

Analytical-2 CS40669

20) Given an array of [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 19, -10, -6, -8, 11, -9] integers sort the following elements using insertion sort using brute-force approach strategy, analyse complexity of algorithm.

Sol: Given

4 -2 5 3 10 -5 2 8 -3 6 7 -4 19 -10
-6 -8 11 -9

1) -2 4 5 3 10 -5 2 8 -3 6 7 -4 19 -10 -6 -8 11 -9

2) -2 3 4 5 10 -5 2 8 -3 6 7 -4 19 -10 -6 -8 11 -9

3) -2 3 4 5 10 -5 2 8 -3 6 7 -4 19 -10 -6 -8 11 -9

4) -5 -2 3 4 5 10 -2 8 -3 6 7 -4 19 -10 -6 -8 11 -9

5) -5 -2 2 3 4 5 10 8 -3 6 7 -4 19 -10 -6 -8 11 -9

6) -5 -2 2 3 4 5 8 10 -3 6 7 -4 19 -10 -6 -8 11 -9

7) -5 -3 -2 2 3 4 5 8 10 6 7 -4 19 -10 -6 -8 11 -9

8) -5 -3 -2 2 3 4 5 6 8 10 7 -4 19 -10 -6 -8 11 -9

9) -5 -3 -2 2 3 4 5 6 7 8 10 -4 19 -10 -6 -8 11 -9

10) -5 -4 -3 -2 2 3 4 5 6 7 8 10 1 9 -10 -6 -8 11 -9

11) -5 -4 -3 -2 1 2 3 4 5 6 7 8 10 9 -1 0 -6 -8 11 -9

12) -5 -4 -3 -2 1 2 3 4 5 6 7 8 9 10 -1 0 -6 -8 11 -9

13) -5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 9 10 0 -6 -8 11 -9

- 14) -5 -4 -3 -2 12 3 4 5 6 7 8 9 10 -6 -8 11 -9
 15) -6 -5 -4 -3 -2 -10 1 2 3 4 5 6 7 8 9 10 -8 11 -9 } 3
 16) -8 -6 -5 -4 -3 -2 -10 1 2 3 4 5 6 7 8 9 10 11 -9 }
 17) -9 -8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 } 2

Insertion sort time complexity:

Best case - $O(n)$, if the list is already sorted, where n is number of elements in the list

Average case - $O(n^2)$, if the list is randomly ordered

Worst case - $O(n^2)$, if the list is in reverse order.

- Q19) Sort the following elements using insertion sort using brute approach strategy, [38, 27, 43, 39, 82, 10, 15, 88, 52, 60, 5] and analyse complexity of the algorithm.

SOL - Given [38, 27, 43, 39, 82, 10, 15, 88, 52, 60, 5]

1) 38 27 43 39 82 10 15 88 52 60 5

2) 27 38 43 39 82 10 15 88 52 60 5

3) 27 38 43 39 82 10 15 88 52 60 5

4) 27 38 39 43 82 10 15 88 52 60 5

5) 82 27 38 39 43 210 15 188 52 60 5

- 4) 3 27 38 43 9 8 2 10 15 88 52 60 5
 5) 3 9 27 38 43 8 2 10 15 88 52 60 5
 6) 3 8 9 27 38 43 2 10 15 88 52 60 5
 7) 2 3 8 9 27 38 43 10 15 88 52 60 5
 8) 2 3 8 9 10 27 38 43 15 88 52 60 5
 9) 2 3 8 9 10 15 27 38 43 88 52 60 5
 10) 2 3 8 9 10 15 27 38 43 52 88 60 5
 11) 2 3 8 9 10 15 27 38 43 52 88 60 5
 12) 2 3 8 9 10 15 27 38 43 52 60 88 5
 13) 2 3 5 8 9 10 15 27 38 43 52 60 88

Insertion Sort time complexity:-

Best case - $O(n)$ if the list is already sorted, where n is number of elements in the list.

Avg case - $O(n^2)$ if the list is randomly ordered.

Worst case - $O(n^2)$ if the list is in reverse order.

