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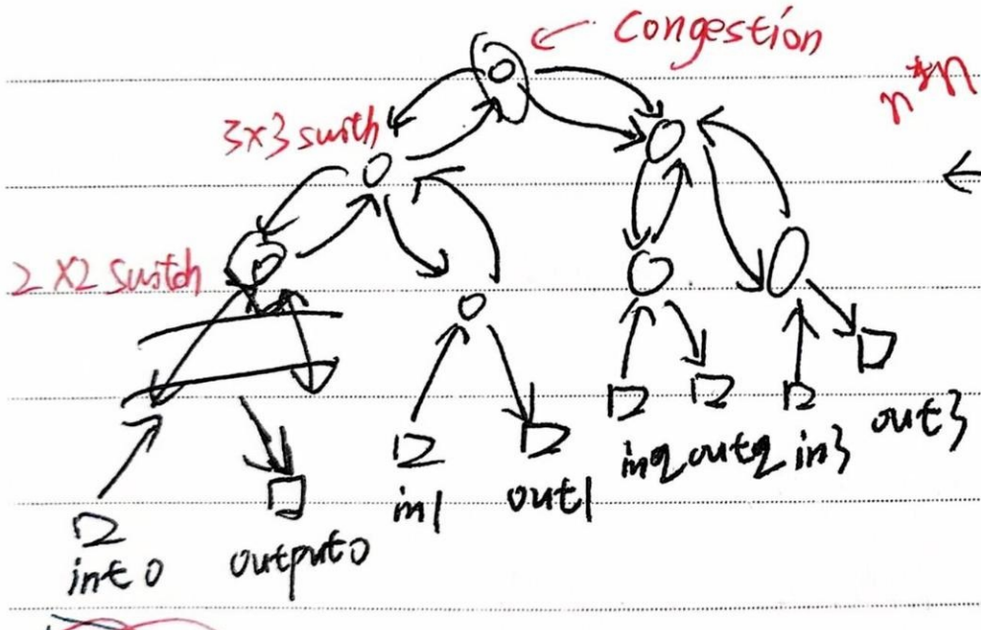
LEC 10 Communication Network 3.6

1. Complete Binary Tree

○ = switch: direct packets through Network

□ = Terminal: ~~soure~~ source & destination of data



n*n

diameter = 6

4*4 network

out in

switch count =

1+2+4+8+...+n = 2n

latency is the time required for a packet to travel an input to an output

diameter of a network is the length of the shortest path between the input and output that are ~~far~~ furthest apart

diameter = 2 · (1 + log N)

↑ the leave to root

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latency, Diameter

(押折)

NxN Network	Diameter	switch size	#Switch	congestion
Binary Tree	2·(1+log N)	3x3	2N-1	N
2D Array	2N	2x2	N^2	2
Butterfly	2+log N	2x2	~~All~~ N(log₂N)	√N or √N/2
~~Banes~~ Benes Network	1+2log N	~~2x2~~ ~~3x3~~	2N log₂N	1

A permutation is a function $\pi : \{0, \cdots n-1\} \rightarrow \{0, \cdots n-1\}$ such that no two numbers are mapped to the same value

