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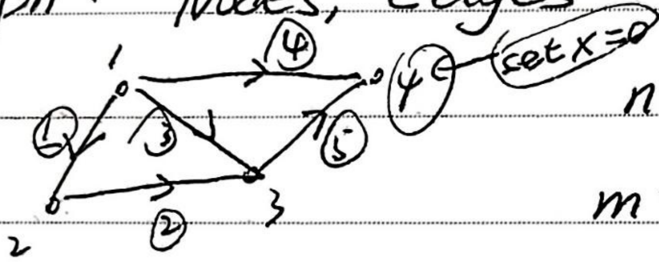
18.06 LEC 12. Graphs & Networks
Incidence Matrices 2.6

more Application (problem 12 has example!)

Graph: Nodes, Edges



$n = 4$ nodes

$m = 5$ edges

Incidence Matrix

$$A = \begin{array}{c} \text{node} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{edge } 1 \leftarrow 1 \text{ to } 2 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}$$

loop

1, 2, 3 ⇒ 1+2=3, loop ~~coor~~ correspond to dependent column

$Ax = 0$

rank = 3

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_3 - x_1 \\ x_4 - x_1 \\ x_4 - x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x = x_1, x_2, x_3, x_4$ as potentials at nodes

$A \downarrow$

$x_2 - x_1$, etc, the potential differences

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$x = \begin{bmatrix}1\\1\\1\\1\end{bmatrix} \cdot c$ ← always have this, that $\dim N(A) = 1$, $\boxed{\begin{matrix} n-r=1 \\ 4-3=1 \end{matrix}}$

why rank = n-1

↑ a basis of nullspace, dimension of $N(A) = 1$

$A^T y = 0$, $N(A^T)$ $\dim N(A^T) = m - r$ ~~$= 4 - 1 = 3$~~

$= 5 - 3 = 2$

the potential differences $\xrightarrow[\text{OHM's Law}]{c}$ currents y_1, y_2, y_3, y_4, y_5 on edges

$\downarrow$

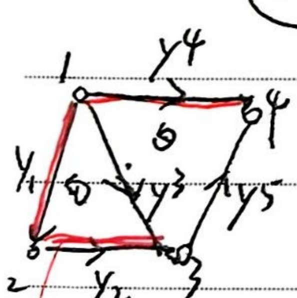
$A^T y = 0$

Kirchoff's CL (KCL)

$A^T y = 0$

$$\begin{array}{cc} & \text{edges} \\ n \times m & \\ \text{nodes} & \begin{bmatrix} -1 & 0 & -1 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(on nodes = 0) $-y_1 - y_3 - y_4 = 0$ $y_1 - y_2 = 0$



$y_2 + y_3 - y_5 = 0$

$y_4 + y_5 = 0$

Basis for $N(A^T)$

$\begin{bmatrix}1\\1\\-1\\0\\0\end{bmatrix}$, $\begin{bmatrix}0\\0\\1\\-1\\1\end{bmatrix}$

~~rows~~ column space of A^T

$$\begin{bmatrix} -1 & 0 & -1 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

column 1+2=3

from the loop

$\dim N(A^T) = m - r = \#\text{loops}$

basis: ↑ ↑   ↑ ⇒ have no loop, a Tree : $= \#\text{edges} - (\#\text{nodes} - 1)$

$m - (n-1)$

(rank $= n-1$) $= m - r$

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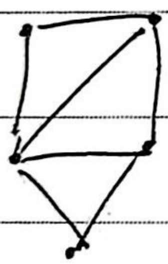
$\#\text{nodes} - \#\text{edges} + \#\text{loop} = 1$ (边 = loop + nodes − 1)

$\boxed{\text{Euler's formula}}$ (a great topology fact)

it works for all graph

Ex:

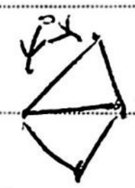


nodes edges loop

$5 - 7 + 3 = 1$

poential

$e = Ax$ $\underline{y = Ce}$ $A^T y = f$



$\Rightarrow \boxed{A^T C A x = f}$ balaence equation

potential difference

always symmetric