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$\boxed{\text{LEC 24 b quiz R review 2,12}}$ +2,13

review for Exam 2

Emphesizes Chapter 3

⟹ go to next page!!!

1. u, v, w non-zero vectors in $\mathbb{R}^7$

what subspace span? 3 dimension

2. 5×3 U r=3 pivots

$$\begin{bmatrix} | & | & | \end{bmatrix}$$ Null (U) = zero vector {0}

3. $B = \begin{bmatrix} U \\ 2U \end{bmatrix}$ echelon form $\longrightarrow \begin{bmatrix} U \\ 0 \end{bmatrix}$

$C = \begin{bmatrix} U & U \\ U & 0 \end{bmatrix} \rightarrow \begin{bmatrix} U & U \\ 0 & -U \end{bmatrix} \rightarrow \begin{bmatrix} U & 0 \\ 0 & +U \end{bmatrix}$

C $\underset{m}{10} \times 6$ $\dim N(C^T) = 10 - 6 = 4$

rank = 3+3 = 6

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x the col 2 of A

4. $Ax = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$ $x = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

what's the dimsion of $N(A)$?

① size of A: (3×3) row = A's colum number

② rank of A: rank is 2, because $\dim N(A) = 2$

③

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -2 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$A \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 0$

from $d\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow A \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$

④ $Ax = b$ can be solve

if b has the form $c \cdot \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ mutiple

Don't forget the other cases. $r=m$, $r=n$

full rank

5. $B^2 = 0 \Rightarrow B = 0$? $\boxed{\text{False}}$

example: $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

6. $n \times n$ independent columns $\Rightarrow r = n = m$

does $Ax = b$ always solved? Yes!

7.

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ basis for $N(B)$

3×4

subspace of R^4

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$N(D) = N(CD)$ if C is invertible

basis for $N(B)$ = $\begin{bmatrix} 1 & -2 \\ -1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\rightarrow -F$, $\rightarrow I$

Because the row operation doesn't change null space (and row space: $C(A^T)$)

Complete solution of $Bx = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$x_p + x_n = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

quiz-1 review, looked wrong !!!!!

this is quiz-2 review

① $Q = [q_1 \cdots q_n]$ $Q^TQ = I$

Projections - least Squares

Gram-Schmidt

② $\det A$ properties 1-3

big formula ($n!$ terms, $\pm$)

Cofactors / A^{-1}

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③ Eigenvectors $Ax = \lambda x$

$\det(A - \lambda I) = 0$

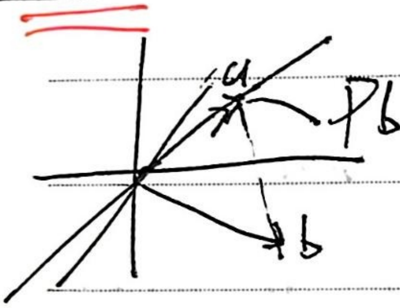
Diagonalige $S^{-1}AS = \Lambda$

↓ Examples

$\boxed{\text{question}}$

1. $a = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ $P = A(A^TA)^{-1}A^T$ ← always

$\dfrac{aa^T}{(a^Ta)}$ ← for one column because a^Ta is a constant, not a vector



$\Rightarrow P = \dfrac{1}{9} \cdot \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} [2\ 1\ 2] = \dfrac{1}{9}\begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{bmatrix}$

$[2\ 1\ 2] \cdot \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = 9$

rank $P = 1$, $\lambda = 0, 0, 1$ ← from trace

$Ax = 0 \cdot x$ → from singular

(have solution)

① because rank is 1

② dim = 2 in null space

③ I can find 2 independent eigenvectors with $\lambda = 0$!

$Ax = 0 \cdot x$

it has two independent solution

what's eigenvector for $\lambda = 1$?

the eigenvector doesn't move $\Rightarrow Ax = x$,

in projection $A \cdot A = A$

so x is a!

$(Pa = a)$

$\Rightarrow \lambda = 1$

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$\Rightarrow$ Solve $u_k = P u_k$, $u_0 = \begin{bmatrix} 9 \\ 9 \\ 0 \end{bmatrix}$

find u_k $\qquad a = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

$u_0 = P \cdot u_0 = \cancel{S \cdot \Lambda S^{-1} u_0}$

$= \underline{a \cdot \frac{a^T u_0}{a^T a}} = a \frac{27}{9} = \underset{\hat{x}}{3} a = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$

$36 + 18 = 54$

$u_k = P^k u_0 = P u_0 = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$

$9 \overline{)54}^{\,6}$

if P is other vector (not a projection)

$u_0 = c_1 x_1 + c_2 x_2 + c_3 x_3 \cdots \rightarrow A^k u_0 = c_1 \lambda^k x_1 + c_2 \lambda^k x_2 + \cdots$

