LEC 11 2024-12-20 graph

general strutgray of the sketch

1. Plot

a) discontinuities (especially in finite)

b) end points ($x \to \pm\infty$)

c). easy points (optional)

2.

a) solve the ~~f(x)~~ $f'(x)=0$

b) Plot crit pts and value

3.

decide whether $f' \lessgtr 0$ on each interval Between critical points discontinuities

4.

$f' \begin{matrix} >0 \\ <0 \end{matrix}$ concave up/down) (\ ) $f''(x)=0$ (inflection point)

5. combine.

EX

$f(x) = \frac{x}{\ln x}$

, $x > 0$

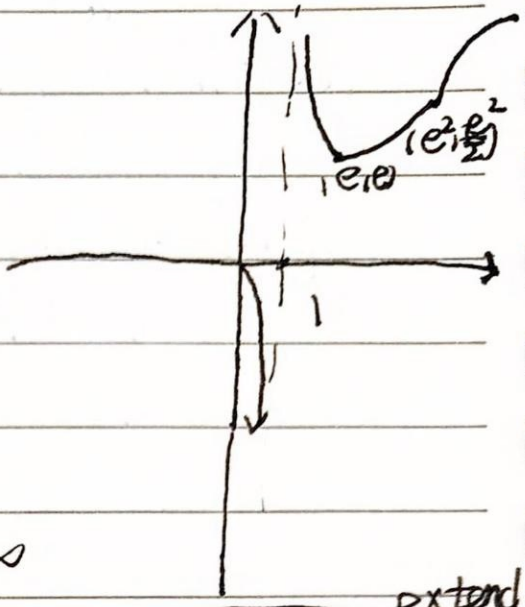
1 a) $f(1^+) = \frac{1}{\ln 1^+} = \frac{1}{0^+} = \infty$

$f(1^-) = \frac{1}{\ln 1^-} = \frac{1}{0^-} = -\infty$

b) ends.

$f(0^+) = \frac{0^+}{\ln 0^+} = \frac{0^+}{-\infty} = 0^-$

$f(x \to \infty) = \frac{x}{\ln x} = \frac{e^x}{x} = \infty$



2.

$f'(x) = \frac{\ln x - x \cdot \frac{1}{x}}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$

as $f'(x) = 0 \Rightarrow x = e$

critical value $f(e) = \frac{e}{\ln e} = e$ (e, e)

extend

$\frac{1}{\ln x} - \frac{1}{\ln^2 x} = 0$

$f'(0^+) \simeq \frac{1}{x^+} - \frac{2}{x^2}$ $(-\infty)^2 (\infty)$ ∞ $-\infty$

$\simeq \frac{1}{x} - \frac{2}{x^2} \simeq 0$

Double check

(3) f is decreasing on $0 < x < 1$, $1 < x < e$

increasing on $e < x < \infty$

$f'(x) = \frac{\ln x - 1}{(\ln x)^2} = \frac{1}{\ln x} - \frac{1}{(\ln x)^2}$

$\frac{-}{+} < 0$, $0 < x < 1$

$\frac{-}{+} < 0$, $1 < x < e$

$\frac{+}{+} > 0$, $e < x$

4, $f''(x) = \frac{2 - \ln x}{x (\ln x)^3}$

$\frac{+}{+} < 0$, concave down $0 < x < 1$

$\frac{+}{+} > 0$, concave up $1 < x < e^2$

$\frac{-}{+} < 0$, concave down $e^2 < x$

Date.

max / min problems

key to find only need to look at CRITICAL POINTS AND END POINTS AND POINTS OF DISCONTINUITY