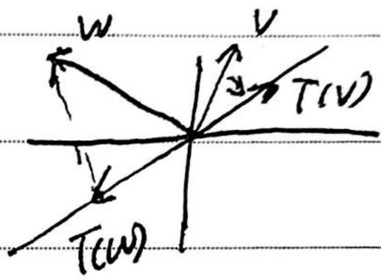
LEC 30 Linear Transformations and their matrices 2.16

without coordinates : no matrix

with coordinate $\Rightarrow$ MATRX

Example: 1. Projection is LT

$T: \mathbb{R}^2 \to \mathbb{R}^2$



also called map or mapping

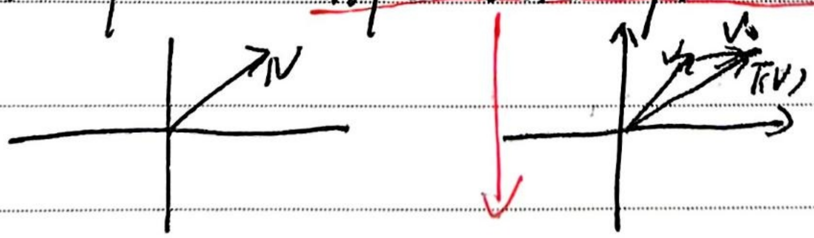
~~linear~~

Linear Transformation

$$\boxed{\begin{cases} T(v+w) = T(v) + T(w) \\ T(cv) = cT(v) \\ T(cv+dw) = cT(v) + dT(w) \end{cases}}$$

$$\boxed{T(0) = 0}$$

Example2: shift whole plane by v_0 not Linear Transf



$T(v) = v + v_0$

no, this is not a linear Transformation $\Rightarrow T(0) = v_0$

$T(2v) \neq 2(v+v_0)$

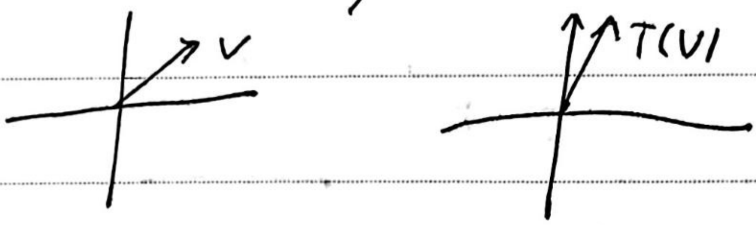
NON Example:

$$T(v) = \|v\| \qquad T: R^3 \to R^1$$

$$T(v \cdot -2) = \|2v\| \neq -2\|v\|$$

Example2:

Rotation by $45°$ $T: R^2 \to R^2$ is LT

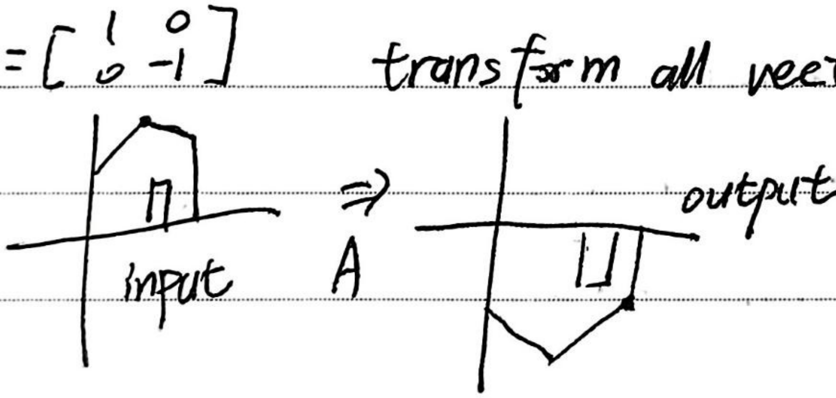


Example3: important !! Matrix A (linear transformation)

$$T(v) = Ax$$

$$T(v+w) = Av + Aw, \quad A(cv) = cAv$$

ex: $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ transform all vectors in plane



the transformation like a kind of abstract description of matrix multiplication

what's our goal?

$\Rightarrow$ understand linear transformations

Start : $T: R^3 \to R^2$

Example : $T(v) = Av$ — 2 by 3 matrix

output in R^2, input in R^3

$\boxed{\text{Information needed to know } T(v) \text{ for all input}}$

$T(V_1), T(V_2), \dots, T(V_n)$ for any basis $v_1, \dots, v_n$ — $\boxed{\text{input}}$

$\Rightarrow$ every $v = c_1 v_1 + \dots + c_n v_n$

then I know $T(v) = c_1 T(v_1) + \dots + c_n T(v_n)$

coordinate : the basis is sure

Come from a basis. — coordinate of V

ex $v = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\boxed{\text{Construct matrix } A \text{ that represent linear transformation } T}$

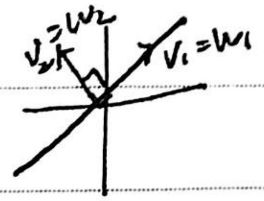
$T: R^n \to R^m$

Choose basis $v_1 \cdots v_n$ for inputs R^n

choose basis $w_1 \cdots w_m$ for outputs in R^m

$\parallel$
want matrix A

ex:



projection what's A $(c_1, c_2) \to (c_1, 0)$

$v = c_1 v_1 + c_2 v_2$ $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$T(v) = c_1 v_1$

eigenvector basis leads to diagonal matrix Λ

Project onto 45° line use stand basis

$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = w_1 \quad v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = w_2$

the matrix $P = \frac{aa^T}{a^Ta} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$

Rule to find Matrix A

1st column of A given $v_1 — v_n$, $w_1 — w_m$

↓

apply $T(v_1) = a_{11}w_1 + a_{21}w_2 + \cdots + a_{m1}w_m$

2nd colum of A:

$T(v_2) = a_{12}w_1 + \cdots + a_{m2}w_m$

$A\begin{pmatrix}\text{input} \\ \text{coordinate}\end{pmatrix} = \begin{pmatrix}\text{out} \\ \text{coordinate}\end{pmatrix}$

$T = \frac{d}{dx}$ Input: $c_1 + c_2x + c_3x^2$. basis: $1, x, x^2$

linear output: $c_2 + 2c_3x^1$ Basis: $1, x$

$$A\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} c_2 \\ 2c_3 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$R^3 \to R^2$ 2×3