LEC 24. 2024.12.28.

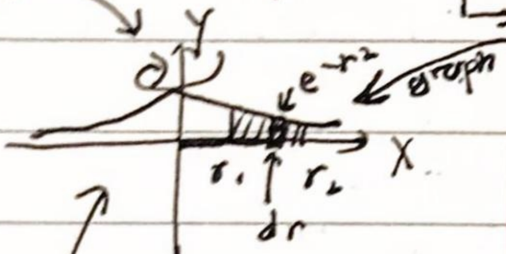
Dart board Example: (飞镖靶)

what's the probability that this guy gets hit by a dart.

$hits = c \cdot e^{-r^2}$ (a kind of normal distribution) (正态分布)

(离距离中心点 r 处的 hits, C为常数)

just like a board



绕 y-axis 旋转一周后即是 board

Shells volume

$$\int_{r_1}^{r_2} \underbrace{2\pi r}_{\text{周长 (长)}} \cdot \underbrace{e^{-r^2}}_{\text{高}} \underbrace{dr}_{\text{宽}} = Part$$

$$= -\pi e^{-r^2} \Big|_{r_1}^{r_2}$$

$$= \pi(e^{-r_1^2} - e^{-r_2^2})$$

$\therefore Part = c\pi(e^{-r_1^2} - e^{-r_2^2})$

WHOLE = $0 \le r < \infty$

$$= c\pi(e^{-0^2} - e^{-\infty^2})$$

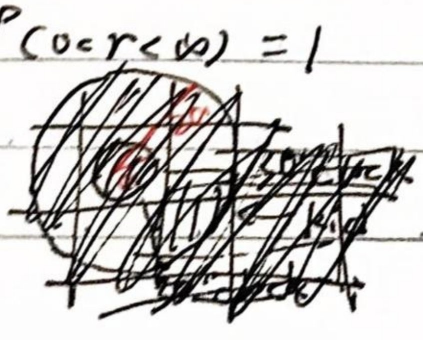
$$= c\pi(1-0) = c\pi$$

$\therefore$ probability $= \frac{c\pi(e^{-r_1^2} - e^{-r_2^2})}{c\pi} = \cancel{e^{-r_1^2} e} = e^{-r_1^2} - e^{-r_2^2}$

in $r_1 < x < r_2$

$P(0 < r < \infty) = 1$

Then

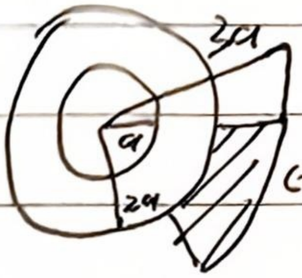


(条件)

equal : $P(0<r<a)=\frac{1}{2}$

$\hookrightarrow e^{-0^2}-e^{-a^2}=\frac{1}{2}$

$e^{-a^2}=\frac{1}{2}$



← the kid $\frac{1}{6}$ of the circle

$\Rightarrow P(\text{kid be hited})=\frac{1}{6}P(2a<r<3a)$

$=\frac{1}{6}\cdot\left(e^{-(2a)^2}-e^{-(3a)^2}\right)$

$=\frac{1}{6}\cdot\left((e^{-a^2})^4-(e^{-a^2})^9\right)$

$=\frac{1}{6}\cdot\left((\frac{1}{2})^4-(\frac{1}{2})^9\right)$

$\approx\frac{1}{6}\cdot\frac{1}{16}\approx\frac{1}{100}=1\%$

weight: $w(r)=2\pi c\, r\, e^{-\frac{r^2}{(\sigma)}}$

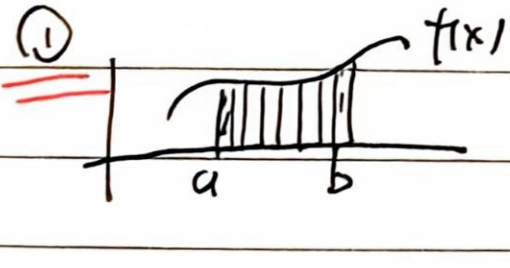
Next

(数值)