- (Q18) Sort the array 64, 25, 12, 22, 11 using selection sort, what is the time complexity of selection sort in the best, worst and avg cases.

Sol:- Given,

64, 25, 12, 22, 11,
 ↑ ↑
 Start Min

64, 25, 12, 22, 11
 ↑ ↑
 start min

64, 25, 12, 23 11
 ↑ ↑
 Start min

64, 25, 12, 22, 11
 ↑ ↑
 Start min.

|| ||
 sorted 25, 12, 23, 64 unsorted
 array start ↑ array
 min min

|| 12 { 25, 23, 64
 { start ↑
 ↑ min min

|| 12 22 { 25, 64
 , start
 min

|| 12 22 25 { 64
 { start min

11	12	22	25	64
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Time complexity:-

Best Case - $O(n^2)$

Avg Case - $O(n^2)$

Worst Case - $O(n^2)$

Q17) Sort the array 64, 34, 25, 12, 23, 11, 90 using bubble sort. What is the time complexity of best/worst case.

Sol:- 64, 34, 25, 12, 23, 11, 90

I

64, 34, 25, 12, 22, 11, 90
 ↑ ↓
 1 2

34, 64, 25, 12, 22, 11, 90

34, 64, 25, 12, 22, 11, 90
↓ ↓
; ;

34, 25, 64, 12, 22, 11, 90
↓ ↓
; ;

34, 25, 12, 64, 22, 11, 90
↓ ↓
; ;

34, 25, 12, 22, 64, 11, 90
↓ ↓
; ;

34, 25, 12, 22, 11, 64, 90
↓ ↓
; ;

34, 25, 12, 22, 11, 64, 90

I
34, 25, 12, 22, 11, 64, 90
↓ ↓
; ;

25, 34, 12, 22, 11, 64, 90
↓ ↓
; ;

25, 12, 34, 22, 11, 64, 90
↓ ↓
; ;

25, 12, 22, 34, 11, 64, 90
↓ ↓
; ;

25, 12, 22, 11, 34, 64, 90
↓ ↓
; ;

25, 12, 22, 11, 34, 64, 90
↓ ↓
; ;

25, 12, 22, 11, 34, 64, 90
↓ ↓
; ;

III
25, 12, 22, 11, 34, 64, 90
↓ ↓
; ;

12, 25, 22, 11, 34, 64, 90
↓ ↓
; ;

12, 22, 25, 11, 34, 64, 90
↓ ↓
i j

12, 22, 11, 25, 34, 64, 90
↓ ↓
i j

12, 22, 11, 24, 34, 64, 90
↓ ↓
i j

12, 23, 13, 24, 34, 64, 90
↓ ↓
i j

12, 22, 11, 24, 34, 64, 90

12, 23, 13, 24, 34, 64, 90
↓ ↓
i j

12, 22, 11, 24, 34, 64, 90
↓ ↓
i j

12, 11, 22, 24, 34, 64, 90
↓ ↓
i j

12, 11, 22, 24, 34, 64, 90
↓ ↓
i j

12, 11, 22, 24, 34, 64, 90
↓ ↓
i j

12, 11, 22, 24, 34, 64, 90
↓ ↓
i j

11, 12, 23, 24, 34, 64, 90
↓ ↓
i j

11, 12, 22, 24, 34, 64, 90
↓ ↓
i j

11, 12, 22, 24, 34, 64, 90
↓ ↓
i j

04, 04, -> i j

) solve
and
52

Time complexity:-

Best case - $O(n)$

Worst case - $O(n^2)$

Avg. case - $O(n^2)$

6) solve the following elements using merge sort and divide and conquer strategy [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5]

