

# LEC9 Graph Algorithm

3.26 [https://github.com/GUMI-21/MIT6.006\\_note](https://github.com/GUMI-21/MIT6.006_note)

## Terminology

$G = (V, E)$   
 $V$  = Vertices  
 $E$  = Edges in  $V \times V$

## Simple Graph

- No self loops
- Every edge is distinct
- $|E| = O(|V|^2)$   
Directed:  
 $|E| \leq 2 \binom{|V|}{2} = O(|V|^2)$   
Undirected  
 $|E| \leq \binom{|V|}{2} = O(|V|^2)$
- *neighbors*  
Adjacent vertices
- *degree*

## Adjacency list

Set maps vertex  $v$  as  $\text{Adj}(v)$ .

May Store  $\text{Adj}(v)$  as direct access array hash table.

## Path

### Model Graph Problems

- $\text{Single\_pair\_reachability}(G, s, t)$ :  
is there a path in  $G$  from  $s$  to  $t$  ?
- $\text{Single\_pair\_shortest\_path}(G, s, t)$ :  
return distance from  $s$  to  $t$  and a shortest path
- $\text{single\_source\_shortest\_paths}(G, s)$ :  
Return shortest distance from  $s$  to all  $t$  plus a shortest path tree

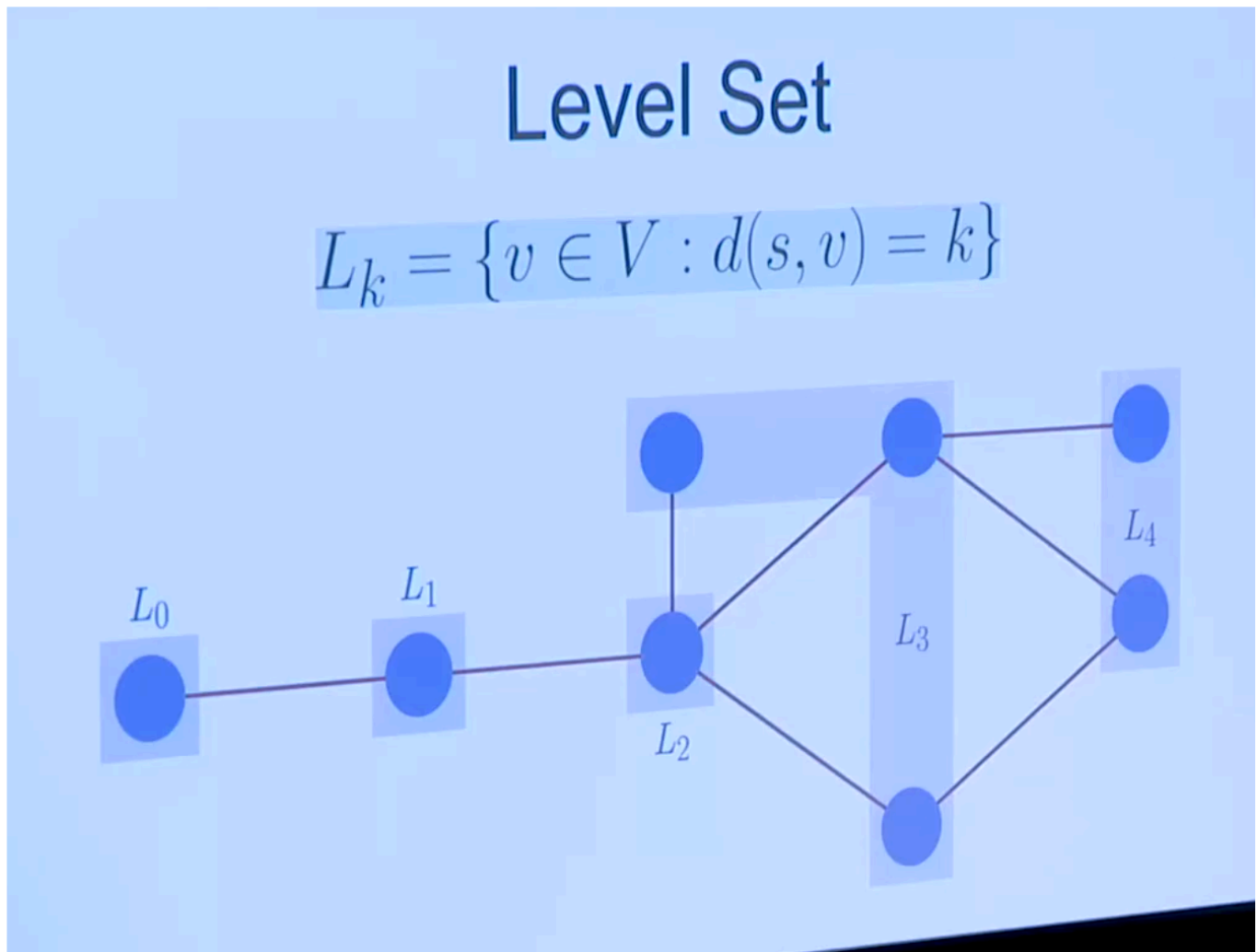
## The Shortest path tree

every vertex just store one thing which is the previous vertex on its shortest path.

- $P(V)$   
previous of  $V$   
but if we change the source or one edge, we may need to renew every  $P(V)$  in the graph.

## Algorithm

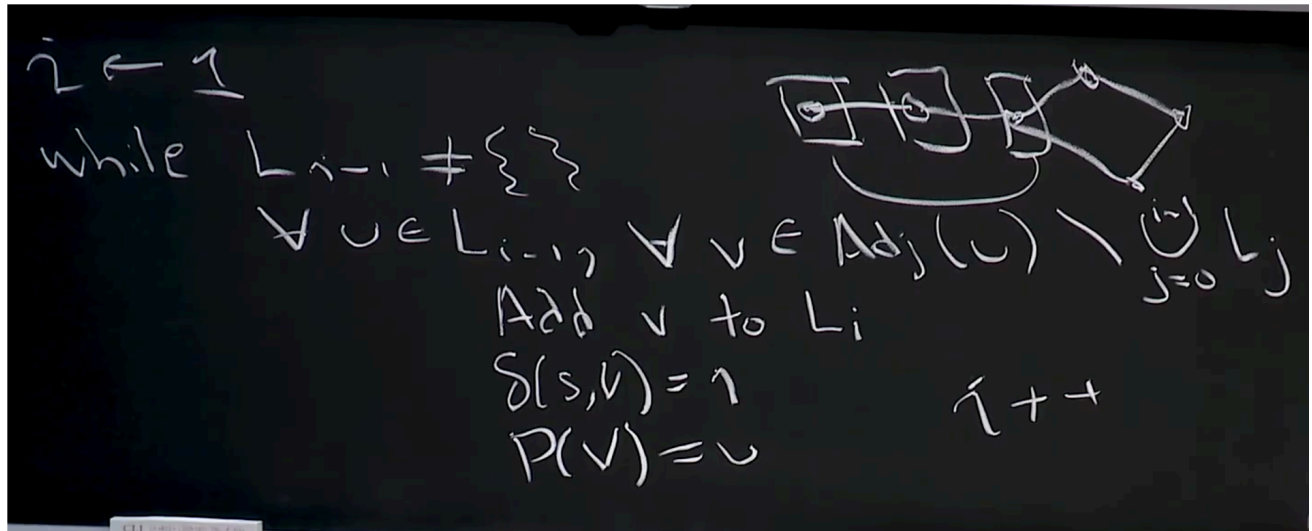
### Level Set



### Breadth-First Search

- init  
 $L_0 \{s\}$   
P array {} *initial vertex length array*  $O(|V|)$   
level set  $\{[L_0]\}$

- then  
 $i = 1$   
 while  $L_{i-1} \neq \{\}$



while loop: we going with the order of distance, means take  $O(|E|)$  time.

so the runtime is  $O(|E| + |V|)$