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## LEC 5. Linear Sorting 3.19

### Problem Session 3 3.20

#### Problem 3-2: (reduction)

- |                                  |   |  |
|----------------------------------|---|--|
| <u>Set (Hashing)</u>             | → | Sequence                               |
| - build $O(n)$ exp.              | ↔ | - build $O(n)$ exp.                    |
| - find $O(1)$ exp.               | ↔ | set-at and get-at $O(1)$               |
| - delete/insert $O(1)$ exp. a.m. |   | - insert/delete $O(n)$ exp.            |
|                                  |   | - insert/delete first/last $O(1)$ e.a. |
- in sequence

Idea: 1. Index = sign key to each item

get-at( $i$ ): find( $i$ ).<sup>value</sup> seq-build( $A$ ): set-build( $\langle \text{key} = i, \text{value} = A[i] \rangle$ )  
 set-at( $i$ ): find( $i$ ).value =  $X$  for  $i = 0, \dots, |A|-1$

insert/delete: iterate all items, insert/delete one  
 rebuild OR use delete/insert of Hashing to  
 shifting sequence

- insert/delete first & last:

insert - last: insert ( $\langle \text{key } |A|, \text{value } X \rangle$ )

insert - first: need shift all items  $\Rightarrow$  not  $O(1)$  time  
 it's  $O(n)$  X



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Invariant 1: keys = {first, first+1, ..., first+len-1}

Idea: store variable first = key of first item (index i)

=> Insert-first: decrement first key to -1, -2, ..., insert (key first, value x)

delete-first: increase first key

delete (key first - 1, value)

all of interfaces of before should plus first to i which initial first is 0

Problem 3.-3: C++ Critter sort

Sort n objects by keys

a) an integer  $x_i$  between  $-n$  and  $n$ .  $[-n, n]$

Radix sorts n ints  $\in \{0, \dots, u-1\}$  in  $\Theta(n \log u)$

$O(\log u + n \log n \log u)$ .

" $O(n)$  is  $u = n^{O(1)}$ "

(b) strings over 26 letters of  $\text{len} \leq 10 \lceil \lg n \rceil$

tuple sort  $\equiv$  radix sort on base 26

(c) integer  $f_i$  under  $i^2$

(d) 1.  $O(n \lg n)$  via merge sort





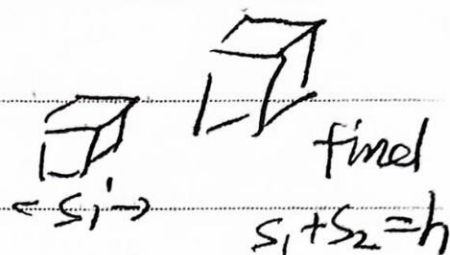
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(a) in  $O(n)$  time

Problem 3-4.

$$S = \{s_0, s_1, \dots, s_{n-1}\}$$

want 2 numbers  $\in S$ , summing to  $h$ (a)  $O(n)$  expected time~~idea~~ idea 2: build hash table on  $S$ idea: call find,  $O(n)$  times.idea: loop over  $S$ for  $s_i \in S$ : find  $s_j \in S$  such that  $s_i + s_j = h$ 

$$\equiv \text{find}(h - s_i) \Rightarrow O(1) \text{ exp}$$

so  $O(n)$  exp(b)  $O(n)$  worst-casefind biggest pair wise sum  $\leq h$ , no  $s_i + s_j = h$ assume  $h \approx 600n^b$  find  $s_1, s_2$  close to  $h$ Idea 1: radix sort  $\dots$  but  $s_i$  could be  $\gg h$  $\Rightarrow$  Idea:  $s_i > h \Rightarrow$  throw away  $\leftarrow$  radix sort $\rightarrow s_i$  is sorted. for  $s_i \in S, i = 0, 1, \dots, n-1$ \* binary search for every  $s_i \Rightarrow O(n \log n)$  $h - s_i$ , bs can tell me most approach number



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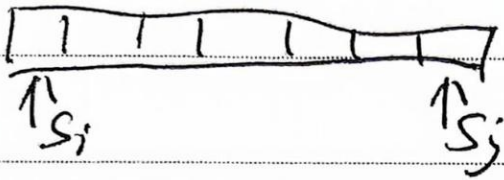
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$\Rightarrow$  return largest candidate<sup>C</sup>

Idea:



two-finger Algorithm

$S_i + S_j \Rightarrow$  how close to  $h$

①  $S_i + S_j > h \Rightarrow j = j - 1$  decrement  $j$

②  $S_i + S_j < h \Rightarrow$  increment  $i$

$\hookrightarrow$  add to candidate<sup>C</sup>

Invariant  $S[i'] + S[j'] \leq h$  or  $> h$  candidate<sup>C</sup>  
for all  $i' \leq i \leq j \leq j'$

$i \geq j$  stop return max candidate<sup>C</sup>

P3-5 cut;  $\downarrow i=2$   
a b c d b c  $\leftarrow D$

c d b c a b

$k=4 \rightarrow$  deal 12 c d b c

$P(D, i, k)$

sort  $b c c d$

①  $P(D, i, k)$

$26 \# \in [0, n]$

$\hookrightarrow$  base  $n+1$ , 26 digits

$(n+1)^{26}$

$\left. \begin{array}{l} a a b a \\ b a a a \\ a b a a \end{array} \right\}$  all same  
 $a c c a b$