LEC 32 2025.1.1

parametric curves continue

$$\begin{cases} x = a\cos t \\ y = a \cdot \sin t \end{cases} \qquad x^2+y^2=a^2$$

$$ds^2 = dx^2+dy^2, \quad ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} \cdot dt$$

$$\therefore \frac{ds}{dt} = \sqrt{(-a\sin t)^2+(a\cos t)^2} \qquad \frac{dx}{dt} = -a\sin t, \ \frac{dy}{dt} = a\cos t$$

$$\therefore \frac{ds}{dt} = a \leftarrow \text{speed}$$

NOTATION

$$\boxed{\begin{aligned} \Delta s^2 &\simeq \Delta x^2 + \Delta y^2 \\ \left(\frac{\Delta s}{\Delta t}\right)^2 &\simeq \left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2 \\ \frac{ds}{dt} &= \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} \end{aligned}}$$

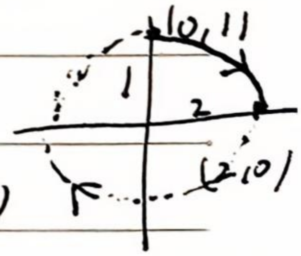
$$\left(\frac{dx}{dt}\right)^2 \neq \frac{dx^2}{dt^2} \text{ NEVER}$$

$$\boxed{\frac{d^2x}{dt^2} = \left(\frac{d}{dt}\right)^2 x = \frac{d}{dt} \cdot \frac{d}{dt} \cdot x}$$

Ex 2, $x = 2\sin t$, $y = \cos t$

$$\frac{1}{4}x^2+y^2=1 \qquad , t=0, (0,1)$$

$$t = \frac{\pi}{2}, (2,0)$$



clock wise

$$\frac{ds}{dt} = \sqrt{(2\cos t)^2+(-\sin t)^2}$$

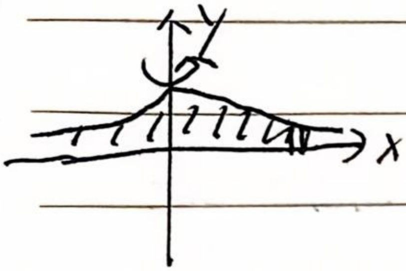
$$\text{Arclength} = \int_0^{2\pi} ds = \int_0^{2\pi} \sqrt{4\cos^2 t + \sin^2 t} \cdot dt$$

↑ can't figure out

MY Interput:

京大过去问

$$\int_{-\infty}^{+\infty} e^{-x^2} dx = a(x)$$



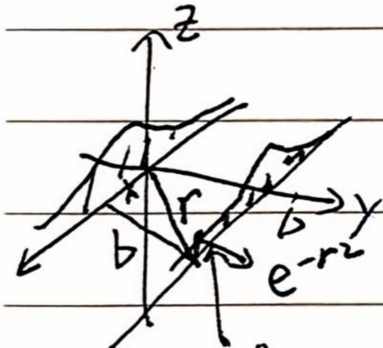
around Y:

$$dV = dx \cdot x \cdot 2\pi \cdot y$$

$$V = \int_0^{\infty} 2\pi x \cdot e^{-x^2} \cdot dx$$

$$= \pi \cdot (-e^{-x^2})\Big|_0^{\infty}$$

$$= \pi$$



$$r^2 = b^2 + x^2$$

$$\text{Area}(b) = \int_{-\infty}^{+\infty} e^{-r^2} dx = \int_{-\infty}^{+\infty} e^{-b^2} \cdot e^{-x^2} \cdot dx$$

$$= e^{-b^2} \cdot a$$

$$V = \int_{-\infty}^{+\infty} \text{Area}(y) \cdot dy$$

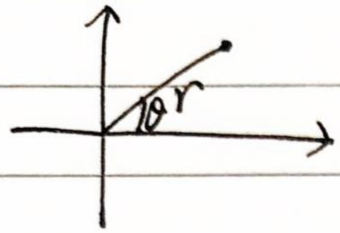
$$= \int_{-\infty}^{+\infty} e^{-y^2} \cdot a \cdot dy = a^2$$

$$\therefore a^2 = V = \pi, \quad a = \sqrt{\pi}$$

$$\therefore \int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$$

continue to LEC 32: $y = \cancel{f(x)}$ we need throw away

POLAR COORDINATES (极坐标)

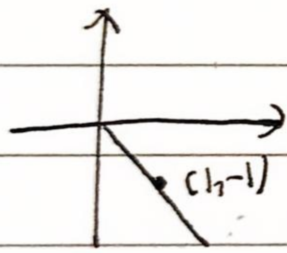


$r =$ the distance to origin

Formulas: $\boxed{x = r\cos\theta \ , \quad y = r\sin\theta}$

$$\begin{cases} r = \sqrt{x^2+y^2} \\ \theta = \tan^{-1} y/x = \tan^{-1}(-y/-x) \end{cases}$$ need look graph

Ex1: $(x, y) = (1, -1)$, in polar coord:



a) $r = \sqrt{2}$, $\theta = \frac{\pi}{4} + \frac{3}{2}\pi = \frac{7}{4}\pi$

b) $r = \sqrt{2}$, $\theta = -\frac{\pi}{4}$

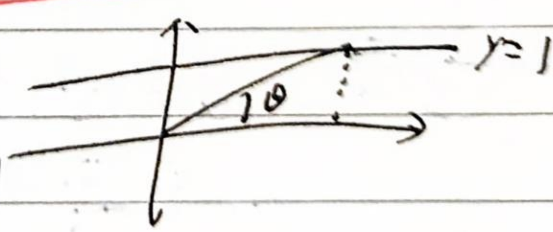
c) $r = -\sqrt{2}$, $\theta = \cancel{\frac{3\pi}{4}} \ \frac{3}{4}\pi$

~~Ex2:~~

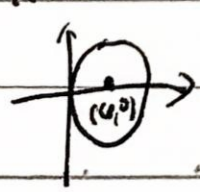
Ex4: $y = 1$

$y = r\sin\theta = 1$, $\boxed{r = \frac{1}{\sin\theta}}$ $r = r(\theta) \leftarrow$ almost

$\theta \in (0, \pi)$



Ex5: off center circle



$(x-a)^2 + y^2 = a^2$, $x^2 - 2ax + a^2 + y^2 = a^2 \rightarrow r^2 - 2ax = 0$

$r^2 - 2a \cdot r\cos\theta = 0$

$r = 2a\cos\theta$ (or $r = 0$)