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Problem Set 2

2-1

(a) $T(n) = 4T(\frac{n}{2}) + O(n)$

① Master Th.M:

$T(n) = 4T(n/2) + O(n)$

$a=4,\ b=2,\ f(n)=O(n)$

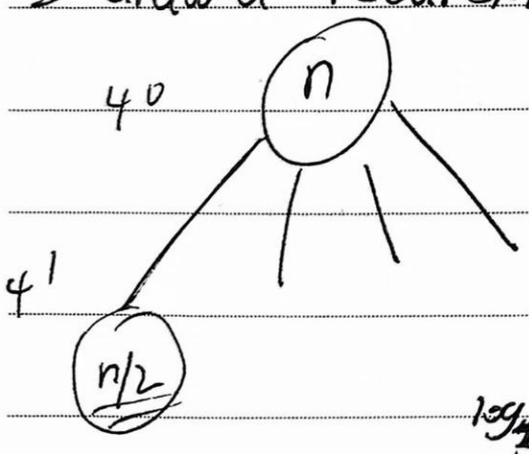
$n^{\log_2 4} = n^2$

case 1, $f(n) = O(n^{\log_2 4 - 1}),\ \varepsilon = 1$

so $T(n) = \Theta(n^2)$

Tree

② draw a recursion tree



$\log_4 n$ levels

each level have 4^i nodes

works of every node: $n/2^i$

$\Rightarrow$ all works $= \sum_{i=0}^{\log_4 n} (4^i) \cdot n/2^i$

$= n \cdot \sum_{i=0}^{\log_4 n} 2^i$ (GS)

$= n \cdot \frac{(1-2^{\log_2 n+1})}{1-2}$

$= n \cdot (2^{\log_2 n+1} - 1)$

$= \Theta(n^2)$

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~~$\log_4 n = \Omega(\log_4 n)^2$~~

(b) $T(n) = 3T(\frac{n}{\sqrt{2}}) + O(n^4)$

① Master THM ~~choose $f(n) = O(n^4)$~~

$a=3,\ b=\sqrt{2},\ f(n) = n^4 \in O(n^4)$

choose $f(n) = \Omega(n^{\log_{\sqrt{2}} 3 + \varepsilon})$ where $\varepsilon = 4 - \log_{\sqrt{2}} 3$

$= O(n^{\log_{\sqrt{2}} 3} \cdot n^{\varepsilon})$

$a \cdot f(n/b) = 3 \cdot O(\frac{n^4}{4}) < c \cdot O(n^4),\ c \in (0,1)$

$\log_{\sqrt{2}} 3 = \frac{\log_2 3}{\log_2 \sqrt{2}} = 2 \cdot \log 3$

$O(n^4) \subseteq \Omega$

$\Rightarrow = \Omega(n^{2 \cdot \log 3 + \varepsilon})$, $0 \le \varepsilon \le 4 - 2\log_2 3$ ~~$\varepsilon \in (0,1)$~~

and $3 \cdot (\frac{n}{\sqrt{2}})^4 \quad c \cdot n^4$

$\| \frac{3}{4} n^4 < c \cdot n^4$ for $\frac{3}{4} < c < 1$

so $T(n) = O(n^4)$

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format of (b) Mast THM

$a=3,\ b=\sqrt{2},\ f(n) = O(n^4)$

$\Rightarrow f(n) = O(n^4) \ge \Omega(n^{\log_{\sqrt{2}} 3 + \varepsilon}),\ 0 \le \varepsilon \le 4 - \log_{\sqrt{2}} 3$