Sol:-

Given,

$0 \quad 1 \quad 2 \quad 3 \cdot 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11$
 38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5

$$M = \frac{l+h}{2} = \frac{0+11}{2} = \frac{11}{2} = 5.5 \approx 6$$

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad | \quad 7 \quad 8 \quad 9 \quad 10 \quad 11$
 38, 27, 43, 3, 9, 82, 10 | 15, 88, 52, 60, 5

$$M = \frac{l+h}{2} = \frac{0+5}{2} = 3$$

$$M = \frac{l+h}{2} = \frac{7+11}{2} = \frac{18}{2} = 9$$

$0 \quad 1 \quad 2 \quad 3 \quad | \quad 4 \quad 5 \quad 6 \quad | \quad 7 \quad 8 \quad 9 \quad | \quad 10 \quad 11$
 38, 27, 43, 3 | 9, 82, 10 | 15, 88, 52 | 60, 5

$$M = \frac{l+h}{2} = \frac{0+3}{2} = 1.5 \approx 2$$

$$M = \frac{l+h}{2} = \frac{10}{2} = 5$$

$$M = \frac{7+9}{2} = 8$$

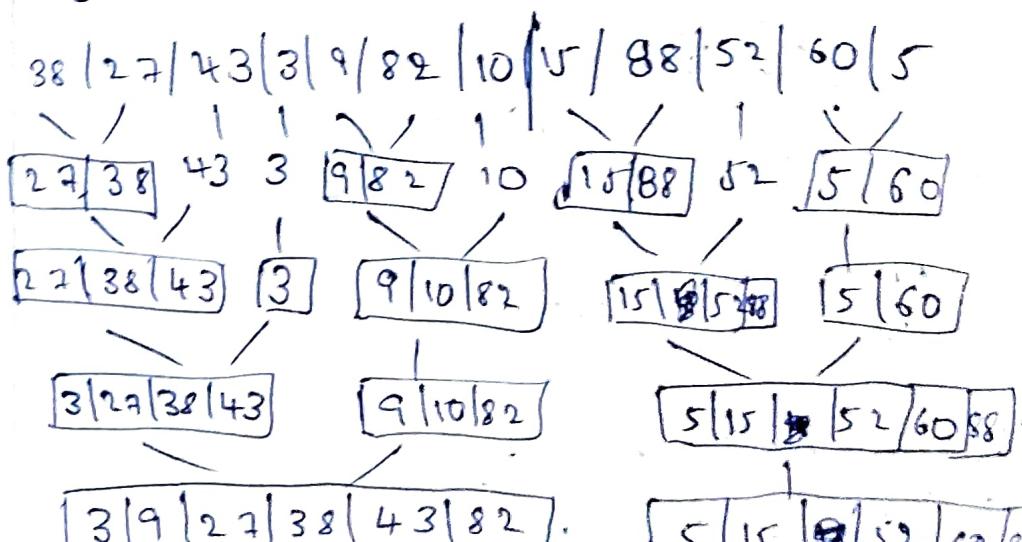
$$M = \frac{l+h}{2} = \frac{0+5}{2} = 2.5 \approx 3$$

$0 \quad 1 \quad 2 \quad | \quad 3 \quad 4 \quad 5 \quad | \quad 6 \quad 7 \quad 8 \quad | \quad 9 \quad 10 \quad | \quad 11$
 38, 27, 43 | 3 | 9, 82 | 10 | 15, 88 | 52 | 60, 5

$$M = \frac{0+2}{2} = 1$$

$0 \quad 1 \quad 2 \quad | \quad 3 \quad 4 \quad 5 \quad | \quad 6 \quad 7 \quad 8 \quad | \quad 9 \quad 10 \quad | \quad 11$
 38, 27 | 43 | 3 | 9 | 82 | 10 | 15 | 88 | 52 | 60 | 5

$$M = 0$$



3 5 9 15 27 38 43 52 60 82 88

Time complexity

Best case - $O(n^2)$

Avg case - $O(n^2)$

Worst case - $O(n^2)$

Q 15) Find the index of the target value 10 using binary search from the following list of elements

[2, 4, 6, 8, 10, 12, 14, 16, 18, 20]

Sol:- $\frac{l+h}{2} = \frac{0+9}{2} = \frac{9}{2} = 4.5 \approx 4$

$\frac{l+h}{2} = \frac{0+9}{2} = \frac{9}{2} = 4.5 \approx 4$

Key = 10.

l	h	Mid	Condition
0	9	4	$A[\text{mid}] > \text{key}$ $\text{high} = \text{mid} - 1$ $A[\text{mid}] \leq \text{key}$. return result

