Lec 10 2024-12-19

$T' = T(1-v^2/c^2)^{\frac{1}{2}} \approx T(1+\frac{1}{2}\frac{v^2}{c^2})$

$\cancel{\frac{\Delta T}{T}} =$ $\cancel{1+rx}$ $(1+x)^r \approx 1+rx$

QUADRATIC APPROX

use this when liner approx is not engouth

$$\boxed{f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2 \quad x \approx 0}$$

why $\frac{1}{2}f''(0)$

$f(x) = ax^2+bx+c \quad f'(x) = 2ax+b \quad f''(x) = 2a$

$f(0) = c \quad f'(0) = b \quad \frac{1}{2}f''(0) = a$

Formula

$$\begin{cases} \sin x \approx x \;; \quad \cos x \approx 1-\frac{1}{2}x^2 \\ e^x \approx 1+x+\frac{1}{2}x^2 \\ \ln(1+x) \approx x-\frac{1}{2}x^2 \\ (1+x)^r \approx 1+rx+\frac{r(r-1)}{2}x^2 \end{cases}$$

x near 0

$\ln(1+x) \approx 0 + 1\cdot x + \frac{-1}{2}x^2$

$= x - \frac{1}{2}x^2$

$(1+x)^r \approx \cancel{r+} 1+rx+\frac{r(r-1)}{2}x^2$

$f'(x)$		$f''(x)$	
$\frac{1}{x+1}$	(1)	$-\frac{1}{(1+x)^2}$	(-1)
$r(1+x)^{r-1}$		$r(r-1)(1+x)^{r-2}$	
(r)		$r(r-1)$	

CURVE SKETCHING

GOAL: Draw graph f ~~@~~ using f', f'',
positive/ negative

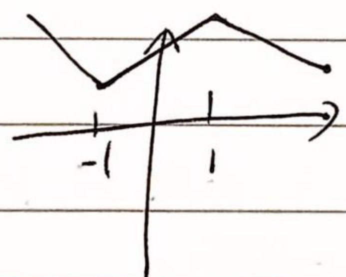
WARING: Don't abandon your common sense

$f' > 0 \Rightarrow f$ is increasing
$f'' > 0 \Rightarrow f'$ is increasing

Ex1

$f(x) = 3x - x^3$ $\quad$ $f'(x) = 3 - 3x^2 = 3(1-x)(1+x)$

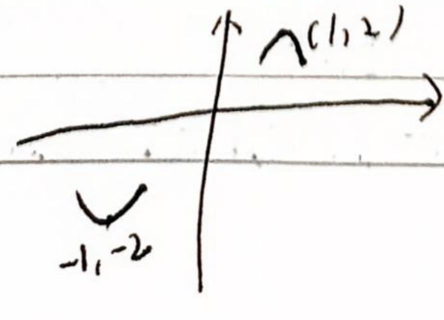
$-1 < x < 1 \Rightarrow f'(x) > 0$, other $f'(x) < 0$



Defin If $f'(x_0) = 0$, we call x_0 a critical point
$y_0 = f(x_0)$ is called a critical value

Ex1: $f'(x) = 0 \Rightarrow (1-x)(1+x) = 0$, $x = 1, -1$

$f(1) = 3 \cdot 1 - 1^3 = 2$ $\quad$ $f(-1) = -2$

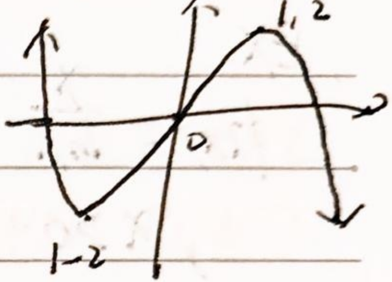


odd: $f(0) = 0$

Ends $x \to \pm\infty$

$f(x) = 3x - x^3$, $x \to -\infty$, $f(x) \to +\infty$

$x \to \infty$, $f(x) \to -\infty$



$f''(x) = -6x$,

$f''(x) < 0$, $x > 0$

$f''(x) > 0$, $x < 0$