$\log_{\sqrt{2}} 3 = 2\log_2 3$ so $\Omega(n^{2\log 3 + \varepsilon})$ &

$a \cdot f(n/b) = 3(\frac{n}{\sqrt{2}})^4 = \frac{3}{4} \cdot n^4 < c \cdot n^4$ where $c \in (\frac{3}{4}, 1)$

so $T(n) = O(n^4)$

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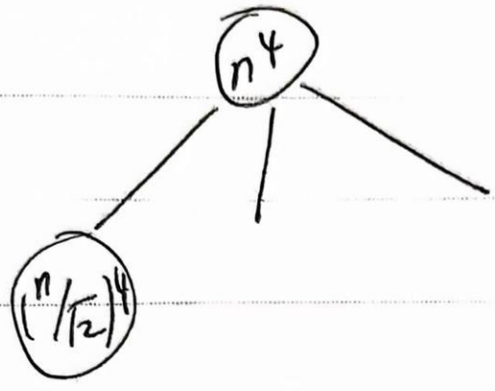
Tree:



leaves: $\log_{\lceil 2 \rceil} n$

nodes: 3^l

work of node: $(n/\lceil 2 \rceil^l)^4$

Total works of Tree: $= \sum_{l=0}^{\log_{\lceil 2 \rceil} n} 3^l \cdot \frac{n^4}{4^l}$

$$= n^4 \sum_{l=0}^{\log_{\lceil 2 \rceil} n} \left(\frac{3}{4}\right)^l$$

$$= n^4 \sum_{l=0}^{2\log_2 n} \left(\frac{3}{4}\right)^l$$

$$= n^4 \cdot \frac{\left(1-\left(\frac{3}{4}\right)^{2\log_2 n+1}\right)}{1-\frac{3}{4}}$$

$$= n^4 \cdot 4 \cdot \left[1-\left(\frac{3}{4}\right)^{2\log_2 n+1}\right]$$

$$< 4n^4 = O(n^4)$$

OR

$3^{2\log n} = n^{2\log 3}$ leaves in tree so $T(n) = \Omega(n^{2\log 3})$

?

$3^{2\log n} = X$

$\log_3 \cdot 2\log n = \log_3 X$

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cc) $T(n) = 2T(\frac{n}{2}) + 5n\log n$

Master TBM:

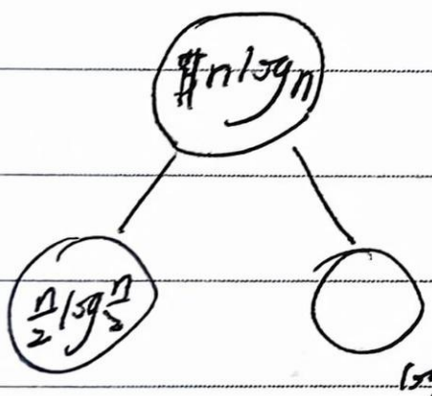
$a=2,\ b=2,\ f(n) = 5n\log n = \Theta(5n\log n)$

use case 2 of Master TBM $= \Theta(n\log n)$

$f(n) = \Theta(n^{\log_2 2} \cdot \log^k n),\ k=1$

$\Rightarrow T(n) = \Theta(n \cdot \log^2 n)$

Recusive Tree:



leaves: $\log n$

nodes: 2^i

work: $\frac{n}{2^i} \cdot \log\frac{n}{2^i}$

so Total work $= \sum_{i=0}^{\log n} 2^i \cdot \left(\frac{n}{2^i} \cdot \log\frac{n}{2^i}\right)$

$= \sum_{i=0}^{\log n} 2^i \cdot n(\log n - i)$

$= n\sum_{i=0}^{\log n} \Theta(\log n \cdot i)$

$= n[(\log n)^2 - \frac{\log(n)}{2}\log(n)] = \Theta(n\log^2 n)$

$= n \cdot [\log n(\log n + 1) - \frac{\log n \cdot (\log n + 1)}{2}]$

$= n[\frac{\log^2 n + \log n}{2}] = \Theta(n \cdot \log^2 n)$

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(d) $T(n) = T(n-2) + \Theta(n)$

