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# LEC 2 INDUCTION 2.28.

1. To prove P is true, we assume P is false (i.e. $\neg P$ is T) & then use that ~~hypsthe hepth~~ hypothesis to devise a falsehood or contradiction
IF $\underbrace{\neg P}_{F} \Rightarrow F$ is true.

Ex: Thm: $\sqrt{2}$ is irrational.

Pf (by Contradiction)

assume for purpose of contradiction that $\sqrt{2}$ is rational

$\Rightarrow \sqrt{2} = a/b$ (~~frat~~ fraction in ~~least~~ lowest terms)

$\Rightarrow 2 = a^2/b^2 \Rightarrow 2b^2 = a^2$, so $a^2$ is even, $a$ is even too. $(2 \mid a) \Rightarrow 4 \mid a^2 \Rightarrow 4 \mid 2b^2 \Rightarrow 2 \mid b^2$, so $b$ is even too

(multiple by 4)

so $a/b$ is not in lowest term

$\Rightarrow$ ~~contr~~ contradiction #

$\Rightarrow \sqrt{2}$ is irrational, check.

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$\boxed{\text{INDUCTION}}$ axiom

Let $P(n)$ be a predicate. If $P(0)$ is true and $\forall n \in \mathbb{N}, (P(n) \Rightarrow P(n+1))$ is true then $\forall n \in \mathbb{N}$, $P(n)$ is true.

by notation

If $P(0)$, $P(0) \Rightarrow P(1)$, $P(1) \Rightarrow P(2)$, ---

then $P(0), P(1), P(2)$ --- are true

$\boxed{\text{Ex1:}}$ Thm: $\forall n \geq 0 \quad 1+2+3+\cdots+n = \frac{n(n+1)}{2}$

$$\sum_{i=1}^{n} i = \sum^{1 \leq i \leq n} i = \sum_{1 \leq i \leq n} n$$

If $n=1$ $\quad 1+2+\cdots+n = 1$

If $n \leq 0$ $\quad 1+2+\cdots+n = 0$

$n=4$ $\quad 1+2+3+4 = 10 = \frac{4(4+1)}{2}$

Pf: by induction

Let $P(n)$ be proposition that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Base Case: $P(0)$ is true, $\sum_{i=1}^{0} i = 0 = \frac{0(0+1)}{2}$ ✓

inductive step For $n \geq 0$, show $P(n) \Rightarrow P(n+1)$ is true

Assume $P(n)$ is true for purposes of induction

(i.e., assume $1+2+\cdots+n = \frac{n(n+1)}{2}$)

need to show $1+2+\cdots+(n+1) = \frac{(n+1)(n+2)}{2}$

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$\Rightarrow \underbrace{1+2+\cdots+n}+(n+1)$

$\forall n \geq 0,$ $\frac{n(n+1)}{2}+(n+1) = \frac{n^2+n+2n+2}{2} = \frac{(n+1)(n+2)}{2}$ ✓

$P(n) \Rightarrow P(n+1)$ , check

$\boxed{\text{EX2}}$ Thm: $\forall n \in \mathbb{N},\ 3 \mid (n^3-n)$

PF by induction let $P(n)$ $3 \mid (n^3-n)$

base case: $n=0$ , $3 \mid (0-0)$ ✓

Inductive step: For $n \geq 0$ show $p(n) \Rightarrow p(n+1)$

Assume $P(n)$ is True, i.e. $3 \mid (n^3-n)$

Examine $(n+1)^3-(n+1) = n^3+3n^2+3n+1-(n+1)$

$= n^3+3n^2+2n$

$= n^3-n+3n^2+3n$

$= \underbrace{(n^3-n)+3(n^2+n)}$

$\Rightarrow 3 \mid (n+1)^3-(n+1)$ ✓, multiple by 3

$\Rightarrow P(n) \Rightarrow p(n+1)$ is true, check ✓

Base case $P(b)$ is true , assume $P(n)$ true/false

Inductive step : $\forall n \geq b,\ P(n) \Rightarrow P(n+1)$

Conclude $\forall n \geq b .\ P(n)$

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EX3. Thm (NOT) All hourse are the same color

PF by induction

P(n) I any set of $\overset{n \geq 1}{n}$ horses, the horses are all the same color

Base case: P(1) true since just one horse

Inductive Step: assume P(n) to prove P(n+1)

n+1 horses $\underbrace{H_1, H_2, \cdots,}_{P(n)\text{ are the same color}} H_{n+1}$

$H_2, \cdots H_{n+1}$ also the same color

⇒ since ($H_1$) color = color($H_2 \cdots H_n$) = color($H_{n+1}$)

⇒ all n+1 are same color ⇒ P(n+1). check ✓

Why? What's wrong? n=1, is a empty set

$P(1) \Rightarrow P(2)$ X $P(2) \Rightarrow P(3) \Rightarrow P(4) \Rightarrow \cdots P(n)$ ✓

base case P(2) is false