Analytical - 3

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) Calculate the Number of ways to achieve a sum of 15 when rolling four six sided dice.

Sol: Given,

$x_1 + x_2 + x_3 + x_4 = 15$

No of possible out comes for 4 dice $= 6^4 = 1296$

As the min value of dice is 1

$x_1 + 1 + x_2 + 1 + x_3 + 1 + x_4 + 1 = 15$

$x_1 + x_2 + x_3 + x_4 = 15 - 4 = 11 \longrightarrow (1)$

The no of non-negative integer solutions $= C(k+n-1, n-1)$

$\Rightarrow C(11+4-1, 4-1) = C(14, 3) = {}^{14}C_3 = \frac{14 \times 13 \times 12}{3 \times 2 \times 1} = 28 \times 13 = 364$

As the value of dice should not exceed 6

put $x_1 = 6$ in (1)

$6 + x_2 + x_3 + x_4 = 11 \Rightarrow x_2 + x_3 + x_4 = 11 - 6 = 5$

The no of non-negative integer solutions $= C(5+3, 3)$

$\Rightarrow {}^8C_3 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56 \Rightarrow$ for four dice so $\Rightarrow 4 \times 56 = 224$

As the value of dice should not exceed 6

put x_1 & $x_2 = 6$ in (1)

$6 + 6 + x_3 + x_4 = 11 \Rightarrow x_3 + x_4 = -1$

-1 are in valid case

Subtract the min value and 6 exceeding numbers

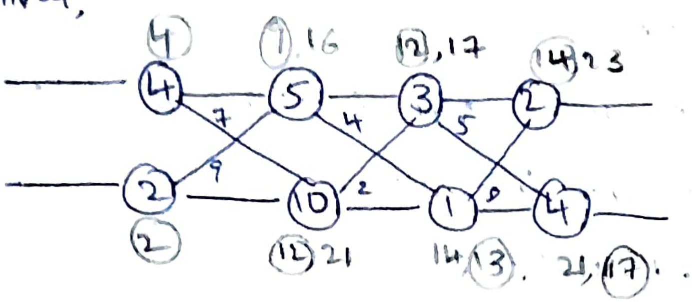
$\Rightarrow 364 - 224$

$\Rightarrow 140$ //

The number of ways to get sum as 15 from 4 dice $= 140$ //

(2) Two assembly lines L1: [4, 5, 3, 2], L2: [2, 10, 1, 4], L1:L2: [7, 4, 5], L2:L1: [9, 2, 8]. Calculate Minimum cost:-

sol: Given,



4	9	12	14
2	12	13	17

1 ←	1 ←	1 ←	1 ←
2	2	2	2

(3) Construct OBST tree for {10, 20, 30, 40} wi = {0.1, 0.2, 0.3, 0.4} Calculate total cost of the tree.

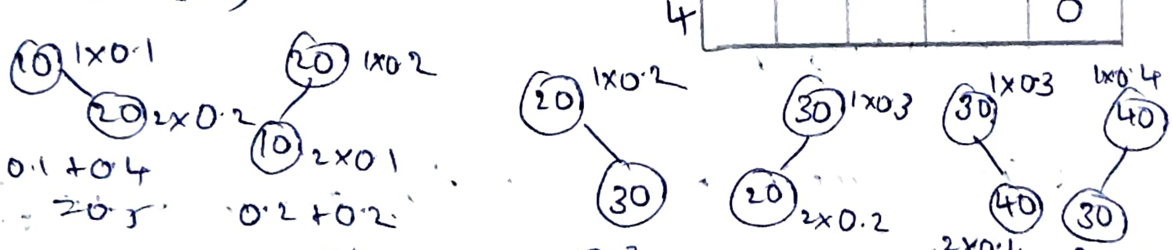
sol:- j-i = 0

0-0 = 0 (0, 0)
1-1 = 0 (1, 1)
2-2 = 0 (2, 2)
3-3 = 0 (3, 3)
4-4 = 0 (4, 4)
5-5 = 0 (5, 5)

j-i = 1
1-0 = 1 (0, 1) = 0.1
2-1 = 1 (1, 2) = 0.2
3-2 = 1 (2, 3) = 0.3
4-3 = 1 (3, 4) = 0.4

	0	1	2	3	4
0	0	0.1	0.4 [2]	1.0 [2]	2.0 [4]
1		0	0.2	0.7 [3]	1.2 [3]
2			0	0.3	1.0 [4]
3				0	0.4
4					0

j-i = 2
2-0 = 2 (0, 2)
3-1 = 2 (1, 3)
4-2 = 2 (2, 4)



0.1 + 0.4 = 0.5

0.2 + 0.2 = 0.4

0.2 + 0.6 = 0.8

0.3 + 0.4 = 0.7

0.3 + 0.8 = 1.1

0.4 + 0.6 = 1.0

$j-i=3$

$3-0=3\ (0,3)\ k=1,2,3$

$4-1=3\ (1,4)\ k=2,3,4$

$$\min\begin{cases}cost(0,0)+cost(1,3)+0.6\\ cost(0,1)+cost(2,3)+0.6\\ cost(0,2)+cost(3,3)+0.6\end{cases}$$

$$\Rightarrow \min\begin{Bmatrix}0+0.7+0.6\\0.1+0.3+0.6\\0.4+0.4+0.6\end{Bmatrix}\Rightarrow \min\begin{Bmatrix}1.3\\ \underline{1.0}\\1.4\end{Bmatrix}$$

$$\Rightarrow \{1.0\}^2$$

$j-i=4$

$4-0=4\ (0,4)\ k=1,2,3,4$

$$\min\begin{cases}cost(0,0)+cost(1,4)+1.0\\ cost(0,1)+cost(2,4)+1.0\\ cost(0,2)+cost(3,4)+1.0\\ cost(0,3)+cost(4,4)+1.0\end{cases}\Rightarrow \min\begin{Bmatrix}0+1.2+1.0\\0.1+1.0+1.0\\0.4+0.4+1.0\\1.0+0+1.0\end{Bmatrix}$$

