Unit 1 : $Ax=b$ and the Four Subspaces

$\boxed{\text{LEC 1 \quad 2025.2.1}}$

the geometry of Linear Equations

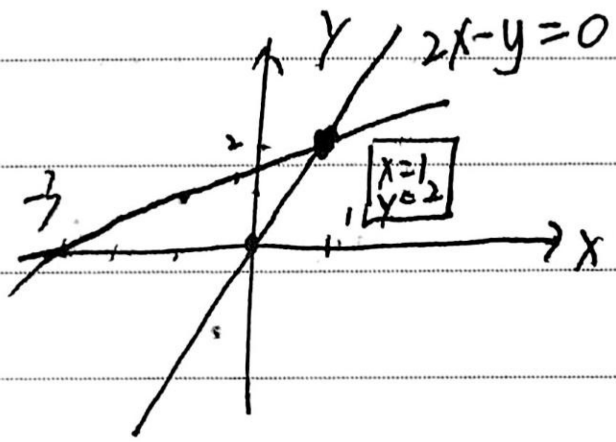
Exame:

$$\begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases} \qquad \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

A matrix X = b

$Ax = b$

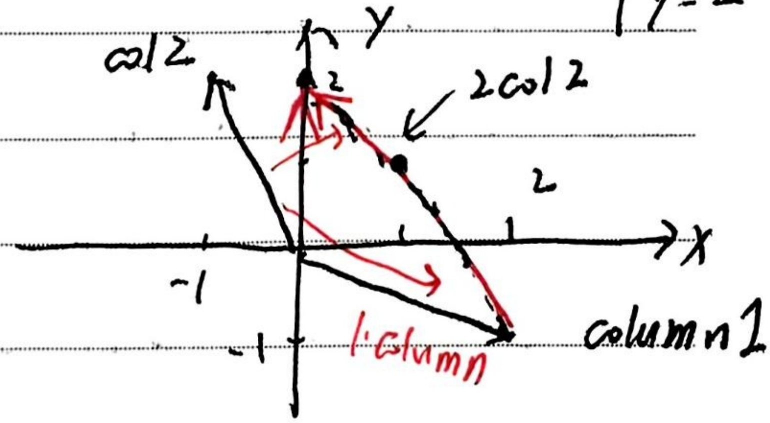
Row picture



$$x\begin{bmatrix} 2 \\ -1 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

linear combination of columns

take $\begin{cases} x = 1 \\ y = 2 \end{cases}$

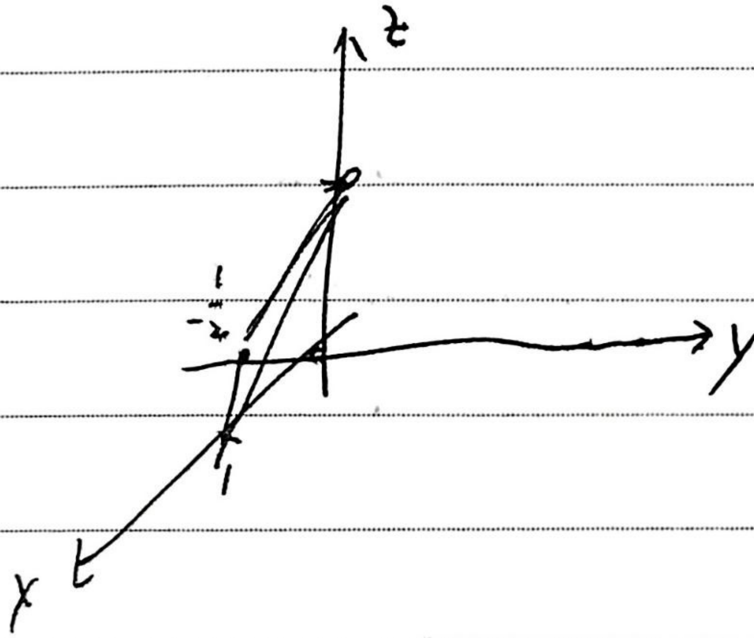


Ex2.

$$2x - y = 0$$
$$-x + 2y - z = -1$$
$$-3y + 4z = 4$$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

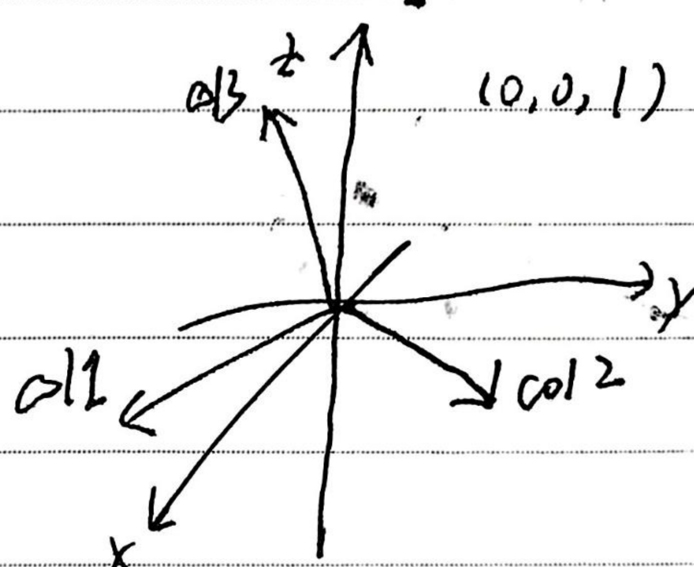
Row picture:



column picture:

$$\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} x + y \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} +$$

$$z \cdot \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$



Can I solve $Ax=b$ for every b? for this the answer is YES.

$Ax = b$ by column

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

meaning: Ax is a combination of A

LEC 2.25.21 An Overview.

Vectors → matrices → subspaces

$u \; v \; w$ $\boxed{\text{lin comb}}$ $x_1 u + x_2 v + x_3 w = b$

↑ scala

$\begin{bmatrix}1\\-1\\0\end{bmatrix} \begin{bmatrix}0\\1\\-1\end{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix}$

comb of cols

the matrix $Ax = \boxed{\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}x_1\\-x_1+x_2\\x_3-x_2\end{bmatrix} = b$

$A\begin{bmatrix}1\\4\\9\end{bmatrix} = \begin{bmatrix}1\\3\\5\end{bmatrix}$

对比

if $b = \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$

solution: $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}b_1\\b_1+b_2\\b_1+b_2+b_3\end{bmatrix} = \begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix} = x$

↑ inverse matrix

$Ax = b$, $x = A^{-1}b$ $\quad S = A^{-1}$

Ex2: $C = \begin{bmatrix}1&0&-1\\-1&1&0\\0&-1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}x_1-x_3\\x_2-x_1\\x_3-x_2\end{bmatrix} = b = \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix} \Rightarrow \boxed{b_1+b_2+b_3 = 0}$

combs in a plane

if $Cx = 0$, only $\begin{bmatrix}c\\c\\c\end{bmatrix}$ $\quad \begin{bmatrix}0\\0\\0\end{bmatrix}$ if $b=0$

C^{-1} can't get back to x

A^TA watch this in course