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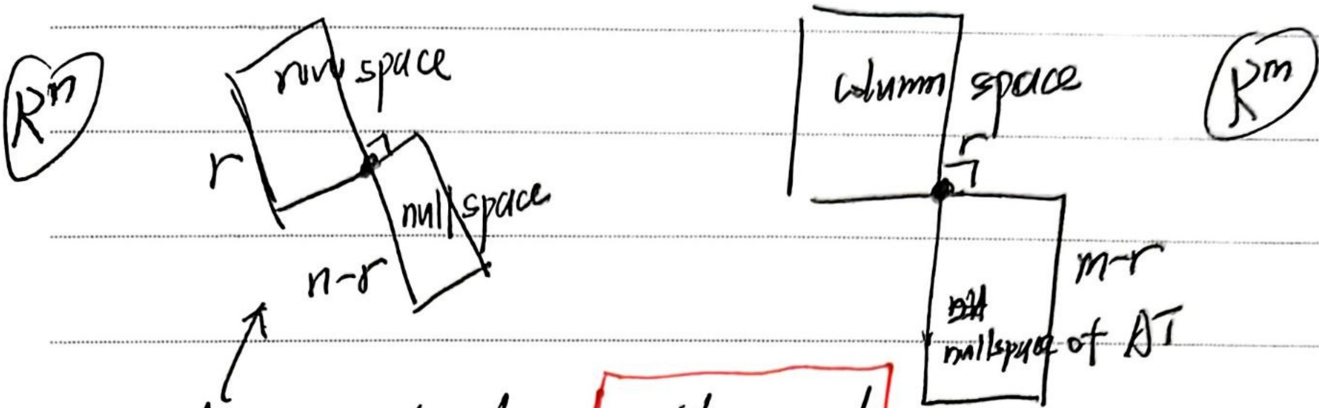
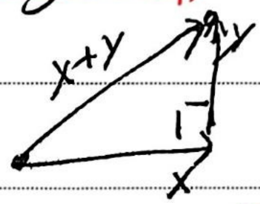
Unit 2.

LEC 14. Orthogonal Vectors and Subspace 2.7.



subspace to be orthogonal

orthogonal vectors



Pythagoras

$x^T y$ is the dot product! $\boxed{x^T y = 0}$

~~[x1, x2]~~

$[x_1, y_1] \cdot \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = x_1 x_2 + y_1 y_2 \rightarrow$ the dot product

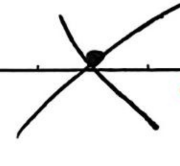
$\|x\|^2 = 14$ $\quad x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\quad y = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ $\quad x+y = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$

$x^T x$ $\quad \|y\|^2 = 5$

$\Rightarrow x^T x + y^T y = (x+y)^T (x+y)$ $\quad \|x+y\|^2 = 19$

$\boxed{\text{Subspace } S \text{ is orthogonal to Subspace } T}$

means: every vector in S is orthogonal to every vector in T



only zero vector in intersection

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row space is orthogonal to nullspace

why?

$$Ax = 0 \qquad \begin{bmatrix} \text{row1 of } A \\ \text{row2 of } A \\ \text{row3 of } A \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$c_1 (\text{row1})^T \cdot x + c_2 (\text{row2})^T \cdot x + \cdots = 0$$

~~null space~~

null space and rowspace are orthogonal Complements in R^n

Coming: $Ax = b$ when there is no solution

$m > n$

$$A^T A \hat{x} = A^T b$$

$A^T A$ ① square $n \times n$

$n \times m \quad m \times n$

$n \times n$

② symmetric

~~③~~ $(A^T A)^T = A^T A$

$$\underset{A^T}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \end{bmatrix}} \underset{A}{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 5 \end{bmatrix}} = \begin{bmatrix} 3 & 8 \\ 8 & 30 \end{bmatrix}$$

← rank = A rank of

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$\boxed{\text{rank of } A^TA = \text{rank of } A}$

A^TA is invertiable exactly if A has independent columns.

↓ $(r>1)$