常微分方程:

一个自变量 x

$F(x, y, y', y'', \cdots y^{(n)}) = 0$

$x + y' = y^3$

偏微分方程

$z = F(x, y) \quad x, y, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

方程的阶: $y', y'', y''' \quad y'' + y' + x = 0$

通解: $y' = 3y^{\frac{2}{3}} \Rightarrow y = (x + c)^3$

constant

特解: 初始条件 $y|_{x=x_0} = y_0$

① 可分离变量的方程

$$\rightarrow \int \boxed{f(x)dx = \int g(y)dy} + C$$

eg. 例1:

$(y+x^2y)y' = x+xy^2$ 求此方程通解

$y' = \frac{dy}{dx}$ $\quad (y+x^2y)\frac{dy}{dx} = x+xy^2$

$$y(1+x^2)\frac{dy}{dx} = x(1+y^2)$$

$$\frac{y}{1+y^2}\cdot dy = x/(1+x^2)\cdot dx$$

$$\Rightarrow \frac{1}{2}\ln(1+y^2) = \frac{1}{2}\ln(1+x^2) + C$$

$$\ln(1+y^2) = \ln(1+x^2) + C$$

$$\therefore 1+y^2 = e^C\cdot(1+x^2)$$

$$\underline{y^2 = C_1(1+x^2)-1}$$ (通解)

例2.

$y' = 3y^{\frac{2}{3}} \Rightarrow \frac{dy}{dx} = 3y^{\frac{2}{3}} \Rightarrow$ $(y=0)$ $dx = \frac{1}{3}y^{-\frac{2}{3}}dy$

$$\Rightarrow x+C = \cancel{\tfrac{1}{3}}\, y^{\frac{1}{3}}$$

$$\begin{cases} y = (x+C)^3 \\ y \equiv 0 \end{cases}$$

$\rightarrow$ 奇解