$\pi(i) = \pi(j)$ iff $i = j$

↑ just means a map

↓ map from one side to other side

Ex. $\pi(i) = N-1-i$

$\pi(i) = i$

permutation Routing Problem

Def: for each i, we want direct the packet at Input i to Output $\pi(i)$: path

Ex: input 0 → output 0 taken is denoted by $P_{i,\pi(i)}$

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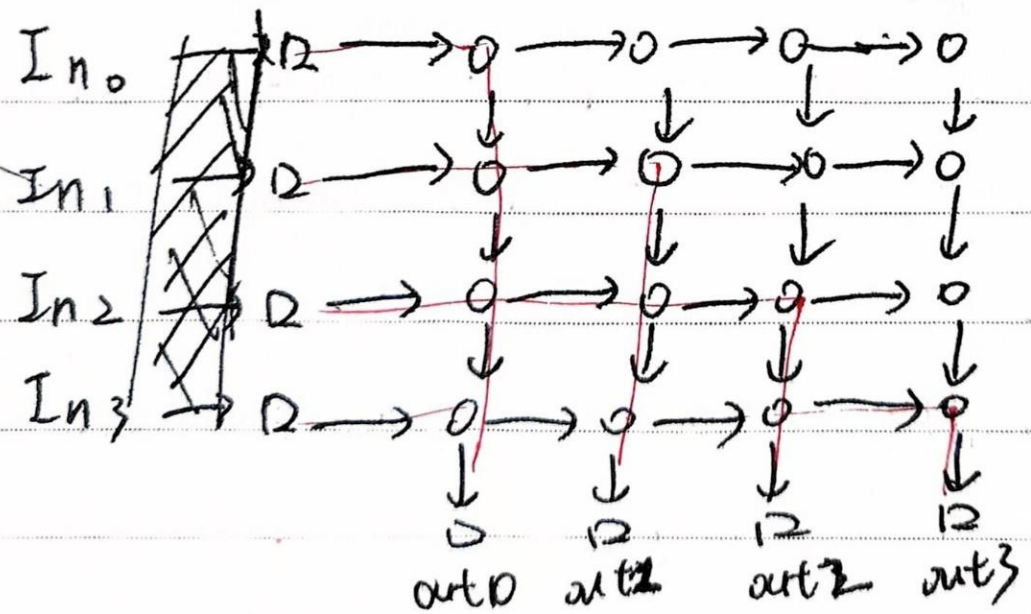
The congestion of paths $P_{0,\pi(0)}, \cdots P_{N-1,\pi(N-1)}$ is equal to the lagest number of path that ~~path~~ pass thorough a single switch

in example : $\pi(i) = N-1-i$ の時, → congestion = 4

the length to not

$$\text{Max congestion} = \max_{\pi} \left\{ \begin{array}{l} \min \text{ congestion } P_{0,\pi(0)}, \cdots P_{N-1,\pi(N-1)} \\ \text{solutions } P_{0,\pi(0)} \cdots P_{n-1,\pi(n-1)} \end{array} \right.$$

2 × 2 Array Network



Theorem Congestion of an N-input array is 2

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Pf let π be a permutation

$P_{i,\pi(i)}$ = path from input i rightward to column $\pi(i)$ and downward to output $\pi(i)$

if switch in row i and column $\pi(i)$ transmits ≤ 2

because the packets only can come from left or up $\Rightarrow$ congestion is ≤ 2