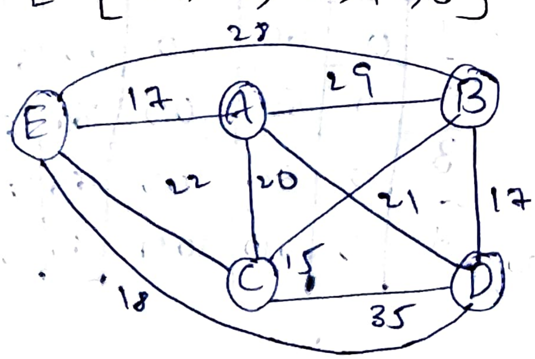
$$=\min\begin{Bmatrix}2.2\\2.1\\ \underline{2.0}\end{Bmatrix}$$

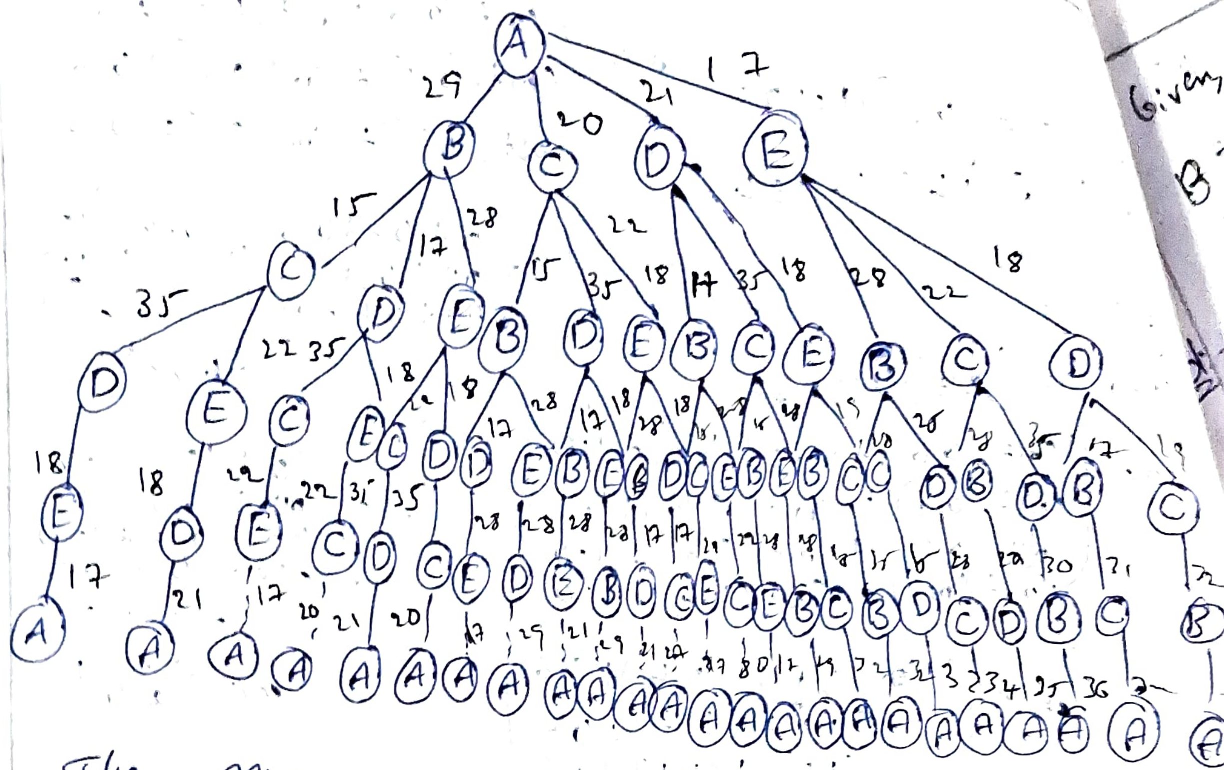
$$\Rightarrow \min\begin{cases}(1,1)+(2,4)+0.6\\ (1,2)+(3,4)+0.6\\ (1,3)+(4,4)+0.6\end{cases}$$

$$\Rightarrow \min\begin{Bmatrix}0+1.0+0.6\\0.2+0.4+0.6\\0.7+0+0.6\end{Bmatrix}=\begin{Bmatrix}1.6\\ \underline{1.2}\\1.3\end{Bmatrix}$$

4) Given, TSP for 5 Citiy

$$\begin{array}{c|c} & A\ \ B\ \ C\ \ D\ \ E\\ A & [0, 29, 20, 21, 17]\\ B & [29, 0, 15, 17, 28]\\ C & [20, 15, 0, 35, 22]\\ D & [21, 17, 35, 0, 18]\\ E & [17, 28, 22, 18, 0]\end{array}$$





The minimum cost of the tree = 87

A – E – D – B – C – A

⑤ Knapsack for weight [10, 20, 30, 40], value = [60, 100, 120, 200], capacity = 50

Sol:

Item	weight	value (p)
1	10	60
2	20	100
3	30	120
4	40	200

	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	160	180	220
4	0	60	100	160	200	260

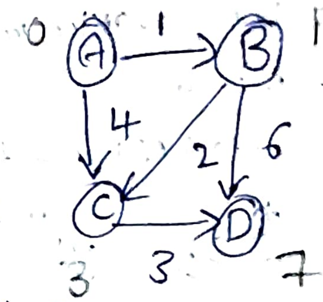
$$V[4,50] = \max\left\{V(3,50),\ V(3,50-40)+200\right\}$$

$$= \max(220,\ 60+200)$$

$$= \max(220,\ 260) = 260$$

1) Given edges A, B, C, D weights A→B = 1, A→C = 4, B→C = 2, B→D = 6, C→D = 3 using BF Algorithm find the shortest path

Sol:- Initial step:-



V	A	B	C	D
d	0	∞	∞	∞
P	-	-	-	-

No of vertices = 4

No of iterations = 4-1 = 3

A→B = 1

A→C = 4

B→C = 2

B→D = 6

C→D = 3

Step ①-

V	A	B	C	D
d	0	1	4	∞
P	-	A	A	C

Step 2:-

V	A	B	C	D
d	0	1	3	7
P	-	A	B	C

Step 3:-

V	A	B	C	D
d	0	1	3	6
P	-	A	B	C

Path	S.D	S.P
A→B	1	A→B
A→C	3	A→C
A→D	6	A→B→C→D

⇒ A → B → C → D = 6 is The shortest path

7) Determine the probability of rolling five dice to get 20.

