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LEC 20 2/10

1. Formular for A^{-1}
2. Cramers Rule for $x = A^{-1}b$
3. $|\det A|$ = volume of box

cofactor of row 2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} C^T$$

C: the cofactor matrix

$\boxed{\text{products of } n\text{-}1 \text{ entries}}$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1}$$

$\boxed{\text{products of } n \text{ entries}}$

check

proof: $AC^T = (\det A)\cdot I$ $\quad \left(A^{-1} = \frac{1}{\det A}\cdot C^T\right)$

$$\begin{bmatrix} \cancel{a_{11}} & \cdots & \cancel{a_{1n}} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} C_{11} & \cdots & C_{n1} \\ C_{12} & & \vdots \\ \vdots & & \vdots \\ C_{1n} & & C_{nn} \end{bmatrix} = \begin{bmatrix} \det A & & 0 \\ & \det A & \\ 0 & & \ddots \ \det A \end{bmatrix}$$

in $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ cofactors: $\begin{bmatrix} a & -b \\ -c & a \end{bmatrix}$

$A_s = \begin{bmatrix} a & b \\ a & b \end{bmatrix}$, $\det A_s = a\cdot b + b\cdot(-a) = 0$

↑ signular this is exactly take a row with another cofactor column (product)

$Ax = b \Rightarrow x = A^{-1}b = \frac{1}{\det A} C^T b \rightarrow C_{11}b_1 + C_{21}b_2 + \dots$

$\det B_1 \leftarrow$ from $C^T b$

CRAMER'S RULE

$x_1 = \frac{\det B_1}{\det A}$

$x_2 = \frac{\det B_2}{\det A}$

$$B_1 = \begin{bmatrix} b_1 & \\ b_2 & n-1 \\ b_3 & \text{columns} \\ b_4 & \text{of } A \\ b_n & \end{bmatrix} = A \text{ with column 1 replaced by } b$$

$\uparrow C_{22}$

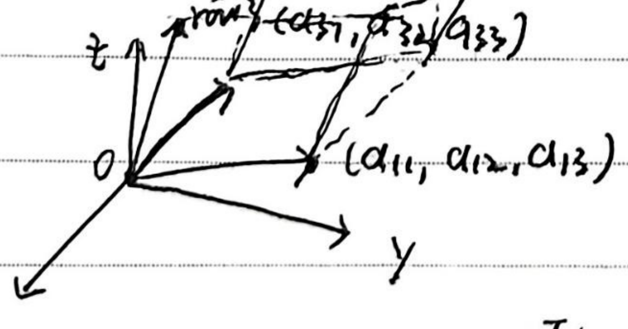
$\Rightarrow x_1 = C_{11} \cdot b_1 / \det A_1,\ x_2 = C_{21} b_2 / \det(A) \dots \dots$

in general: $B_j = A$ with column j replaced by b

$\Rightarrow x_j = \cancel{B_j} \cdot \frac{\det B_j}{\det A}$

3x3

$\boxed{|\det A| = \text{volume of box}}$



if $A = I$

$A = Q$ (orthogonal matrix)

$Q^T Q = I \Rightarrow \det Q^T Q = 1$

$\Rightarrow |Q^T||Q| = 1$

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$\Rightarrow |Q|^2 = 1 \Rightarrow |Q| = \pm 1$

if a take edge 1 to double $\Rightarrow V' = 2V$

det A also be 2 det A, so volume

satisfies this property 3a.

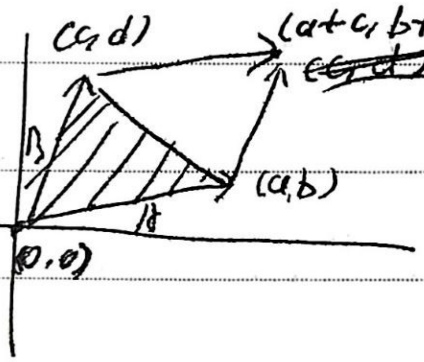
$|\det A|$ = volume of box ①, ②, ③a, 3b

I ~~2~~

inverse, volume doesn't change

③b:

$$\begin{vmatrix} a+a' & b+b' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}$$



area = $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= ad - cb$ for parallelogram

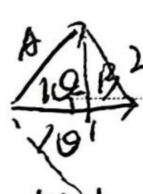
$\frac{1}{2}(ad-bc)$ for triangular

$(a,b)\cdot(c,d) = ac+bd$

$A\cdot B = |A||B|\cdot\cos\theta$

$S = \frac{1}{2}|A|\cdot|B|\cdot\sin\theta$

$\theta' = 2\pi - \theta$



$2S = A'\cdot B = |A'|\cdot|B|\cdot\sin\theta$

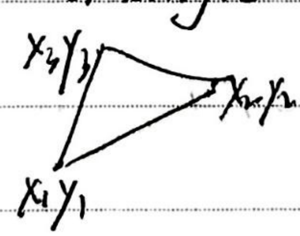
$A' = (-b_1, a_1)$

$2S = A'\cdot B = (-b, a)\cdot(c,d)$

$= ad - bc$

$2S \to S$

triangle



area $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

move triangular to origin