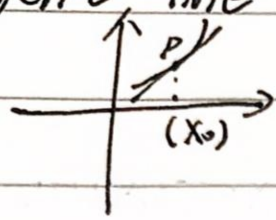
Prof. David Jerison

18.01 Derivatives and intergration

<< session 1 >> $\boxed{\text{<<clec1>>}}$

tangent line

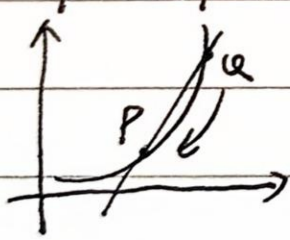


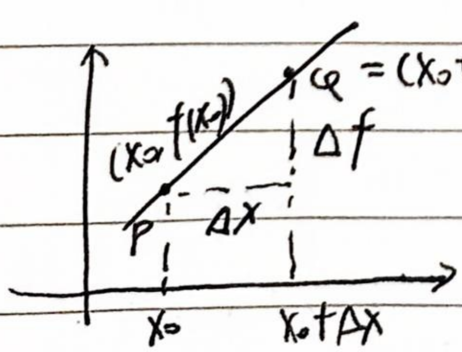
~~y=~~ $y-y_0=m(x-x_0)$

slope $= m = f'(x_0)$ ~~calcus~~ calculus

DEFIN $f'(x)$ the derivative of $f(x_0)$, is the slope of the ~~slope of the~~ tagent line to $y=f(x)$ at ~~th~~ the point, (P)

~~tag~~ tangent line = Limt of secant Line PQ as $Q \to P$ (P fixed)





$Q = (x_0+\Delta x, f(x_0+\Delta x))$

slope $= \frac{\Delta f}{\Delta x}$ of the secant

$m = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$ slope of the tangent

$\cancel{\underset{m}{f'(x_0)} = \lim_{\Delta \to 0}}$

formula m

$$\boxed{f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}}$$

example:

$\frac{1}{x}$

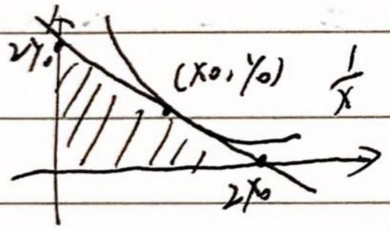
$$\frac{\Delta f}{\Delta x} = \frac{\frac{1}{x_0+\Delta x} - \frac{1}{x_0}}{\Delta x} = \frac{1}{\Delta x}\left(\frac{x_0-(x_0+\Delta x)}{(x_0+\Delta x)x_0}\right)$$

$$= \frac{1}{\Delta x}\left(\frac{-\Delta x}{(x_0+\Delta x)x_0}\right)$$

$$= -\frac{1}{(x_0+\Delta x)\cdot x_0}$$

$$\xrightarrow{\Delta x \to 0} = -\frac{1}{x_0^2}$$

$\frac{1}{x}$ $\therefore$ $\boxed{f'(x_0) = -\frac{1}{x_0^2}}$



area

计算三角形面积

由上知: $m = -\frac{1}{x_0^2}$

$\therefore$ 直线: $y - y_0 = -\frac{1}{x_0^2}(x - x_0)$

令 $y=0$ $\therefore -\frac{1}{x_0} = -\frac{1}{x_0^2}(x - x_0)$

$\therefore \frac{x}{x_0^2} = \frac{2}{x_0}$ $\quad \therefore x = 2x_0$

反转 (x, y) $\therefore y = 2y_0$

$\therefore S = 2x_0 \cdot 2y_0 \cdot \frac{1}{2} = 2x_0 y_0$

$= 2x_0 \cdot \frac{1}{x_0}$

$= 2$

symmetry explain

$y = \frac{1}{x} \Leftrightarrow xy = 1 \Leftrightarrow y = \frac{1}{x}$

(符号)

notations

$$f' = \frac{df}{dx} = \underbrace{\frac{dy}{dx} = \frac{d}{dx}f = \frac{d}{dx}y}_{\text{Leibnize}}$$

$\uparrow$

Newton's

~~Expl~~ Example2:

$f(x) = x^n$, $n=1,2,3\cdots$

$\frac{dx^n}{dx} = ?$

binomia theorem (二项式定理)

because $\Delta x \to 0$

$(x+\Delta x)^n = (x+\Delta x)\cdots(x+\Delta x)$

$= x^n + n\cdot\Delta x\cdot x^{n-1} + \text{junk}$ (垃圾)

$O((\Delta x)^2)$

$\left(\frac{n!}{k!(n-k)!}\right)$

$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$

terms $(\Delta x)^2$, $(\Delta x)^3 \cdots$

$$\frac{d}{dx}x^n = \cancel{\frac{x^n-(x+\Delta x)^n}{\Delta x}} \quad \frac{(x+\Delta x)^n - x^n}{\Delta x}$$

$$= \frac{1}{\Delta x}\left(x^n + n\cdot\Delta x\cdot x^{n-1} + O((\Delta x)^2) - x^n\right)$$

$$= \frac{1}{\Delta x}\left(n\cdot\Delta x\cdot x^{n-1} + O((\Delta x)^2)\right)$$

$$= n\cdot x^{n-1} + O(\Delta x)$$

$\Delta x \to 0$

$= n\cdot x^{n-1}$ ∴ $\boxed{\frac{d}{dx}x^n = n\cdot x^{n-1}}$ formula

Date.

$$\frac{d}{dx} x^{n} = -\frac{n}{x^{n+1}}$$