Lec3

~~Devi~~

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

Derivative Formulas

Derivative Formulas

specific $f(x)$ $(f(x) = x^n, \frac{1}{x})$

general $(u+x)' = u' + x'$

$(cu)' = cu'$ c: constant

$\frac{d}{dx}\sin x$ $\frac{d}{dx}\cos x$

$$= \frac{\sin(x+\Delta x) - \sin x}{\Delta x} = \frac{\sin x \cos \Delta x + \cos x \cdot \sin \Delta x - \sin x}{\Delta x}$$

$$= \sin x \left(\frac{\cos \Delta x - 1}{\Delta x}\right) + \cancel{\sin \Delta x} \cdot \cancel{\cos x} \cdot \cos x \cdot \frac{\sin \Delta x}{\Delta x}$$

A B

$\Delta x \to 0 = \sin x \cdot 0 + \cos x \cdot 1$

$= \cos x$

$\frac{d}{dx}\sin x = \cos x$ specific famula $\frac{d\sin x}{dx} = \cos x$

$$\frac{\cos(x+\Delta x) - \cos x}{\Delta x} = \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x}$$

$$= \cos x \left(\frac{\cos \Delta x - 1}{\Delta x}\right) - \sin x \left(\frac{\sin \Delta x}{\Delta x}\right)$$

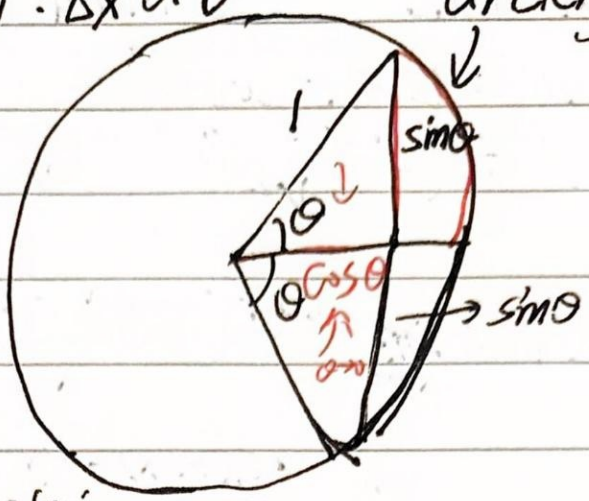
$= -\sin x$

$\frac{d}{dx}\cos x = -\sin x$ speelfic formula

(B) Why $\frac{\sin x}{x} = 1$ x in radians (在弧度中)

geometric proof: $\Delta x \sim \theta$

θ替换为arclength好处便看清楚在哪向



无限近 θ与sinθ相近(equal) 同价

bow string

$\frac{2\sin\theta}{2\theta}$ ← bow (arclength θ)

$= \frac{\sin\theta}{\theta}, \theta \to 0 = 1$?

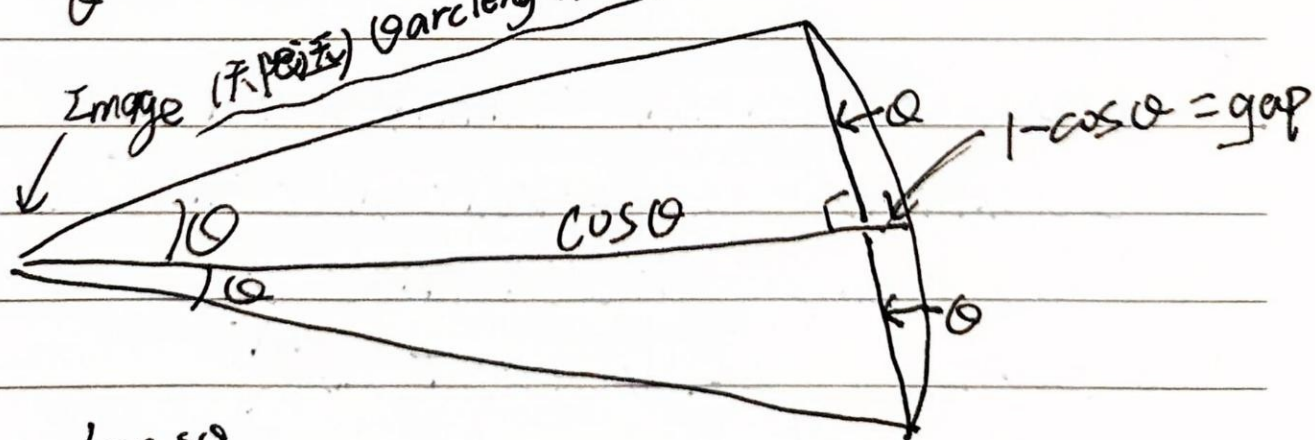
$\sin\theta = 0$

(合为一点)

(A) Why $\frac{\cos x - 1}{x} = 0$ x in radians (在弧度中)

无限近, θ比cosx-1大很多, θ高价

$\frac{1-\cos\theta}{\theta}$



$\frac{1-\cos\theta}{\theta}, \theta \to 0 = 0$

(高阶比低阶)

~~Proof~~

General rule | Formula

$$(uv)' = u'v + u \cdot v'$$

$$(u/v)' = \left(\frac{u'v - uv'}{v^2}\right) \quad v \neq 0$$

$$(u+v)'(x) = \lim_{\Delta x \to 0} \frac{(u+v)(x+\Delta x) - (u+v)(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x)}{\Delta x} + \lim_{\Delta x \to 0} \frac{v(x+\Delta x) - v(x)}{\Delta x}$$

$$= u' + v'$$