ex: $\pi(0)=0$, $\pi(N-1)=N-1$ $\Rightarrow$ congestion is 2 □

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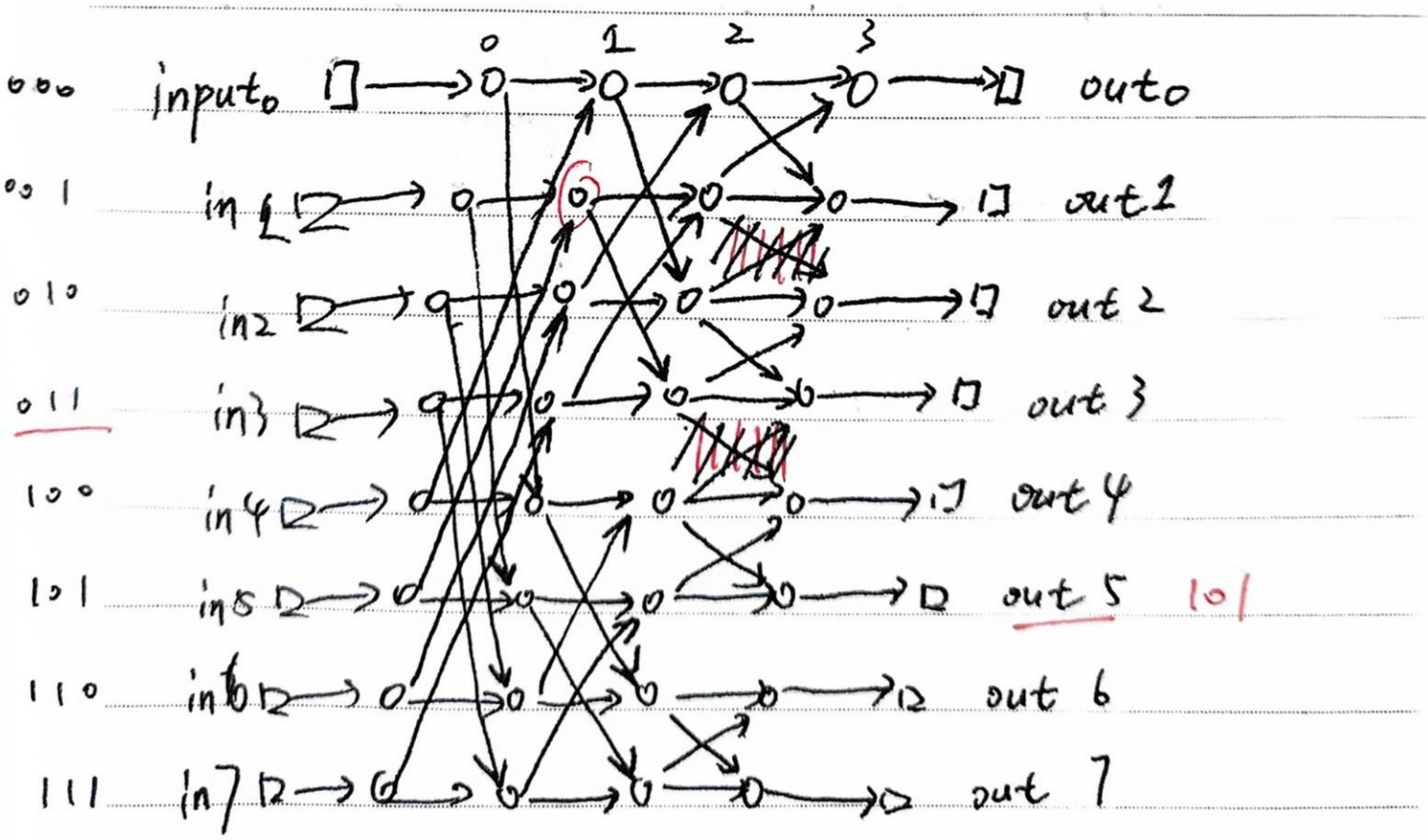
3. ~~Input 0 □~~ Butterfly network



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Switch is uniquely identified by it's row & column

$(b_1, \cdots, b_{\log N}, l)$ → $(b_1, \cdots, b_{l+1}, \cdots, b_{\log N}, l+1)$

bit ↑ level

$(b_1, \cdots, \bar{b}_{l+1}, \cdots, b_{\log N}, l+1)$ the sequence of binary

~~component~~ complement

Ex:

switch $(x_1, \cdots x_{\log N}, 0) \rightarrow (y_1, x_2, x_3 \cdots x_{\log N}, 1)$

$(y_1, y_2, x_3, \cdots x_{\log N}, 2)$

$(y_1, \cdots y_{\log N}, \log N)$

in example N=8 so the binary length is $\log_2 8 = 3$

switch ex (011)

ex: input 011 → output 5 = 101

how to route?

