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LEC 18 Determinants detA 2.9

properties 1, 2, 3, 4 - 10 + signs

Determinants $\det A = |A|$

$\boxed{\text{properties}}$

① $\det I = 1$

② exchange rows: reverse sign of det

$\Rightarrow \det P = 1$ or $-1$

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \Rightarrow \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

~~term~~ LINEAR EACH ROW

③ ⓐ $\begin{vmatrix} ta & tb \\ c & d \end{vmatrix} = t \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ ⓑ $\begin{vmatrix} a+a' & b+b' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}$

④ 2 equal rows $\rightarrow$ det $= 0$

why: Exchange rows $\rightarrow$ get same matrix, so the determinant must be 0

⑤ Substract $l \times$ row $i$ from row $k$, then the determinant doesn't change

$\rightarrow \begin{vmatrix} a & b \\ c-la & d-lb \end{vmatrix} \overset{③b}{=} \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \underbrace{\begin{vmatrix} a & b \\ -la & -lb \end{vmatrix}}_{③a} = 0$

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⑥ Row of Zeros → $\det A = 0$

$$\begin{vmatrix} 0 & 0 \\ c & d \end{vmatrix} = \begin{vmatrix} 0\cdot 0 & 0\cdot 0 \\ c & d \end{vmatrix} \overset{③a}{=\!=} 0 \cdot \begin{vmatrix} 0 & 0 \\ c & d \end{vmatrix} = 0$$

⑦ $U = \begin{bmatrix} d_1 & * & * & * \\ 0 & d_2 & * & * \\ 0 & 0 & \ddots & * \\ 0 & 0 & 0 & d_n \end{bmatrix}$ $\det(U) = d_1 \cdot d_2 \cdots d_n$

product of pivots

by elimination

⑤, ③a, ①

⑧ $\det(A) = 0$ exactly when $A$ is singular

$\det(A) \,!= 0$, when $A$ is invertible

⑨ $\det(AB) = \det(A) \cdot \det(B)$

? $\det(A^{-1})$ $A^{-1}A = I \Rightarrow \det(A^{-1}) = \frac{1}{\det(A)}$

~~⑩~~ $\det(A^2) = \det A^2$

$\det 2A = \boxed{2^n} \det A$ ③a

⑩ $\det A^T = \det A$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$$ ⇒ exchange column change sign.

#10 $|A^T| = |A|$

↓

Proof of 10: $|LU| = |U^T L^T|$

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lower triangular

$$\Rightarrow |U^T|\overset{\substack{1\\ \shortparallel}}{|L^T|} = \overset{\substack{1\\ \shortparallel}}{|L|}\,|U| \quad ✓$$

$$\Rightarrow \underset{\substack{\shortparallel \\ d_1 \cdots d_n}}{|U^T|} = \underset{\substack{\shortparallel \\ d_1 \cdots d_n}}{|U|} \quad ✓$$