LEC 3. Sets and Sorting 3.16

Problem Session 2. 3.18

MASTER THM: $\boxed{T(n) : aT(n/b) + f(n)}$

① $f(n) = O(n^{\log_b a - \varepsilon})$ for some $\varepsilon > 0$

$\Rightarrow T(n) = \Theta(n^{\log_b a})$

② $f(n) = \Theta(\boxed{n^{\log_b a}} \log^k n)$, k often be 0

for some $k \geq 0$

$\Rightarrow T(n) = \Theta(n^{\log_b a} \cdot \log^{k+1} n)$

③ $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some $\varepsilon \geq 0$ &

$af(n/b) < cf(n)$ for some $c \in (0, 1) \Rightarrow$

$T(n) = \Theta(f(n))$

Problems:

ⓐ $T(n) = 2T(n/2) + O(\sqrt{n})$ (method 1)

$a = 2$, $b = 2$, $f(n) = O(\sqrt{n})$

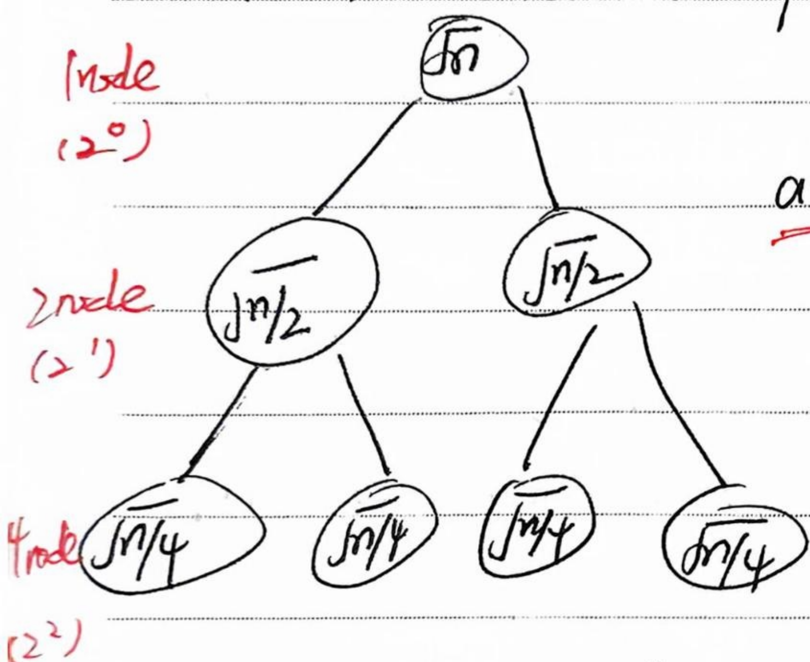
$n^{\log_b a} = n^{\log_2 2} = n$

$f(n) = O(n^{\frac{1}{2}}) = O(n^{1 - 1/2}) = O(n^{\log_b a - \varepsilon}) \Rightarrow \varepsilon = \frac{1}{2} > 0$

$= O(n^{\log_2 2 - \frac{1}{2}})$

so the problem (a) is case 1 $\Rightarrow T(n) = \Theta(n)$ ✓

(method 2) draw a ~~compar~~ recursion tree



$a=2$, $b=2$ $\Rightarrow$ a edges

b vetrics

$\log_2 n$ levels, each level l has ~~[illegible]~~

$\text{work}_l = 2^l \cdot \sqrt{n \cdot 2^{-l}}$ (sum of work runtime)

Then the total work: $\sum_{l=0}^{\log_2 n} 2^l \sqrt{n \cdot 2^{-l}}$

$$= \sqrt{n} \cdot \sum_{l=0}^{\log_2 n} \underbrace{2^{l - \frac{1}{2}l}}_{(\sqrt{2})^l} \qquad = \sqrt{n} \cdot \frac{(\sqrt{2})^{\log_2 n+1} - 1}{\sqrt{2} - 1} \quad \text{GS}$$

$\left(\text{or } \frac{a_1(1-q^{n+1})}{1-q}\right)$

$\frac{x^{n+1}-1}{x-1}$

Just simply this expression $\qquad \frac{1}{2}\log_2 n + \frac{1}{2}$

$$= \sqrt{n} \cdot \frac{1}{\sqrt{2}-1} \cdot \left(2^{\log_2 \sqrt{n} + 1/2} - 1\right) = \sqrt{n} \cdot \frac{1}{\sqrt{2}-1} \left(\sqrt{n} \cdot \sqrt{2} - 1\right)$$

$$= \Theta(n)$$

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(b) $T(n) = 8\,T(n/4) + \underline{O(n\sqrt{n})}$

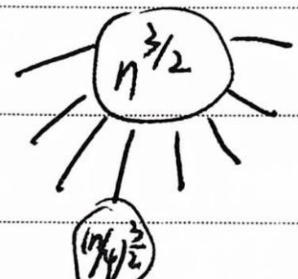
$n^{3/2}$ master thm

$n^{\log_b a} = n^{\log_4 8} = n^{3/2}$

↳ case 2 $k=0$ $\Rightarrow T(n) = \cancel{\Theta}\,O(n^{3/2} \log n)$

↑ ~~also can be Θ~~

~~Recurse~~ recurrence Tree



level l : 8^l nodes

work each node $= (n \cdot 4^{-l})^{\frac{3}{2}}$

levels: $\log_4 n$

$$\text{work} \lesssim c \cdot \sum_{l=0}^{\log_4 n} \underline{8^l \cdot (n \cdot 4^{-l})^{3/2}}$$

$$= c \sum_{l=0}^{\log_4 n} n^{3/2} \quad \boxed{\sum_{l=0}^{\log_4 n} 1}$$

$$= c \cdot n^{\frac{3}{2}} \cdot \underline{(\log_4 n + 1)}$$

$\boxed{\text{the good way to visualize a recursive alg}}$ $= O(n^{3/2} \log n)$

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$= \cancel{\log_b n \cdot \log_a}$

$2 \cdot \log_4 n = \log_2 n$

$\cancel{n = 2n}$

$$\log_4 n = \frac{\log_2 n}{\log_2 4}$$

$\log_b a = \log_1 a / \log_1 b$

basis doesn't matter