$\therefore D \cdot x^n = 0$

lee5.

implicit differentiation

$m-1 \to m + \frac{m}{n} \quad \frac{m}{n} - 1$

before: $\frac{d}{dx}x^a = ax^{a-1}$

Today: $a = m/n$, m & n: integers

Example:

$y = x^{m/n}$, $y^n = x^m$

$$\frac{d}{dx}y^n = \frac{d}{dx}x^m$$

$$\left(\frac{dy^n}{dx}\right)\cdot\frac{dy}{dx} = mx^{m-1}$$

$$n \cdot y^{n-1}\frac{dy}{dx} = m \cdot x^{m-1}$$

$$\frac{dy}{dx} = \frac{m \cdot x^{m-1}}{n \cdot y^{n-1}}$$

$$\frac{dy}{dx} = \frac{m}{n}\frac{x^{m-1}}{\left((x)^{\frac{m}{n}}\right)^{n-1}}$$

$$= \frac{m}{n} \cdot x^{\frac{m}{n}-1}$$

$$= a \cdot x^{a-1}$$

Ex2: $x^2+y^2=1$ $\quad y^2=1-x^2,\ y=\pm\sqrt{1-x^2}$

chain rule (positive branch)

① $y' = \left[(1-x^2)^{\frac{1}{2}}\right]' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$

$\cancel{= 2(1-x^2)^{\frac{1}{2}}}$ $\quad = \dfrac{-2x}{2\sqrt{1-x^2}} = -\dfrac{x}{\sqrt{1-x^2}}$

explicit method (显式方法)

② $\boxed{\begin{array}{l}\text{implicit (隐式)}\\ x^2+y^2=1\\ D\,x^2+y^2 = D1,\quad 2x+2y\cdot y'=0\end{array}}$

$y' = -\dfrac{x}{y} = -\dfrac{x}{\pm\sqrt{1-x^2}}$

find

~~Ⓓ~~ Inverse Function

$y=f(x),\quad g(y)=x \quad g(f(x))=x,$

$y=f^{-1} \quad f=g^{-1}$

$\cancel{f(x)}\ f^{-1}(x)$

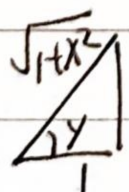
example: $y=\tan^{-1}x$, 求 $y' \Rightarrow (\tan y)'=(x)'$

$\dfrac{d}{dy}\tan y \Rightarrow \dfrac{\sin y}{\cos y} = \dfrac{\cos y\cdot\cos y+\sin y\sin y}{\cos^2 y} = \dfrac{1}{\cos^2 y} = \sec^2 y$

$\dfrac{d}{dy}\tan^{-1}x = ?\quad D(\tan y = x) \to \dfrac{1}{\cos^2 y}\, y' = 1$

$\downarrow$ $\quad y' = \cos^2 y = \dfrac{1}{1+x^2}$

$= \dfrac{1}{1+x^2}$



$x\ (\tan y = x)$

$y' = \cos^2(\tan^{-1}x)$

correct but two complicated

$\cos y = \dfrac{1}{\sqrt{1+x^2}}$

$\therefore \cos^2 y = \dfrac{1}{x^2+1}$

$\therefore y' = \dfrac{1}{1+x^2}$, $\quad \therefore \dfrac{d}{dy}\tan^{-1}x = \dfrac{1}{1+x^2}$

example2. $y = \sin^{-1}x$

$\sin y = x \rightarrow \cos y \cdot y' = 1 \quad y' = \frac{1}{\cos y}$

~~$y' = \frac{1}{\cos(\sin^{-1}x)}$~~

$y' = \frac{1}{\sqrt{1-\sin^2 y}}$

$= \frac{1}{\sqrt{1-x^2}}$

myself:

$y = \sin^{-1}x \qquad \sin y = x \qquad \cos y \cdot y' = 1$

$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$