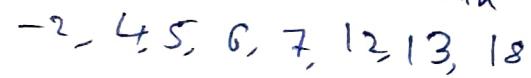
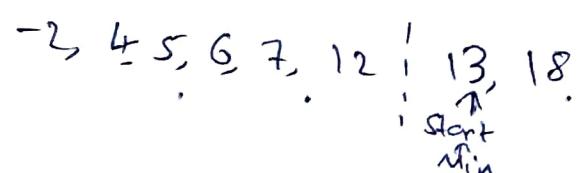
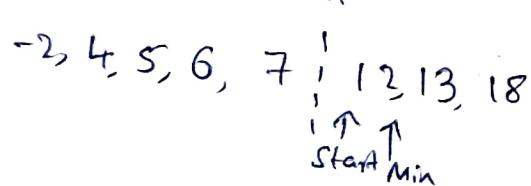
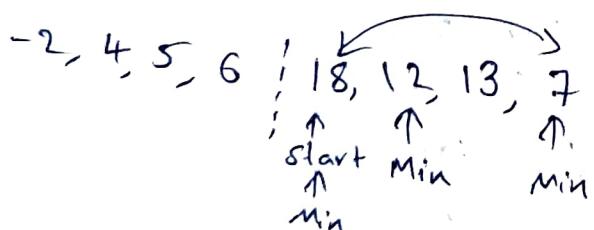
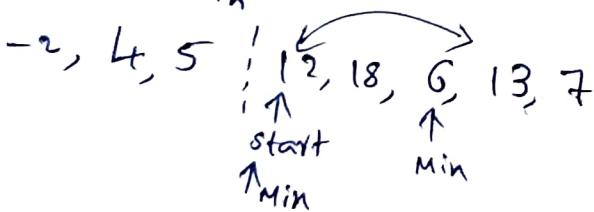
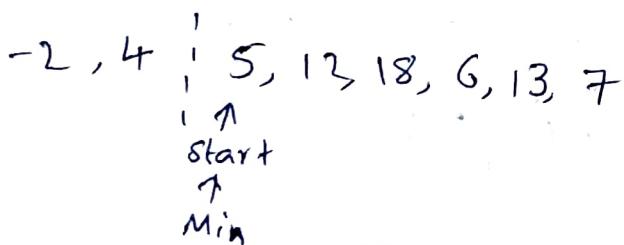
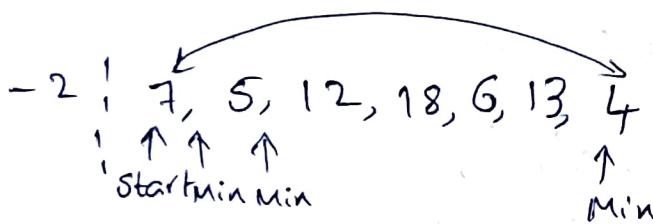
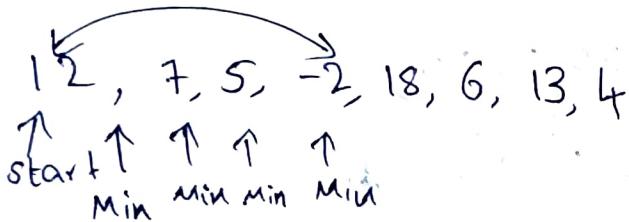
$$\therefore B = \frac{l+h}{2} = \frac{0+9}{2} = 4.5 \approx 4$$

key value = 10

Index 4 = 10. //

Find the no. of items to perform swapping for selection sort. Also estimate the time complexity for the order of notation set $S(12, 7, 5, -2, 18, 6, 13, 4)$

Sol: Given, $S(12, 7, 5, -2, 18, 6, 13, 4)$



Time complexity:-

Best case:- $O(n^2)$

Avg. case:- $O(n^2)$

Worst case:- $O(n^2)$

(iii) Apply merge sort and order the list of elements {45, 62, 12, 5, 22, 30, 50, 20}

Data set - (45, 62, 12, 5, 22, 30, 50, 20) Set up a recursive relation for the number of key comparison made in merge sort.

