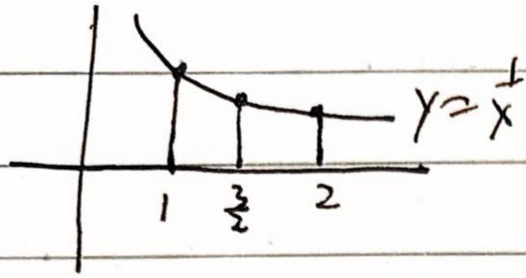
LEC25 2024.12.28

two intervals



trapezoidal rule:

$\Delta x(\frac{1}{2}y_0 + y_1 + \frac{1}{2}y_2)$ $b=2, a=1, \Delta x = \frac{b-a}{n} = \frac{1}{2}$

$= \frac{1}{2}(\frac{1}{2}\cdot 1 + \frac{2}{3} + \frac{1}{2}\cdot\frac{1}{2}) \approx 0.96$

$\cancel{= \frac{1}{4}(}$

Simpson's rule:

$\frac{\Delta x}{3}(y_0 + 4y_1 + y_2) = \frac{1}{6}(1 + 4\cdot\frac{2}{3} + \frac{1}{2}) \approx 0.69444$

$\boxed{\text{Simpson's - Exactans} \approx (\Delta x)^4}$

(助记)

~~Men~~ Mnemonic device:

check: $f(x)=1$

$\Delta x\cdot(\frac{1}{2} + n-1 + \frac{1}{2}) = \Delta x\cdot n$

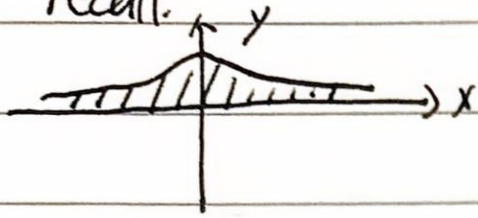
$\Delta x = \frac{b-a}{n}$ $= b-a$

$\int_a^b 1\,dx = b-a$

Date.

Amazing!

recall:



$y = e^{-t^2}$

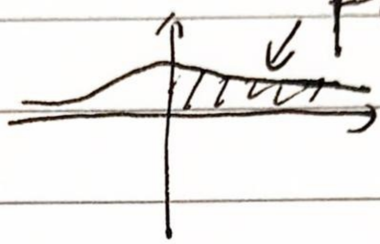
$Q = \int_{-\infty}^{\infty} e^{-t^2}\,dt$

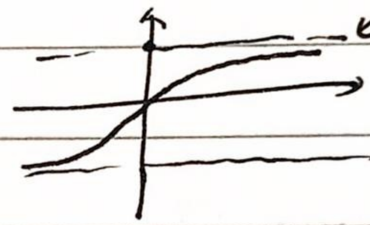
~~$V = Q^2$~~ (in lec24) $V = \pi$ $\boxed{Q^2 = V}$, $Q = \sqrt{\pi}$ (next)

compute V by slices:

$F(x) = \int_0^x e^{-t^2}\,dt$

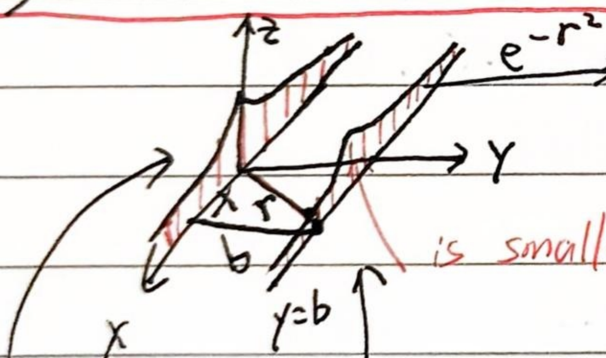
$F(\infty) = \int_0^\infty e^{-t^2}\,dt$





