~~LEC 11 Independence, Basis, and Dimension~~ 2.6

LEC 12 Matrix Spaces ; Rank 1 ; 2.6
Small World Graphs

Basis of new vector spaces → M = all 3 by 3 matrices
↳ subspace Symmetric 3x3

Rnak one matrices

Small world graphs

Basis for M = all 3x3's ; $\underline{\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&1&0\\&0&\end{bmatrix}, \begin{bmatrix}0&0&1\\&0&\end{bmatrix} \cdots \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}}$

dimension = 9 standard basis

the dimension of symmetric metric of 3x3 = 6,

a subspace of M (dim S = 6)

$S \cap U$ = symmetric and upper triangluar

= diagonal 3x3's dim $(S \cap U) = 3$

$S \cup U$ = any element of S + any element of U

= all 3+3's dim $(S \cup U) = 9$

$\Rightarrow \boxed{\overset{6}{\dim S} + \overset{6}{\dim U} = \overset{9}{\dim (S \cup U)} + \overset{3}{\dim (S \cap U)}}$

U: the upper triangular

ex:

$\frac{d^2y}{dx^2} + y = 0$, $y = \underbrace{\cos x, \sin x}_{\text{basis}}, \cancel{e^{ix}}$

complelete solution: $y = c_1 \cos x + c_2 \sin x$

dimension: 2

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~~Rah~~ Rank ONE Matrices

$$2\times3 \quad A=\begin{bmatrix}1 & 4 & 5\\ 2 & 8 & 10\end{bmatrix} \leftarrow \text{basis}$$

the dimension of $C(A)$

$= \text{rank} = \dim C(A^T) = 1$

$$= \begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}1 & 4 & 5\end{bmatrix}$$

$1\times2 \quad 3\times1 \quad = 2\times3$

$\Rightarrow$ Rank 1 matrix $A = u\cdot v^T$

Ex: M = all 5×17 matrices (with rank 4 (can break down to rank 1 matrices)

subset of rank ④ $\rightarrow 2$ matrices not a subspace (rank can > 4)

Ex: In $\mathbb{R}^4$, $v=\begin{bmatrix}v_1\\v_2\\v_3\\v_4\end{bmatrix}$, S = all v in $\mathbb{R}^4$ with $v_1+v_2+v_3+v_4=0$

"nullspace of $Av=0$", $A=[1,1,1,1]$

rank of $A=1$

$\dim N(A) = n-r = 3$

basis for S ($N(A)$): $\begin{bmatrix}-1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\0\\1\end{bmatrix}$

$3+1=4=n$

dimension$(A)=1$ $\quad$ (r) $\dim C(A^T)=1$, $\dim(N(A^T))=0$

$C(A) = \mathbb{R}^1$, $N(A^T) = [0]$ $\quad$ comes from A

$r=m=1$ $\quad$ $[1\ 1\ 1\ 1]$

$\text{of }1 = 1 = m$ $\quad$ free variables

$x_1+x_2+x_3+x_4=0$

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why? subspace: 1. contain zero vector, 2. closed under addition and scalar multiplication. 2 rank 4 matrices add $\Rightarrow$ rank can be 5, or multiple 0, it can change rank

if I add 2 rank 4 matrices, it's sum must be in the subset of rank 4 matrices if the subset is a subspace! obviously, two rank 4 matrices sum's ~~rank~~ rank can be $\overset{(\neq)}{>} 4$! and it ≤ 8

Graph = {nodes, edges}