$y=\sin^{-1}x \qquad \sin y = x \qquad \cos y\cdot y' = 1$

$$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$\boxed{\text{LEC 6}}$

exponentials and logarithms

(指数) (对数)

$$a^{x_1+x_2} = a^{x_1}\cdot a^{x_2}$$

$$a^{x_1x_2} = (a^{x_1})^{x_2} \qquad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

Goal: $\frac{d}{dx}a^x = ?$

$$\lim_{\Delta x\to 0}\frac{a^{x+\Delta x}-a^x}{\Delta x} = \frac{a^x(a^{\Delta x}-1)}{\Delta x} = \cancel{\lim}\ a^x\cdot \underbrace{\lim_{\Delta x\to 0}\frac{a^{\Delta x}-1}{\Delta x}}_{M(a)}$$

(a=e)

as $M(e)=1$

$$\frac{d}{dx}e^x = a^x\cdot 1 = e^x$$

$$\frac{d}{dx}e^x\Big|_{x=0} = 1$$

Date.

proof why e exist:

$$f(x)=2^x,\quad f'(0)=M(2)$$

$$f(kx)=2^{kx}=(2^k)^x=b^x \qquad b=2^k$$

$$\frac{d}{dx}b^x=\frac{d}{dx}f(kx)=k\cdot f'(kx)$$

$$\frac{d}{dx}b^x\Big|_{x=0}=k\cdot f'(0)=k\cdot M(2)$$

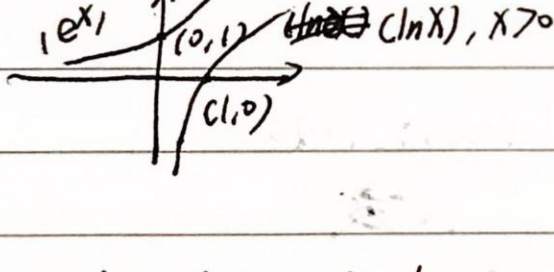
$b=e$, when $k=\frac{1}{M(2)}$

Nature log

$w=\ln x \qquad y=e^x \Leftrightarrow \ln y=x$

$$\ln(x_1x_2)=\ln x_1+\ln x_2$$

$\ln(1)=0 \qquad \ln(e)=1 \qquad \cancel{e^{\ln(e)}=e}$



The Derivative of log

$w=\ln x \Leftrightarrow e^w=x$

$\frac{d}{dx}e^w=\frac{d}{dx}x=1$, $\boxed{\frac{d}{dw}e^w}\left(\frac{dw}{dx}\right)=1$, $e^w\cdot\frac{dw}{dx}=1$,

$\| \; e^w$ (pre page) $\quad \frac{dw}{dx}=\frac{1}{e^w}=\frac{1}{x}$

$$\boxed{\frac{d}{dx}\ln x=\frac{1}{x}}$$

$\lim_{\Delta x \to 0} \cancel{a^{x+\Delta x}} \frac{a^{\Delta x}-1}{\Delta x} = \ln a$

Back to $\frac{d}{dx}a^x = M(a)\cdot a^x$

To differentiate any exponential: two methods

①:

$\boxed{\frac{d}{dx}a^x = ? \quad \text{USE BASE } e;\ a^x = (e^{\ln a})^x = e^{x\ln a}}$

$\frac{d}{dx}a^x = \frac{d}{dx}e^{x\ln a} = (\ln a)e^{x\ln a}$

$\boxed{\frac{d}{dx}a^x = (\ln a)\cdot a^x}$ $\qquad (e^{\ln a})^x = a^x$

my:

$\frac{d}{dx}a^x \quad a^x = (e^{\ln a})^x = (e^{x\cdot\ln a})' = (\ln a)\cdot e^{x\ln a}$

$= (\ln a)\cdot a^x$

$M(a) = \ln a$

$\frac{d}{dx}2^x = \ln 2\cdot(2^x) \qquad \frac{d}{dx}10^x = 10^x\cdot\ln(10)$

②:

Logarithmic differentiation

$\frac{d}{dx}u = ??$, $\quad \frac{d}{dx}\ln u = \frac{d\ln u}{du}\cdot\frac{du}{dx} = \frac{1}{u}\frac{du}{dx}$ easier

$\boxed{D\ln u = \frac{u'}{u}}$

$\frac{d}{dx}a^x = ?\ /\ u = a^x \qquad \ln u = x\cdot\ln a$

$(\ln u)' = \ln a$

$\therefore \frac{u'}{u} = (\ln u)' = \ln a$

$\therefore u' = u\cdot\ln a = a^x\cdot\ln a$

EXAMPLE: moving exponent

$v = x^x$

$\ln v = x \cdot \ln x$

$(\ln v)' = \ln x + 1$

$\frac{v'}{v} = \ln x + 1$, $v' = (\ln x + 1) \cdot x^x$

$\frac{d}{dx} x^x = x^x \cdot (1 + \ln x)$

(法 ①: 对 x 取 e 为底，② 两边同时取 ln)

example

2. $\lim_{n \to \infty} (1 + \frac{1}{n})^{n}$ → moving

$\ln (1+\frac{1}{n})^n = n \cdot \ln (1+\frac{1}{n})$

as $\Delta x = \frac{1}{n} \to 0$

$= \frac{1}{\Delta x} \ln (1 + \Delta x) \boxed{- \ln 1}$

$\left\{ \begin{array}{l} -\ln 1 \\ -\cos\frac{\pi}{2} \\ -\sin 0 \end{array} \right.$

$= \frac{\ln(1+\Delta x) - \ln 1}{\Delta x}$

$\frac{d}{dx} \ln x \Big|_{x=1} = 1$

$\therefore \lim_{n \to \infty} (1+\frac{1}{n})^n = e^1$

$\lim_{n \to \infty} (1+\frac{1}{n})^n = e^{\lim_{n \to \infty} \ln (1+\frac{1}{n})^n} = e^1$

$e \approx (1+\frac{1}{100})^{100}$