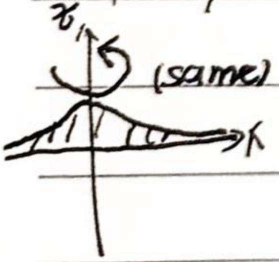
$\therefore Q = 2F(\infty)$, $F(\infty) = \frac{\sqrt{\pi}}{2}$

why $Q^2 = V$: (use slices)



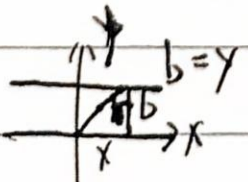
$\to A(b)$, area of slice $y=b$,

is smaller than which $y=0$



the formula $V = \int_{-\infty}^{\infty} A(y)\cdot dy$

the way: fix $y=b$, calculate $A(b)$

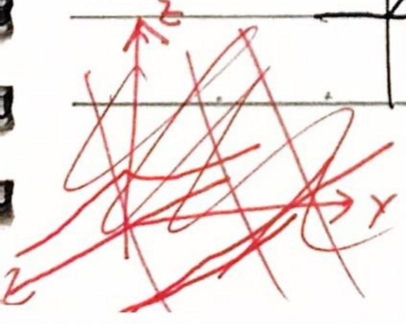


x is virable

$r^2 = x^2 + b^2$, hight $= e^{-(b^2+x^2)}$

$= e^{-b^2} \cdot e^{-x^2}$

why, tend r back x



$\therefore$ hight $= e^{-r^2}$
$r^2 = x^2 + b^2$

A(b) area under | hight $= e^{-b^2} \cdot e^{-x^2}$

$= c \cdot e^{-x^2}$

$$A(b) = \int_{-\infty}^{\infty} e^{-b^2} \cdot e^{-x^2} \cdot dx$$

$$= e^{-b^2} \int_{-\infty}^{\infty} e^{-x^2} dx$$

$$= e^{-b^2} \cdot Q$$

$\boxed{b \to 0,\ A(b) = Q \quad b \to \infty,\ A(b) = 0}$

$\boxed{Q = \int_{-\infty}^{+\infty} e^{-t^2} \cdot dt}$

the area of e^{-t^2} under

$$V = \int_{-\infty}^{\infty} A(y) \cdot dy = \int_{-\infty}^{\infty} e^{-y^2} \cdot Q \cdot dy$$

$$= Q \cdot \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$= Q \cdot Q = Q^2$$

~~V 基本就是 A(y)~~ V's slice = A(y)

Exam Question

1. Calculate Definite Integrals
   (VIA FTC 1 & SUBSTITUTION)
2. NUMERICAL APPROX:
   - RIEMANN SUM
   - TRAPEZOADAL RULE
   - SIMPSON'S RULE
3. AREAS / VOLUMES
4. Other cumulative sums (AVG)
5. SKETCH $f(x) = \int_a^x f(t)\, dt$

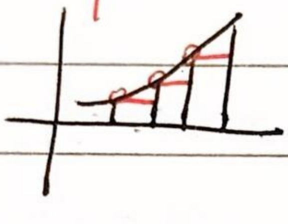
REVIEW

Riemann Sum $y=\frac{1}{x}$

in this case lower sun = right-hand sum
upper sun = left-hand Sum

decreasing

so if func is ~~in~~ increasing, the lower sun = left-hand Sum

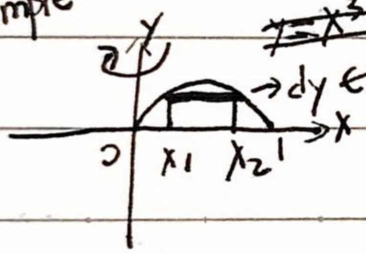


upper sun = right-hand Sum

AREAS / VOLS → ① 2D

of revolation ② dx or dy? $d\cancel{x}: \int_0^1 2\pi x(x-x^3)dx = V$

example:



~~$y=x^3-x$~~ $y=x-x^3$

use washer (圆) $(\pi x_2^2-\pi x_1^2)$

$V=\int \pi\cdot(x_2^2-x_1^2)\cdot dy$

x_1 & x_2, solve $x-x^3=y$ → correct but too diffcult

don't use this