Sol:- Given,

$x_1 + x_2 + x_3 + x_4 + x_5 = 20$

No of possible out comes for 5 dices $= 6^5$
$= 7776$

The min value of dice is 1

$x_1 + 1 + x_2 + 1 + x_3 + 1 + x_4 + 1 + x_5 + 1 = 20$

$x_1 + x_2 + x_3 + x_4 + x_5 = 20 - 5 = 15$

The no. of non-negative solutions = C(15+4, 4)

$\Rightarrow {}^{19}C_4 = \frac{19\times18\times17\times16}{4\times3\times2\times1} = 19\times6\times17\times2 \Rightarrow 19\times12\times17$

$= 3876$

As the maximum of dice should not exceed 6

let $x_1 = 6$

$6 + x_2 + x_3 + x_4 + x_5 = 15 \Rightarrow x_2 + x_3 + x_4 + x_5 = 9$

$\Rightarrow {}^{13}C_4 \Rightarrow \frac{11\times10\times9\times8}{4\times3\times2\times1} \Rightarrow \frac{\cancel{4950}}{24} = \cancel{330}$ $\frac{9\times8\times7\times9}{4\times3\times2\times1}$

126 $9\times14 = 126$

for all 5 dice $= 5\times\cancel{330} = \cancel{1650}$

615

As the maximum of dice should not exceed 6 for

Let $x_1, x_2 = 6$

$6 + 6 + x_3 + x_4 + x_5 = 15 \Rightarrow x_3 + x_4 + x_5 = 15 - 12$

$= 3$

${}^{7}C_4 \Rightarrow 75$

$\Rightarrow 5\times35 = 350$

Invalid cases must be subtracted

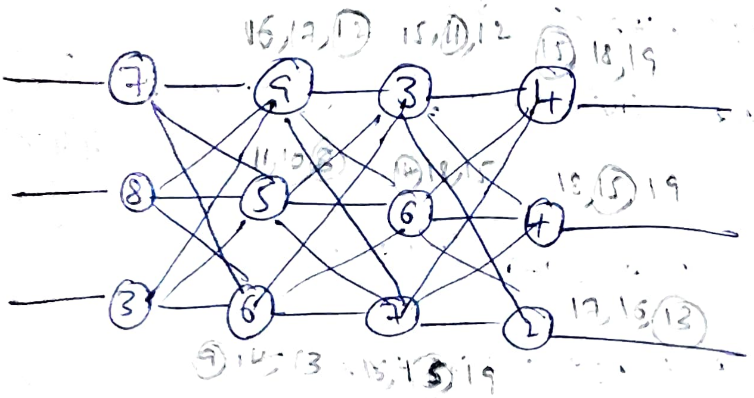
$\Rightarrow 3876 - \cancel{1650} - 350$ (651)

$\Rightarrow 2226 - \cancel{1575}$

$= 651$

(2) L1 = [7, 9, 3, 4], L2 [8, 5, 6, 4],

L3: [3, 6, 7, 2] find minimum cost for the lines



7	12	11	15
8	8	12	15
3	9	15	13

1	3	2	1
2	3	2	1
3	3	2	1

(9) Given, wi {15, 25, 35, 45, 55}, {0.05, 0.15, 0.4, 0.25, 0.15}
find the OBST tree

Sol:- Given,

{15, 25, 35, 45, 55}

{0.05, 0.15, 0.4, 0.25, 0.15}

j-i = 2

2-0 = 2 (0,2) (1,2)

3-1 = 2 (1,3) (3,3)

4-2 = 2 (2,4) (3,4)

5-3 = 2 (3,5) (4,5)

	0	1	2	3	4	5
0	0	0.05	0.25 [2]	0.85 [3]	1.35 [3]	1.80 [3]
1		0	0.15	0.70 [3]	1.20 [3]	1.80 [4]
2			0	0.4	0.90 [3]	1.35 [3,4]
3				0	0.25	0.55 [4]
4					0	0.15
5						0

j-i = 3

(0,3)

(1,4) min $\begin{Bmatrix} 0.70 \\ 0.45 \\ 0.25 \end{Bmatrix}$ + 0.6

(2,5) (0,3)

$\Rightarrow$ min $\begin{Bmatrix} 1.30 \\ 1.08 \\ 0.85 \end{Bmatrix}$ = 0.35

min $\begin{Bmatrix} 1.70 \\ 1.20 \\ 1.50 \end{Bmatrix}$ ~~80~~ = 1.20 (3)

min $\begin{Bmatrix} 1.35 \\ 1.35 \\ 1.70 \end{Bmatrix}$ = 1.35

$j-i=4$

$(0,4)$

$(1,5)$

$1,4 = \min \begin{Bmatrix} 2.05 \\ 1.80 \\ 1.35 \\ 1.70 \end{Bmatrix} = 1.35$

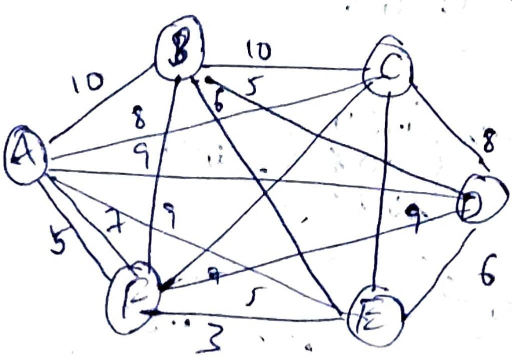
$\min \begin{Bmatrix} 2.30 \\ 2.45 \\ 1.80 \\ 2.15 \end{Bmatrix} = 1.80$

$j-i=5$

$\Rightarrow (0,5) = \min \begin{Bmatrix} 2.80 \\ 2.65 \\ 1.80 \\ 2.00 \\ 2.35 \end{Bmatrix} = 1.80$ //

