Memo No.

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LEC34 Final Review 2.18

1\. Given $Ax = \begin{bmatrix}1\\0\\0\end{bmatrix}$ no solution $\rightarrow r<m$

$m \times n$ $Ax = \begin{bmatrix}0\\1\\0\end{bmatrix}$ has exactly 1 solution

↳ null space $\{0\}$

~~m~~ $n-r=0$

① $\boxed{m > n = r \quad 3}$

one example $m=3,\ n=r=1$

$A = \begin{bmatrix}0 & 0\\1 & 0\\0 & 1\end{bmatrix}$

$n=r=2$

only if A is square $\det(AA^T)=\det(A$

② True or False for A

$m \times m$ $\quad$ $n \times n$ $\quad$ 0

✓ ~~1.~~ $\det(A^T A) = \det(AA^T)$ ✗

$n \times n$

✓ 2. $A^T A$ is invertiable if $r=n$: independ cols of A

✗ 3. AA^T is positive definite

3×3 is semi positive definite

symmetric $\Rightarrow$ λ's >0, pivots >0, $\det(\ )>0$

rank $= 2$

4\. $A^T y = c$ proof : at least 1 solution for every c

in fact ∞ solutions

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Mo Tu We Th Fr Sa Su

$A^T y = c$, $r = n \Rightarrow$ full row rank

$n \times m$ so at least have one solution

$m > r = n \Rightarrow \dim N(A^T) = m - r > 0$

2. $A = \begin{bmatrix} v_1 & v_2 & \overset{\text{cols}}{v_3} \end{bmatrix}$

① solve $Ax = v_1 - v_2 + v_3$, $x = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

② suppose $v_1 - v_2 + v_3 = 0$, then the solution is not unique $N(A) \neq \{0\}$

③ if v_1, v_2, v_3 are orthonormal, then the comb $0 v_1 + 0 v_2$ is closest to v_3

3.

$$A = \begin{bmatrix} .2 & .4 & .3 \\ .4 & .2 & .3 \\ .4 & .4 & .4 \end{bmatrix}$$

col1 + col2 = 2 (col3)

$\lambda_1 = 0$, $\lambda_2 = 1$ (markov), $-.2 = \lambda_3$

$u_k : A^k u(0)$, $u(0)$ given $\begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$

↳ what's approach $= c_1 \lambda_1^k x_1 + c_2 \lambda_2^k x_2 + c_3 \lambda_3^k x_3$

$\qquad 0 \qquad 1 \qquad -.2$

$\lambda = 1$

$$\begin{bmatrix} -.8 & .4 & .3 \\ .4 & -.8 & .3 \\ .4 & .4 & -.6 \end{bmatrix} \overset{x_2}{\begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$k \to \infty \to u_k = c_2 \cdot x_2$

4. $\underline{2\times 2}$

① Projection onto $a=\begin{bmatrix}4\\-3\end{bmatrix}$

$P=\dfrac{aa^T}{a^Ta}$ $\qquad \cancel{0,\begin{bmatrix}1\\2\end{bmatrix} \quad 3,\begin{bmatrix}2\\1\end{bmatrix}}$

5. $0, \begin{bmatrix}1\\2\end{bmatrix}, \quad 3\begin{bmatrix}2\\1\end{bmatrix}$ $\qquad A=\underset{S}{\begin{bmatrix}1&2\\2&1\end{bmatrix}}\underset{\Lambda}{\begin{bmatrix}0&0\\0&2\end{bmatrix}}\underset{S^{-1}}{\begin{bmatrix}1&2\\2&1\end{bmatrix}^{-1}}$

2 independent

6. $A\neq B^TB$ for any B, what's A (not square) orthogonal eigenvectors but not symmetric?

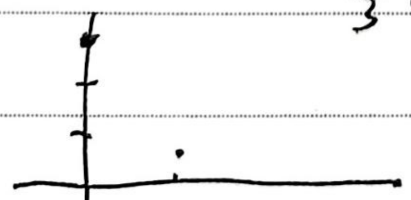
$\Rightarrow$ skew-symmetric $\begin{bmatrix}0&1\\-1&0\end{bmatrix}$

orthogonal complex matrix

7. least square $\begin{bmatrix}1&0\\1&1\\1&2\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}=\begin{bmatrix}3\\4\\1\end{bmatrix}$ ← b

$\begin{bmatrix}\hat{c}\\\hat{d}\end{bmatrix}=\begin{bmatrix}11/3\\-1\end{bmatrix}$ ① what the projection P of b onto column space of A is?

$\frac{11}{3}\times \text{col}1 - 1\cdot\text{col}2$



$A\hat{x}=P$ $\qquad A^T\cdot e=0$

$A^T(b-A\hat{x})=0$

$A^TA\hat{x}=A^Tb$