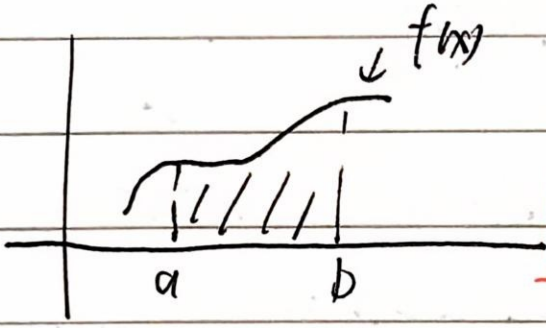
LEC 18 2024.12.25

UNIT 3 INTRO TO INTEGRATION

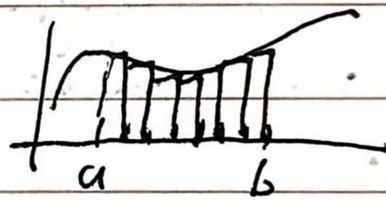
TODAY DEFINITE INTEGRALS



FIND AREA UNDER A CURVE $= \int_a^b f(x)dx$

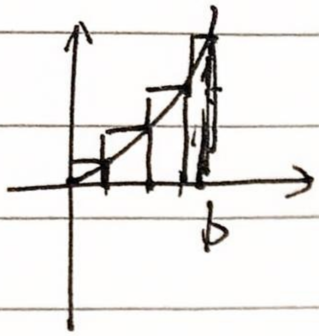
To compute this area:

1. divide into "rectangles" (矩形)
2. add up areas
3. taking a limit as rectangles get thin



Date.

Example 1. $f(x)=x^2$; $a=0$ b arbitrary (可变)



base length: b/n (all equal intervals)

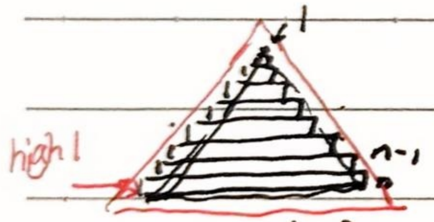
x	$f(x)$
b/n	$(b/n)^2$
$2b/n$	$(2b/n)^2$
$3b/n$	$(3b/n)^2$

Total areas of □'s

$$\left(\frac{b}{n}\right)\cdot(b/n)^2+\left(\frac{b}{n}\right)\left(\frac{2b}{n}\right)^2 \cdots \left(\frac{b}{n}\right)\left(\frac{nb}{n}\right)^2 = \sum_{i=1}^{n}\frac{b}{n}\left(\frac{ib}{n}\right)^2 = \left(\frac{b}{n}\right)^3\sum_{i=1}^{n} i^2$$

$$=\left(\frac{b}{n}\right)^3(1^2+2^2+3^2+4^2+\cdots+n^2)$$

$$=b^3\,\frac{1^2+2^2+\cdots+n^2}{n^3}$$



$$V=\frac{1}{3}(\text{base})\cdot(\text{ht})=\frac{1}{3}n^2\cdot n$$

$$\frac{1}{3}n^3<1^2+2^2+\cdots+n^2<\frac{1}{3}(n+1)^2(n+1)$$

$$\frac{1}{3}<\frac{1+2^2+\cdots+n^2}{n^3}<\frac{1}{3}\left(1+\frac{1}{n}\right)^3$$

$$\downarrow n\to\infty \to \frac{1}{3}$$

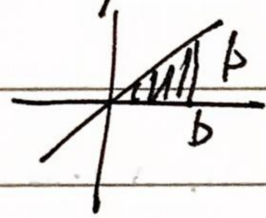
$$\therefore \frac{1+2^2+\cdots+n^2}{n^3}\to\frac{1}{3}\quad(n\to\infty)$$

$$\boxed{\int_a^b x^2\,dx=\frac{1}{3}b^3}\quad (a=0)$$

$$\boxed{\sum_{i=1}^{n} a_i=a_1+a_2+\cdots+a_n \quad \Sigma\ \text{sigma}}$$

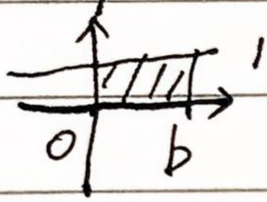
$$\frac{1}{n^3}\sum_{i=1}^{n} i^2\to\frac{1}{3},\ n\to\infty$$

Example 2 $f(x)=x$



Area $= \frac{1}{2} b^2$

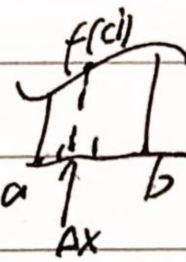
Example $f(x)=1$



Area $= b \cdot 1 = b$

Notation (Rieman Sums) (li: man ji he)

$$\sum_{i=1}^{n} \underbrace{f(c_i)}_{ht} \cdot \underbrace{\Delta x}_{base}$$



$(\Delta x \to 0)$ $\int_a^b f(x)\,dx$