10) Find TSP

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 10 & 8 & 9 & 7 & 5 \\ 10 & 0 & 10 & 5 & 6 & 9 \\ 8 & 10 & 0 & 8 & 9 & 7 \\ 9 & 5 & 8 & 0 & 6 & 5 \\ 7 & 6 & 9 & 6 & 0 & 8 \\ 5 & 9 & 7 & 5 & 8 & 0 \end{bmatrix} \end{array}$$



Assembly line station)

knapsack problem, capacity = 60

Item 1	20	100	10	500
Item 2	30	120	20	100
Item 3	10	60	30	

	0	10	20	30	40	50	60
0	0	0	0	0	0	0	0
1	0	60	60	60	60	60	60
2	0	60	100	160	160	160	160
3	0	60	100	160	180	220	280

$v((3,60)) = \max(v(i-1, w), v(i-1, w - w[i] + value[i]))$

$= \max(v(3,60), v(2, 60-30) + 120$

$= \max(160, 160+120) = 280$

12) Given vertices, $V_1, V_2, V_3, V_4, V_5 \rightarrow$

$V_1 \rightarrow V_2 = 3$

$V_1 \rightarrow V_3 = 8$

$V_2 \rightarrow V_3 = 2$

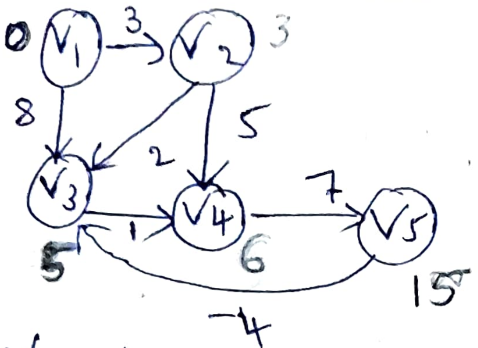
$V_2 \rightarrow V_4 = 5$

$V_3 \rightarrow V_4 = 1$

$V_4 \rightarrow V_5 = 7$

$V_5 \rightarrow V_3 = -4$

Sol:-



Initialization

V	V_1	V_2	V_3	V_4	V_5
d	0	∞	∞	∞	∞
p	–	–	–	–	–

$V_1 \to V_2 = 3$
$V_1 \to V_3 = 8$
$V_2 \to V_3 = 2$
$V_2 \to V_4 = 5$
$V_3 \to V_4 = 1$
$V_4 \to V_5 = 7$
$V_5 \to V_3 = -4$

No. of vertices = 5
No. of iterations = 5 − 1 = 4

Step ①:-

V	V_1	V_2	V_3	V_4	V_5
d	0	3	8	∞	∞
p	–	V_1	V_1	V_3	V_4

Step ②:-

V	V_1	V_2	V_3	V_4	V_5
d	0	3	~~8~~ 5	8	∞
p	–	V_1	V_2	V_2	V_4

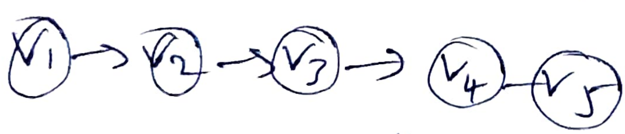
Step ③:-

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	15
p	–	V_1	V_5	V_3	V_4

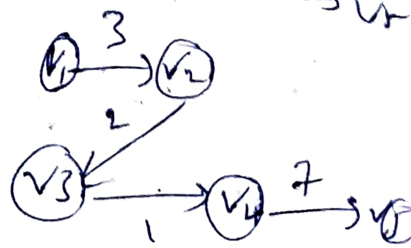
Step ④:-

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	13
p	–	V_1	V_5	V_3	V_4

path	S.D	S.P
$V_1 \to V_2$	3	$V_1 \to V_2$
$V_1 \to V_3$	5	$V_1 \to V_2 \to V_3$
$V_1 \to V_4$	6	$V_1 \to V_2 \to V_3 \to V_4$
$V_1 \to V_5$	13	$V_1 \to V_2 \to V_3 \to V_4 \to V_5$



= 13 is the shortest path



3) two eight-sided dice, number of ways to achieve a sum of 10.

Sol: Given,

$x_1 + x_2 = 10$, The possible out comes $= 8^2 = 64$

As the minimum value is 1

$x_1 + x_2 + = 10 - 2$

$= 8$

$^{8+3}C_3 \Rightarrow {}^{11}C_3 \Rightarrow \frac{11 \times 10 \times 9}{3 \times 2 \times 1} \Rightarrow 11 \times 15 = 165$

for 2 dices $= 2 \times 165 = 330$

$^{8+1}C_1 \Rightarrow {}^9C_1 \Rightarrow 9$ for 2 dice $= 2 \times 9 = 18$

As the maximum value is 8, it should not exceed 8

let $x_1 = 8$

$8 + x_2 = 10 \Rightarrow x_2 = 2$

(2, 8), (3, 5), (4, 6)

$^{2+1}C_1 \Rightarrow {}^3C_1 = 3 \Rightarrow$ for 2 dice $= 2 \times 3 = 6$

$\Rightarrow$ The no of ways $= 18 - 6 = 12$ //

$\Rightarrow 8^3 = 64 \times 8 = 512$

$^{8+2}C_2 \Rightarrow {}^{10}C_2 \Rightarrow \frac{10 \times 9}{2 \times 1} = 36$

for 3 dice $= 108$

$^{2+2}C_2 \Rightarrow {}^4C_2 \Rightarrow \frac{4 \times 3}{2 \times 1} = 6$ for 3 dice $= 18$

$\Rightarrow 120 - 18 \Rightarrow 102$

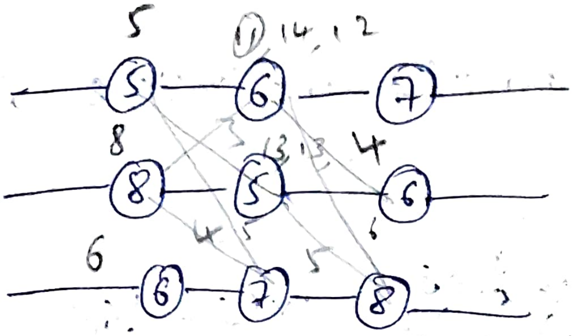
$\therefore$ for two dice $= 7$ ways

for three dice $= 25$ ways

(14) Line1: [5, 6, 7], L2 [8, 5, 6] ; L3 [6 7, 8]

