2024.12.23

LEC 16 (微分方程)

DIE DIFFERENTIAL EQUATION

Ex1

$\frac{dy}{dx} = f(x)$ $\quad y = \int f(x)\cdot dx$

Ex2

$\left(\frac{d}{dx} + x\right) y = 0$

annihilation operator

$\frac{dy}{dx} = -xy$

$\downarrow$

$\int \frac{dy}{y} = \int -x\,dx$

$\ln y = -\frac{1}{2}x^2 + C$

$y = e^{-\frac{1}{2}x^2 + C}$

$= A\cdot e^{-\frac{1}{2}x^2}$, $A \in \mathbb{R}$

$\frac{dy}{dx} = f(x)\cdot g(y) \Rightarrow \frac{dy}{g(y)} = f(x)\cdot dx$

$H(y) = \int \frac{dy}{g(y)}$; $F(x) = \int f(x)\cdot dx$

$H(y) = F(x) + C$, C is constant

↑ implicit $\quad y = H^{-1}(F(x) + C)$

Remake

Ex $\ln|y| = -\frac{x^2}{2} + C \quad (y \neq 0)$

$|y| = A\cdot e^{-\frac{x^2}{2}}$

$y = \pm A\cdot e^{-\frac{x^2}{2}} \quad (A > 0)$

leaves out $y = 0$

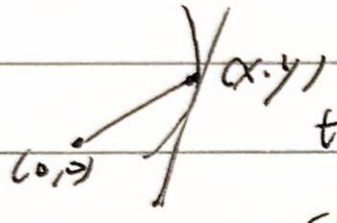
Ex1 → continue separate of vars

$$\frac{dy}{dx} = f(x)$$

$$dy = f(x)\cdot dx$$

$$y = \int dy = \int f(x)\cdot dx = \frac{1}{2}f^2(x)$$

Ex 3,



the slope of the tagent line = twice slope of ray from origin

$$\rightarrow \frac{dy}{dx} = 2\frac{y}{x} \qquad \int \frac{dy}{y} = \int \frac{dx}{x}\cdot 2, \quad x, y \neq 0$$

$$\ln|y| = 2\ln x + C$$

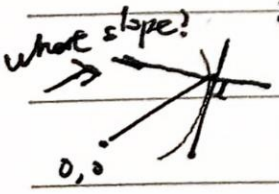
~~$y = Ax^2 + B$~~ X

$$e^{\ln y} = e^{2\ln x + C}$$

$$y = x^2 \cdot e^C = A\cdot x^2$$

Warning: $\frac{dy}{dx} = \frac{2y}{x}$ ← undifined at $x = 0$

Ex4. the (tp线的垂线)



$$\frac{dy}{dx} = \frac{-1}{\text{the slope of the tagent line}}$$

associate Ex3

$$\frac{dy}{dx} = \frac{-1}{2(y/x)} = -\frac{x}{2y}$$

$$\rightarrow \int 2y\cdot dy = \int -x\cdot dx$$

$$y^2 = -\frac{x^2}{2} + C$$

$$\frac{x^2}{2} + y^2 = C$$

$$= y^2$$

at $y=0$, slope is not exist

Unit 2 EXAM

1. LINEAR / Quadratic Approx
2. sketch A graph
3. Max / Min
4. RELATED RATES
5. ANTI DERIVATIVE + SOLVE
6. MVT