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LEC 24.

Markov Matrix

steady state

Fourier sen Projections

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$$A = \begin{bmatrix} .1 & .01 & .3 \\ .2 & .99 & .3 \\ .7 & 0 & .4 \end{bmatrix}$$

steady state: $\lambda = 1$

Markov Martix

① All entries ≥ 0

② All columns add to 1

The key points:

1. $\lambda = 1$ is an eigenvalue

2. All other $|\lambda_i| < 1$

$u_k = A^k u_0 = c_1 \lambda_1^k x_1 + c_2 \lambda_2^k x_2 + \cdots$ / ...

$\lambda_1 = 1$, $\lambda_2 < 1 \Rightarrow$ go disapear

→ x_1 part of u_0 is the steady state ≥ 0

3. the eigenvector $x_1 \geq 0$

$$A - 1I = \begin{bmatrix} -.9 & .01 & .3 \\ .2 & -0.01 & .3 \\ .7 & 0 & -0.6 \end{bmatrix}$$, all cols add to 0

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→ $A-I$ is singular ($\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $a+b+c=0$, so $a=-b-c$, a is b&c's linear combination)

because $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is in the nullspace of A^T

↳ $= m-r$, it means $m-r > 0$, $m=n$

then X_1 are in $N(A)$

corresponding to the eigenvalue 1

this guy is the steady state

(eigenvalues of A $\det(A-\lambda I)=0$

= eigenvalues of A^T are the same

$\det(A^T-\lambda I)=0$ $\det(A)=\det(A^T)$)

$$A-I = \begin{bmatrix} -.9 & +.01 & .3 \\ .2 & -0.01 & .3 \\ .7 & 0 & -0.6 \end{bmatrix} \cdot \begin{bmatrix} .6 \\ 33 \\ .7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

eigenvector $X_1 > 0$

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where the Markov Martix comes from?

$u_{k+1} = A u_k$, A is a Markov Matrix

$$\begin{bmatrix} u_{cal} \\ u_{mass} \end{bmatrix}_{t=k+1} = \begin{bmatrix} .9 & .2 \\ .1 & .8 \end{bmatrix} \begin{bmatrix} u_{cal} \\ u_{mass} \end{bmatrix}_{t=k} \qquad u_0 = \begin{bmatrix} 0 \\ 1000 \end{bmatrix}$$

people number

$$\begin{bmatrix} .9 & .2 \\ .1 & .8 \end{bmatrix} \qquad \lambda_1 = 1 \qquad \lambda_2 = .7$$

$$\begin{bmatrix} -.1 & .2 \\ .1 & -.2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

x_1

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$\lambda_2 = .7, \begin{bmatrix} .2 & .2 \\ .1 & .1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

x_2

after 100 steps

$u_k = c_1 1^k \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 (.7)^k \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$u_0 = \begin{bmatrix} 0 \\ 1000 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$c_1 = \frac{1000}{3}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \frac{2000}{3}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

~~Projection~~ with Orthonormal basis

$q_1, \cdots, q_n \rightarrow$ orthonormal basis/expension

any $v = x_1 q_1 + x_2 q_2 + \cdots + x_n q_n \rightarrow [q_1 \cdots q_n]\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = v$

$q_1^T v = x_1 q_1^T q_1 + 0 \cdots + 0$ $\quad Qx = v$

$\Rightarrow x_1 = q_1^T \cdot v$ $\quad x = vQ^{-1}$

Fourier series: $f(x) = f(x + 2\pi)$ $\quad = Q^T v$

$f(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \cdots$

dot Product

vectors $v^T w = v_1 w_1 + \cdots + v_n w_n$

function $f^T g = \int_0^{2\pi} f(x) g(x) dx$ $\quad \int_0^{2\pi} \sin x \cos x \, dx = 0$

$= 0$, the functions are orthogonal

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$f(x) = a_0 \frac{1}{2} + a_1 \cos x + b_1 \sin x + a_2 \cos 2x \cdot \frac{1}{}$

find a_1 the left $\rightarrow \int_0^{2\pi} f(x) \cos x \, dx$ inner product with $\cos x$

$f(x)$ multiply $\cos x$ and integral

the right : $\int_0^{2\pi} a_1 \cos^2 x \, dx$, other antire is disappear

$\Rightarrow [0, 2\pi] \ni x$

$= a_1 \pi = \int_0^{2\pi} f(x) \cos x \, dx$

$a_1 = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos x \, dx$

it is an expansion in an orthonormal basis

$\frac{1}{2\pi}$

expansion in an orthonormal basis