lec 20. 2024. 12.25

FTC1 If $F' = f$ then

$\underline{\int_a^b f(x)dx = F(b) - F(a)}$

used to evaluate integrals

Today $F(b) - F(a) = \int_a^b f(x)dx$

use f to understand F

$\quad \| F'$

Information About F' => ABOUT F

Compare FTC1 with MVT

$\Delta F = F(b) - F(a)$, $\Delta x = b - a$

$\Delta F = \int_a^b f(x) \cdot dx$ (FTC1)

$\boxed{\frac{\Delta F}{\Delta x} = \frac{1}{b-a}\int_a^b f(x)dx}$ average(f)

$\downarrow$ FTC1

$(\min F')\Delta x \le \Delta F = \text{Ave}(F') \cdot \Delta x \le (\max F') \cdot \Delta x$

$\underline{\min}(F')\Delta x \le \Delta F = F'(c) \cdot \Delta x \le (\max F') \cdot \Delta x$

$\int$ over (a,b) $\uparrow$ MVT $\rightarrow$ vague, some c , $a < c < b$

that FTC1 is more complex than MVT, we can drop MVT now.

Ex: $F'(x) = \frac{1}{1+x}$, $F(0) = 1$

$A < F(4) < B$, what A and B

ΔY $\qquad\qquad$ ΔX

① MVT

$$F(4)-F(0) = F'(c)\cdot(4-0)$$

$$= \frac{1}{1+c}\cdot 4$$

$0 \le c \le 4$ $\quad$ rang $\frac{1}{1+0}\cdot 4,\ \frac{1}{1+4}\cdot 4$ $\qquad$ conclusion.

$$\frac{4}{5} < F(4)-F(0) < 4$$

$$\therefore \frac{9}{5} < F(4) < 5$$

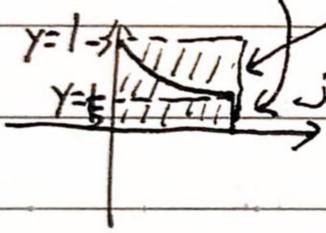
② FTC1

$$F(4)-F(0) = \int_0^4 F'(x)\,dx$$

$$= \int_0^4 \frac{1}{1+x}\,dx = \ln(1+x)\Big|_0^4 = \ln(5)$$

$$\int_0^4 \frac{1}{5}\,dx < \quad < \int_0^4 dx \Rightarrow \frac{4}{5} < F(4)-F(0) < 4$$

$$\frac{9}{5} \to \frac{4}{5}+1 < F(4) < 5$$



just upper and lower of the Riemann Sum

FTC2

If f is continuous, and $G(x) = \int_a^x f(t)\,dt;\ a \le t \le x$

Then $G'(x) = f(x)$ $\qquad$ don't mix up t and x

$G(x)$ solves the differential equation $\begin{cases} y' = f \\ y(a) = 0 \end{cases}$

Example.

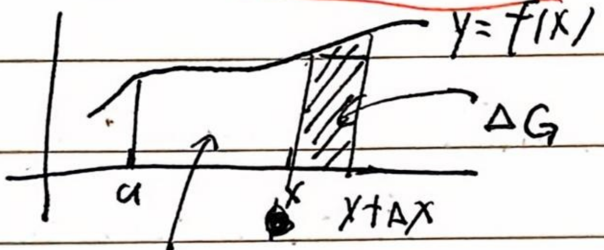
$$\frac{d}{dx}\left[\int_1^x \frac{dt}{t^2}\right] = \frac{1}{x^2}$$

getback $\quad$ ↳ $G(x)$ $\quad$ $G'(x) = f(x)$, $f(t) = \frac{1}{t^2}$, $f(x) = \frac{1}{x^2}$

$$-\frac{1}{t}\Big|_1^x = \left(-\frac{1}{x}+1\right)'$$

$$= \frac{1}{x^2}$$

PROOF OF FTC 2



$$G(x) = \int_a^x f(t)\,dt = \int_a^x f(t)\,dt$$

$$\Delta G \approx \underbrace{\Delta x}_{\text{base}} \cdot h \approx \Delta x \cdot f(x)$$

$$\lim_{\Delta x \to 0} \leftarrow \frac{\Delta G}{\Delta x} = f(x)$$

$$G'(x) = f(x)$$

PROOF OF FTC 1

start $F' = f$, assume f is continuous

Define $G(x) = \int_a^x f(t)\,dt$

FTC2: $G'(x) = f(x)$ $\therefore F' = G'(x) \overset{\text{MVT}}{\Rightarrow} F(x) = G(x) + C$

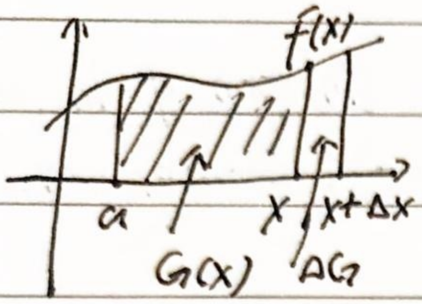
$\therefore F(b) - F(a) = G(b) + C - (G(a) + C)$

$$= G(b) - G(a) = \int_a^b f(x)\,dx - \underbrace{\int_a^a f(x)\,dx}_{0}$$

same as t $\therefore = \int_a^b f(x)\,dx$

My PROOF

FTC2. If $G(x) = \int_a^x f(t)\,dt$, $G'(x) = f(x)$



$\lim_{\Delta x \to 0} \to \Delta G = \Delta x \cdot f(x)$

$\therefore \lim_{\Delta x \to 0} \frac{\Delta G}{\Delta x} = f(x) \quad \therefore G'(x) = f(x)$

FTC1

$F' = f$ Define $G(x) = \int_a^x f(t)\,dt$

from FTC2 $G'(x) = f(x)$

$\therefore G'(x) = F'(x)$ ~~+C~~ $, G(x) = F(x) + C$

~~G(b) - G(a) = F(b)~~ $\therefore F(x) = G(x) + C$

$\therefore F(b) - F(a) = G(b) - G(a) = \int_a^b f(t)\,dt - \int_a^a f(t)\,dt$

$= \int_a^b f(t)\,dt = \int_a^b f(x)\,dx$

↑ t as x, is same

$y' = e^{-x^2}, \quad y(0) = 0$

$F(x) = \int_0^x e^{-t^2}\,dt$

$\boxed{\begin{array}{l} G(x) = \int_0^{g(x)} f(x)\,dx \\ G'(x) = f(g(x)) \cdot g'(x) \\ \text{if } g(x) = x \\ G'(x) = f(x) \end{array}}$