$$(u+v)'(x) = \lim_{\Delta x \to 0} \frac{(u+v)(x+\Delta x) - \ldots}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{u(x+\Delta x) - u(x)}{\Delta x} + \lim_{\Delta x \to 0} \frac{v(x+\Delta x) - v(x)}{\Delta x}$$

$$= u' + v'$$

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lec 4

$$\boxed{\begin{aligned}(cu)' &= c u' \\ (c+u)' &= c' + u'\end{aligned}}$$

$\frac{d}{dt}(cu) = c\frac{du}{dt}$ c: contem

$\frac{d}{dx}(c+u) = \frac{dc}{dx} + \frac{dc}{du}$ c. func

$c, +, \div, \times, \circ$; & Higher derivatives

$$\boxed{(cu)' = c'u + cu'}$$

Proof

$$\Delta(uv) = u(x+\Delta x)\cdot v(x+\Delta x) - u(x)v(x)$$

$$= [u(x+\Delta x) - u(x)]\cdot v(x+\Delta x) + u(x)[v(x+\Delta x) - v(x)]$$

$$= (\Delta u)\cdot v(x+\Delta x) + u(x)(\Delta v)$$

$$\therefore \frac{\Delta(uv)}{\Delta x} = \frac{\Delta u}{\Delta x}\cdot v(x+\Delta) + u(x)\frac{\Delta v}{\Delta x}$$

$$\Delta x \to 0 \downarrow \quad \frac{d(uv)}{dx} = \frac{du}{dx}\cdot v(x) + u(x)\cdot\frac{dv}{dx}$$

My proof:

$$\Delta(uv) = u(x+\Delta x)\cdot v(x+\Delta x) - u(x)\cdot v(x)$$

$$= [u(x+\Delta x) - u(x)]\cdot v(x+\Delta x) + u(x)\cdot v(x+\Delta x) - u(x)\cdot v(x)$$

$$= [u(x+\Delta x) - u(x)]\cdot v(x+\Delta x) + u(x)[v(x+\Delta x) - u(x)]$$

$$= \Delta u \cdot v(x+\Delta x) + u(x)\cdot \Delta v$$

$\Delta x \to 0$

$$\frac{\Delta(uv)}{\Delta x} = \frac{duv}{dx} = \frac{du}{dx}\cdot \underset{\downarrow\ \text{contsn}}{v(x+\Delta x)} + \frac{u(x)}{\Delta x}\,\Delta v$$

$$= \frac{du}{dx}\cdot v(x) + u(x)\cdot \frac{dv}{dx}$$

$$= u'\cdot v(x) + u(x)\cdot v'$$

$$\therefore (uv)' = u'v + u\cdot v'$$

quotient rule

$$\boxed{\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}}$$

$$\Delta\left(\frac{u}{v}\right) = \frac{u+\Delta u}{v+\Delta v} - \frac{u}{v} = \frac{v\cdot(u+\Delta u) - u\cdot(v+\Delta v)}{(v+\Delta v)\cdot v}$$

$$= \frac{v\cdot\Delta u - u\cdot\Delta v}{(v+\Delta v)\cdot v} \qquad \frac{\Delta\left(\frac{u}{v}\right)}{\Delta x} = \frac{v\cdot\frac{\Delta u}{\Delta x} - u\cdot\frac{\Delta v}{\Delta x}}{(u+\Delta v)\cdot v}$$

$$\Delta x \to 0 \qquad = \frac{v\cdot\frac{du}{dx} - u\cdot\frac{dv}{dx}}{v\cdot v}$$

$v\cdot v \to$ (∵ v is ~~contin~~ continuous)

∴ $\Delta v \to 0$

$$= \frac{v\cdot u' - u\cdot v'}{v^2}$$

$$\therefore \boxed{\frac{d}{dx}\left(\underset{\downarrow\ x^{-n}}{\frac{1}{x^n}}\right) = \cancel{-x^{2n}}\ \frac{-1\cdot n x^{n-1}}{x^{2n}} = -\frac{n}{x^{n+1}} = -n\cdot x^{-n-1}}$$

Composition rule

$y = (\sin t)^{10} = \sin^{10} t$

method: use new variable name

as $x = \sin(t)$, $y = x^{10}$ $\quad y' = 10x^{9}$ $\quad x' = \cos(t)$

$y'(t) = 10\sin^{9}(t) \cdot \cos(t)$

proof: $\frac{\Delta y}{\Delta t} = \frac{\Delta y}{\Delta x} \cdot \frac{\Delta x}{\Delta t}$

$\downarrow \Delta t \to 0$

$$\boxed{\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}}$$ formula

chain Rule

Higher Derivative

$u = u(x) \quad u' = u'(x) \quad u'' = (u')'$

Ex $(\sin x)'' = \cos' x = -\sin(x)$

operate, applied to func

Other notation: $u' = \frac{du}{dx} = \left(\frac{d}{dx}\right) u = Du$ $\quad \boxed{D = \frac{d}{dx}}$

$u'' = \frac{d}{dx}\frac{du}{dx} = \left(\frac{d}{dx}\right)^2 u = D^2 u$

$= \frac{d^2 u}{dx^2}$

$\hookrightarrow$ Not $d(x^2)$

Just different notation to same thing

ex: $u''' = \frac{d^3 u}{dx^3} = D^3 u$

Example: $D^n x^n = ?$ $n = 1, 2, 3 \cdots$

$D x^n = (n \cdot x^{n-1})$

$D^2 x^n = n \cdot (n-1) \cdot x^{n-2}$

$D^3 x^n = n(n-1)(n-2) \cdot x^{n-3}$

$\vdots$

$D^{n-1} x^n = (n(n-1) \cdots\cdots 2) x^1$

$D^n x^n = \underbrace{[n \cdot (n-1) \cdots 1] \cdot 1}_{\hookrightarrow n!} = n!$

$\therefore D^n x^n = n!$, costant!

$\therefore D^{n+1} \cdot x^n = 0$