Memo No.

Date / /

LEC6 Graphy Theory and Coloring 3.4

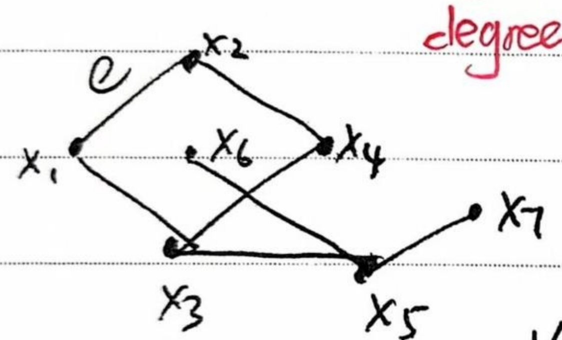
Claim (U Chicago) on average, men have 74% more opposite-gender partners than woman

claim (ABC News) 233% 234%

Graphy

degree(x_1) = 7

degree(x_5) = 3



Def: A graph G is a pair of sets (V, E) where V is a ~~set~~ nonempty set of items called vertices or nodes

& E is a set of 2-item subsets of V called edges

$V = \{x_1, x_2, x_3, \cdots x_7\}$

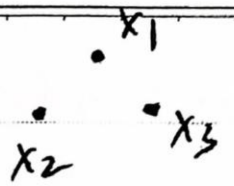
$E = \{\{x_1, x_2\}, \{x_1, x_3\}, \{x_3, x_4\}, \cdots \{x_5, x_7\}\}$

$\hookrightarrow x_1 - x_2$, $x_1 \rightarrow x_2$ also

we don't agree with empty graph, but can only graph with nodes and no edges

Mo Tu We Th Fr Sa Su
Memo No.
Date / /

Ex1



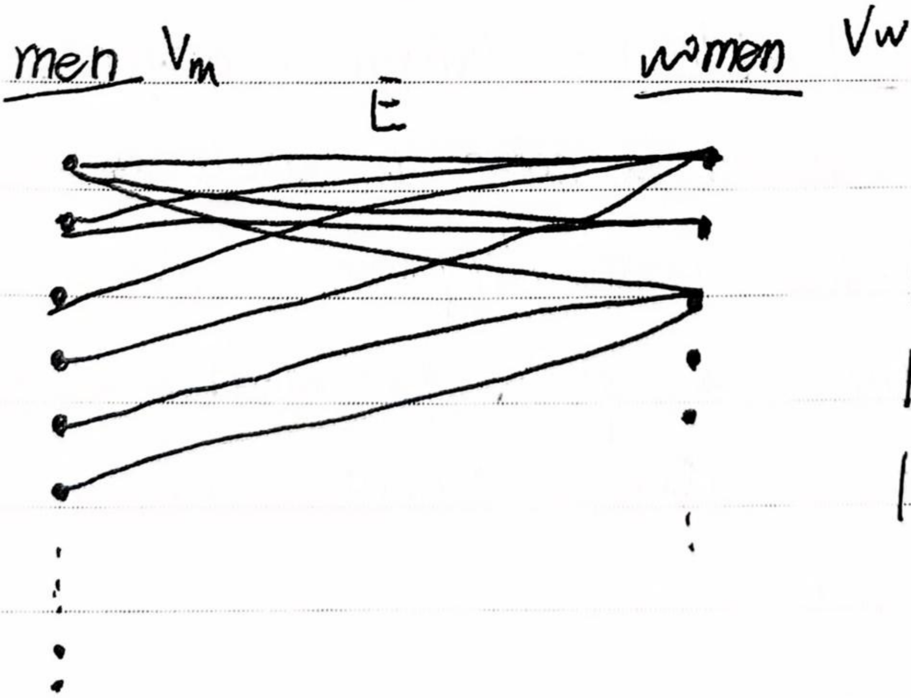
$G=(V,E)$, $V=\{x_1, x_2, x_3\}$, $E=\phi$

Def: Two nodes x_i & x_j are adjacent if $\{x_i,x_j\} \in E$

Def: An edge $e=\{x_j, x_i\}$ is incident to x_i & x_j

Def: the number of edges incident to a node is the degree of the node

Def the graph is simple if it has no loops or multiple edges



$|V| \approx 300$ M

$|V_m| \approx 147.6$ m

$|V_w| \approx 152.4$ m

$|E| = ??$

Memo No.
Date / /

Def A_m = average # of oppsite gender partners for ~~mam~~ men

A_w = —— for woman

What is A_m / A_w ?