L1: L2 [3, 4], [4, 5], [5, 6] · Calculate min time

Sol:-



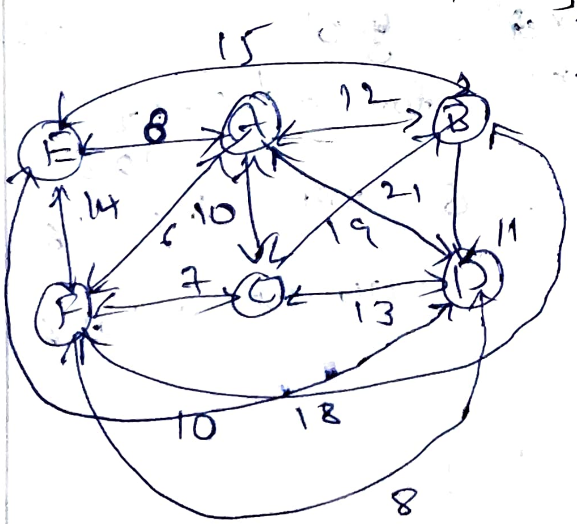
(15) Given keys [5, 15, 25, 35, 45, 55]

[0.1, 0.05, 0.3 0.25, 0.3, 0.1], construct OBT Tree

Sol:-

15) Given TSP for 7 cities in the matrix form

	A	B	C	D	E	F
A	0	12	10	19	8	6
B	12	0	21	11	15	10
C	10	21	0	13	5	7
D	19	11	13	0	18	8
E	8	15	5	18	0	14
F	16	10	7	8	14	0



Find the expected sum of the outcomes when rolling three four sided dice

Sol: $\text{sum} = 3 \; (1+1+1)$

$\text{Sum} = 4 = \frac{3}{64} \; (1+1+2, \; 1+2+1, \; 2+1+1)$

$5 = \frac{6}{64}$

$6 = \frac{10}{64}$; $7 = \frac{12}{64}$

$8 = 12/64$, $9 = 10/64$, $10 = 6/64$; $11 = 3/64$ +

$12 = 1/64$

$\Sigma (\text{sum} * \text{probability})$

$= \left(3 \times \frac{1}{64}\right) + \left(4 \times \frac{3}{64}\right) + \left(5 \times \frac{6}{64}\right) + \left(6 \times \frac{10}{64}\right) + \left(7 \times \frac{12}{64}\right) +$

$\left(8 \times \frac{12}{64}\right) + \left(9 \times \frac{10}{64}\right) + \left(10 \times \frac{6}{64}\right) + \left(11 \times \frac{3}{64}\right) + \left(12 \times \frac{1}{64}\right)$

$= \frac{480}{64} = 7.5$ //

12) keys $[10, 20, 30]$

probabilities $[0.2, 0.5, 0.3]$ find the optimal BST Total cost.

Sol:-

$k = \{10, 20, 30\}$

$v = \{0.2, 0.5, 0.3\}$

$j - i = 3$

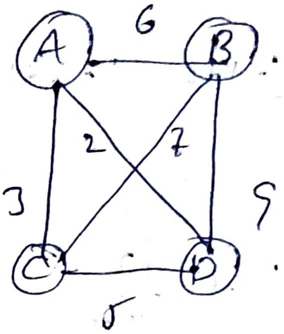
$3 - 0 = (0, 3)$

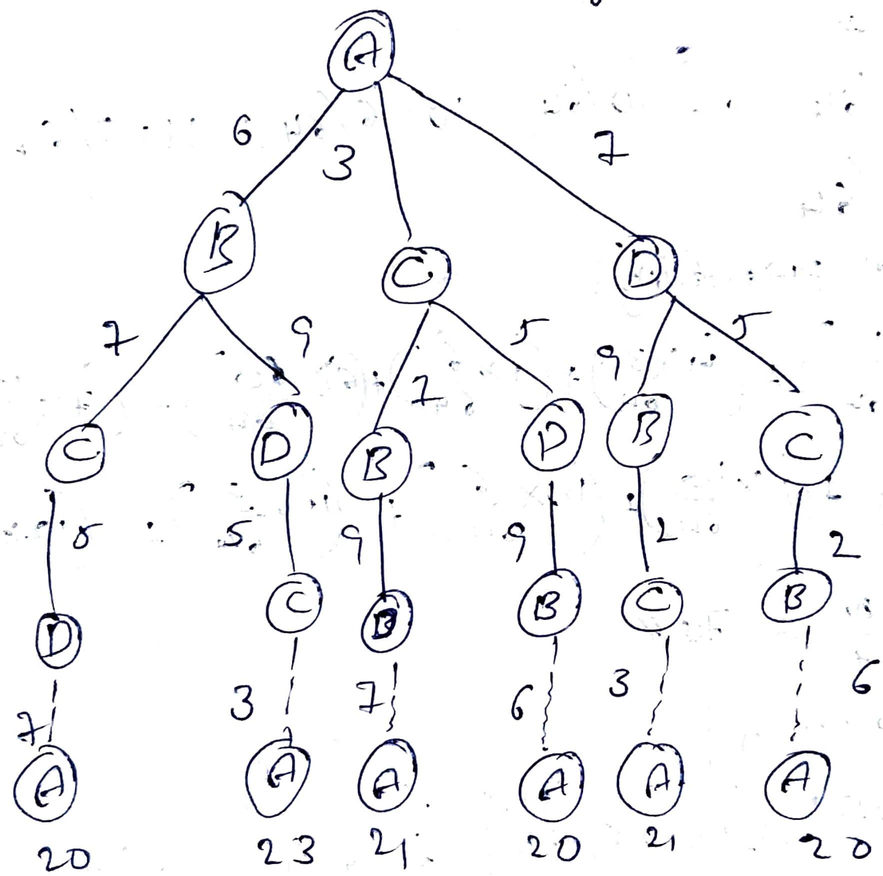
$\text{cost}_{(0,3)} = \left(\min \begin{Bmatrix} 2.1 \\ 1.5 \\ 1.1 \end{Bmatrix}\right) = (1.1)^3$

