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LEC9 Linear independence 2.5

Spaning a Space

BASIS And dimension

a bunch of vectors

Suppose A is m by n with $\boxed{m<n}$

then there are some non-zero solutions to $Ax=0$

(more unknown than equations)

the reason is there will be free variables

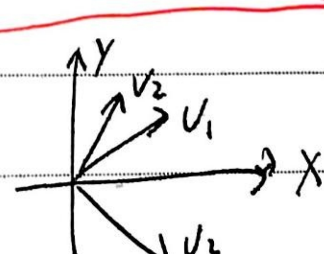
at least one!!

Independence

Vectors $x_1, x_2, \cdots$ are independent if no combination gives the zero vector (except the zero combination ($c=0$))

$\Rightarrow c_1x_1 + c_2x_2 + c_3x_3 + \cdots c_nx_n \neq 0$

definition of independence



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v_1, v_2, v_3 are dependent !

$$A = \begin{array}{ccc} v_1 & v_2 & v_3 \\ \end{array} \begin{bmatrix} 2 & 1 & 2.5 \\ 1 & 2 & -1 \end{bmatrix} \Rightarrow m=2, n=3$$

$m<n$

if $\begin{bmatrix} 2 & 1 & 2.5 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

it has a null-space $\Rightarrow$ $\underline{r \le m < n}$

so there will be some (c_1, c_2, c_3) make $Ax=0$

(non-zero)

Repeat when $v_1, \cdots v_n$ are columns of A

They are independent if nullspace of A is the {zero vector}

They are dependent if $Ac=0$ for some non-zero c.

rank $=n$ → no free variables $N(A)=\{0\}$

rank $<n$ → yes free variables

$\boxed{\text{Span a Space}}$

Vectors $v_1, \cdots v_e$ span a ~~s~~ subspace means the space consists of all combinations of those vectors. (the smallest space)

if $v_1, \cdots v_n$ are independent, I've got the right number of vectors $\Rightarrow$

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Basis for a vector space is a sequence of vectors $v_1, v_2, \cdots v_d$ with 2 properties

① they are independent
② they span the space

Example:

Space is R^3, One basis is $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$

standard basis

R^n, if n vectors given basis if $n \times n$ matrix is invertible $(r = m = n)$

a great fact: every basis about (for) a given space, has the same number of vectors.

$\boxed{\text{Definition}}$: the dimension of the space

R^n have n vectors ; n dimension

Example:

rank = 2 = # pivot column = $\boxed{\text{dimension}}$ of the column space

Space $C(A)$

$\begin{bmatrix}1 & 2 & 3 & 1\\1 & 1 & 2 & 1\\1 & 2 & 3 & 1\end{bmatrix}$

①, ②, not independent

$N(A)$

$\begin{bmatrix}-1\\-1\\1\\0\end{bmatrix}$

one of a basis for column space: column 1 & 2

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$\Leftarrow$ not the dimension of A is the dimension of the column space of A take attention

another basis for the column space $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 6 \end{bmatrix}$

$\uparrow$ has to be independent and span the space

so another great fact:

① the dimension = the rank of the column of column space

what about null space: $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(the number)

dimension $N(A)$ = # free variable.

$= n - r$

two free ~~...~~ variables

two special solution

② the dimension of null space = the ~~...~~ number of free variables $= n - r$