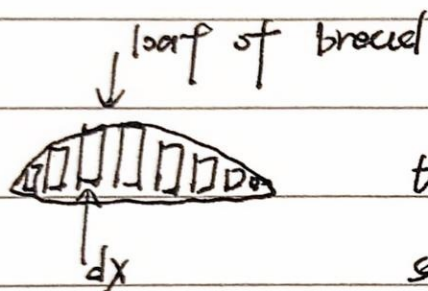Try only use note

Session 57: How to Calculate Volumes

2024. 12.27

Suppose you have a loaf of bread and want to find the volume of the loaf

loaf of bread

dx

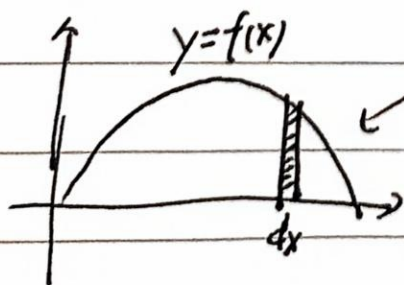the face of the slice (the part you spread butter on) is $A(x)$

so $\quad \Delta V = A(x) \cdot \Delta x$

in the limit $\quad dV = A(x) \cdot dx$
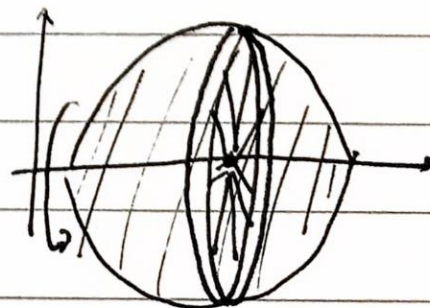
$\therefore V = \int A(x) \cdot dx$

in riemann sum approaching this volume looks like $\sum_{i=1}^{n} A_i \cdot \Delta x$ if the loaf has n slices
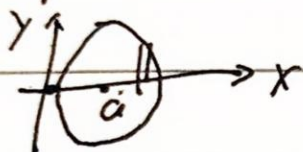
SOLIDS OF REVOLUTION (旋轉術)

$y = f(x)$

revolve around x-axis

dx

Method 1.

Method of disk:

area of face: $\pi \cdot f^2(x) = \pi y^2$

disk: $\therefore dv = \pi y^2 \cdot dx$

Example volume radius a

$$dV = \pi y^2 dx$$

$$(x-a)^2 + y^2 = a^2$$

$$y^2 = a^2 - (x-a)^2$$

$$= -x^2 + 2ax$$

$$V = \int_0^{a\cdot 2} \pi(2ax - x^2)dx$$

$$V = \pi\left(ax^2 - \frac{1}{3}x^3\right)\Big|_0^{a2}$$

$$= \pi\left(4a^3 - \frac{1}{3}8a^3\right)$$

$$= \frac{4}{3}\pi a^3 \quad \leftarrow \text{the volume of ball}$$

$$v = \pi\left(4ax^2 - \frac{1}{3}x^3\right)\Big|_a^b$$

Method 2
Method of shells (凡壳)

← what's the volume

$y = a$
$y = x^2$
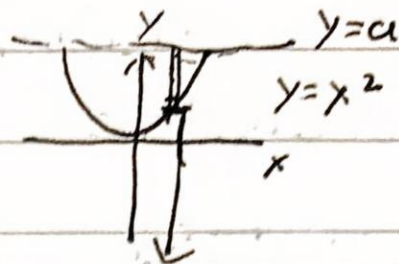
revoled around y-axis
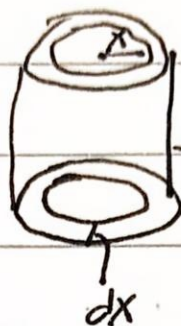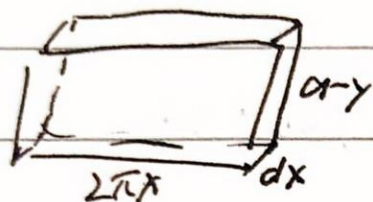
thickness = dx

height = $a - y = a - x^2$

$a - y$

$2\pi x$ dx

because revoled

$$dV = 2\pi x \cdot (a-y) \cdot dx = 2\pi x (a - x^2) dx \downarrow$$

$$= 2\pi (ax - x^3) dx, \quad 0 < x < \sqrt{a}$$

$$V = \int_0^{\sqrt{a}} 2\pi (ax - x^3) dx = 2\pi \left(\frac{a}{2} x^2 - \frac{1}{4} x^4 \right) \Big|_0^{\sqrt{a}}$$

$$= 2\pi \left(\frac{a^2}{2} - \frac{1}{4} a^2 \right)$$

$$= \frac{a^2}{2} \pi$$

Beware of Units

dy

→ 70°C

outside 0°C

100°C   (热度)

(fire)

how much heat need to do it all 100°C

$T = 100 - 30y$ only can do in disk Method