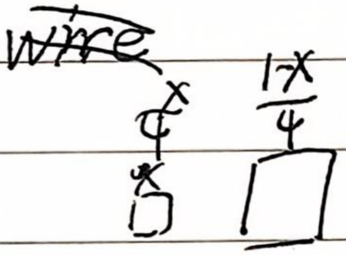
OF DISCONTINUITY

LEC 12   2024.12.20 $\cancel{f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2}$

MAX / MIN



$A = \left(\frac{x}{4}\right)^2 + \left(\frac{1-x}{4}\right)^2$

$A' = \frac{x}{8} - \frac{(1-x)}{8} = 0$, $x = 1-x$

$x = \frac{1}{2}$

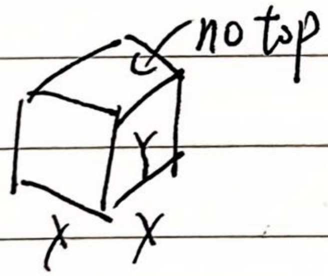
$A\left(\frac{1}{2}\right) = \frac{1}{32}$

Ends: $0 < x < 1$, $A(0^+) = 0 + \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$A(1^-) = \left(\frac{1}{4}\right)^2 + 0 = \frac{1}{16}$

min point $\left(\frac{1}{2}, \frac{1}{32}\right)$

Ex2 ~~consid~~ find the box without a top



$V = x^2 y$

$A = x^2 + 4xy$

$y = \frac{V}{x^2}$

$A = x^2 + 4x \cdot \left(\frac{V}{x^2}\right)$

$= x^2 + \frac{4V}{x}$

$A' = 2x - \frac{4V}{x^2} = 0 \qquad 2x^3 = 4V$

$x^3 = 2V$

$x = \sqrt[3]{2} \cdot \sqrt[3]{V}$

$= 2^{\frac{1}{3}} \cdot V^{\frac{1}{3}}$

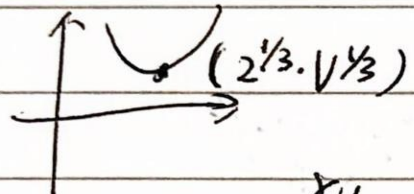
$\boxed{\text{V is fixed}}$

Ends: $0 < x < \infty$

$y = \frac{V}{x^2} > 0$

$A(0^+) = x^2 + \frac{4V}{x} \Big|_{x=0^+} = \infty$

$A(\infty) = x^2 + \frac{4V}{x} \Big|_{x=\infty} = \infty$

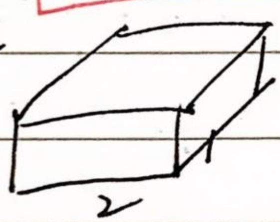


double check $A'' = 2 + \frac{8V}{x^3} > 0$, concave up ✓

$x = 2^{\frac{1}{3}} V^{\frac{1}{3}}, \quad y = 2^{-\frac{2}{3}} \cdot V^{\frac{1}{3}}$

$\boxed{x/y = 2}$ the best answer to problem

shape



~~tagg~~ lagest box (MAX V)

EX2 by implicit diff

$V = x^2 y, \quad A = x^2 + 4xy$

$\frac{d}{dx}(V = x^2 y) \qquad 0 = 2x \cdot y + x^2 \cdot y'$

$y' = -2xy/x^2 = -\frac{2y}{x}$

$\frac{d}{dx} A \qquad 0 = 2x + 4y + 4xy'$

$\cancel{= 2x}$

$2x + 4y - 8y = 0$, $2x = 4y$, $\boxed{x/y = 2}$ Faster

implicit diff

disadvantage: did not check whether this critical point is max min or neither