LEC 29. 2024.12.31

continue to method of integration

$\frac{P(x)}{Q(x)}$ rational function

USE PARTIAL FRACTIONS (部分分式)

Ex1.

easy

$$\int\left(\frac{1}{x-1}+\frac{3}{x+2}\right) = \ln|x-1| + 3\ln|x+2| + C$$

$$\frac{1}{x-1}+\frac{3}{x+2} = \frac{4x-1}{(x-1)(x+2)} = \frac{4x-1}{x^2+x-2}$$

disguised (伪装)

Detect "easy" part — cover up method

Ex1 ⇒

$$\frac{4x-1}{x^2+x-2} \overset{①}{=} \frac{4x-1}{(x-1)(x+2)}$$

$$\overset{②}{=} \frac{A}{x-1}+\frac{B}{x+2}$$

③ Solve for A & B

④ $\frac{4x-1}{x+2} = A + \frac{B}{(x+2)}(x-1)$ $(x \xrightarrow{\text{tends}} 1)$

now: plug in $x=1$ ⇒ $\frac{4-1}{1+2} = A = 1$

⑤ $\frac{4x-1}{x-1} = \frac{A\cdot(x+2)}{x-1} + B$

now: plug in $x=-2$ ∴ $B = \frac{-8-1}{-3} = 3$

∴ ② $= \frac{1}{x-1}+\frac{3}{x+2}$

① Facter Q (denominator)

② Set up

③ cover up

where:

(fully)

Q (x) can be $\boxed{\text{compeletly factored}}$ ~~+Q(x)7x~~

and degree P < degree Q (幂次)

$$\underbrace{\frac{4x-1}{(x-1)(x+2)}}_{①} \overset{②}{=} \frac{A}{x-1} + \frac{B}{x+2}$$

③ $\frac{4x-1}{x+2} = A$ $(x=1)$, $B = \frac{4x-1}{x-1}$, $x=-2$

For instance:

$$\frac{x^2+3x+8}{(x-1)(x-2)(x+5)} \overset{②}{=} \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+5}$$

Example 2: (deg P < deg Q)

$$\frac{x^2+7}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

$$B = \frac{1+2}{1+2} = 1, \quad C = \frac{4+2}{(-3)^2} = \frac{2}{3}$$

test

$$\frac{A}{(x-1)^2} + \frac{C}{x+2} = \frac{x^2+1}{(x-1)^2(x+2)}$$

$A = \frac{2}{3}$

$C = \frac{5}{9}$ ~~$\frac{5}{9}$~~

$$\frac{\frac{2}{3}}{(x-1)^2} + \frac{\frac{5}{9}}{x+2}$$

$$\frac{3x+6+9x^2-18x+9}{(x-1)^2(x+2)}$$

For A, plug your favorte $x=0$ (number any)

$$\frac{0+2}{1\cdot 2} = -A + B + \frac{C}{2} = -A + 1 + \frac{1}{2}\times\frac{2}{3} \;\; \frac{1}{3}$$

$$A = 1 + \frac{1}{3} - 1 = \frac{1}{3}$$

$$\therefore u = \frac{\frac{1}{3}}{x-1} + \frac{1}{(x-1)^2} + \frac{2/3}{x+2}$$

Date.

Example 3, $\deg P < \deg Q$,

Q has a quadratic factor.

$$\frac{x^2}{(x-1)(x^2+1)} \underset{②}{=} \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

①

cover up for A, $\frac{1}{1+1} = A = \frac{1}{2}$

For B and C: clear denominator

$$x^2 = A(x^2+1) + (Bx+C)\cdot(x-1)$$

x^2 coefficients: $1 = \frac{1}{2} + B$, $\therefore B = \frac{1}{2}$, x^0 term: left 0 right $\frac{1}{2} - C$

$\therefore C = \frac{1}{2}$

What if $\deg P \geq \deg Q$

easy ↓ ↓ use cover up

$$\frac{x^3}{(x-1)(x+2)} \underset{\text{inverse}①}{=} \frac{x^3}{x^2+x-2} = (x-1) + \frac{3x-2}{x^2+x-2}$$

$$\begin{array}{r|l} & x-1 \quad \text{— quotient} \\ x^2+x-2 & x^3 \\ & x^3+x^2-2x \\ \hline & -x^2+2x \\ & -x^2-x+2 \\ \hline & +3x-2 \quad \text{remainder} \end{array}$$

Date.