LEC30. 12/31. 2024

STEP0 LONG DIVISION

INTEGRATION BY PARTS (分部積分)

$(uv)' = u'v + u \cdot v'$

$uv' = (uv)' - u'v$ $\quad \therefore \int uv' \cdot dx = uv - \int u'v dx$

$\Downarrow$

$\int_a^b uv'dx = uv|_a^b - \int_a^b u'v dx$

Ex1. $\int \underbrace{\ln x}_{u} \cdot dx$ $\quad u \cdot v'$

$u = \ln x, \quad u' = \frac{1}{x}$

$v = x, \quad v' = 1$

then $u \cdot v|_a^b - \int_a^b u'v dx$

$= x \cdot \ln x - \int \frac{1}{x} \cdot x dx$

$= x \ln x - x + C$

Ex2: $\int (\ln x)^2 dx$

$u = (\ln x)^2 \quad u' = 2(\ln x) \cdot \frac{1}{x}$

$= (\ln x)^2 \cdot x - \int 2(\ln x) \frac{1}{x} \cdot x \, dx$

$= x(\ln(x))^2 - 2\int \ln x \cdot dx$

$= x(\ln x)^2 - \underbrace{2(x \ln x - x)}_{\text{from ex1}} + C$

Ex3 (Reduction Formula)

$$\int (\ln x)^n \cdot dx$$

$$= x\cdot(\ln x)^n - \int n(\ln x)^{n-1}\cdot \frac{1}{x}\cdot x\, dx$$

$$F_n(x) = \int (\ln x)^n dx$$

$$\boxed{F_n(x) = x\cdot(\ln x)^n - n\,F_{n-1}(x)}$$

$$F_0(x) = \int (\ln x)^0 dx = x$$

~~Ex2.~~

Ex1: $F_1(x) = x\cdot\ln x - 1\cdot F_1(x) = x\cdot\ln x - x + C$

Ex2: $F_2(x) = x\cdot(\ln x)^2 - 2\cdot F_1(x) = x\cdot(\ln x)^2 - 2(x\cdot\ln x - x) + C$

Ex4: $\int x^n e^x dx$

$u \quad v'$

$$u' = n\cdot x^{n-1}$$

$$= x^n\cdot e^x - \int n\cdot x^{n-1}\cdot e^x dx$$

$$G_n(x) = \int x^n e^x dx$$

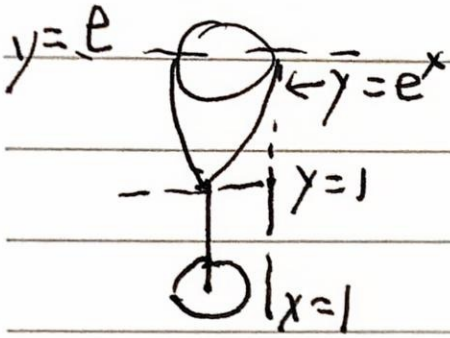
$$\boxed{G_n(x) = x^n e^x - G_{n-1}(x)}$$

$$G_0(x) = e^x ; \quad G_1(x) = x\cdot e^x - e^x \quad (+C)$$

only by experience

Ex (Application)

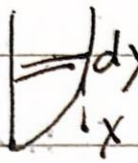
Find the V of an



~~rotted~~

rotated ~~about~~ around y-axis

a) horizontal disk



$\int_1^e \pi x^2 \cdot dy \quad , x = \ln y$

$= \pi \int_1^e (\ln y)^2 \cdot dy$

$u = (\ln y)^2 \quad v = y$

$u' = 2(\ln y) \cdot \frac{1}{y}$

$= \left((\ln y)^2 \cdot y \Big|_1^e - \int_1^e \frac{2\ln y}{y} \cdot y \cdot dy \right) \pi$

$= \left((\ln e)^2 \cdot e - (\ln 1)^2 \cdot 1 \right) \cdot \pi$

$= \left(e - \int_1^e 2 \ln y \, dy \right) \cdot \pi$

$\ln y = t$

$\frac{1}{y} dy = dt$

$2\int_1^e \ln y \, dy = 2\left(\ln y \cdot y \Big|_1^e - \int_1^e \frac{1}{y} \cdot y \, dy \right)$

$= 2 \cdot (e - (e-1))$

$= 2 \cdot 1 = 2$

$= (e-2) \cdot \pi$

b) by shells

$V_{01} = \int_0^1 (e-y) \cdot 2\pi x \, dx$

$= 2\pi \int_0^1 (e - e^x) \cdot x \, dx$

$= 2\pi \left(\int_0^1 e \cdot x \, dx - \int_0^1 e^x \cdot x \, dx \right)$ → $= G_1(x) = e^x(x-1)\Big|_0^1$

$= 2\pi \cdot \frac{e}{2} + 2\pi \cdot e = 2\pi \left(\frac{e}{2} + 1 \right)$

$= +1$

$0 + 1$

↑ the area of triangle $= \pi(e-2)$