U Chicago = 1.74 ABC New = 3.33

$$A_m = \frac{\sum_{x \in V_m} deg(x)}{|V_m|} = \frac{|E|}{|V_m|}$$

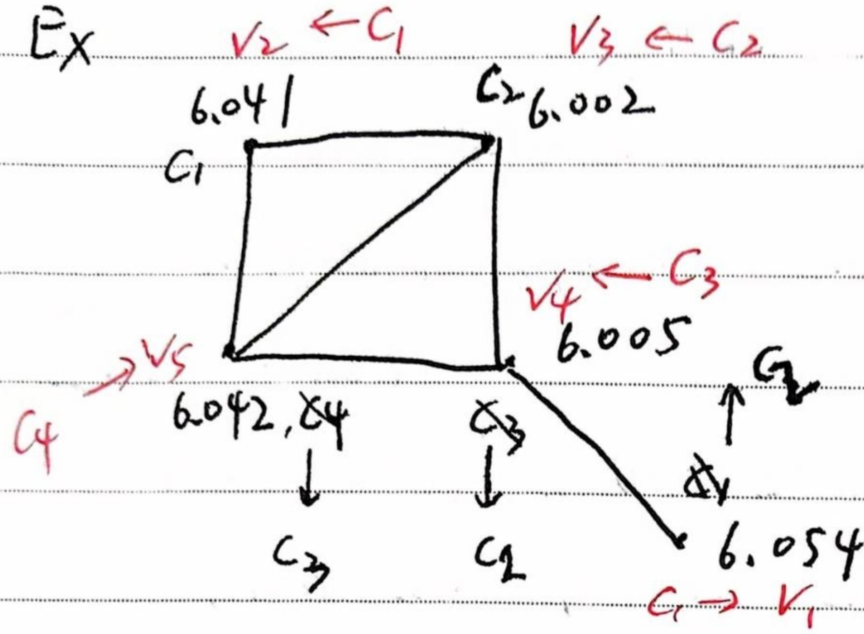
$$A_w = \frac{\sum_{x \in V_w} deg(x)}{|V_w|} = \frac{|E|}{|V_w|}$$

$$A_m / A_w = \frac{|E|/|V_m|}{|E|/|V_w|} = \frac{|V_w|}{|V_m|} = 1.0325$$

Graph Coloring Problem: Given a graph G and k colors, assign a color to each node, so adjacent nodes get different colors.

Df the minimum value of k for which such a coloring exists is the chromatic number of the graph $\chi(G)$

Ex



① ordering

slots

wed	5-7 pm	C_1
"	7-9 pm	C_2
"	9-11 pm	C_3
"	11-1 am	C_4
"	1-3 am	C_5

Basic Coloring Alg for $G(V, E)$

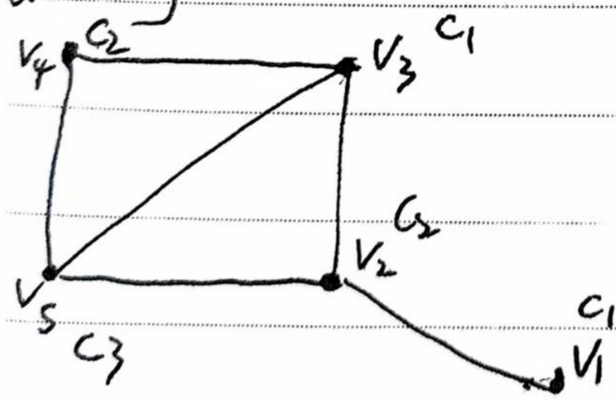
1. order the nodes $V_1, V_2, \cdots, V_n$
2. order the colors $C_1, C_2, \cdots$
3. For $i = 1, 2 \cdots n$, assign the lowest legal color to V_i

