LEC36 2025.1.3

Unit 5. DEALING WITH ∞

If $\begin{cases} f(x) \to \infty \\ g(x) \to \infty \\ f'(x)/g'(x) \to L \end{cases}$ as $x \to a$ THEN $f(x)/g(x) \to L$ $(x \to a)$

$a = \pm\infty$, $L = \pm\infty$, ok

$f(x) \ll g(x)$ means the $\frac{f(x)}{g(x)} \to 0$, $x \to \infty$

$(x \to \infty)$, $(f, g > 0)$

~~$\ln x \gg x^p$~~ $\ln x \ll x^p \ll e^x \ll e^{x^2}$ $\to \infty$

RATE OF DECAY $\frac{1}{\ln x} \gg \frac{1}{x^p} \gg e^{-x} \gg e^{-x^2}$, $p > 0$

IMProper Integrals 广义积分

DEFN: $\int_a^\infty f(x)dx = \lim_{N \to \infty} \int_a^N f(x) dx$

converges or diverges (收敛/发散)

The integral converges if limit Exist → area is finite

diverges if limit not exist → area is infinite

Ex1. $\int_0^\infty e^{-kx} \cdot dx$, $\boxed{k > 0}$

$$\int_0^N e^{-kx} dx = -\frac{1}{k} e^{-kx} \Big|_0^N$$

$$= -\frac{1}{k} \cdot e^{-Nx} + \frac{1}{k}$$

$$= \frac{1}{k}(1 - e^{-Nx})$$

As $N \to \infty$, $= \frac{1}{k}$ $\therefore \int_0^\infty e^{-kx} dx = \frac{1}{k}$

Ex1

the number of the decay in a radiotive substance in time $0 \le t \le T$

$$\int_0^T A \cdot e^{-kt} \cdot dt$$

Ex2.

$$\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$$

Ex3 $\int_1^\infty \frac{dx}{x}$

$= \ln x \Big|_1^\infty = \ln\infty \to \infty$

$\therefore$ diverges

$$\int_1^\infty \frac{dx}{x^p} = \frac{x^{-p+1}}{-p+1}\Big|_1^\infty$$

$$= \frac{(\infty)^{-p+1}}{-p+1} - \frac{1}{-p+1}$$

$p<1$: $\infty \to \infty$

$p>1$: $0 \to \frac{1}{p-1}$

$\therefore \int_1^\infty \frac{dx}{x^p}$, $p \le 1$ diverges

$p>1$ converges $= \frac{1}{p-1}$

LIMIT COMPARISON

IF $f(x) \sim g(x)$ as $x \to \infty$, $\frac{f(x)}{g(x)} \to 1$ $(x \to \infty)$,

THEN $\int_a^\infty f(x)dx$ and $\int_a^\infty g(x)dx$

either both converge or both diverge

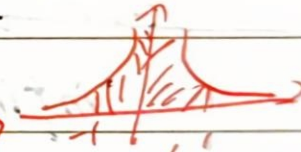
Ex: $\int_1^\infty \frac{dx}{\sqrt{x^2+10}} \sim \int_1^\infty \frac{dx}{x}$ $\therefore$ diverges

$\sqrt{x^2+10} \sim \sqrt{x^2} = x$

Ex: $\int_0^\infty \frac{dx}{\sqrt{x^3+3}} \sim \int_1^\infty \frac{dx}{x^{\frac{3}{2}}}$, converges

Ex:

$\int_{-1}^{1} \frac{1}{x^2}\,\cancel{dx} = -x^{-1}\cancel{\Big|_{-1}^{1}} = \cancel{-2}$



wrong , because $(-1,1)$, $\frac{1}{x^2}$ is diverge