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Memo No. _____

Date / /

LEC 2

2025.2.2

Elimination with
Matrices.

A

$$x + 2y + z = 2$$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

$$3x + 8y + z = 12$$

$$\begin{bmatrix} 3 & 8 & 1 \end{bmatrix} \xrightarrow{(2,1)}$$

$$\begin{bmatrix} 0 & 12 & -2 \end{bmatrix} \rightarrow -2$$

$$4y + z = 2$$

$$\begin{bmatrix} 0 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 & 1 \end{bmatrix}$$

Elimination

$$\xrightarrow{(3,2)}$$

$$\begin{bmatrix} 11 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 12 & -2 \end{bmatrix} \rightarrow -2$$

$$\begin{bmatrix} 0 & 0 & 15 \end{bmatrix}$$

pivots

Back-substitution

$$b: \begin{bmatrix} 2 \\ 12 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ 6 \\ -10 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x + 2y + z = 2 \\ 2y - 2z = 6 \\ 5z = -10 \end{cases}$$

$$x = 2$$

$$2y - 2z = 6$$

$$y = 1$$

$$5z = -10$$

$$z = -2$$

Elimination matrices

step 1

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} \quad (2,1)$$

↑ Ex 1



$$\text{ex: } [1, 2, 3] \cdot \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} = \begin{matrix} 1 \times \text{row 1} \\ + \\ 2 \times \text{row 2} \\ 3 \times \text{row 3} \end{matrix}$$

combining the rows

~~subtract~~

(3, 2), step 2, subtract:

$$E_{32} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

$$E_{32}(E_{21}A) = U \Rightarrow (E_{32}E_{21})A = U$$

(can change ()'s order)

permutation exchange rows 1 and 2

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

matrix

Inverses

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$E^{-1} \quad E \quad I$