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MIT 6.006

Problem set 0.

0-2

$A = \{ i + \binom{5}{i} \mid i \in \mathbb{Z}$ and $0 \le i \le 4\}$ and $B = \{3i \mid 1 \le i \le$
$\{1, 2, \cancel{3}, 4, 5\}\}$.

(a) $A \cap B$

$i + \binom{5}{i} = \{1, 6, 12, 13, 9\} = A$

$0 + \frac{5!}{0!(5-0)!} = 1$ $\quad B = \{3, 6, \cancel{9}, 12, 15\}$

$1 + \frac{5!}{1!(4)!} = 6$

$2 + \frac{5!}{2!(3!)} = 12$

$3 + \frac{5!}{3!(2!)} = 13$

$4 + \frac{5!}{4!(1)} = 9$

So $A = \{6, 12\}$

$A \cap B = \{6, 12\}$

(b) $|A \cup B| = |\{3, 6, 9, 12, 15\}|$

$|A \cup B| = \{1, 3, 6, 9, 12, 13, \cancel{9}\, 15\} = 7$

(c) $|A - B| = |\{1, 9, 13\}| = 3$

Problem 02

(a)

$E[X] = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = 1.5$

$E[Y] = \frac{(\cancel{x}! + 6!)/x^2}{2} =$

$= ((6!)/6!)^2 = (3.5)^2 = 12.25$

$E[X+Y] = 13.75$

30 120
$\times$ 3
360
720
+ 720
12) 1440
12
24

Problem 03

$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$ for any integer $n \geq 1$

Proof: (by inductive) $P(n) = \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

Base cases ~~i=1~~, $n=1$, $P(1) = 1 = \frac{1(1+1)}{2} = 1$ ✓

Inductive step: Assume $P(n)$ is true to proof $P(n+1)$

$\cancel{P(n+1) = \sum_{i=1}^{n+1} i^3 = \sum_{i=1}^{n} i^3 + (1+n)^3 = \left[\frac{n(n+1)}{2}\right]^2 + n^3}$

$= \frac{n^2(n^2+2n+1)}{4} + n = \frac{n^4+2n^3+n^2+4n^3}{4} = \frac{n^4+6n^3+n^2}{4}$

$F \quad \frac{n+1}{4}$

$\cancel{(n+1)(n+2) = (n^2+3n+2)^2 = (n^2+3n+2)(n^2+3n+2)}$

$\cancel{= n^4+3n^3+6n+3n^3+6n}$

$\cancel{(n+1)(n+2) = (n^2+3n+2)^2 = n^4+3n^3+2n^2+3n^3+9n^2+6n+2n^2}$

$+6n+4$

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Inductive Step: Assume $P(n)$ is true

$P(n+1) = \sum_{i=1}^{n+1} i^3 = \sum_{i=1}^{n} i^3 + (n+1)^3 = [\frac{n(n+1)}{2}]^2 + (n+1)^3$

$= \frac{(n+1)^2 \cdot n^2 + 4 \cdot (n+1)^3}{4} = \frac{(n+1)^2(n^2+4n+4)}{4}$

$= \frac{[(n+1)(n+2)]^2}{4} = [\frac{(n+1)(n+2)}{2}]^2$

So $P(n+1)$ ✓, 12

Problem 05.

Prove by induction that every connected undirected Graph $G=(V,E)$ for which $|E| = |V| - 1$ is a acylic.

Proof: (By inductive)

Base Case: $G = (1, 0)$ is acylic ✓

Inductive Step: Assume $P(n)$, $G = (n, n-1)$ is a acylic Graph.

$P(n+1)$: $G(n+1, n)$

G is connected, Avg degree $= \frac{2n}{n+1} < 2$,

so there is a vetex v with degree 1, connect

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to exactly one vertex u, Removing v and the edge connecting v to u yields a Graph G' on ~~k~~ n ~~vec~~ vertices and m-1 edges that is also connected from P(n) we know G' is acylic, then put back v and edge to u,

Vertex v can not be cycle of G ✓ 17

so G is acylic.

↑ because degree of a circle at least 2, So G contains a circle only G' have a cycle, from P(n) we know G' is acylic, So G is also acylic, ✓ 17