Mo Tu We Th Fr Sa Su

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9 Euler 方程. 欧拉方程

可化为二阶

$$x^n y^{(n)} + \underbrace{P_1}_{\text{constant}} x^{n-1} y^{n-1} + \cdots + P_{n-1} \cdot x \cdot y' + P_n = f(x)$$

$n=2$ $\quad x^2 \cdot y'' + Pxy' + qy = f(x) \longrightarrow$ 二阶

换元 令 $x = e^t$, $t = \ln x$

$$\boxed{\frac{dy}{dx}} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \boxed{\frac{dy}{dt} \cdot \frac{1}{x}}$$

$$y'': \frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{d\left(\frac{1}{x}\frac{dy}{dt}\right)}{dx} = \frac{-\frac{1}{x^2}dx \cdot \frac{dy}{dt} + \frac{1}{x} \cdot d\left(\frac{dy}{dt}\right)}{dx}$$

$$= -\frac{1}{x^2} \cdot \frac{dy}{dt} + \frac{1}{x} \cdot \frac{d\left(\frac{dy}{dt}\right)}{dx}$$

$$= -\frac{1}{x^2}\frac{dy}{dt} + \frac{1}{x} \cdot \underbrace{\frac{d\left(\frac{dy}{dt}\right)}{dt}}_{=\frac{d^2y}{dt^2}} \cdot \frac{dt}{dx} \to \frac{1}{x}$$

$$= -\frac{1}{x^2}\frac{dy}{dt} + \frac{1}{x^2}\frac{d^2y}{dt^2}$$

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算子 : $\frac{d}{dt}$ = D $\qquad \frac{dt^2}{dt} = 2t$

$\frac{dy}{dt} = Dy$, $Dt^2 = \frac{dt^2}{dt} = 2t$

$D = \frac{d}{dt}$, $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{1}{x} = Dy \cdot \frac{1}{x}$

$y' = D(y) \cdot \frac{1}{x}$, $xy' = Dy$

$y'' = -\frac{1}{x^2}\frac{dy}{dt} + \frac{1}{x^2}\frac{dy^2}{dt^2} = -\frac{1}{x^2}Dy + \frac{1}{x^2}D^2y$

$x^2y'' = D^2y - Dy = D(D-1)y$

$x^2y'' + xy' + qy = f(x)$

$\Rightarrow D(D-1)y + pDy + qy = f(e^t)$

$D^2 - Dy + pDy + qy = f(e^t)$

$$\boxed{y''_t + (p-1)y'_t + qy = f(e^t)}$$

二阶常系数线性微分方程

$$\left(\int Q(x)\cdot e^{-\int p(x)dx}dx + C\right)\cdot e^{\int p(x)dx}$$

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$$\boxed{\begin{aligned} xy' &= Dy = (y_t)' = y_t' \\ x^2y'' &= D(D-1)\,y = D^2y - Dy = (y')_t' - y_t' \\ x^3y''' &= D(D-1)(D-2)\,y \\ &\vdots \\ x^ny^n &= D(D-1)(D-2)\cdots(D-(n-1))\,y \end{aligned}}$$

ex, $x^2y'' - xy' + y = x\ln x$

$\searrow e^t\cdot t$

1, $\boxed{\text{令 } x = e^t}$ $\qquad$ $\cancel{dx} = \ln x = t$

$x^2y'' = D\cdot(D-1)y \qquad D = \frac{d}{dt}$

$xy' = Dy$

$D(D-1)y - Dy + y = t\cdot e^t$

$\Rightarrow D^2y - Dy - Dy + y = t\cdot e^t$

$\underline{y''_t - 2y'_t + y = te^t}$

Step1:

$\lambda^2 - 2\lambda + 1 = 0 \qquad \lambda_1 = \lambda_2 = 1$

$Y = c_1e^t + \cancel{c_2e}\ c_2xe^t$

Step2: $y^* = \cancel{z(x)e^t}\ z(t)e^t$

$p = -2,\ q = 1,\ r = 1$

$z'' + (0)z' + 0(z) = t \quad \therefore z = \frac{1}{6}t^3$

$2r+p \qquad r^2+pr+q \qquad \therefore y = c_1e^t + c_2te^t + \frac{1}{6}t^3e^t$

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$$y = C_1 x + C_2 \ln x \, X + \frac{1}{6}(\ln x)^3 X$$

$\boxed{\text{全微分方程}}$

<u>偏导</u>

一元函数 $f(x) = x^2$ $y' =$

二元函数 $f(x,y) = xy + 3x^2y$

$$\begin{cases} \frac{\partial z}{\partial x} = y + 6xy + 3y^2 \end{cases}$$