LEC19. 2024.12.25

Fundamental theorem of calculus [FTC1]

ftc1 (微积分基本定理)

$$\boxed{\text{If } F'(x) = f(x), \text{ then } \int_a^b f(x)\,dx = F(b)-F(a)}$$

$$F = \int f(x)\,d(x)$$

New notation $F(b)-F(a) = F(x)\Big|_a^b = F(x)\Big|_{x=a}^{x=b}$

Ex $F(x) = x^3/3$ $F'(x) = x^2 \Rightarrow \int_a^b x^2\,dx = x^3/3\Big|_{x=a}^{x=b}$

$= \frac{b^3}{3} - \frac{a^3}{3}$

$$\int_0^b x^2\,dx = \frac{x^3}{3}\Big|_0^b = \frac{b^3}{3}$$

(a=0)

Ex

$$\sum_{i=1}^{n} v(t_i)\cdot\Delta t \doteq \int_a^b v(t)\,dt = x(b)-x(a)$$

Riemann sum

$\underbrace{1\text{sec}}_{} = \Delta t$

distance in 1 second

Extend integration to the case $f<0$

Example $\int_0^{2\pi} \sin x\,dx = -\cos x\Big|_0^{2\pi} = -\cos 2\pi - (-\cos 0)$

$= -1 + 1 = 0$

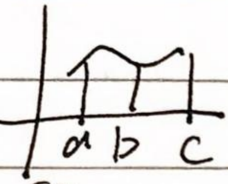
the area up + miner

$$\boxed{\begin{array}{l}\text{Properties of Integrals}\\ 1.\ \int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx\end{array}}$$

2. $\int_a^b c f(x)dx = c \cdot \int_a^b f(x)dx$

c doesn't depend on x

3. $a<b<c$ ← not ness



$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

(OK with 5)

4. $\int_a^a f(x)dx = 0 = F(a) - F(a) = 0$

☆5. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\quad \| \; F(b) - F(a) = -(F(a) - F(b))$

6. (Estimation)

If $f(x) \leq g(x)$ the $\int_a^b f(x)dx \leq \int_a^b g(x)dx$ $\quad a<b$

Example of estimation

$e^x \geq 1, \; x \geq 0$

$\int_0^b e^x dx \geq \int_0^b 1dx$, $\quad e^x \Big|_0^b = e^b - 1$

$x \Big|_0^b = b - 0 = b$

$e^b - 1 \geq b \Leftrightarrow e^b \geq b+1, \; (b \geq 0)$

Repeat $\int_0^b e^x dx \geq \int_0^b (1+x)dx$

$e^b - 1 \quad b + b^2/2 \Big|_0^b \Rightarrow e^b \geq b + \frac{b^2}{2} + 1 \; (b \geq 0)$

$e^b \simeq 1 + f'(0) b + \frac{f''(0)}{2} b^2$

$= 1 + b + \frac{1}{2} b^2$

(liner approach)

chage of variables (=substitutson)

$$\int_{u_1}^{u_2} g(u)du = \int_{x_1}^{x_2} g(u(x))\cdot dx \quad u=u(x) \quad du=u'(x)dx$$

$$u_1=u(x_1), \quad u_2=u(x_2$$

only work is u' does not chage sign

Example

$$\int_1^2 (x^3+2)^5 x^2 dx$$

$\Rightarrow u = x^3+\cancel{8}2,$ $\quad (1,2) \Rightarrow \cancel{(4,11)} \ (3,10$

$du=3x^2dx \Rightarrow \int_3^{10} u^5 \frac{1}{3} du = \frac{1}{18} u^6 \Big|_3^{10}$

$= \frac{(10^6-3^6)}{18}$

Warning

$$\int_{-1}^{1} x^2 dx \neq \int_1^1 u\,du = 0$$

$u=x^2, \ du=2xdx, \ u_1=(-1)^2, \ u_2=1^2=1$

$$u=x^2, \ u'=2x \begin{cases} >0, & x>0 \\ <0, & x<0 \end{cases}$$