LEC 21 2024.12.26

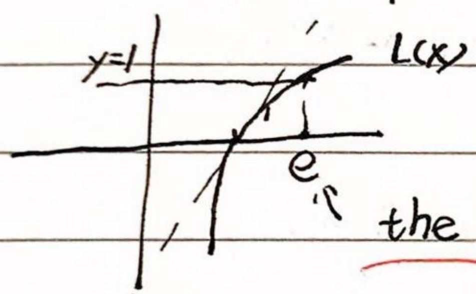
FTC $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

$L'(x) = \frac{1}{x}$ $L(x) = \int_1^x \frac{dt}{t}\,dt$ ← defination of ln function

$L(1) = \int_1^1 \frac{dt}{t}dt = 0$, $L''(x) = -\frac{1}{x^2}$ ← $<0$, concave down

$L(1) = 0$, $L'(1) = \frac{1}{1} = 1$

sketch of graph




$L(e) = 1$

the DEFNATION of e: ~~L(e)=1~~

why is $L(x) < 0$ on $0 < x < 1$

① $L(1) = 0$ & L is increasing

② $L(x) = \int_1^x \frac{dt}{t} = -\int_x^1 \frac{dt}{t} < 0$, when $x < 1$

Claim $L(ab) = L(a) + L(b)$ ?? ln's property

$\int_1^{ab} \frac{dt}{t} = \int_1^a \frac{dt}{t} + \int_a^{ba} \frac{dt}{t}$

$\int_a^{ab} \frac{dt}{t}$

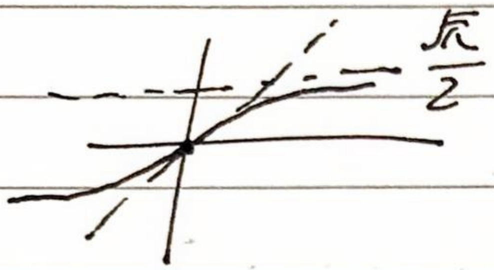
as $t = a\cdot u$ $dt = a\cdot du$ $\int_a^{ab} \frac{dt}{t} = \int_1^b \frac{a\cdot du}{au} = \int_1^b \frac{1}{u}du$

$u \in (1, b)$ $= L(b)$

Ex2. $F(x) = \int_0^x e^{-t^2} \cdot dt$

$\Rightarrow F'(x) = e^{-x^2}$, $F(0) = 0$, $F'(x) > 0$, $F'(0) = 1$

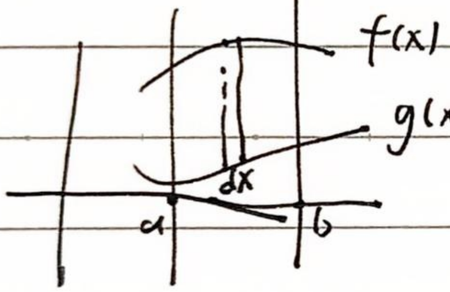
$F''(x) = -2x \cdot e^{-x^2} \begin{cases} < 0, & x > 0 \\ > 0, & x < 0 \end{cases}$




$\lim_{x \to \infty} F(x) = \frac{\sqrt{\pi}}{2}$

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt = \frac{2}{\sqrt{\pi}} F(x)$$

## AREAS BETWEEN CURVES

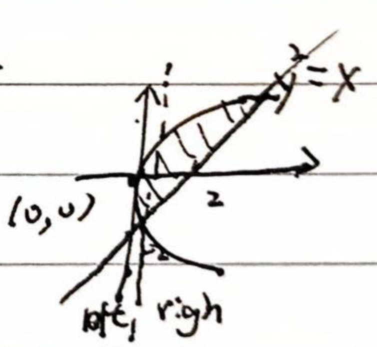



$$\underset{\uparrow \text{ limits, a\&b}}{\int_a^b} \underbrace{(f(x) - g(x))}_{\text{hight}} \cdot \underbrace{dx}_{\text{base}} = \text{Area}$$

intergrand

Ex. between $x = y^2$ and $y = x - 2$

step1. draw a picture




$y = y^2 - 2$, $\begin{cases} y = -1, & x = 1 \\ y = 2, & x = 4 \end{cases}$

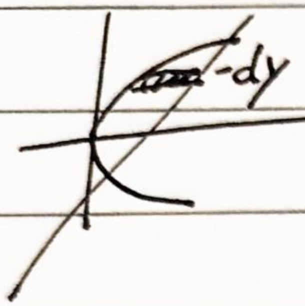
top: $y = \sqrt{x}$, buttom: $y = -\sqrt{x}$, right: $y = x - 2$

$\boxed{\text{Mthod 1}}$ Area $= \int_0^1 (\sqrt{x} - (-\sqrt{x}))dx + \int_1^4 (\sqrt{x} - (x-2))dx$

$= \int_0^1 2\sqrt{x}\, dx + \int_1^4 (2 + \sqrt{x} - x)dx$

$\boxed{\text{Method 2}}$ (better)

use horizontal slice.



left $x=y^2$  right $x=y+2$

$$A=\int_{-1}^{2}(y+2-y^2)\cdot dy$$

$$=\left(\frac{y^2}{2}+2y-\frac{y^3}{3}\right)\Big|_{-1}^{2}=\dots=\frac{9}{2}$$