ex: 011 → cross level 1 → cross level 2 → straight level → output 5

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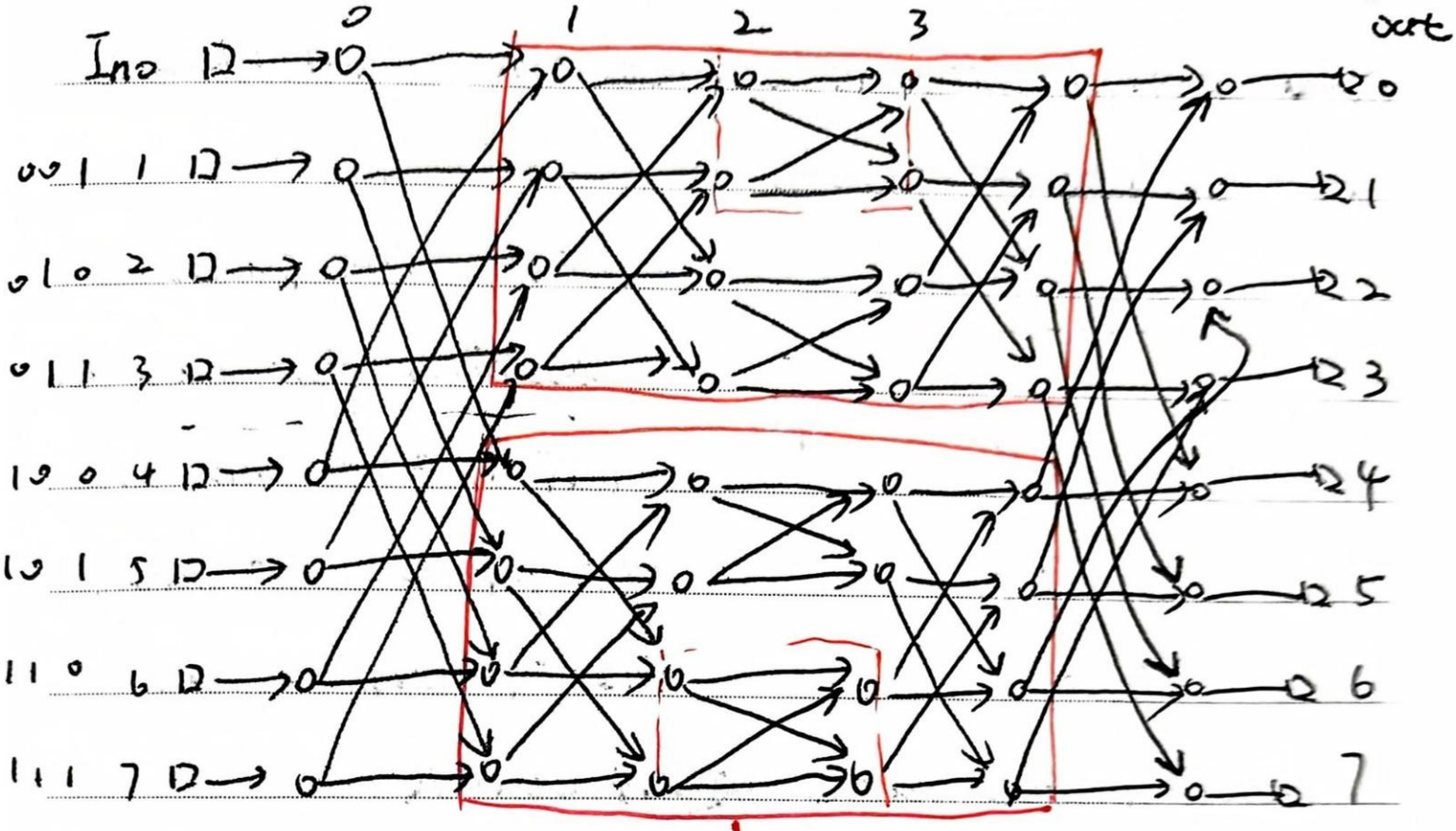
Benes Network



It's Butterfly Network plus self

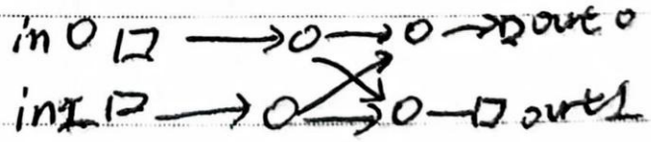
Symmetric!

again a Benis Network

Theorem: The congestion of the N-input Benis Network is actually 1, when $N=2^a$ for some $a \geq 1$

Pf By induction on a: P(a) = "the theorem is true for a"

Basecase $N=2^1$



case: $\pi(0)=0$, $\pi(1)=1$ ✓ , $\pi(0)=1$, $\pi(1)=0$ ✓

inductive Step! Assume P(a) is true

Ex: $\pi(0)=1$, $\pi(1)=5$, $\pi(2)=4$, $\pi(3)=7$, $\pi(4)=3$, $\pi(5)=6$, $\pi(6)=0$, $\pi(7)=2$

