2024-12-19

UNIT 2 APPLICATIONS OF ~~DEREV~~ DIFFerentions

Linear approximation

Lec 9

Formula $\boxed{f(x) \approx f(x_0) + f'(x_0)(x-x_0)}$

curve $y=f(x)$ $\approx$ $y=f(x_0)+f'(x_0)(X-X_0)$

the tagent line

the curve Line approoximate to the tagent line

$y=\ln x$ as $x_0=1$ $\ln(x) \approx \ln 1 + (\frac{1}{x})|_{x=1} \cdot (x-1$

$\approx x-1$

Δx, small

$\boxed{\frac{\Delta f}{\Delta x} \approx f'(x_0)}$.

$(\Delta x = x - x_0)$

$\Rightarrow \Delta f \approx f'(x_0) \cdot \Delta x$

$\Leftrightarrow f(x) - f(x_0) \approx f'(x_0)(x-x_0)$

$\Rightarrow f(x) \approx f(x_0) + f'(x_0)(x-x_0)$

x nearby x_0

as $x_0=0$

$\boxed{f(x) \approx f(0) + f'(0)x}$ $x \approx 0$

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$x \to 0$

$f(x) \approx f(0) + f'(0) \cdot x$ $\quad \frac{f(x) - f(0)}{x} = f'(0)$

$\sin x \quad \cos x \quad e^x \quad f(x) \approx f(x_0) + f'(x_0) \cdot (x - x_0)$

$$\boxed{\begin{aligned} &\begin{cases} f(x) \approx \sin 0 + \cos 0 \cdot x \approx x & \sin x,\ x \to 0 \\ f(x) \approx \cos 0 + (-\sin 0) \cdot x \approx 1 & \cos x,\ x \to 0 \\ e^x \approx e^0 + (e^0) \cdot x \approx x + 1 & e^x,\ x \to 0 \end{cases} \\ &(1+x)^r \approx 1^r + r \cdot (1+x)^{r-1} \cdot x = 1 + r \cdot x \end{aligned}}$$

Ex1 $\ln(1+x) = \ln(1) + \frac{1}{x+1}\Big|_{x=0} \cdot x = x$

as $1+x = u$, $\ln u \approx u - 1$, $\therefore \ln(1+x) \approx x$, $x \to 0$

Ex2 $\cancel{\ln 1.1 \approx 1}$

$\ln 1.1 \approx \frac{1}{10} \rightarrow \ln(1+0.1) = \ln(1+x) \approx x$

Hard easy $\quad 0.1 \to 0 \quad \therefore \ln 1.1 \approx \frac{1}{10}$

Ex3, Find linear approx" near $x=0$". $(x \approx 0)$ ~~(x→0)~~

of $\frac{e^{-3x}}{\sqrt{1+x}}$

$$\frac{e^{-3x}}{\sqrt{1+x}} = e^{-3x} \cdot (1+x)^{-\frac{1}{2}} \approx (-3x+1) \cdot [1 + (-\tfrac{1}{2})x]$$

$$= 1 - 3x + \tfrac{3}{2}x^2 - \tfrac{1}{2}x$$

$$\approx 1 - \tfrac{7}{2}x$$

x^2 is negative

we drop. x^3 and higher

Ex 4,

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QUADRATIC Approximation (二次近似)

$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2$$

(why $\frac{1}{2}$ after)

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\ln(1.1) = \ln(1+\tfrac{1}{10}) \approx \tfrac{1}{10} - \tfrac{1}{2}(\tfrac{1}{10})^2$$

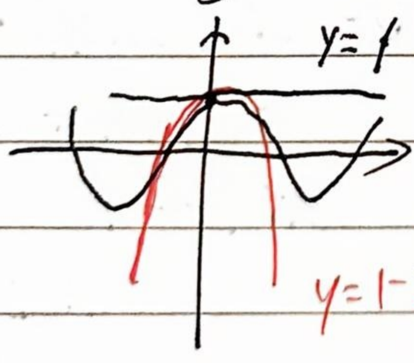
$$= 0.095\ldots$$

$$x_0 = 0 \quad f(x) \approx f(0) + f'(0)\cdot x + \frac{f''(0)}{2}\cdot x^2$$

$$\begin{cases} \sin x \approx x \\ \cos x \approx 1 - \frac{1}{2}x^2 \\ e^x \approx 1 + x + \frac{1}{2}x^2 \end{cases}$$

f''	$f''(0)$
$-\sin x$	0
$-\cos x$	-1
e^x	1

Geometric significance



(抛物线)

$y = 1 - \frac{1}{2}x^2$ "best" fit parabola

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