for A is a projection vector

$\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 1$

$\Rightarrow A^k u_0 = c_3 \cdot 1^k x_3 = c_3 x_3$

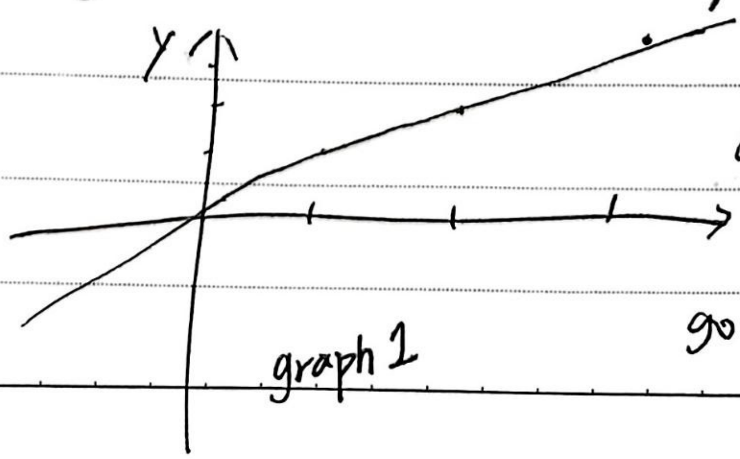
$\boxed{\text{QUESTION 2.}}$

fitting a straight line to these points

$t = 1 \quad y = 4$

$t = 2 \quad y = 5$

$t = 3 \quad y = 8$



$y = Dt$

go thorough origin

graph 1

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$$\begin{cases} 1 \cdot D = 4 \\ 2 \cdot D = 5 \\ 3 \cdot D = 8 \end{cases} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} D = \begin{bmatrix} 4 \\ 5 \\ 8 \end{bmatrix}$$

$$A x = b$$

$\underline{A^T A \hat{x} = A^T b}$ to find the best D

$\cancel{A^T}$

$$e = b - p, \quad (b - p) \cdot \cancel{A} A^T = 0 \qquad p = A \hat{x}$$

$$\cancel{b} = b \cdot A^T - A^T A \hat{x} = 0, \quad A^T A x = \cancel{b \cdot A^T} A^T \cdot b$$

$$\Rightarrow 14 \hat{D} = 38 \Rightarrow \hat{D} = \frac{38}{14} = \frac{19}{7}, \quad y = \frac{19}{7} t$$

projecting b onto column space of A (the line)

~~b~~

question 3

$a_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad a_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ Gram-Schmit

plane = col of A, find $\boxed{\text{orthonormal}}$ basis

start the first vector, make second vector B perp.. to A

$(b - P \cdot b)$ (e)

$\Rightarrow A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{a_1^T b}{a_1^T a_1} \cdot a_1$ (b: a_2) minus components of B in A

$\boxed{B - P b}$ projection to A

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{6}{14} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{4}{7} \\ \frac{1}{7} \\ -\frac{4}{7} \end{bmatrix}$$

$\frac{3}{7}$

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Third Question 3

4×4 $\quad \lambda_1, \lambda_2, \lambda_3, \lambda_4$

(a) Invertible $\Rightarrow$ $Ax = 0$ no solution exact

$x = \{0\} \Rightarrow$ no zero eigenvalue.

(b) $\det A^{-1} = \left(\frac{1}{\lambda_1}\right)\left(\frac{1}{\lambda_2}\right)\left(\frac{1}{\lambda_3}\right)\left(\frac{1}{\lambda_4}\right)$, $\det A = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4$

(c) trace of $A+I$: $= (\lambda_1 + 1) + (\lambda_2 + 1) + \cdots$

$= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + 4$

$A_4 = \begin{bmatrix} & 1 & & \\ 1 & & 1 & \\ & 1 & & 1 \\ & & 1 & 1 \end{bmatrix}$ $\qquad A_4 = \begin{bmatrix} 1 & 1 & & \\ 1 & 1 & 1 & \\ & 1 & 1 & 1 \\ & & 1 & 1 \end{bmatrix}$

Question 4

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad D_n = \det A_n$$

cof=1

use cofactors to show $D_n = \underline{1}\, D_{n-1} + \underline{-1}\, D_{n-2}$

$\cancel{D}\det A = D_3 + (-1)\cdot$

$$\det \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = \underset{n-2}{A_2} + 0 \Rightarrow A_4 = A_3 - A_2$$

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$D_1 = 1$, $D_2 = 0$,

$$\begin{bmatrix} D_n \\ D_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} D_{n-1} \\ D_{n-2} \end{bmatrix}$$

find the eigenvalue

$$\begin{vmatrix} 1-\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - \lambda + 1 = 0$$

i plane

$\lambda = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}\, i}{2}$, $|\lambda|$

$r = 1$, $z = 1 \cdot e^{i\theta}$ $\lambda_1 = e^{3}$

$|\lambda| = \frac{1}{4} + \frac{3}{4} = 1$ ⇒ on the unit circle

$z = \cos\theta + \sin\theta\, i$ ⇒ $\frac{1}{2} + \frac{\sqrt{3}}{2}$ $\theta = \frac{\pi}{3}$

$\cos\theta$, $\sin\theta$, $\theta = \frac{\pi}{3}$ $e^{i\frac{\pi}{3}}$, $e^{-i\pi/3}$

take the six powers: $(e^{i\frac{\pi}{3}})^6 = e^{2\pi i}$, $e^{-2\pi i}$

$\boxed{\cos\frac{\pi}{3} = \frac{\cancel{\sqrt{3}}\,1}{2}}$ $z = 1$ 1 steady state

⇒ $A^6 = I$ λ ⇒ $A^6 = I$, $u_k = A^6 \cdot u_0$

Question 5

$$A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 3 & 0 \end{bmatrix} = A_4^T \qquad A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

① $P = A(A^TA)^{-1}A^T$

② eigenvectors and eigenvalues

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$$A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

$$|A_3 - \lambda I| = \begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 2 \\ 0 & 2 & -\lambda \end{vmatrix} = -\lambda^3 + 5\lambda = 0$$

$$\lambda(-\lambda^2 + 5) = 0$$

$$\Rightarrow \lambda = 0, \sqrt{5}, -\sqrt{5}$$