LEC2 Unit1 2024.12.19

EXPONENTS

$$\frac{d}{dx}x^r = r\cdot x^{r-1}$$

$$x^r = (e^{\ln x})^r = e^{r\ln x}$$

$$\frac{d}{dx}x^r = D(e^{r\ln x}) \cancel{= r\ln x\, e^{r\ln x}}$$

$$= e^{r\cdot\ln x}\cdot(r\cdot\ln x)' = \frac{r}{x}\cdot e^{r\cdot\ln x}$$

$$= x^r\cdot\frac{r}{x}$$

$$= r\cdot x^{r-1}$$

log diff Method2

$u = x^r$, $(\ln u)' = (r\cdot\ln x)'$, $\frac{u'}{u} = \frac{r}{x}$

$$u' = \frac{r}{x}\cdot u \cancel{= x^r\cdot\frac{r}{x}}$$

$$= r\cdot x^{r-1}$$

$\boxed{\text{REVIEW OF UNIT 1}}$

General Formulas,

$(u+v)'$, $(cu)'$, $(uv)'$, $(u/v)'$

$\frac{d}{dx}f(u) = f'(u)\cdot u'(x)$ $[u = u(x)]$ chain rule

Implicit diff inverse, log diff

$\frac{\sin x}{\cos x}$ $\frac{\cos^2 x -}{\cos^2 x}$

Specific funs

$(\cos x)$ $(-\sin x)$ $\left(\frac{1}{\cos^2 x}\right)$ $(\sec x \cdot \tan x)$

x^r, $\sin x$, $\cos x$, $\tan x$, $\sec x$

e^x, $\ln x$ $\left(\frac{1}{x}\right)$

$\frac{1}{\sin x}$ $\frac{\sin x - \cos x}{\sin^2 x}$

$\tan^{-1}x$, $\sin^{-1}x$

$\left(\frac{1}{1+x^2}\right)$ $\left(\frac{1}{\sqrt{1-x^2}}\right)$ $(\cos^{-1}x)' = \frac{-1}{\sqrt{1-x^2}}$

Ex

$$\frac{d}{dx}\sec x = \frac{d}{dx}(\cos x)^{-1}$$
$$= -(\cos x)^{-2} \cdot \sin x$$
$$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \tan x = \sec x \cdot \tan x$$

$$\frac{d}{dx}\ln(\sec x) = \frac{\sec x \cdot \tan x}{\sec x} = \tan x$$

Ex

$$\frac{d}{dx}(x^{10}+8x)^6 = 6(x^{10}+8x)^5 \cdot (10x^9+8)$$

Ex $\frac{d}{dx} e^{x \cdot \tan^{-1}x} = e^{x \cdot \tan^{-1}x} \cdot (x \cdot \tan^{-1}x)'$

$$= e^{x \cdot \tan^{-1}x} \cdot \left(\tan^{-1}x + \frac{x}{1+x^2}\right)$$

$\cancel{\sin^{-x}}$ $\sin^{-1}x = y$, $(x)' = (\sin y)'$ $\cos^2 y + \sin^2 y = 1$

$y' \cos y = 1$ $\cos y = \sqrt{1-\sin^2 y}$

$y' = \frac{1}{\cos y}$ $= \sqrt{1-x^2}$

$= \frac{1}{\sqrt{1-x^2}}$

Date.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{u \to 0} \frac{e^{u}-1}{u} = \frac{d}{du} e^{u} \Big|_{u=0} = 1$$

Derive formule for

$(\sin^{-1} x)'$, $(\ln x)'$ in implicit diff

$y = \sin^{-1} x$, $\sin y = x$

$$y' \cdot \cos y = 1 , \quad y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 x}} = \frac{1}{\sqrt{1-x^2}}$$

$y = \tan^{-1} x$ $\qquad \tan y = x$

$$y' \cdot \tan' y = x$$

$$\therefore y' \cdot \frac{1}{\cos^2 y} = \cancel{x}\, 1 \quad | \quad y' = \cos^2 y = \frac{1}{1+x^2}$$

$\frac{\sin y}{\cos y} = x$

$$\sin^2 y = 1 - \cos^2 y = x^2 \cdot \cos^2 y$$

$$\cos^2 y = \frac{1}{1+x^2} \quad \therefore (\tan^{-1} x)' = \frac{1}{1+x^2}$$