$\frac{1}{x^2}$ x $\frac{-1}{x} = -\frac{1}{x^2}$ $-2x$

$$\frac{d}{dx}x^n = -\frac{n}{x^{n+1}}$$

«lec2»

Last time: derivative = SLOPE of the tegent line

$\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$ $\frac{d}{dx}x^n = n\cdot x^{n-1}$

Rate of change

$$\frac{\Delta y}{\Delta x} \longrightarrow \frac{dy}{dx}$$

avg change (瞬时) instanteneous chage

Limit And Continuity

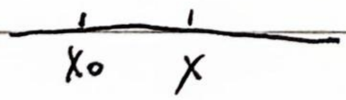
derivatives ~~alw~~ always harder:

$$\lim_{x\to x_0} \frac{f(x_0+\Delta x)-f(x_0)}{x-x_0}$$

$x = x_0$, gives $\frac{0}{0}$ (~~alter~~ always need cancellation)

$\lim_{x\to x_0^+} f(x)$ = right-hand limit

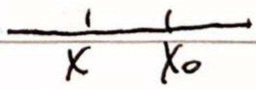
$\begin{bmatrix} x\to x_0 \\ x>x_0 \end{bmatrix}$



(右极限)

$\lim_{x\to x_0^-} f(x)$ = left-hand limit

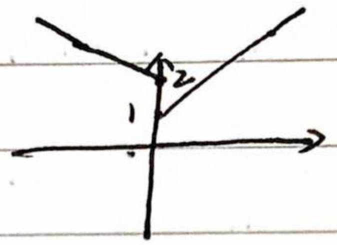
$\begin{bmatrix} x\to x_0 \\ x<x_0 \end{bmatrix}$



(左极限)

Example 1:

$$f(x)=\begin{cases} x+1, & x>0 \\ -x+2, & x<0 \end{cases}$$



$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} x+1 = 1$, $\lim_{x\to 0^-} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} -x+2 = 2$

DEF'N f is continuous at x_0 means $\to \lim_{x\to x_0} f(x) = f(x_0)$

连续的条件 可~~微~~微) 导 $\cancel{\lim_{x\to x_0} f(x_0)} = \lim_{x\to x_0} f(x) = f(x_0)$

cts at x_0:

1. $\lim_{x\to x_0} f(x)$ exist $\left(\begin{matrix}\text{from } L+R \\ L=R\end{matrix}\right)$
2. $f(x_0)$ is defined
3. they are equal $[\lim_{x\to x_0} f(x_0) = f(x_0)]$

Example: (跳跃不连续

① Jump discontinuity: lim from L +R is exist but not equal

② REMOVABLE DISCONTINUITY (可去不连续):

lim from left ~~&~~ & right is equal ~~t~~ at

Examle:

$g(x) = \frac{\sin x}{x}$; $g(0) = ??$, $\lim_{x\to 0} \frac{\sin x}{x} = 1$

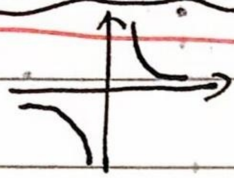
$h(x) = \frac{1-\cos x}{x}$; $h(0) = ??$, $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

} remvable discontinuity at $x=0$

Date.

③ Infinite discontinuity (极限不连续)

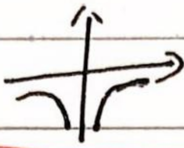
$y = \frac{1}{x}$



odd (奇) ①

$\lim_{x \to 0^+} \frac{1}{x} = \infty$; $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ ~~$\lim_{x \to 0} \frac{1}{x} = \infty$~~

$\lim_{x \to 0} \frac{-1}{x^2} = -\infty$ ✓



even (偶) ②

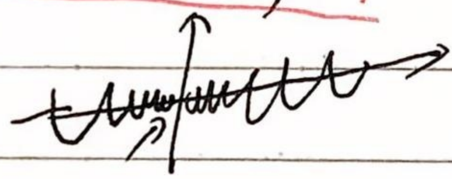
①+②: if you take a derivative of an odd function, you always get an even function

对一个奇函数求导就会得到一个偶函数

④ OTHER (UGLY) Discontinuity

example:

$y = \sin\frac{1}{x}$ as $x \to 0$



no left or right limit

Theorem (定理)

(DIFF ⇒ CTS) (differentiable ⇒ continues)

$\frac{\Delta y}{\Delta x}$ 可微 ⇒ 连续

If f is differentiable at x_0, then f is continues at x_0

PROOF: $\lim_{x \to x_0} f(x) - f(x_0) \stackrel{?}{=} 0$

$$\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}(x-x_0) = f'(x)\cdot 0 = 0$$

cts $\lim_{x\to x_0} f(x) = f(x_0)$ $f'(x)$ exist

微分：Δy，dy，导数：$f'(x)$，$\frac{dy}{dx}$ 即"可微"/"可导"

可微通常是可导的另一种描述 $\Rightarrow$ "连续"

{ differentiable (differentiate) 可以理解为"可导"、"可微"。侧重于函数的性质
derivative 是具体的导数值，更侧重于具体的数值

~~tee~~ problem

$$f(x)=\begin{cases} ax+b & x>0 \\ \sin 2x, & x\le 0 \end{cases}$$

when ~~x~~ a and b can make the function cts (continuous) and not differentiable?

1. continuous:

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$ $\lim_{x\to 0^-} f(x) = \sin 2x = 0$

$\lim_{x\to 0^+} f(x) = ax+b = b$

$\therefore b=0$

2. not differentiable:

$f'(0^-) = 2\cos 2x = 2$ $f'(0^+) = a$

$\therefore$ when $a\ne 2$, $b=0$ the function is cts but not diffarable.