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LEC 33. Left and Right Inverse; Pesudo inverses; 4 subspace

2.17

① $\boxed{\text{2-sided inverse}}$

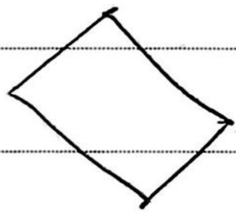
$AA^{-1} = I = A^{-1}A$ $\quad r=m=n$ full rank

② $\boxed{\text{left inverse}}$

full column rank $r=n$

nullspace $= \{0\}$ independent columns

0 or 1 solutions to $Ax=b$



$\underset{n\times m \,\cdot\, m\times n}{A^TA}$ is same as

$\Rightarrow n\times n$ symmetric

$$\boxed{\underbrace{(A^TA)^{-1}A^T}_{\hookrightarrow A^{-1}\text{left}}}$$

because $\underbrace{(A^TA)^{-1}A^T}_{n\times m}\cdot \underset{m\times n}{A} = I$

$\Rightarrow \underset{n\times m}{A^{-1}\text{left}} \cdot \underset{m\times n}{A} = I_{n\times n}$

multiply on left

$P = A(A^TA)^{-1}A^T$, it's projection to A column space

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③ Right Inverse

full row rank $r=m<n$

$n(A^T)=\{0\}$ independent rows

$Ax=b$ free ~~vari~~ varibles $n-r$

∞ solutions

$$\boxed{A\,\underline{A^T(AA^T)^{-1}} = I \qquad A\cdot A^{-1}_{right} \rightarrow \text{right inverse}}$$

when $r=m<n$

multiple A on right

$C(A^T)$

$P_1 = A^T(AA^T)^{-1}A$ this is projection onto rowspace

this is important review

Summery:

$n\times n \cdot \frac{n\times m}{m\times n} = n\times m \cdot m\times n$

$n\times m\ m\times n$

$= n\times n$

left ~~row~~ inverse: $A^{-1}_{left} = (A^TA)^{-1}A^T$

$r=n<m$ $\quad (A^TA)^{-1}A^T\cdot A = I \rightarrow n\times n$

$A\cdot(A^TA)^{-1}A^T = P$

right inverse: $A^{-1}_{right} = A^T(AA^T)^{-1} \rightarrow m\times m$

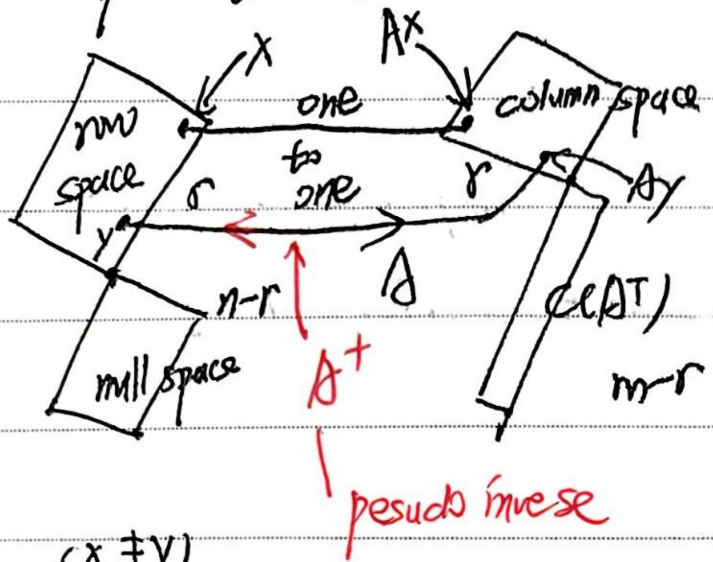
$r=m<n$ $\quad A\cdot A^T(AA^T)^{-1} = I \rightarrow m\times m$

$A^T(AA^T)^{-1}\cdot A = P$ of $C(A^T)$

if left inverse exist and equal to right inverse

$\Rightarrow$ A is invertiable ($r=m=n$)

the pesudoinverse



r dimsion space

$(x \neq y)$

IF x, y in row space then $Ax \neq Ay$ (in column space)

$y = A^+(Ay)$,

pesudoinverse: column space maps to row space

Proof: Suppose $Ax = Ay$

$\Rightarrow A(x-y) = 0$, but x, y in row space and difference, this means $x-y$ in nullspace!

so $x-y$ must in row space

so $Ax \neq Ay$

Can we find,

How to find the persudoinverse A^+

① start from SVD: $A = U \cdot \Sigma \cdot V^T$

$$\Sigma = \begin{bmatrix} \sigma_1 & & & 0 \\ & \ddots & & \\ & & \sigma_r & \\ 0 & & & 0 \end{bmatrix}$$ n cols, m rows

$$\Sigma^+ = \begin{bmatrix} 1/\sigma_1 & & & 0 \\ & 1/\sigma_2 & & \\ & & \ddots & \\ 0 & & 1/\sigma_r & 0 \end{bmatrix} \quad n \times m$$

$\underset{m \times n}{r = r}$

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$$\underset{n\times m}{\Sigma^{+}}\ \underset{m\times n}{\Sigma} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ & & 1 \\ 0 & & 0 \end{bmatrix} \leftarrow \begin{matrix} n\times n \\ \cancel{m\times m} \end{matrix}$$

$$\begin{bmatrix} \equiv \end{bmatrix}\begin{bmatrix} | & | \end{bmatrix} = \begin{bmatrix} | & | \\ | & | \end{bmatrix}$$

3×1 $\cancel{\ }$

$$\Sigma\Sigma^{+} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \ 0 \end{bmatrix} \leftarrow m\times m$$

$$\boxed{A^{+} = V\Sigma^{+}U^{T}}$$

$\uparrow (V^{T})^{-1}$ $\uparrow (U)^{-1}$