Numerical Intergral

1. Riemann Sums. 大 $S=\sum_{i=1}^{n} f(x_i)\cdot\Delta x$
2. trapezoidal rule (梯形法则) (相比与黎曼sum 使用梯形来近似) $T=\sum_{i=1}^{n}\frac{f(x_{i-1})+f(x_i)}{2}\cdot\Delta x$
3. Simpsion's rule (辛普森法则)

①



$a=x_0<x_1<\cdots<x_n=b$

$\Delta x=x_i-x_{i-1}$

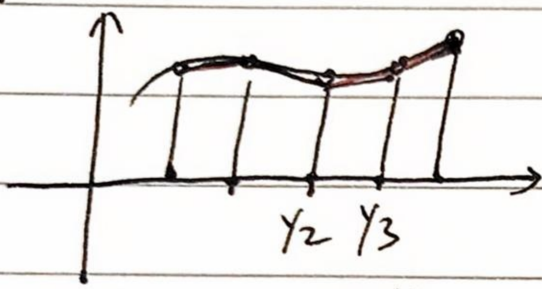
$y_0=f(x_0),\ y_1=f(x_1),\ \cdots,\ y_n=f(x_n)$

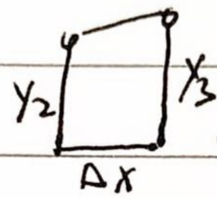
Date.

Riemann Sum:

$$\boxed{\begin{array}{ll} (y_0+y_1+\cdots+y_{n-1})\cdot\Delta x & \text{(left hand)} \\ (y_1+\cdots\cdots+y_n)\cdot\Delta x & \text{(right hand)} \end{array}}$$

②,





(梯形)

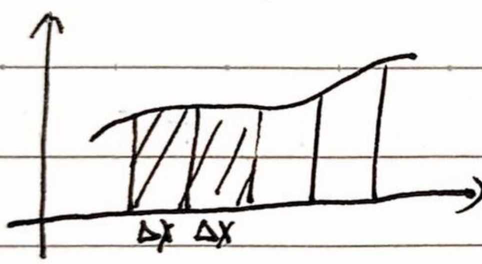
area of Trapezoid

$$S=\Delta x\left(\frac{y_2+y_3}{2}\right)$$

$$\Delta x\cdot\left(\frac{y_0+y_1}{2}+\frac{y_1+y_2}{2}+\cdots+\frac{y_{n-1}+y_n}{2}\right)$$

$$\boxed{=\Delta x\cdot\left(\frac{y_0}{2}+y_1+\cdots+y_{n-1}+\frac{y_n}{2}\right)}=\frac{\text{LEFT RS}+\text{Right RS}}{2}$$

③: SIMPSON'S RULE:

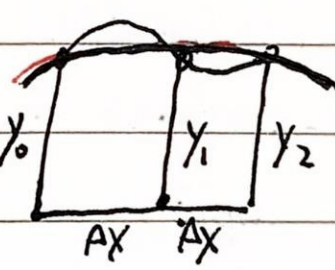


(偶)

(needs n even)

(用抛物线)

use parabola



AREA UNDER PARABOLA

$$\underbrace{2\Delta x}_{\text{base}}\cdot\underbrace{\left(\frac{y_0+4y_1+y_2}{6}\right)}_{\text{averagehight}}$$

oh problem set

$$\therefore S=\frac{2\Delta x}{6}\cdot\left((y_0+4y_1+y_2)+(y_2+4y_3+y_4)+\cdots+(y_{n-2}+4y_{n-1}+y_n)\right)$$

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1 4 1
    1 4 1
        1 4 1
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1 4 2 4 2 4 1
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$$=\frac{\Delta x}{3}\cdot(y_0+4y_1+2y_2+4y_3+2y_4+\cdots+2y_{n-2}+4y_{n-1}+y_n$$