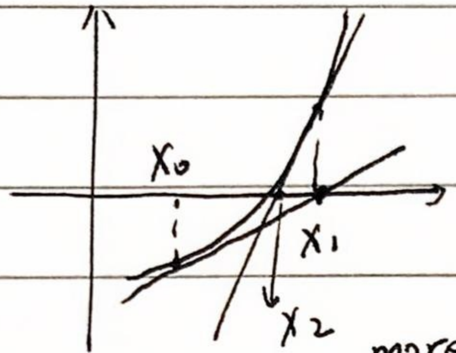
LEC 14 2024.12.21

NEWTON'S METHOD CONTINUE



more and more approch to $(X, 0)$

$X_1 \approx f(0) \approx X_0 - \frac{f(X_0)}{f'(X_0)}$

$X_1 = X_0 - \frac{f(X_0)}{f'(X_0)}$ repeat

$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$ $X_3 = X_2 - \frac{f(X_2)}{f'(X_2)}$

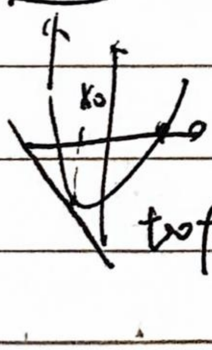
ERROR ANALYSIS

E_1

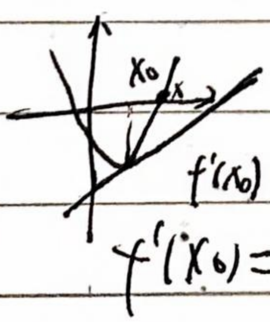
E_2

~~$f'(x)$~~ $|f'|$ not too small ~~$|f''|$~~ $|f''|$ not too big, X_0 is near

By X

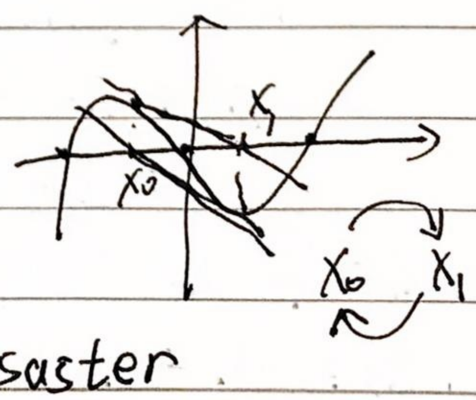


too far



$f'(X_0)$ too small

$f'(X_0) = 0$ is a disaster



MEAN VALUE ~~THEREON~~ THEOREM (MVT)

(中值定理)

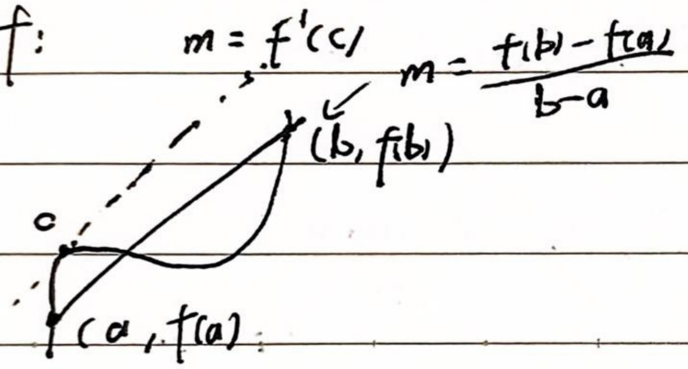
(*)

$$\frac{f(b)-f(a)}{b-a} = f'(c) \text{ for some } c,\ a<c<b$$

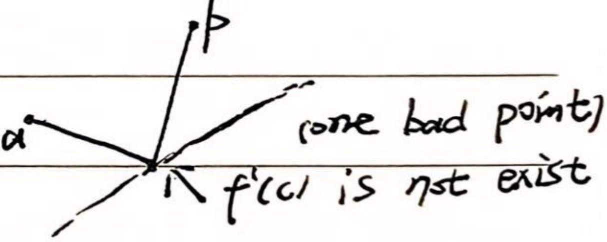
provided f is diff'able in $a<x<b$

and is ~~continute~~ continuous in $a \le x \le b$

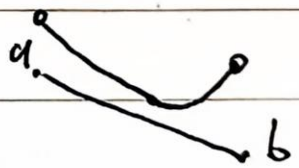
Poof:



why f is differentable :



why $a \le x \le b$ is continuous:



this situation

Applications to graphy

1. if $f(x)>0$, then f is increasing
2. if $f(x)<0$, then f is decreasing
3. If $f'=0$, the f is constant

because of the MVT

KROOF:

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

$$f(b)-f(a) = f'(c)\cdot(b-a)$$

$$\boxed{f(b) = f(a) + f'(c)\cdot(b-a)}$$

$a<b$

$(b-a)>0$

1. ~~f'(x)~~ $f'(c)>0 \Rightarrow f(b)>f(a)$
2. $f'(c)<0 \Rightarrow f(b)<f(a)$
3. $f'(c)=0 \Rightarrow b(b)=f(a)$

$$\boxed{\min f' \leq \frac{f(b)-f(a)}{b-a} = f'(c) \leq \max f'}$$

on $a \leq x \leq b$

MVT

Ex, $e^x \geq x+1$ on $x \geq 0$

~~$f' = e^x$~~

as $f(x) = e^x - (x+1)$

$f'(x) = e^x - 1 \geq 0,\ x \geq 0$, $f(0)=0$

$\therefore f(x) \geq 0,\ e^x-(x+1) \geq 0,\ e^x \geq x+1$ on $x \geq 0$