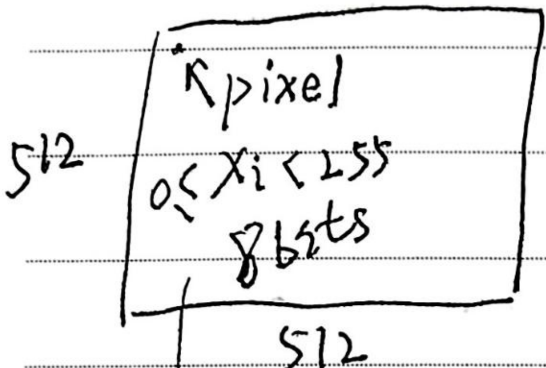
LEC31. Change of Basis; Image Compression. 2.16

(压缩)

Change of Basis.



$x \in \mathbb{R}^n \quad n = (512)^2$

JPEG : is change of basis

$x = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$

standard basis:

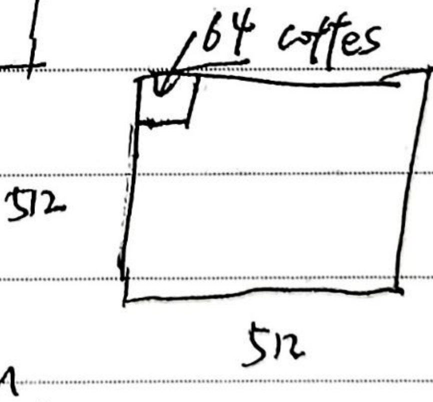
$\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$

better basis:

$\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ -1 \\ \vdots \\ -1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

JPEG: Fourier basis 8x8

$\begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ w \\ w^2 \\ \vdots \\ w^{n-1} \end{bmatrix}$



Signal x

lossless $\downarrow$ change basis

~~coff~~ coeffs c

lossless $\downarrow$ compression

$\hat{c}$

$\hat{x} = \sum \hat{c}_i v_i$

Wavelets $\mathbb{R}^8$

$$\begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\\-1\\-1\\-1\\-1\end{bmatrix}\begin{bmatrix}1\\1\\-1\\-1\\0\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\0\\0\\0\\1\\1\\-1\\-1\end{bmatrix}\begin{bmatrix}1\\-1\\0\\0\\0\\0\\0\\0\end{bmatrix} \qquad \begin{bmatrix}P_1\\ \vdots\\ P_8\end{bmatrix}$$

single P

$$P = c_1 w_1 + c_2 w_2 + \cdots + c_8 w_8$$

$$P = \begin{bmatrix} & & W & & \end{bmatrix}\begin{bmatrix}c_1\\ \vdots\\ c_8\end{bmatrix}$$

$$P = Wc$$

$$c = W^{-1}P \Rightarrow$$ a good basis means fast inverse

Good Basis:

① Fast FFT, FWT

② a few is enough

it's application

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$\boxed{\text{Change of basis}}$

columns of W = new basis vectors

$[x]$ old basis $\longrightarrow$ $[c]$ new basis

$$\boxed{x = Wc}$$

T with respect to $v_1 \dots v_8$ it has matrix $\underline{A}$

with respect to $w_1 \cdots w_8$ it has matrix $\underline{B}$

it's ~~simi~~ similar

$$B = M^{-1}A\underline{M}$$ change of basis matrix

What is A? using basis $v_1 \cdots v_8$.

Know T completely from $\underline{T(v_1)}, \underline{T(v_2)}, \cdots, \underline{T(v_8)}$

Because every $x = c_1v_1 + c_2v_2 + \cdots + c_8v_8$

Then $T(x) = c_1T(v_1) + c_2T(v_2) + \cdots$

Write $T(v_1) = a_{11}v_1 + a_{21}v_2 + \cdots + a_{81}v_8$

$T(v_2) = a_{12}v_1 + a_{22}v_2 + \cdots$

$$[A] = \begin{bmatrix} a_{11} & a_{21} & \cdots \\ \vdots & \vdots & \\ a_{18} & a_{28} & \end{bmatrix}$$

1st input is $v_1 = \lambda_1 v_1$

Eigenvector basis

$T(v_i) = \lambda_i v_i$ what is A

$$A = \begin{bmatrix} \lambda_1 & 0 & & \\ 0 & \lambda_2 & & \\ 0 & 0 & \ddots & \\ 0 & 0 & & \\ 0 & \vdots & & \ddots \\ 0 & & & \lambda_n \end{bmatrix} = \Lambda$$