	0	1	2	3
0	0	0.2	$(0.8)^1$	$(1.1)^3$
1		0	0.5	$(1.1)^3$
2			0	0.3
3				0

18) Given, vertices (V_1, ~~V_2, V_3, V_4, V_5~~) A, B, C, D find TSP of 4 cities

$$\begin{array}{c|c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 6 & 3 & 7 \\ 6 & 0 & 2 & 9 \\ 3 & 2 & 0 & 5 \\ 7 & 9 & 5 & 0 \end{bmatrix} \end{array}$$





A → B → C → D → A = 20

A → D → C → B → A = 20 } min optimal path

Given vertices A, B, C, D, E, F

$A \to B = 6$
$A \to D = 7$
$B \to C = 5$
$B \to E = 4$
$B \to D = 8$
$C \to B = 2$
$D \to C = 3$
$D \to E = 9$
$E \to F = 7$
$F \to C = 2$

V	A	B	C	D	E	F
d	0	∞	∞	∞	∞	∞
P	-	-	-	-	-	-

1)

V	A	B	C	D	E	F
d	0	6	4	7	2	9
P	-	A	D	A	B	E

2)

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

3)

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

4)

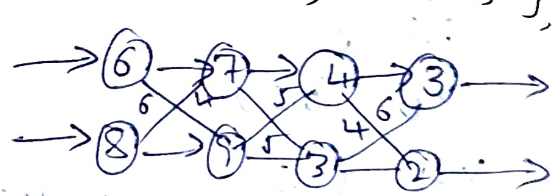
V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

5)

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

V	S.D	S.P
A	0	A
B	2	A-D-C-B
C	4	A-D-C
D	7	A-D
E	2	A-D-C-B-E
F	9	A-D-C-B-E-F

(20) Calculate Two assembly lines L1: [6,7,4,5], L2: [8,9,3,2] $l_1 : l_2$ [6,5,4] from (l_2 to l_1]



	1	2	3	4
F[i]	6	13	17	22
F[j]	8	17	20	22

	1	2	3	4
l_1	1	1	1	1
l_2	2	2	2	2

(4) keys {5, 15, 25, 35, 45, 55} with query possibilities

{0.1, 0.05, 0.2, 0.25, 0.30, 0.1}

	0	1	2	3	4	5	6
0	0	0.1	(0.2)	$(0.35)^3$	$(0.71)^4$	$(1.2)^4$	(1.4)
1		0	0.05	$(0.3)^3$	(0.6)	$(1.25)^4$	$(1.5)^4$
2			0	0.2	$(0.64)^4$	$(0.8)^4$	$(1.6)^4$
3				0	0.25	$(0.8)^4$	(1.35)
4					0	0.30	(0.5)
5						0	0.1
6							0

$j-i = 2$

(2,0) = (0,2)

(3-1) = (1,3)

(4-2) = (2,4)

(3,5) = (3,5)

(6-4) = (4,6)

1) 5 1×0.1
15 2×0.01
= 0.2

15 1×0.05
5 2×0.1
= 0.25

2) 15 1×0.05
25 2×0.2
= 0.45

25 1×0.2
15 2×0.05
= 0.3

3) 25 1×0.2
35 2×0.25
= 0.7

35 1×0.25
25 2×0.2
= 0.65

4) 35 1×0.25
45 2×0.3
= 0.85

45 1×45
0.25 2×25
= 0.8

5) 45 1×0.3
55 2×0.1
= 0.5

55 1×0.1
45 2×0.3
= 0.7

$j-i = 3$

3-0 = (0,3)

4-1 = (1,3)

5-2 = (2,5)

6-3 = (3,6)

$$\min \begin{cases} cost(0, 1-1) + cost(1,3) \\ cost(0, 2-1) + cost(2,3) \\ cost(0, 3-1) + cost(3,3) \end{cases} + 0.35$$

$$= \min \begin{cases} 0.65 \\ 0.65 \\ 0.55 \end{cases} = 0.55$$

1.)

J1: $W_1 = 25$; $V = 80$

J2: $W_2 = 35$; $V = 90$

J3: $W_3 = 45$, $V = 120$

J4: $W_4 = 30$; $V = 70$

	0	25	35	45	30	70
0	0	0	0	0	0	0
1	0	80	80	80	80	80
2	0	80	90	90	90	90
3	0	80	90	120	120	90
4	0	80	90	120	120	150

23) Given,

$A \to B = 1$
$A \to C = 4$
$B \to C = 3$
$B \to D = 2$
$B \to E = 2$

$D \to B = 1$
$D \to C = 5$
$E \to D = 3$

V	A	B	C	D	E
d	0	∞	∞	∞	∞
P	–	–	–	–	–

1)

V	A	B	C	D	E
d	0	-1	4	∞	∞
P	–	A	A	–	–

2)

V	A	B	C	D	E
d	0	-1	4	∞	1
P	–	A	A	–	B

3)

V	A	B	C	D	E
d	0	-1	4	3	1
P	–	A	A	E	B

4)

V	A	B	C	D	E
d	0	-1	4	3	1
P	–	A	A	E	B

V	S.D	S.P
A	0	A
B	-1	$A \to B$
C	4	$A \to C$
D	3	$A \to E \to D$
E	1	$A \to B \to C$

$$\underset{k=(2,3,4)}{cost(i,j)} = \min \begin{Bmatrix} cost(1,1) + cost(2,4) \\ cost(1,2) + cost(3,4) \\ cost(1,3) + cost(4,4) \end{Bmatrix} + 0.5$$

$$= \min \begin{Bmatrix} 1.15 \\ 0.95 \\ 0.8 \end{Bmatrix}$$

$$\underset{k=3,4,5}{cost(i,j)} = \min \begin{Bmatrix} cost(2,3-1) + cost(3,5) \\ cost(2,4-1) + cost(4,5) \\ cost(2,5-1) + cost(5,5) \end{Bmatrix} + 0.75$$

$$= \min \begin{Bmatrix} 1.55 \\ 1.25 \\ 1.4 \end{Bmatrix}$$

$$\underset{k=2,3,4,5}{cost(i,j)} = \min \begin{Bmatrix} cost(0) + 1.25 \\ 0.05 + 0.8 \\ 0.55 + 0.3 \\ 0.8 + 0 \end{Bmatrix} + 0.8 = \begin{Bmatrix} 2.05 \\ 1.65 \\ 0.85 \\ 1.6 \end{Bmatrix}$$

$$\underset{k=1,2,3,4,5}{cost(0,5)} = \min \begin{Bmatrix} 0 + 0.85 \\ 0.1 + 1.25 \\ 0.2 + 0.8 \\ 0.55 + 0.3 \\ 1.05 + 0 \end{Bmatrix} + 0.9 = \min \begin{Bmatrix} 1.75 \\ 2.35 \\ 1.9 \\ 1.35 \\ 1.95 \end{Bmatrix}$$

