Memo No.

Date / /

MIT 6042J 2010 Fall

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Proof

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A proof is a method for ascertaining (verify) the truth

A mathematical proof is a verification of a proposition by a chain of logical deductions from a set of axioms

① A Proposition (命題) is a statement that is either True or False

Ex: 1. $2+3=5$

2. $\forall n \in N$, $n^2+n+41$ is a prime number

predicate: proposition whose truth depends on the value of vars

$N$: {0, 1, 2, 3 ...} Nature number

prime: ↑ not (質數)

| n | $n^2+n+41$ | Prime |
|---|---|---|
| 0 | 41 | ✓ |
| 1 | 43 | ✓ |
| 2 | 47 | ✓ |
| 3 | 53 | ✓ |

but $n=41$, $\times$ prime

so it's false

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3. $a^4+b^4+c^4=d^4$ has no positive integer solutions

$\exists\ a,b,c,d \in N^+,\ \underbrace{a^4+b^4+c^4=d^4}_{\text{predicate}}$

$\{1,2,\cdots\}$

4. four color theorem (famous !)

5. every ~~inter~~ even integer but 2 is the sum of 2 primes

ex: $24=11+13$

↑ still not proved!

6. $\forall n \in Z,\quad n \geq 2 \Rightarrow n^2 \geq 4$

↑ implies

Def of imply: An implication $p \Rightarrow q$ is true if p is F or q is false

Truth Table

| p | q | $p \Rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

pig fly $\Rightarrow$ I am king

It's true.

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Ex: $\forall n \in \mathbb{Z},\ n \geq 2 \Leftrightarrow n^2 \geq 4$

F: $n = -3$

$\Leftrightarrow$: $\Rightarrow$, $\Leftarrow$

↑ iff (if only if)

| P | q | $P \Rightarrow q$ | $q \Rightarrow P$ | $P \Leftrightarrow q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

(上次)

Def A [axiom] is a proposition that is "assumed" to be true

Ex: if $a=b$, $b=c$, then $a=c$

Axioms should:

1. consistent 2. complete

Def: A set of axioms is [consistent] if no proposition can be proved T & F.

Def: A set of axioms is [complete] if it can be used every proposition is T or F