Use Substitution

$T(n) = T(n-2) + \Theta(n)$

$= T(n-4) + 2\Theta(n)$

$= T(n-6) + 3\Theta(n)$

$= T(n-2i) + i\Theta(n)$, $i \in [0, n]$

$\Rightarrow T(n) = n \cdot \Theta(n) = \Theta(n^2)$

check By induction:

Basecase $T(0) = 0$, $T(2) = \Theta(2) + T(0) \in \Theta(4)$

Induction Step: Assume $T(n) = \Theta(n^2)$

~~$T(n+1) = T(n-1) + \Theta(n+1)$~~ we know $n = 2 \cdot 2$

$T(n+2) = T(n) + \Theta(n+2)$

$= \Theta(n^2) + \Theta(n+2) \in \Theta(n^2)$. 17

why:

$\lim_{n \to \infty} \frac{n^2+n+2}{n^2} < \infty$

$0 <= 3 <$

MASTER THM

① $f(n) = O(n^{\log_b a - \varepsilon})$, $\varepsilon > 0 \Rightarrow T(n) = \Theta(n^{\log_b a})$

② $f(n) = \Theta(n^{\log_b a} \cdot \log^k n)$, $k \geq 0$, $\Rightarrow T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$

③ $f(n) = \Omega(n^{\log_b a + \varepsilon})$, for some $\varepsilon > 0$ & $a \cdot f(n/b) \leq c \cdot f(n)$ for $c \in (0,1) \Rightarrow T(n) = \Theta(f(n))$

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Problem 2-2

Choose String Algorithm

selection Sort, Insertion Sort, merge sort

(a) 1) n items

1. get_at(i) in worst-case $O(1)$ time

2. set_at(i, x) $O(n \log n)$ time 3) in-place

Insertion Sort performs $O(n^2)$ get_at and $O(n^2)$ set_at operations.

$O(n \cdot (n \cdot O(1) + n \cdot O(n \log n) \cdot 1))$

$= n^2 O(1) + n^2 O(n \log n) = O(n^3 \log n)$

~~2)~~ Selection Sort:

~~$O(n^2)$~~ $O(n \cdot (n \cdot O(1) + 2 \cdot O(n \log n))$

$= O(n^2 + O(n^2 \log n))$

$= O(n^2 \log n) < O(n^3 \log n)$

So choose Selection Sort.

(b) A n comparable objects. $O(\log n)$ to Compare

increasing merge sort $n(\log^2 n)$

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(c) sorted array A n

loglogn

selection sort / merge sort do not depend on input

P2-3 Friend Finder

P2-4. MixBookTube.tw Chart

	k	n all viewers
build(V)	n=|V|	O(nlogn)
send(v,m)	m	O(logn)
recent(k)	k	O(k) ✓
ban(v)	V	O(n_v + logn)

ID v is v banned v-m, m removed

Because the message need to be show in time order, use dynamic to array to sort message, recent(k) is O(k) ✓

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also use ~~link~~ ~~dynamic~~ static array to store viewer, but in item every item store the viewer key and a link-list store the viewer's message.

build(v): init a room with v ⇒ O(n) ∈ O(nlgn) ✓ and sort O(nlgn)

no message in initial.

send(v,m), find v first, because is sorted array, take O(lgn) time, And add message to link list tail also O(1) time, add to message array, it take O(1) time ⇒ O(lgn) ✓ (first from static array find is O(1))

recent(k) ⇒ O(k) ✓ itera the array of message

ban(v) ⇒ find v, O(1), itera link list (sort keys of dynamic array, O(nv) and delete message in dynamic array take O(n) ⇒ O(n) ≤ O(nv+lgn) ✓

~~P2-5 Beaver Boking.~~

2-4. ① a double-List contain the sequence of all undeleted message in time order

② a sorted array S of pair(v, Pv) keyed on v, v is id, Pv is a pointer to a viewer-specific single listed Lv store pointers to all m∈ in L