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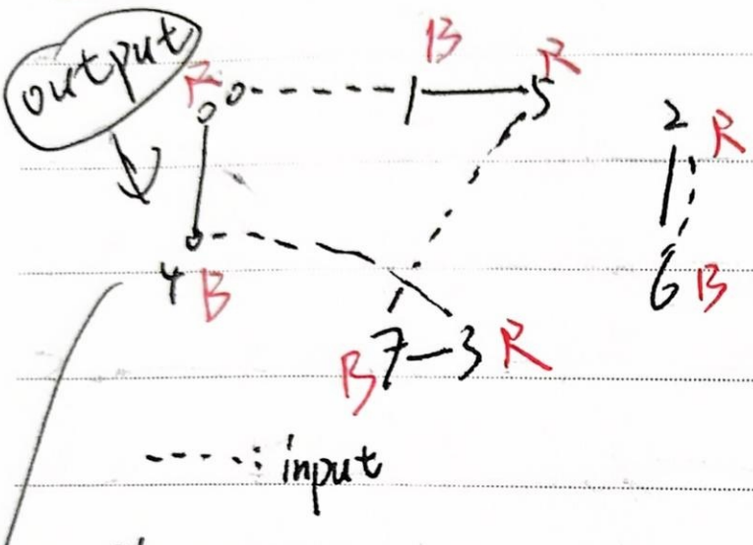
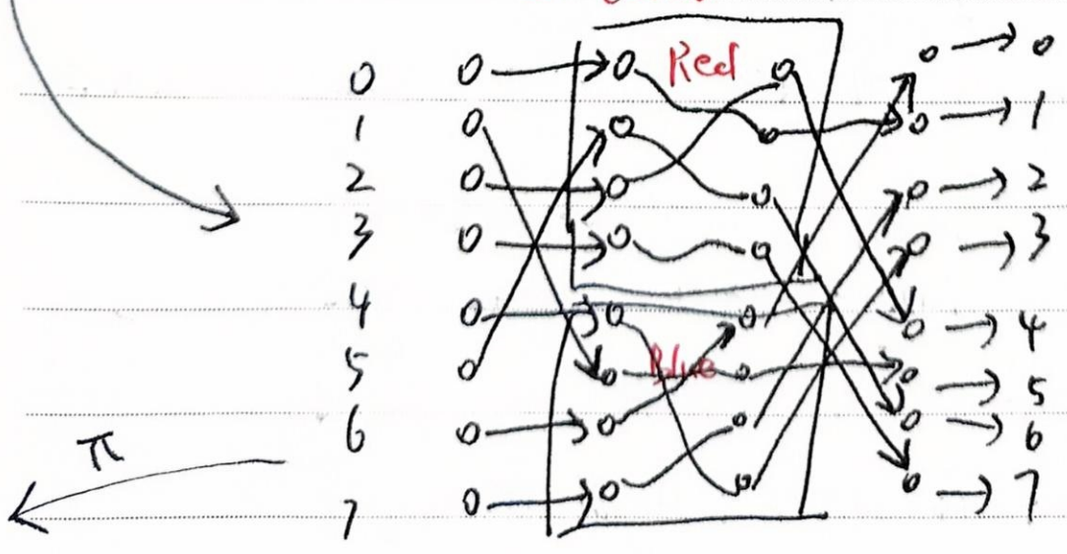
Constrant graph: If two packets must pass ~~thru~~ through different subnetworks, then there is an edge between them.



Ex: The packet destined for output. ($\pi(6) = 0$) and also the packet for outy $\pi(2) = 4$ Cannot pass through the same ~~subnetwork~~ subnetwork

key inside is A 2-coloring of the constrant graph (blue and red subnetwork)

2-color



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Rerrangement in Benes Network.

↑重排

every different mappmey of input and output in Benes Network Need a Rerrongement to maint ain congestion to 1