Mo | Tu | We | Th | Fr | Sa | Su   Date / /

LEC3   Strong Induction   2.28

Good Proofs are

- correct
- complete
- clear
- in order
- brief
- elegant
- well ~~orig~~ organized

good proofs are very like good codes!

Ex: <u>Problem</u>

Find a ~~sequence~~ square of moves to go from

A	B	C
D	E	F ↓
H	G	

to

A	B	C
D	E	F
G	H	

legal moves: slide a letter into adjacent blank ~~sequence~~ square

<u>Thm:</u> There is no sequence of legal moves to invert G & H and return all other ~~&~~ letters to their original position

Memo No.
Date / /

Lemma 1: A row move doesn't change the order of the items. in natural number 1, 2, 3 ... 9

Proof

In a row move, we move an item from i into an adjacent cell $i+1$, nothing else move. Hence the order of items is preserved. check

Column moves:

Lemma 2: A column move changes the relative order of precisely 2 pairs of items

Proof: In a column move, we move an item in cell i to a blank spot in cell ~~i-30, i+30~~ $i+3$, $i-3$

when an item ~~w~~ moves 3 positions, it changes order with 2 items ($i-1$, $i-2$ or $i+1$, $i+2$)

Def A pair of Letters L_1 & L_2 is an inversion (also, inverted pair) if L_1 proceedes L_2 in alphabet. but L_1 is after L_2 in puzzle

Mo | Tu | We | Th | Fr | Sa | Su

Memo No. ______

Date / /

<u>Lemma3:</u> during the move, the # of inversions can only increase by 2, decrease by 2 or stay same

Pf: Row move: no changes (by lemma 1)

~~Row~~ column move: 2 pairs change order (lemma 2)

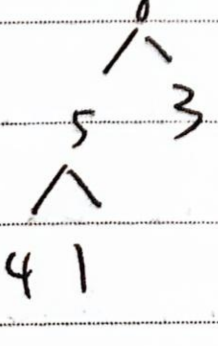
cases

~~A. both pairs in order ⇒ # inverse~~

$\boxed{\text{Strong induction}}$ Axiom

Let $P(n)$ be any predicate, if $P(0)$ is true, & $\forall n$ $(P(0) \wedge P(1) \wedge \cdots \wedge P(n)) \Rightarrow P(n+1)$ is true. then $\forall n\ P(n)$ is true.

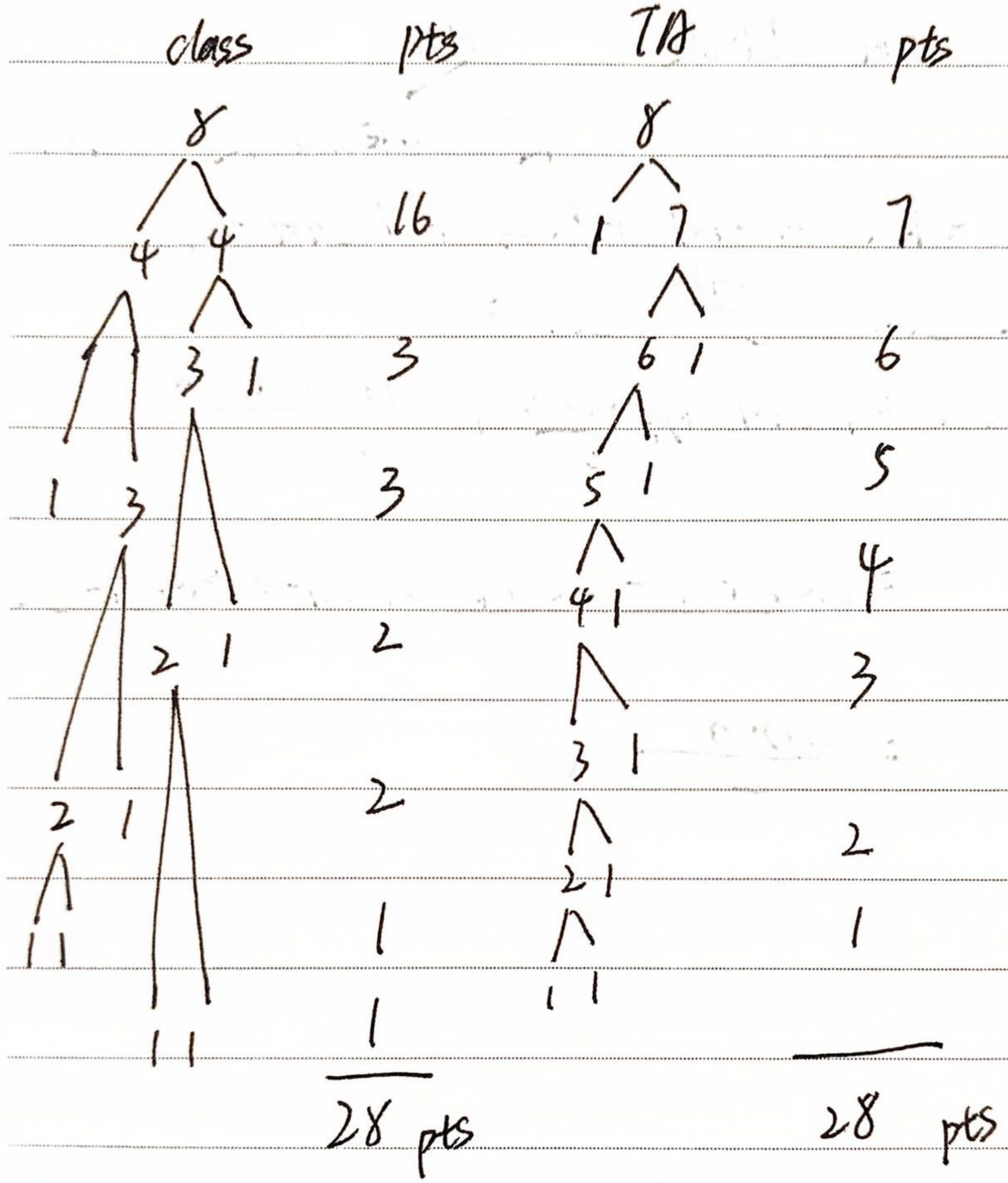
Ex: Unstacking Game



Mo Tu We Th Fr Sa Su

Memo No. ________

Date / /



Thm: All strategies for the n-block games produce the same score $S(n) = \frac{n(n-1)}{2}$

Ex: $S(8) = 28$.

Pf: By strong induction, Base case: $n=1$, $S(1) = 0$ ✓

IH $P(n)$

Inductive step: assume $P(1), P(2) \cdots, P(n)$ to prove

Mo | Tu | We | Th | Fr | Sa | Su

Memo No. ______

Date / /

look at n+1 block

$$n+1 \to k,\ n+1-k \qquad 1 \le k \le n$$

$s(n+1)$

$$\text{score} = k(n+1-k) + P(k) + P(n+1-k)$$

guess $S(n) = \frac{n(n-1)}{2}$, $S(0) = 0$ ✓

$$\Rightarrow P(n+1) = k(n+1-k) + \frac{k(k-1)}{2} + \frac{(n+1-k)(n-k)}{2}$$

$$= \frac{\cancel{2kn} + \cancel{2k} - 2k^2 + k^2 \cancel{-k} + n^2 + n \cancel{-kn} \cancel{-k} \cancel{-kn} + k^2}{2}$$

$$= \frac{(n+1)\cdot n}{2}$$ ✓