{B} = 1\cdot 3 + 2\cdot 4 = 11$$

Three dimensional vectors:

$$\vec{A}\cdot\vec{B} = a_1b_1 + a_2b_2 + a_3\cdot b_3$$

$$\vec{A}\cdot\vec{B} = |\vec{A}|\cdot|\vec{B}|\cdot\cos\theta$$

Mo | Tu | We | Th | Fr | Sa | Su

Memo No.
Date / /

Ex: $\vec{A} = (4,3,6)$, $B = (-2,0,8)$, $C = (1,5,0)$

show they are the vetices of a right triangle

$\vec{AC} = (-3,2,-6)$ $\vec{AB} = (-6,-3,2)$

$\vec{AC} \cdot \vec{AB} = 18 - 6 - 12 = 0$

$\therefore |\vec{AC}| \cdot |\vec{AB}| \cdot \cos\theta = 0$, $\therefore \theta = \frac{\pi}{2}$

orthogonal:

$\vec{A} \perp \vec{B} \Leftrightarrow \vec{A} \cdot \vec{B} = 0$

Problems:

1. a) $\langle 1,2,-4 \rangle \cdot \langle 2,3,5 \rangle$

$= 2 + 6 - 20 = -12$

b) between those two vectors acute, obtuse