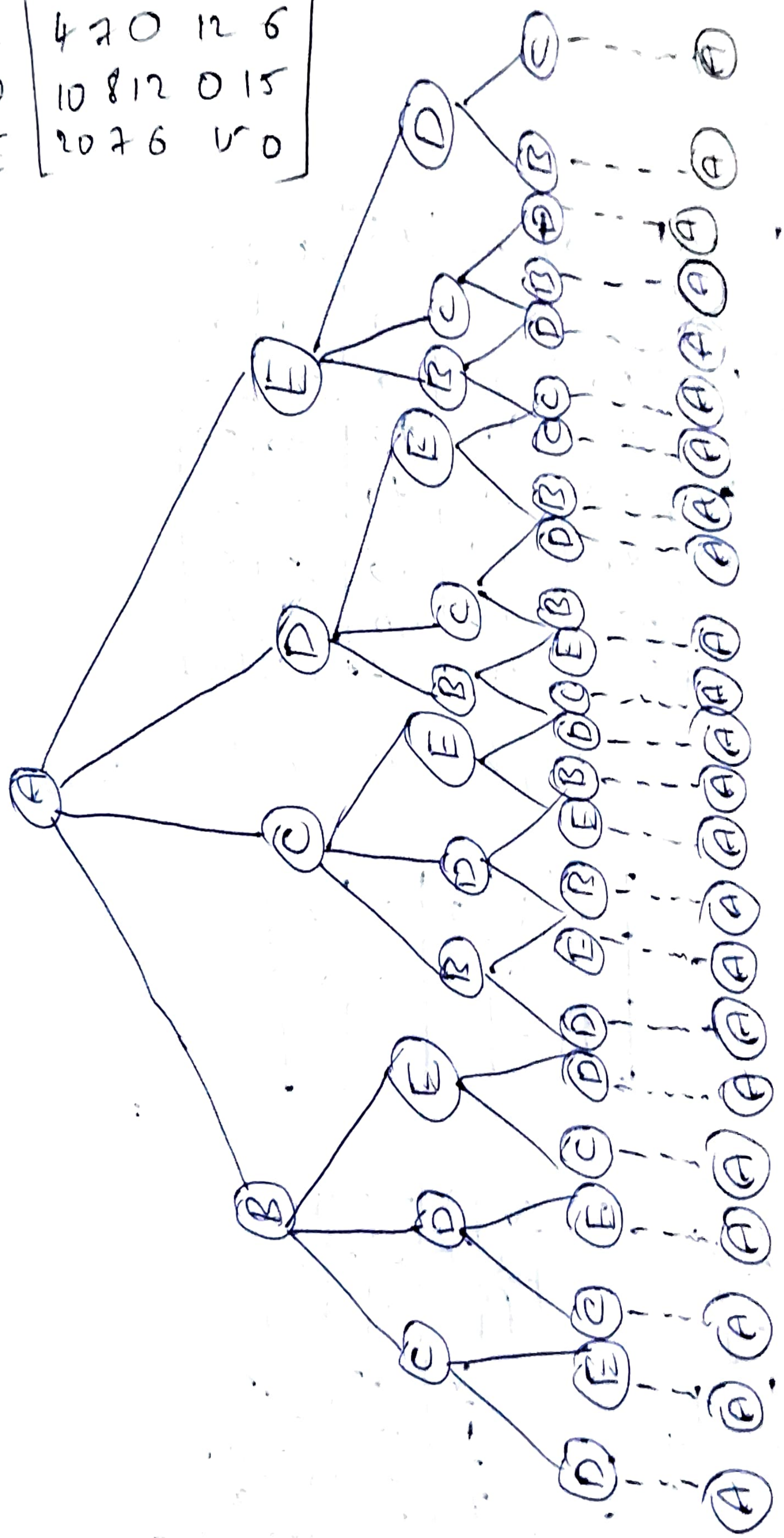
$$\underset{k=2,3,4,5,6}{cost(1,6)} = \min \begin{Bmatrix} 0 + 1.45 \\ 0.05 + 1 \\ 0.3 + 0.5 \\ 0.8 + 0.1 \\ 0.85 + 0 \end{Bmatrix} + 0.9$$

$$= \min \begin{Bmatrix} 2.45 \\ 1.95 \\ 1.7 \\ 1.8 \\ 1.75 \end{Bmatrix}$$

$$\Longrightarrow \min \begin{Bmatrix} 2.45 \\ 1.95 \\ \cancel{1.7} \\ 1.8 \\ 1.75 \end{Bmatrix}$$

TSP. for 5 cities

	A	B	C	D	E
A	0	14	4	10	20
B	14	0	7	8	7
C	4	7	0	12	6
D	10	8	12	0	15
E	20	7	6	15	0



16) Find a knapsack with a capacity of [illegible] with weight and values

[illegible] $w = 10$ $v = 10$
[illegible] $w = [illegible]$ $v = 40$
[illegible] $w = [illegible]$ $v = [illegible]$
[illegible] $w = [illegible]$ $v = 40$
[illegible] $w = [illegible]$ $v = 40$

[illegible]	0	10	[illegible]	20	25	[illegible]	[illegible]
0	0	0	0	0	0	0	0
1	0	10	50	10	10	30	10
2	0	10	10	40	40	40	40
3	0	60	40	40	40	40	40
4	0	10	40	70	70	70	100
5	0	10	40	50	50	10	100
6	0	10	40	70	50	70	160

17) Given a directed graph with vertices 1,2,3,4 & edges:

1→2 = 4
1→3 = 5
2→3 = [illegible]
3→4 = 3
4→[illegible] = [illegible]

V	1	2	3	4
d	0	∞	∞	∞
P				

1)

V	1	2	3	4
d	0	4	5	∞
P		1	1	–

2)

V	1	2	3	4
d	0	4	5	∞
P		1	2	–

3)

V	1	2	3	4
d	0	4	5	8
P		1	2	3

V	S.P	S.P
1	0	1→1
2	4	1→2
3	5	1→2→3
4	8	1→2→3→4

Roll-six-sided dice. Determine the no. of ways to get a sum of 18 ensuring that at least one dice shows a 6.

