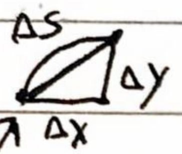
LEC 31 ~~2024~~ 2025.1.1

ARCLENGTH "s"



$\Delta s^2 \approx \Delta x^2 + \Delta y^2$

$ds^2 \approx dx^2 + dy^2$

$\boxed{ds = \sqrt{dx^2+dy^2}}$ $\boxed{ds = \sqrt{1+(dy/dx)^2} \cdot dx}$

$= \sqrt{1+(dy/dx)^2} \cdot dx$

$\therefore$ ARCLENGTH $= \int_a^b \sqrt{1+(dy/dx)^2} \cdot dx = \int_{s_0}^{s_1} ds$

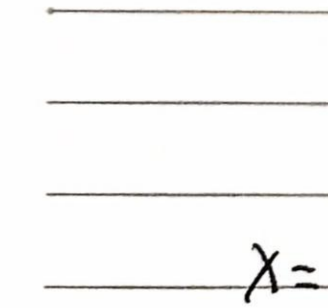
$= \int_a^b \sqrt{1+f'(x)^2} \cdot dx$

Ex1. $y=mx$

$y'=m$ $ds = \sqrt{1+(y')^2} \cdot dx = \sqrt{1+m^2}\, dx$

$[0,10]$ length $\int_0^{10} \sqrt{1+m^2}\, dx = 10\sqrt{1+m^2}$

Ex2. $y=\sqrt{1-x^2}$



My: $y' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot -2x$

$= \frac{-x}{\sqrt{1-x^2}}$ $ds = \sqrt{1+(y')^2} \cdot dx$

$\therefore = \int_0^a \sqrt{1+\frac{x^2}{1-x^2}} \cdot dx$

$= \int_0^a \sqrt{\frac{1}{1-x^2}}\, dx$

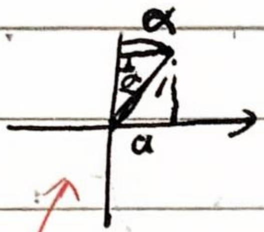
$= \int_0^a (1-x^2)^{-\frac{1}{2}}\, dx$

$x = \cancel{\cos\theta}\ \sin\theta$ $\cancel{dx = -\sin\theta\, d\theta}$ $dx = \cos\theta \cdot d\theta$

$= \int_0^{\sin^{-1} a} \frac{\cos\theta}{\cos\theta}\, d\theta$

$\theta = \sin^{-1} a$ $= \int_0^{\sin^{-1} a} d\theta = \theta \Big|_0^{\sin^{-1} a} = \sin^{-1} a$

$\therefore \sin\alpha = a \longrightarrow$

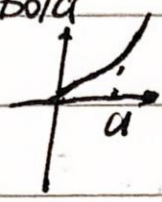


α = radians = 1

$\sin\alpha = a$

the formula of arclengh: $r \cdot \theta$ just $1 \cdot \sin^{-1} a = 1 \cdot \alpha$

EX3. length of ~~半~~ a parabola



$y = x^2, \quad y' = 2x$

$ds = \sqrt{1+4x^2} \cdot dx$

$[0, a]$. arclength $= \int_0^a \sqrt{1+4x^2} \cdot dx$

$\cancel{x = 2\tan\theta}$

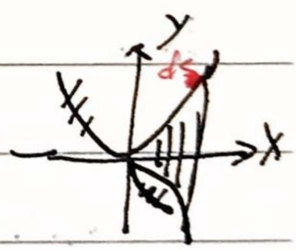
$x = \frac{1}{2}\tan u \qquad dx = \frac{1}{2}\sec^2 u \, du$

$\cdots = \left(\frac{1}{4}\ln(2x + \sqrt{1+4x^2}) + \frac{1}{2}x\sqrt{1+4x^2}\right)\Big|_0^a$

$\left(\frac{1}{4} \middle| \frac{1}{2}\left(\frac{1}{4}+x^2\right) \quad \left|\frac{1}{2}\right|^2 + x^2, \text{ so } x = \frac{1}{2}\tan u \right)$

长 宽

Next: Surface area (length · width)



Example:

$y = x^2$ rotated around the x-axis.

$y' = 2x \qquad dA = (2\pi y)(ds)$

dS ← surface area, also used as dA

$S = \int_a^a 2\pi x^2 \cdot \sqrt{1+4x^2} \cdot dx =$

↑

$x = \frac{1}{2}\tan u$

Ex 2:

the surfacearea of a sphere (立求)

$y=\sqrt{a^2-x^2}$, radius a

$y' = \dfrac{-x}{\sqrt{a^2-x^2}}$

$1+(y')^2 = 1+\dfrac{x^2}{a^2-x^2} = \dfrac{a^2}{a^2-x^2}$

area $\int_{x_1}^{x_2} 2\pi \cdot y\, ds = \int_{x_1}^{x_2} 2\pi\sqrt{a^2-x^2}\cdot\sqrt{\dfrac{a^2}{a^2-x^2}}\cdot dx$

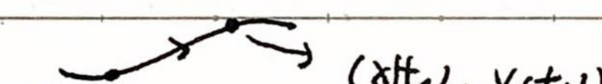
$= \int_{x_1}^{x_2} 2\pi a\, dx$

$= 2\pi a\cdot(x_2-x_1)$

PARAMETRIC CURVES (参数曲线)

$x=x(t)$

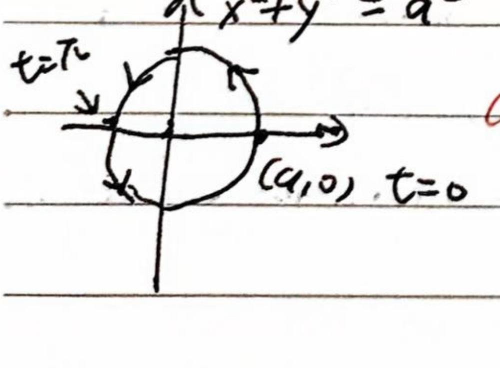
$y=y(t)$



Ex 1:

~~y=acost~~ , $x=a\cos t$, $y=a\sin t$

$x^2+y^2=a^2$ (circle)



counterclockwise (逆时针)