is know as a greedy Algorithm

so ⇒ find a good ordering

from the highest degree node is a good ~~feat~~ idea

② ordering



Thm: If every node in G has degree $\leq d$, then Basic Alg uses at most $d+2$ colors for G

in this case: most colors is $3+1 = 4$

Mo Tu We Th Fr Sa Su
Memo No.
Date / /

Pf: By induction

I.H. $P(n)$ n-nodes graph G

Base Case: $n=2 \Rightarrow$ 0 edges $\rightarrow$ degree $=0$

$\rightarrow$ 1 color $\overset{(d+1)}{=} 0+1$ ✓

Inductive Step: assume $P(n)$ is true for induction

let $G=(V,E)$ be any $n+1$ nodes graph

let d be the max degree in G

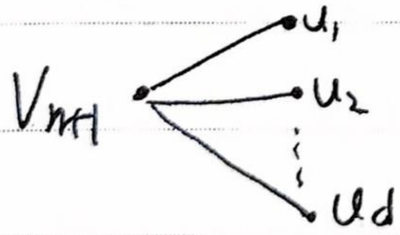
Order the nodes $\underbrace{V_1, V_2, \cdots V_n}, V_{n+1}$

$\Rightarrow$

Remove V_{n+1} from G, to create $G'=(V', E')$

, G' has max degree $\leq d$ & n nodes, so $P(n)$

says basic Algorithm use $\leq d+1$ Colors for $V_1 \cdots V_n$



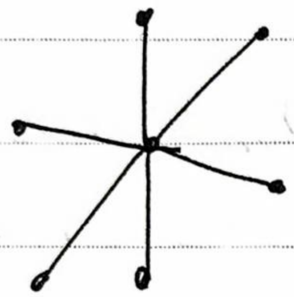
$\leq d$ neighbors

V_{n+1} has $\leq d$ neighbors $\Rightarrow$ so $\exists$ color in $\{C_1, C_2 \cdots$

$\cdots, C_{n+1}\}$ not used by any neighbor Given V_{n+1} that

Color

$\Rightarrow$ Basic Alg uses $\leq d+1$ colors on $G \Rightarrow P(n+1)$, check ex worse situation

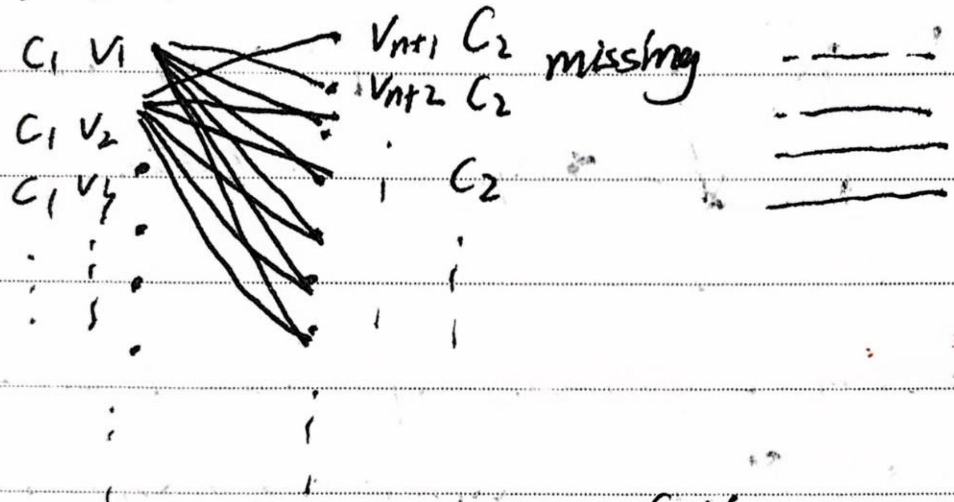


degree = n-1

$\chi(G) = 2$

but Basic coloring Alg is good in this case

a nasty graph:



find a bad ordering:

C_1 V_1	V_2 C_1
C_2 V_3	V_4 C_2
C_3 V_5	V_6 C_3
C_4 V_7	V_8 C_4
⋮	⋮

Important to remember: Bipaitite

Def: A graph $G=(V,E)$ is Bipaitite if V can be split into V_L, V_R so that all the edges connect a node in V_L to a node in V_R.