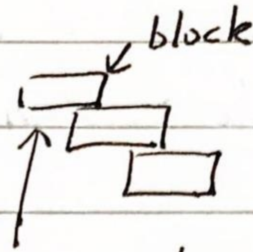
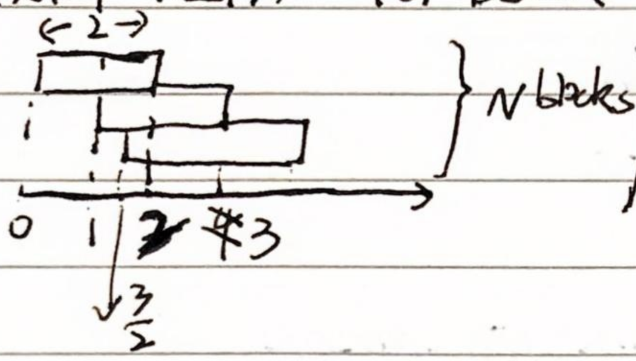
2025.1.3 LEC 38.



START WITH TOP BLOCK



does this fall down

C_{N+1} ← center of mass of N+1 blocks

x-coordinate

$C_{N+1} = \cancel{NC_N+1(C_N+1)}\quad \frac{NC_N+1(C_N+1)}{N+1} = \frac{(N+1)C_N+1}{N+1}$

$= C_N + \frac{1}{N+1}$

just set up → $= \underbrace{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{N}}_{S_N} + \frac{1}{N+1}$

$\ln N < S_N < \ln N + 1$ ∴ ~~I con~~ ∴ S_N is diverge

To get arcoss the table, need 26 units, 26-2=24

so it's need $\boxed{\ln N = 24}$ N: blocks

$N = e^{24}$

POWER SERIES

proof of: $1+x+x^2+x^3+\cdots = \frac{1}{1-x}$, $|x|<1$

$1+x+x^2+x^3\cdots = S$

$-\quad x+x^2+x^3\cdots = S\cdot x$

$1 = S(1-x)$

∴ $S = \frac{1}{1-x}$

$x\cdot(1+x+x^2+\cdots = S)\,x$

$-\quad x+x^2+x^3+\cdots = \cancel{S\cdot x}$

$1 = S - Sx = S(1-x)$

∴ $S = \frac{1}{1-x}$

→ continue

but it requires S is exist (not infinit)

just when S is converge

General Power Series

$a_0 + a_1x + a_2x^2 + \cdots$

$= \sum_{n=0}^{\infty} a_n x^n$, ~~A~~

$|x| < R$ (radius of converge)

$-R < x < R$, where the series converge

$|x| > R$, $\sum_{n=0}^{\infty} a_n x^n$ is diverge

and $|x| = R$ not used by us.

$g(x)$, $f(x)$ $\qquad \frac{d}{dx} f(x)$, $\int f(x)dx$

- $\frac{d}{dx}(a_0 + a_1x + a_2x^2 + a_3x^3 \cdots)$

$= 0 + a_1 + 2a_2x + 3a_3x^2 + \cdots$

$\int (a_0 + a_1x + a_2x^2 + \underbrace{(\cdots)}_{\text{series}}) = C + a_0x + \frac{1}{2} \cdot a_1x^2 + \frac{1}{3}a_2x^3 + \cdots$

what it is

continue

Taylor's formula

$$f(x) = \sum_{i=0}^{\infty} \boxed{\frac{f^{(n)}(0)}{n!}} \cdot x^n$$

$\downarrow a_n$

$f(x) = a_0 + a_1 x + a_2 x^2 \cdots$

$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 \cdots$

$f''(x) = 2a_2 + 6a_3 x \cdots$

$f'''(x) = \cancel{\ldots} + 3 \cdot 4 a \cdots$

$\to 2 \cdot 3 a_3 \cdots$

$f'''(0) = 3 \cdot 2 a_3$

$\frac{f'''(0)}{3 \cdot 2 \cdot 1} = a_3$ $\quad n!$

$\to f^n(0) \cdot x^n$

$$\boxed{\text{In general} \quad a_n = \frac{f^{(n)}(0)}{n!}}$$

($n!$ approx)

$\boxed{0! = 1}$ ← use this

Example: $\quad \frac{f^{(n)}(0)}{n!} \cdot x^n$

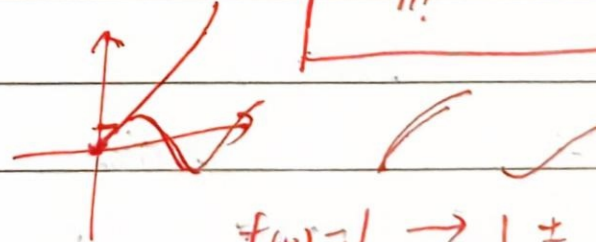
$$e^x = e^0 \cdot x^0 + \frac{e^0}{1!} \cdot x^1 + \frac{e^0}{2!} \cdot x^2 + \frac{e^0}{3!} \cdot x^3 \cdots$$

$$= 1 + x + \frac{1}{2} x^2 + \frac{1}{3!} \cdot x^3 + \frac{1}{4!} x^4 + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$\boxed{\frac{f^n(x)}{n!} \cdot x_n}$$



$f(0) = 1 \to 1 \pm$

$f(0) = 0 \to x \pm$