LEC 6 Column Space and Nullspace 2.4

Vector space arguement

$v+w$ and cv are in the space

all combs $\boxed{c \cdot v + dw}$ are in the space

Subspace

some vectors inside a vector space

(plane through (0,0) in $\mathbb{R}^3$)

Ex:

2 subspace: P and L (line L not in Plane P)

$P \cup L$ = all vectors in P or L or both

union (is this is a subspace?)

no!,

$P \cap L$ = all vectors in both P and L

intersect (is this is a subspace?)

general question: ~~Substract~~

Subspace S and T, intersection $S \cap T$ is a subspace

Mo | Tu | We | Th | Fr | Sa | Su

Memo No.
Date / /

Column space of A

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \Rightarrow \text{a subspace of } R^4 \quad (A\ 4\times3)$$

$C(A)$ ↓ $(1,2,3,4),(1,1,1,1)$

$C(A)$: all linear combinations of columns | $(2,3,4,5)$. three vectors ' $\boxed{\text{linear combinations}}$

$C(A)$ is a subspace of R^4, because the linear combinations of three columns can't fill out R^4, ~~it's not the~~ it's a smaller space

Let's connect $Ax=b$ ⇒

Does $Ax=b$ have a solution for every b?

↑ no, which b do?

$$Ax = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$\boxed{\text{which } b\text{'s allow this system to be solved?}}$

example: $b=(1,2,3,4)$, $x=(1,0,0)$

b ~~in~~ is the combinations of columns! exactly

$\boxed{b \text{ is in the column space } C(A)}$

Memo No. ______
Date / /

Mo Tu We Th Fr Sa Su

$$A=\begin{bmatrix}1&1&2\\2&1&3\\3&1&4\\4&1&5\end{bmatrix}$$

col1 + col2 = col3

so col3 is depended

called pivot columns

not a pivot column

(or take columns 2 and 3)

the Null space of A = all solutions $x=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$

~~= not right~~ to $Ax=0$, $b=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

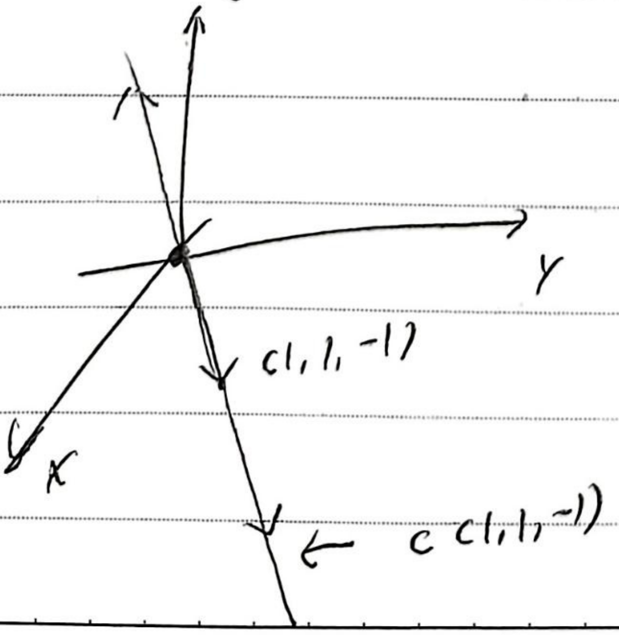
$x=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ in R^3 (n) ($C(A)$ in R^4) (m)

Null space $N(A)$ contains $\begin{bmatrix}0\\0\\0\end{bmatrix}$, $\begin{bmatrix}1\\1\\-1\end{bmatrix}$, $\begin{bmatrix}c\\c\\-c\end{bmatrix} = c\begin{bmatrix}1\\1\\-1\end{bmatrix}$

$$Ax=\begin{bmatrix}1&1&2\\2&1&3\\3&1&4\\4&1&5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

1. zero vector

a line in R^3



Memo No.
Date / /

Mo Tu We Th Fr Sa Su

Cheek that the solution to $Ax = 0$

always give a subspace

$\Rightarrow$ IF $Ax = 0$, and $Ax^* = 0$ then $A(x + x^*) = 0$

x and x^*, $x + x^*$ is in null space of A

if $b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, do x from a subspace?

no, because the zero vector can't be a solution!

the solution is a plane or a line doesn't go through the origin

$\boxed{\text{Subspace must go through the origin}}$