LEC 28. 2024.12.30.

$\sec = \frac{1}{\cos}$, $\csc = \frac{1}{\sin}$, $\tan = \frac{\sin}{\cos}$ ~~ctn~~

$ctn = \frac{\cos x}{\sin x}$

$\sec^2 x = \frac{1}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \tan^2 x$

$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = \sec^2 x$

$\frac{d}{dx} \sec x = \frac{d}{dx} \frac{1}{\cos x} = \tan x \cdot \sec x$

$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$, $u = \cos x$ $du = -\sin x \cdot dx$

$= \int \frac{-du}{u} = -\ln(\cos x) + C$

$\int \sec x \, dx = \int \frac{u'}{u} = \int \frac{d}{dx} \ln(u) = \int \frac{d}{dx} \ln(\sec x + \tan x)$ $= \ln(\sec x + \tan x) + C$

$\frac{d}{dx}(\sec x + \tan x) = (\sec x \cdot \tan x + \sec^2 x)$

$= \sec x (\tan x + \sec x)$

as $u = \sec x + \tan x$, $u' = \sec x \cdot u \cdot dx$, $dx = \frac{u'}{\sec x \cdot u}$

$\frac{d}{dx}(\sec x + \tan x) = \sec x \cdot \tan x + \sec^2 x = \sec x (\tan x + \sec x)$

Ex: $\int \sec^4 x \, dx = \int (1 + \tan^2 x) \cdot \sec^2 x \, dx$, $u = \tan x$, $du = \sec^2 x \, dx$

$= \int (1 + u^2) \cdot du = u + \frac{1}{3} u^3 + C$

$= \tan x + \frac{1}{3} \tan^3 x + C$

Date.

Ex. $\int \frac{dx}{x^2\sqrt{1+x^2}}$ → try to write at square

$\sec^2 x = 1 + \tan^2 x$

as $x = \tan\theta$, $1+x^2 = \sec^2\theta$, $dx = \sec^2\theta \, d\theta$

$= \int \frac{\sec^2\theta \, d\theta}{\tan^2\theta \cdot \sec\theta}$

Rewrite in $\sin x, \cos x$

$= \int \frac{\cos^2\theta}{\cos^3\theta \cdot \sin^2\theta} \cdot d\theta$

$= \int \frac{\cos\theta}{\sin^2\theta} d\theta$

$(A)' = B$, so use sub (I want to get rid of the $\sin^2\theta$) so $\sin\theta = u$

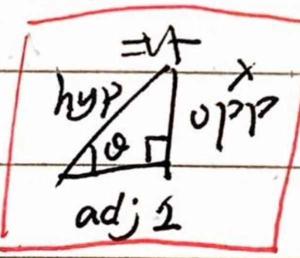
$u = \sin\theta$, $du = \cos\theta \cdot d\theta$

$= \int \frac{du}{u^2} = -\frac{1}{u} + C = -\frac{1}{\sin\theta} + C$

$= \cancel{-\csc} = \underline{\csc\theta + C} = -\frac{1}{\sin(\arctan x)} + C$

↑ correct

Undoing trig sub:



$\tan\theta = \frac{opp}{adj} = \frac{x}{1} = x$

$hyp = \sqrt{x^2+1}$

$\therefore \csc\theta = \frac{hyp}{opp} = \frac{\sqrt{x^2+1}}{x}$

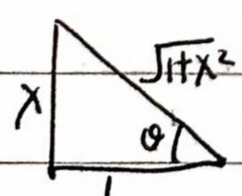
$= -\frac{\sqrt{x^2+1}}{x} + C$

↑ nicer

$\theta = \tan x$

⇓

$\sin(\arctan x)$:



$\sin\theta = \frac{x}{\sqrt{1+x^2}}$, ~~$\sec\theta =$~~ $\csc = \frac{\sqrt{1+x^2}}{x}$

Summary of Trig ~~Substitution~~ Substitution

If integrals contains	make substitution	to get
$\sqrt{a^2-x^2}$	$x=a\cos\theta$ or $x=a\cdot\sin\theta$	$a\sin\theta$ / $a\cdot\cos\theta$
$\sqrt{a^2+x^2}$	$x=a\cdot\tan\theta$ $(\sec^2\theta=1+\tan^2\theta)$	$a\cdot\sec\theta$
$\sqrt{x^2-a^2}$	$x=a\cdot\sec\theta$ $\;=a^2(\sec^2\theta-1)=a^2\tan^2\theta$	$a\cdot\tan\theta$

$\sec^2\theta=1+\tan^2\theta$

Next: Completing the squre

Ex: (~~imaa~~, imazzing)

$$\int\frac{dx}{\sqrt{x^2+4x}} \Rightarrow \text{rewrite to } (x+a)^2+c$$

$$x^2+4x=(x+2)^2-4=(x+2)^2-2^2$$

~~Dix~~ $\boxed{u=x+2}$ ①, $du=dx$

$$\Rightarrow\int\frac{dx}{\sqrt{x^2+4x}}=\int\frac{du}{\sqrt{u^2-2^2}}$$ as $\boxed{u=2\sec\theta}$ ①, $du=2\sec\theta\cdot\tan\theta\, d\theta$

$$\sqrt{u^2-2^2}=\sqrt{2^2(\sec^2\theta-1)}=\boxed{\sqrt{2^2\cdot\tan^2\theta}=2\tan\theta}$$ ③

$$\Rightarrow=\int\frac{2\sec\theta\cdot\tan\theta\cdot d\theta}{2\tan\theta}=\int\sec\theta\cdot d\theta$$

$$=\int\frac{dt}{t}=\ln|t|+c$$

$d(\sec x+\tan x)=\sec x(\tan x+\sec x)$

as $t=\tan\theta+\sec\theta$, $dt=t\cdot\sec\theta\cdot d\theta$

~~$=\ln|\tan$~~

$$=\ln(\sec\theta+\tan\theta)+C$$

$$=\ln\left(\frac{u}{2}+\frac{\sqrt{u^2-4}}{2}\right)+C \quad ①②$$

$$=\ln\left(\frac{x+2}{2}+\frac{\sqrt{x^2+4x}}{2}\right)+C \quad ③$$