$x + x^2 + x^3 + x^4 + x^5 + x^6$

$\Rightarrow x(1 + x + x^2 + x^3 + x^4 + x^5)$

$\Rightarrow \frac{x(1 - x^6)}{1 - x}$

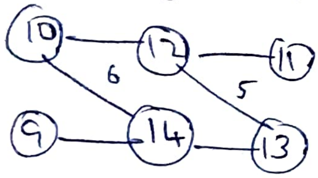
for six dice

$\Rightarrow x\left(\frac{x(1-x)^6}{1-x}\right)^6 \Rightarrow \left(x\left(\frac{(1-x)}{1-x}\right)\right)^6 = x^6(1-x)^6(1-x)^{-6}$

$= x^{18}$

$= 340$

line 1: [10, 12, 11], l_2: [9, 14, 13], and transfer time b/w line [6, 5]. Calculate min assembly line



Before reduction

10	6	5
9	30	30

After red

11	4	6
12	28	27

For keys (8, 12, 16, 20, 24) with access possibilities {0.2, 0.05, 0.4, 0.25, 0.1}

$K = \{8, 12, 16, 20, 24\}$

$V = \{0.2, 0.05, 0.4, 0.25, 0.1\}$

$j - i = 0$

$j-i=2$

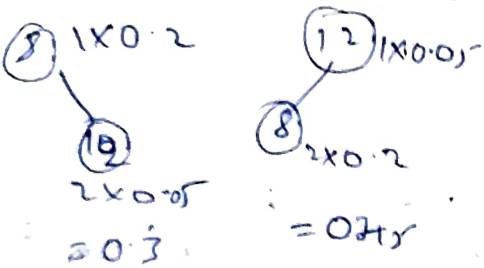
$2-0=(0,2]$

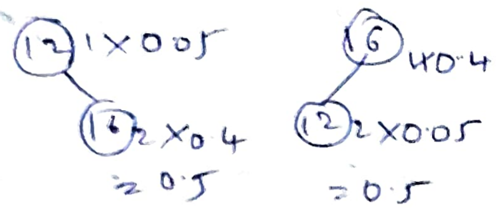
$3-1=[1,3]$

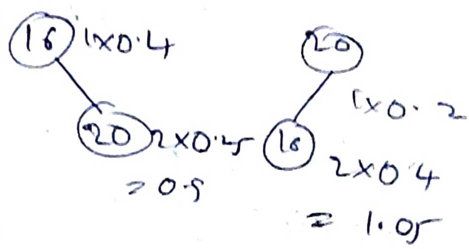
$4-2=[2,4]$

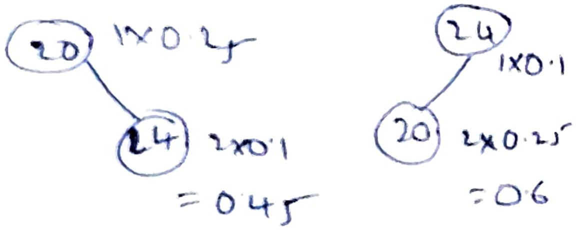
$5-3=(3,5]$

	0	1	2	3	4	5
0	0	0.2	$[0.3]^1$	$[0.95]$	$[1.45]^1$	$[1.65]^5$
1		0	0.05	$[0.5]^3$	$[1]^3$	$[1.3]^3$
2			0	0.4	$[0.9]^3$	$[1.2]^3$
3				0	0.25	$[0.5]^4$
4					0	0.1
5						0

1)


2)


3)


4)


$j-i=3$

$3-0=(0,3)$

$4-1=(1,4)$

$5-2=(2,5)$

$$\Rightarrow \min\begin{Bmatrix} 0+0.5 \\ 0.2+0.4 \\ 0.3+0 \end{Bmatrix} + 0.65$$

$$\Rightarrow \min\begin{Bmatrix} 1.15 \\ 1.25 \\ 0.95 \end{Bmatrix}$$

$\text{cost}(1,4)$ $k=2,3,4$

$$=\min\begin{Bmatrix} 0+0.9 \\ 0.05+0.25 \\ 0.5+0 \end{Bmatrix} + 0.7$$

$$=\begin{Bmatrix} 1.6 \\ \cdot \\ 1.2 \end{Bmatrix}$$

$\text{cost}(2,5)$
$(k = 3,4,5)$ $= \min \left\{ \begin{array}{l} \text{cost}(2,k-1) + \text{cost}(k,5) \\ 0.4 + 0.1 \\ 0.9 + 0 \end{array} \right\} + 0.2$

$= \min \left\{ \begin{array}{l} 1.2 \\ 1.25 \\ 1.65 \end{array} \right\}$

$j - i = 4$

$4 - 0 = (0,4)$
$5 - 1 = (1,5)$ $\quad \min \left\{ \begin{array}{l} 0 + 1 \\ 0.2 + 0.9 \\ 0.3 + 0.25 \\ 0.95 + 0 \end{array} \right\} + 0.9 = \min \left\{ \begin{array}{l} 1.9 \\ 2 \\ 1.45 \\ 1.85 \end{array} \right\}$

$j - i = (1,5)$
$k = 2,3,4,5$ $\quad = \min \left\{ \begin{array}{l} 0 + 1.2 \\ 0.05 + 0.45 \\ 0.5 + 0.1 \\ 1 + 0 \end{array} \right\} + 0.8 = \min \left\{ \begin{array}{l} 2 \\ 1.3 \\ 1.4 \\ 1.8 \end{array} \right\}$