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LEC 19. Lin 29

Formula for det A, cofactor formula

Tridiagonal matrices

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

① $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

② $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

③ linear combination

$$= \begin{vmatrix} a & 0 \\ c & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & d \end{vmatrix}$$

$$= \begin{vmatrix} a & 0 \\ c & 0 \end{vmatrix} + \begin{vmatrix} a & 0 \\ 0 & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & 0 \end{vmatrix} + \begin{vmatrix} 0 & b \\ 0 & d \end{vmatrix}$$

$0 \quad \overset{\text{①}}{ad} \quad \overset{\text{①②}}{-cb} \quad 0$

$$= ad - cb$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & 0 & 0 \\ 0 & 0 & a_{23} \\ 0 & a_{32} & 0 \end{vmatrix} + \cdots$$

$$= a_{11}a_{22}a_{33} + (-a_{11}a_{23}a_{32}) -$$

$$a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}$$

$$- a_{13}a_{22}a_{31}$$

$$\begin{vmatrix} 0 & a_{12} & 0 \\ a_{21} & 0 & 0 \\ 0 & 0 & a_{33} \end{vmatrix} + \begin{vmatrix} 0 & a_{12} & 0 \\ 0 & 0 & a_{23} \\ a_{31} & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & a_{13} \\ a_{21} & 0 & 0 \\ 0 & a_{32} & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & 0 \\ a_{31} & 0 & 0 \end{vmatrix}$$

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BIG Formula for det (18)

$$\det A = \sum \pm a_{1\alpha} a_{2\beta} a_{3\gamma} \cdots a_{nw}$$

↑ $n!$ items

$(\alpha, \beta, \gamma, \cdots w)$ = permutation of $(1, 2, \cdots n)$

Example:

$$\begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix}$$

~~row2 + row3~~

$(4, 3, 2, 1) \to +1$, $(3, 2, 1, 4) \to -1$

Cofactor formula

3×3

$$\det = a_{11}(a_{22}a_{33} - a_{23}a_{32}) + a_{12}(a_{23}a_{31} - a_{21}a_{33}) + a_{13}(\cdots)$$

$$\begin{vmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} 0 & a_{12} & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & 0 & a_{33} \end{vmatrix} + \cdots$$

Cofactor of a_{ij} = $\pm \det$ ($n-1$ matrix with row i erased, col j)

= C_{ij}

$i+j$ → even: $+$, odd: $-$

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Cofactor formula:

$\det A = a_{11}C_{11} + a_{12}C_{12} + a_{1n}\cdot C_{1n}$ (along row 1)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad + b\cdot c\cdot(-1)^{2+1}$$

Ex

A_4 =

$$\begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

$|A_1| = 1 \quad |A_2| = 0$

$|A_3| = -1$

$|A_4| = 1\cdot|A_3| - 1\cdot|A_2| = -1$

$|A_n| = |A_{n-1}| - |A_{n-2}|$

$|A_5| = 0,\ |A_6| = 1,\ |A_7| = 1$

$\Rightarrow 1\ 0\ -1\ 1\ 0\ 1\ 1$