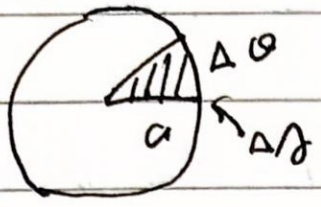
LEC 33 2025.1.2

POLAR COORDINATES

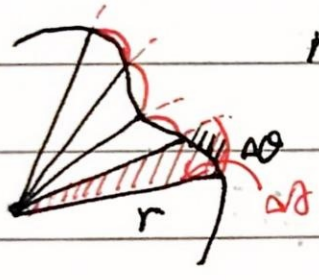
AREA



total $A = \pi a^2$

$\Delta f = \frac{\Delta\theta}{2\pi} \cdot \pi a^2 = \frac{a^2}{2} \cdot \Delta\theta$

variable Pie:



$r = r(\theta)$

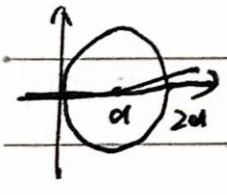
$\pi r^2, \quad \frac{\Delta\theta}{2\pi} \cdot \pi r^2 = \frac{r^2}{2} \cdot \Delta\theta$

$\Delta A \approx \frac{r^2}{2} \cdot \Delta\theta$

$dA = \frac{r^2}{2} \cdot d\theta$

$S = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \cdot d\theta \quad (r = r(\theta))$

Ex: $r = 2a\cos\theta \longleftarrow (x-a)^2 + y^2 = a^2$



$\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$dA = \frac{r^2}{2} \cdot d\theta = 2a^2 \cdot \cos^2\theta \cdot d\theta$

$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2a^2 \cos^2\theta \cdot d\theta$

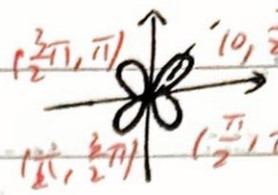
$= 2a^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2} d\theta$

$= 2a^2 \left(\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right)_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$= a^2 \cdot \left(\theta + \frac{\sin 2\theta}{2} \right)_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = a^2 \cdot \pi$

Ex 2. (drawing).

$r = \sin 2\theta$



θ	r
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0

$T = \frac{2\pi}{2} = \pi$

Ex3. $r = \frac{1}{1+2\cos\theta}$

$r + r2\cos\theta = 1.$

$r = 1 - r \cdot 2\cos\theta = 1 - 2x$

$r^2 = (1-2x)^2$

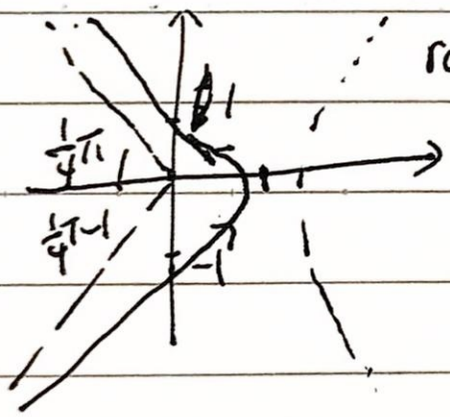
$x^2 + y^2 = (1-2x)^2 = 4x^2 - 4x + 1$

$y^2 = 3x^2 - 4x + 1$ (hyperbora)

双曲线

θ	r
0	$\frac{1}{3}$
$\frac{\pi}{2}$	1
$-\frac{\pi}{2}$	1

$\theta = \pm\frac{2}{3}\pi$ 时, $r \to \infty$



$\frac{dA}{dt} = \frac{1}{2}r^2 \cdot \frac{d\theta}{dt} = \text{constant}$

(角动量守恒) angular momentum

we did not do arclength in polar coordinates

EXAM.

1. Techniqus of integration
   - ① trigle substution
   - ② integration by parts (分部积分)
   - ③ partial fractions (部分分数)
2. parametric curves
   - ① arclength ② area of surface of revolution

$\int_0^{\circ} \llcorner dt$

3. polar coordinates
   - ① including area

Date:

$$\int \frac{x^2}{1+x^2}\,dx$$

don't use trigle substution. is slow

use long division faster

$$= \int \left(1 - \frac{1}{1+x^2}\right) dx$$

$$= x - \tan^{-1}x + C$$

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L' Hospital's rule

a convenient way to calculate lim including new ones.

$x \ln x$, $x \to 0^+$