Mo Tu We Th Fr Sa Su

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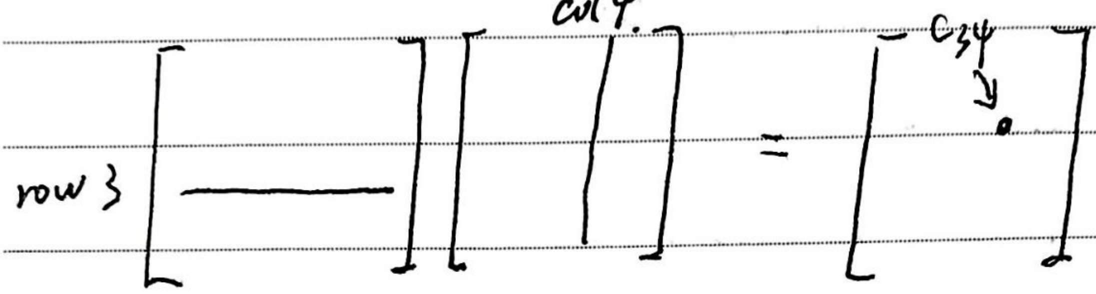
LEC #3 2025.2.2.

Multiplication and inverse Matrices



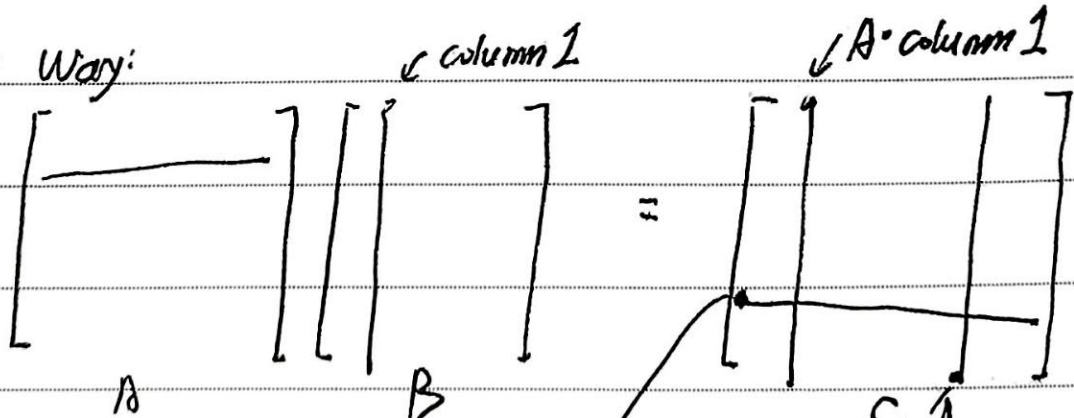
A $m \times n$ B $n \times p$ $C = AB$ $m \times p$

c_{34} = (row 3 of A) · (col 4 of B)

$$c_{34} = a_{31}b_{14} + a_{32}b_{24} + a_{33}b_{34} + \cdots$$

$$= \sum_{k=1}^{n} a_{3k} \cdot b_{k4}$$

another way:



rows of C are combinations of rows of B

columns of C are combinations of columns of A

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columns of A × ~~columns~~ rows of B

$m \times 1$ $1 \times p$

AB = sum of (cols of A) × (rows of B)

$$\begin{bmatrix} 2 & 7 \\ 3 & 8 \\ 4 & 9 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \cdot [1, 6] + \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}[0\ 0]$$

Block

$A_1B_1 + A_2B_3$

$$\underset{A}{\left[\begin{array}{c|c} A_1 & A_2 \\ \hline A_3 & A_4 \end{array}\right]}\underset{B}{\left[\begin{array}{c|c} B_1 & B_2 \\ \hline B_3 & B_4 \end{array}\right]} = \left[\begin{array}{c|c} \ & \ \\ \hline \ & \ \end{array}\right]$$

Inverse

$$A^{-1} \cdot A = I = A \cdot A^{-1}$$

if this exits, invertible / nonsingular

Singular case (No inverse)

ex: $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

lies on a same line

no combination can be the I

$$\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

I can find a vector x with $Ax = 0$ (expect $x = 0$)

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$$\underbrace{\begin{bmatrix}1&3\\2&7\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}a&c\\b&b\end{bmatrix}}_{A^{-1}}=\underbrace{\begin{bmatrix}1&0\\0&1\end{bmatrix}}_{I}$$

A

~~too~~ idea

$\boxed{\text{Gauss - Jordan}}$ (solve 2 equations at once)

$$\begin{bmatrix}1&3\\2&7\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$$

$$\begin{bmatrix}1&3\\2&7\end{bmatrix}\begin{bmatrix}c\\b\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$$

$$\left[\begin{array}{cc|cc}\underbrace{\begin{matrix}1&3\\2&7\end{matrix}}_{A}&&\underbrace{\begin{matrix}1&0\\0&1\end{matrix}}_{I}\end{array}\right]\rightarrow\left[\begin{array}{cc|cc}1&3&1&0\\0&1&-2&1\end{array}\right]$$

$$\rightarrow\left[\begin{array}{cc|cc}\underbrace{\begin{matrix}1&0\\0&1\end{matrix}}_{I}&&\underbrace{\begin{matrix}7&-3\\-2&1\end{matrix}}_{A^{-1}}\end{array}\right]$$

why? :

$$\Rightarrow \boxed{\begin{matrix}E\,[A\ I]=[I\ \ E]\\ EA=I \text{ tell us } E=A^{-1}\end{matrix}}$$