LEC 22 Eigenvalues and Eigenvectors

$\det[A-\lambda I]=0$ 2.10

$TRACE = \lambda_1 + \lambda_2 + \cdots + \lambda_n$

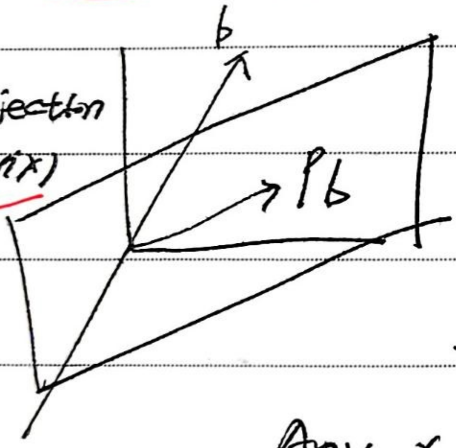
Eigenvectors

Ax parallel to x $\Rightarrow$ $\boxed{Ax=\lambda x}$

(λ can be negative or 0)

if A is singular, then $\lambda=0$ is an eigenvalue

(projection matrix)



what are x's and λ's for a projection ~~matrices~~ matrix?

$\Rightarrow$ Eigenvectors so $Px // x$

$\Downarrow$

Any x in the plane : $Px = x$, $\lambda=1$

Any x perpendicular to the plane: $Px = 0x$, $\lambda=0$

(permutation matrix)

$A=\begin{bmatrix}0&1\\1&0\end{bmatrix}$ $x=\begin{bmatrix}1\\1\end{bmatrix}$ $Ax=\begin{bmatrix}1\\1\end{bmatrix}=x$ $\lambda=1$

$x=\begin{bmatrix}-1\\1\end{bmatrix}$ $Ax=\begin{bmatrix}1\\-1\end{bmatrix}=-x$ $\lambda=-1$

a neat fact: the sum of λ's $= a_{11}+a_{22}+\cdots+a_{nn}$

Memo No
Date / /

and the number of Eigenvalues = n (dim of A)

$\boxed{\text{how to solve } Ax = \lambda x}$ ⇒ X nonzero

Rewrite $(A-\lambda I)x = 0$

must be ~~sign~~ singular why (X can be nonzero)

⇒ $\det(A-\lambda I) = 0$

Find λ First

if I can solve this equation it must be singular means A is invertiable (it has nullspace nonzero)

Ex $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

1. $\det(A-\lambda I) = \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix}$

$= (3-\lambda)^2 - 1$

$= \lambda^2 - 6\lambda + 8 = (\lambda-4)(\lambda-2)$

⇒ $\lambda_1 = 4, \ \lambda_2 = 2$

2. $A - 4I = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ $\lambda_1 = 4, \ x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

pivot

$A - 2I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\lambda_2 = 2, \ x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

~~$A_1 = I$~~ compare with this $A' : \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \boxed{A' + 3I}$

⇒ $\lambda_1 = \lambda_1' + 3, \ \lambda_2 = \lambda_2' + 3$, X's don't change!

Memo No.
Date / /

$\Rightarrow$ if $Ax = \lambda x$, $(A+3I)x = (\lambda+3)x$

Not so great $A+B$, AB

if $Ax = \lambda x$, B has eigenvalues α_1,

$Bx_1 = \alpha x_1$

~~$(A+B)x = (\lambda+\alpha)x$~~

Example Q rotation

$90°$ rotation $Q = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

trace : $0+0 = \lambda_1 + \lambda_2$

det $= 1 = \lambda_1 \cdot \lambda_2$

$\det(Q-\lambda I) = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 > 0$

$= 0$

bad thing $\longrightarrow$ $\lambda_1 = i$, $\lambda_2 = -i$

Q is far away from symmetric

Suppose $A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$ $\det(A-\lambda I) = \begin{vmatrix} 3-\lambda & 1 \\ 0 & 3-\lambda \end{vmatrix}$

$= (3-\lambda)(3-\lambda)$

$\lambda_1 = 3 = \lambda_2$

go to eigenvectors: $(A-\lambda I)x = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$

$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $x_2 =$ no second independent x