LEC23 2024.12.27

AVERAGE VALUE

continuous avg

$$\frac{y_1+y_2+\cdots+y_n}{n} \xrightarrow{n\to\infty} \frac{1}{b-a}\int_a^b f(x)\,dx$$

avg of $f(x)$

$$\Delta x = \frac{b-a}{n}$$

Riemann sum

$$(y_1+\cdots+y_n)\cdot\Delta x \xrightarrow{\Delta x\to 0} \int_a^b f(x)\cdot dx$$

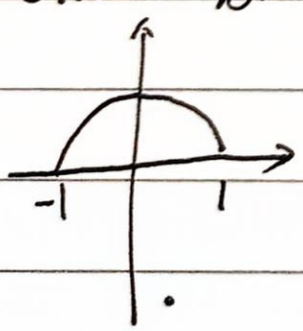
$$\frac{\Delta x}{b-a} = \frac{1}{n} \xrightarrow{n\to\infty} 0 \quad (\Delta x \to 0)$$

Ex1 $f(x)=C$

$$\frac{1}{b-a}\int_a^b C\,dx = C \qquad \text{Ave}(C) = C$$

Ex3 Average ht with arclength θ

$y=\sin\theta$



$\theta: 0\to\pi$

$$\frac{1}{\pi}\int_0^\pi \sin\theta\,d\theta = \frac{1}{\pi}(-\cos\theta)\Big|_0^\pi = \frac{1}{\pi}\cdot 2 = \frac{2}{\pi}$$

↑ by θ

↓ by x

$\frac{2}{\pi} < \frac{\pi}{4}$ ← the avg's relation

Ex 2

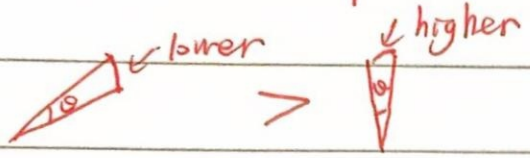
Average ht with x:

$$\frac{1}{1-(-1)}\cdot\int_{-1}^{1}\sqrt{1-x^2}\,dx = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}$$

↑ is the S of the half circle

$\pi r^2 = \pi\cdot 1 = \frac{\pi}{2}$

why: the lower part is more important for θ



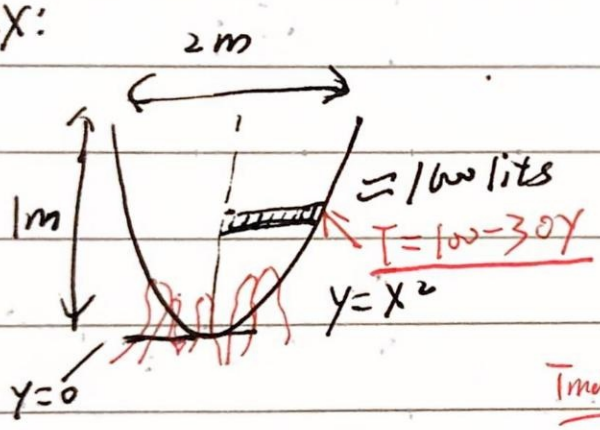
Weight AVERAGE (加权平均)

$$\frac{\int_a^b f(x)w(x)dx}{\int_a^b w(x)dx}$$

shares (股票)

$$\frac{10w_1 + 20w_2 + 30w_3}{w_1+w_2+w_3}$$

OX:



Initial Tmp = 0°C

0°C to (100-30y)°C

T = 100 - 30y, how much energy need

$\frac{T_{max}+T_{min}}{2} = \frac{100+70}{2} = 85$ ↑ the ordinary avg

USE disk Method

$$\int_0^1 T\cdot\pi\cdot x^2\cdot dy = \int_0^1 (100-30y)\cdot y\pi\, dy$$

$$= 50\pi y^2 - 10\pi y^3 \Big|_0^1$$

$$= 40\pi \ deg\cdot cm^3 \quad deg = °C$$

$$= 40\pi\cdot\left(\frac{1cal}{deg\cdot cm^2}\right)\cdot\left(\frac{100\,cm^3}{m}\right)$$

$$= 40\pi\cdot 10^6\ cal$$

$$40\pi\cdot 10^3\ kcal$$

$deg = \frac{kcal}{cal}$

Avg Final temperture

$$\frac{\int_0^1 T\cdot\pi y\,dy}{\int_0^1 \pi y\,dy} = \frac{\int_0^1 T\cdot x^2\pi\,dy}{\int_0^1 \pi x^2\,dy} = \frac{40\pi}{\pi} = 80°C$$

← all temperture of every water curb

↑ the volume of water

the weighting avg (加权平均) $w(y) = \pi y$ (每个加权是每一个 dy 的 disk)

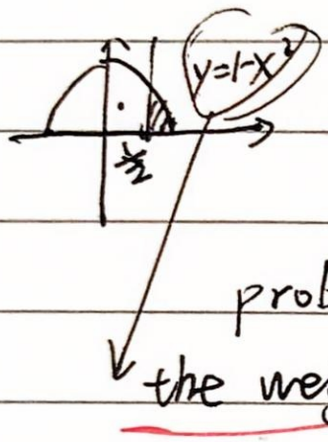
the ordinary avg = 85 > the weighting avg = 80

Because the water is more on the top

as the example of x and θ

Probability (概率论)

Pick a point at random in $0<y<1-x^2$



(比例) proportional to area $x>\frac{1}{2}$

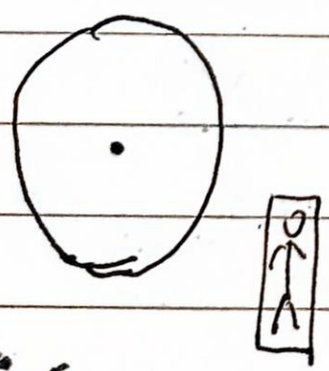
$$\text{probability} = \frac{\text{PART}}{\text{WHOLE}} = \frac{\int_{\frac{1}{2}}^{1}(1-x^2)\,dy}{\int_{-1}^{1}(1-x^2)\,dy} \cdots = \frac{5}{8}$$

the weight

General formula

$$a \le x_1 < x_2 \le b$$

$$P(x_1<x<x_2) = \frac{\int_{x_1}^{x_2} w(x)\,dx}{\int_a^b w(x)\,dx} = \frac{\text{Part}}{\text{whole}}$$



next time