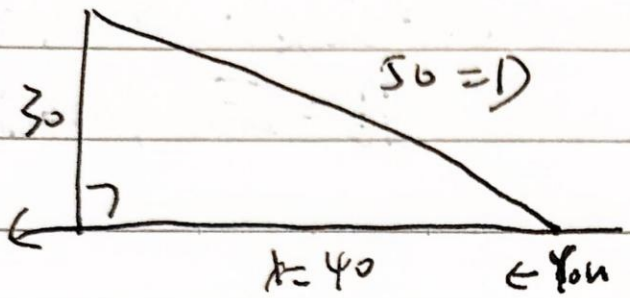
Related Rates

2024.12.21

LEC 13

Police



平均速度 ↓ (瞬时速度)

Are you speeding over $\frac{dD}{dt} = -80$ ft/sec ?

$x^2 + 30^2 = D^2$, $\frac{dD}{dt} = -80$

(could solve for $x = \sqrt{D^2 - 30^2}$) use implicit diff:

$$\frac{dx}{dt} \cdot 2x = 2D \cdot \frac{dD}{dt}$$

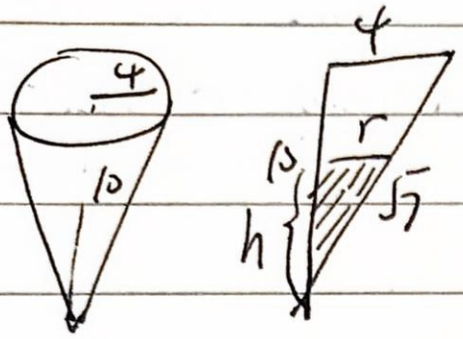
$$2 \cdot 40 \cdot \frac{dx}{dt} = 2 \cdot 50(-80)$$

$$\frac{dx}{dt} = -100 \text{ ft/sec}$$

(圆锥形水箱) (半径)

Ex2 A conical tank with top of radius 4ft depth 10 ft. is being filled at 2 cubic feet (立方英尺) per minute

How fast is the water rising when it is at depth 5 ft



$\frac{r}{h} = \frac{4}{10}$

$V = \frac{1}{3}\pi r^2 \cdot h$ (water)

$\frac{dV}{dt} = 2$ ~~cubic ft/sec~~

quest: $\frac{dh}{dt}$? when $h=5$

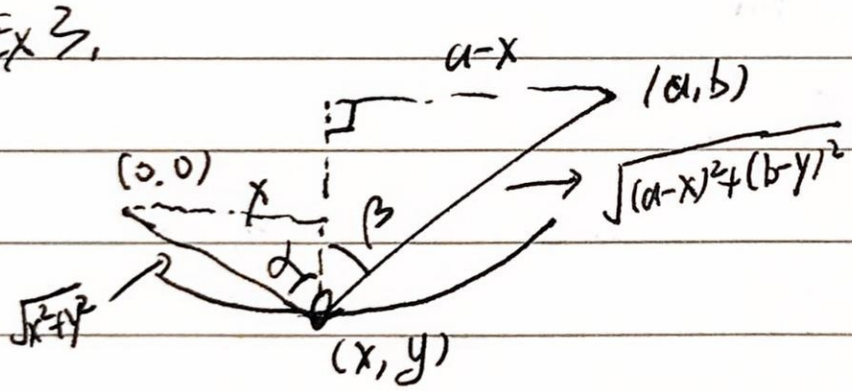
$r = \frac{2}{5}h \quad V = \frac{1}{3}\pi(\frac{2}{5}h)^2 \cdot h$

$2 = \frac{dV}{dt} = \frac{\pi}{3}(\frac{2}{5})^2 3h^2 \cdot \frac{dh}{dt}$

$\therefore 2 = \frac{\pi}{3}(\frac{2}{5})^2 \cdot 3 \cdot 5^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{2\pi}$ ft/~~sec~~min

Ex 3.



The ratio $\sqrt{(a-x)^2+(b-y)^2}$

$\sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} + \sqrt{(a-x)^2+(b-y)^2} = L$ (constant)

Find the least y $\quad y = f(x)$

$y' = 0$ (critical point)

Implicit diff / $\frac{x+yy'}{(x^2+y^2)^{1/2}} \cdot \frac{2}{2} - \frac{(a-x)+(b-y)\cdot y'}{\sqrt{(a-x)^2+(b-y)^2}} = 0$

Date.

as $y'=0$

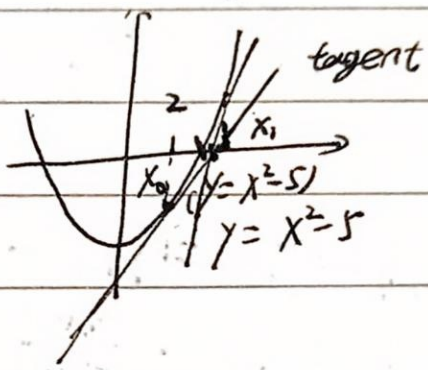
$$\frac{x}{\sqrt{x^2+y^2}} = \frac{a-x}{\sqrt{(a-x)^2+(b-y)^2}}$$

$$\sin\alpha = \sin\beta \quad \therefore \alpha=\beta$$

NEWTON'S METHOD

Example Solve $x^2=5$

$f(x)=x^2-5=0$



tangent line $y-y_0=m(x-x_0)$

x_1 is the x-intercept (截点)

as $y=0$, $0-y_0=m(x_1-x_0)$

$$\frac{y_0}{m}=x_0-x_1 \qquad x_1=x_0-\frac{y_0}{m}$$

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=x_0-\frac{f(x_0)}{f'(x_0)}$$

formula

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

$\frac{f(x)}{f'(x)}$ $f(x-\cdot)=f'(x)(x-$

$x_0=2 \quad f(x)=x^2-5 \quad f'(x)=2x$

$$x_1=x_0-\frac{x_0^2-5}{2x_0}$$

$$=\frac{1}{2}x_0+\frac{5}{2x_0}$$

$x_1=\frac{9}{4}$ $x_2=\frac{1}{2}\cdot\frac{9}{4}+\frac{5}{2}\cdot\frac{4}{9}=\frac{161}{72}$

$x_3=$

n	$\sqrt{5}-x_n$
0	2×10^{-1}
1	10^{-2}
2	4×10^{-5}
3	4×10^{-10}

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