CHAPTER-9
TRIANGLES

1 Exercise 9.2

Q4. **P** is a point in the interior of a parallelogram $ABCD$.Show that

1. $ar(APB) + Ar(PCD) = \frac{1}{2}ar(ABCD)$
2. $ar(APD) + ar(PBC) = ar(APB) + ar(PCD)$

Symbol	**Value**	**Description**
a	4	AB
b	2	AD
θ	$60°$	∠A
$\mathbf{A}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Vertex A
$\mathbf{B}$	$\begin{pmatrix} a \\ 0 \end{pmatrix}$	Vertex B
$\mathbf{D}$	$b\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$	Vertex B
$\mathbf{C}$	$\mathbf{B}+\mathbf{D}$	Vertex C

Table 1: Parameters

2 Solution

As the area of a parallelogram with adjacent sides a and b is,

$$\text{Area of parallelogram} = \|\mathbf{a} \times \mathbf{b}\| \tag{1}$$

And, area of a triangle with adjacent sides p and q is,

$$\text{Area of triangle} = \frac{1}{2}\|\mathbf{p} \times \mathbf{q}\| \tag{2}$$

So,

$$\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{B} - \mathbf{C}\| \tag{3}$$

Consider $\triangle APD$

$$Ar(APD) = \frac{1}{2}\|(\mathbf{A} - \mathbf{D}) \times (\mathbf{A} - \mathbf{P})\| \tag{4}$$

Consider $\triangle PBC$

$$Ar(BPC) = \frac{1}{2}\|(\mathbf{B} - \mathbf{C}) \times (\mathbf{P} - \mathbf{B})\| \tag{5}$$

On adding (4) and (5),

$$Ar(APD) + Ar(PBC) = \frac{1}{2}\left\|(\mathbf{A}-\mathbf{D})\times(\mathbf{A}-\mathbf{P})\right\| + \frac{1}{2}\left\|(\mathbf{B}-\mathbf{C})\times(\mathbf{P}-\mathbf{B})\right\| \tag{6}$$

From equation (3),

$$Ar(APD) + Ar(PBC) = \frac{1}{2}\left\|(\mathbf{A}-\mathbf{D})\times(\mathbf{A}-\mathbf{P})\right\| + \frac{1}{2}\left\|(\mathbf{A}-\mathbf{D})\times(\mathbf{P}-\mathbf{B})\right\| \tag{7}$$

$$\implies Ar(APD) + Ar(PBC) = \frac{1}{2}\left\|(\mathbf{A}-\mathbf{D})\times[(\mathbf{A}-\mathbf{P})+(\mathbf{P}-\mathbf{B})]\right\| \tag{8}$$

Here, AP and PB are adjacent sides of $\triangle\ APB$
From Triangle law of vector addition,
$(\mathbf{A}-\mathbf{P})+(\mathbf{P}-\mathbf{B})=(\mathbf{A}-\mathbf{B})$

$$\implies Ar(APD) + Ar(PBC) = \frac{1}{2}\left\|(\mathbf{A}-\mathbf{D})\times(\mathbf{A}-\mathbf{B})\right\| \tag{9}$$

Since, $(\mathbf{A}-\mathbf{D})$ and $(\mathbf{A}-\mathbf{B})$ are adjacent sides of paralleogram ABCD
From (2),

$$Ar(ABCD) = \left\|(\mathbf{A}-\mathbf{D})\times(\mathbf{A}-\mathbf{B})\right\| \tag{10}$$

From (9) and (10)

$$\therefore Ar(APD) + Ar(PBC) = \frac{1}{2}Ar(ABCD) \tag{11}$$

Similarly, we can prove that,

$$Ar(APB) + Ar(PBD) = \frac{1}{2}Ar(ABCD) \tag{12}$$

On Comparing (11) and (12),

$$Ar(APD) + Ar(PBC) = Ar(APB) + Ar(PCD) \tag{13}$$

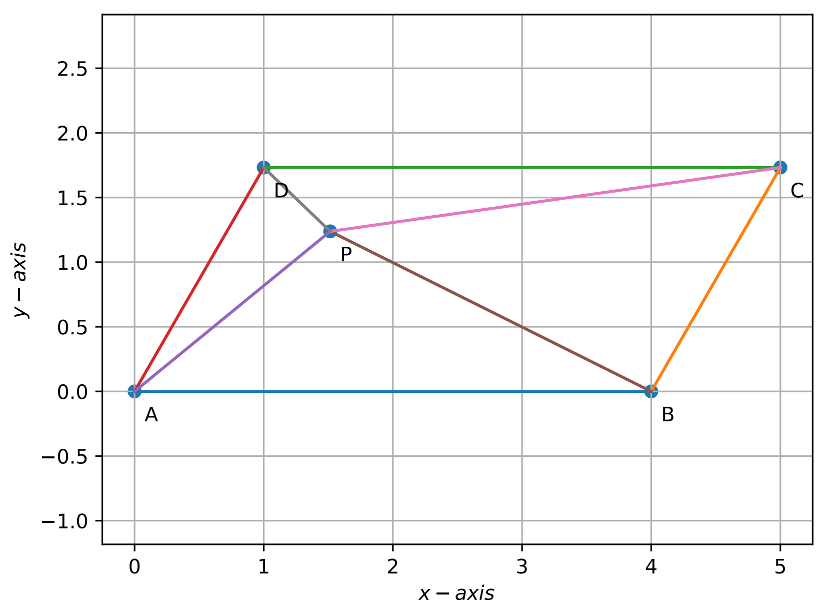

Figure 1: Parallelogram ABCD with interior point P