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Complex function's knowledge may be useful

1. $z = x + yi$ $i^2 = -1$ x 是实部, y 是虚部

2. 极坐标形式: $z = r(\cos\theta + i\sin\theta) = \underline{r \cdot e^{i\theta}}$

$r = |z|$ $\underline{\underline{e^z}} = r \cdot e^{i\theta}$

$e^{ix} = \cos x + i\sin x$ $\frac{d}{dx} e^x = e^x$

6. $\oint_r f(z)\,dz = 2\pi i \sum$ (围绕曲线的极点)

Unit 3

~~线~~.

LEC 25. Symmetric Matrices and Positive Definiteness 2.14

$A = A^T$ symmetric matrices

① The eigenvalues are REAL

② The eigenvectors are PERPENDICULAR

can be chosen $= \boxed{Q\Lambda Q^T}$

usual case: $A = S\Lambda S^{-1}$ symmetric case: $A = Q\Lambda Q^{-1}$

(I have orthonormal eigenvectors)

→ cols of Q

most famous theorems in LA:

$\boxed{\text{symmetric Matrix: } A = Q\Lambda Q^{-1}}$

$$(\alpha\Lambda\alpha^T)^T = (\alpha\Lambda\alpha^T) \checkmark$$

$\boxed{\text{Why real eigenvalue?}}$ think if x, λ is complex

$$Ax = \lambda x \xrightarrow{\text{always}} \bar{A}\bar{x} = \bar{\lambda}\bar{x}$$

conjugate $(\overline{a+ib} = a - ib)$

(共轭)

if A is ~~read~~ real:

$$A\bar{x} = \bar{\lambda}\bar{x}$$

it says, if A is real, and it has a complex λ, it will also have a pair eigenvalue which is real: $\bar{\lambda}$ and the pair of eigenvectors: x and $\bar{x}$

complex real

number

$\Rightarrow \bar{x}^T A^T = \bar{x}^T \bar{\lambda}$ then A is symmetric

$\bar{x}^T A = \bar{x}^T \bar{\lambda}$ $\cancel{\Rightarrow x}$

another operation $\bar{x}^T A x = \bar{\lambda} \cdot \bar{x}^T \cdot x$ ← both sides

$Ax = \lambda x \Rightarrow$ $\bar{x}^T A x = \lambda \bar{x}^T x$

multiply x

multiply $\bar{x}^T$

so $\bar{\lambda}\bar{x}^T \cdot x = \lambda \bar{x}^T \cdot x \Rightarrow \bar{\lambda} = \lambda$ $(x \neq 0)$

$\Rightarrow$ this tell us λ is real!

$$\bar{x}^T \cdot x = [\bar{x}_1 \ \bar{x}_2 \cdots \bar{x}_n] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \bar{x}_1 x_1 + \bar{x}_2 x_2 + \cdots + \bar{x}_n x_n$$

$(a+ib)(a-ib) = a^2 + b^2$

$i^2 = -1$

$\Rightarrow \bar{X}^T X$ is length square

if A isn't real

$\boxed{\text{Good matrices}}$: real λ's, perpendiculer X's

$\hookrightarrow A = \bar{A}^T$

one more

$A = Q \Lambda Q^T$ ($A = A^T$ symmetric)

break down $= \begin{bmatrix} q_1 & q_2 & \cdots \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{bmatrix} \begin{bmatrix} q_1^T \\ q_2^T \\ \vdots \\ q_n^T \end{bmatrix}$

colums * rows

$= \lambda_1 \underbrace{q_1 q_1^T}_{\text{projection matrix}} + \lambda_2 q_2 q_2^T + \cdots$

Every symmetric matrix is a comb of perp projection matrices

$A = A^T$

Signs of pivots are same as signs of λ's

pivots (positive) = # positive λ's

$\Downarrow$ so we can shift the matrix by $7 \cdot I$,

then take pivots, so we can find how many λ's is above 7 or blew 7.

and det(A) = product of eigenvalues

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Positive definite symmetric Matrix

2.14

① all eigenvalues are positive

② so all the pivots are positive

$15-4=\lambda 11$

$$\begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix}$$

pivots : $5, \frac{11}{5}$

product of pivots = det(A)

eigenvalues:

$(64-44)/4$

$\lambda^2-8\lambda+11=0$, $\lambda = 4 \pm \sqrt{5}$

$\boxed{\text{determinant}}$

$$\begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$$

this example is fail

③ all subdeterminants are positive

definite in Positive symmetric Matrix

① all eigenvalues are positive

② all the pivots are positive

③ all subdeterminants are positive

!!!! !! symmetric!! !!!!

this bring everything about matrix together!!