Sol. Given

ds (45, 62, 12, 5, 22, 30, 50, 20)

$\frac{0}{45}, \frac{1}{62}, \frac{2}{12}, \frac{3}{5}, \frac{4}{22}, \frac{5}{30}, \frac{6}{50}, \frac{7}{20}$

$$n = \frac{14h}{2} = \frac{0+7}{2} = \frac{7}{2} = 3.5 \approx 4$$

$\frac{0}{45}, \frac{1}{62}, \frac{2}{12}, \frac{3}{5}, \frac{4}{22} | \frac{5}{30}, \frac{6}{50}, \frac{7}{20}$

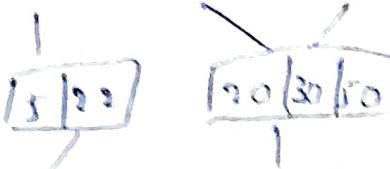
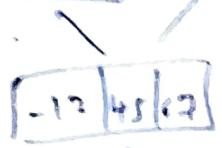
$$n = \frac{14h}{2} = \frac{0+4}{2} = \frac{4}{2} = 2 \quad n = \frac{14h}{2} = \frac{5+7}{2} = \frac{12}{2} = 6$$

$\frac{0}{45}, \frac{1}{62}, \frac{2}{12} | \frac{3}{5}, \frac{4}{22} | \frac{5}{30}, \frac{6}{50}, \frac{7}{20}$

$$n = \frac{14h}{2} = \frac{0+2}{2} = 1$$

$45|62|12|5|22|30|50|20$

$45|62|12|5|22|30|50|20$



Recurrence relation:-

$$T(n) = 2T\left(\frac{n}{2}\right) + c(n)$$

$$a=2$$

$$b=2 \quad k=1 \quad P=1$$

$$\log_2^2 = 1$$

$$\log_2 \frac{b}{a} = k$$

$$P > -1 \quad O(n^{k \log_n^{P+1}})$$

$$O(n^{1 \log_{n}^{1+1}})$$

$$= O(n \log n) = O(n \log n) //$$

Q12) Demonstrate Binary Search method to search key = 23 from the array arr[] = [25, 8, 12, 16, 23, 38, 56, 72, 91]

Sol:-

Given

$$\text{arr[]} = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10] \\ [25, 8, 12, 16, 23, 38, 56, 72, 91]$$

key = 23

$$B = \frac{0+9}{2} = \frac{19}{2} = 4.5 \approx 5$$



l	h	mid	Condition
0	9	5	$A[\text{mid}] = \text{key}$ return result

$$\therefore A[\text{mid}] = \text{key}$$

$$\therefore \text{Index of } 23 = 5 //$$

Q11) Given an array of $\{4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 10, 9, -1, 0, -6, -8, 11, -9\}$ integers, find max and min prod that can be obtained by multiplying 2 integers from array.

Sol:- Given,

$$\text{arr} [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -10, -6, -8, 11, -9]$$

Maximum no's:-

$$\text{Two Max no's} = 11, 10 = 11 \times 10 = 110 \\ (\text{largest})$$

$$\text{Two smallest no's} = -9 \times -8 = 72$$

Product = 110 is the highest (maximum)

Minimum no's:-

$$\text{Two Min no's} = 11 \times -9 = -99$$

$$\text{Two Min no's} = 11 \times -8 = -88$$

Product = -99 is the (minimum).

Q10) Solve the following recurrence relations and find the order of growth for solution.

$$T(n) = 4T(n/2) + n^2, T(1) = 1$$

Sol:- $a = 4$

$$b = 2 \quad k = 2 \quad p = 1$$

$$\log_6^9$$

$$\log_2^4 \Rightarrow \log_2^2 \Rightarrow 2 \log_2^1 = 2(1) = 2$$

$$\log_6^9 = k$$

$$P \geq -1 \quad O(nk \log^{P+1} n)$$

$$O(n^2 \log^2 n) \Rightarrow O(n^2 \log n)$$

9) Determine whether $h(n) = n \log n + n$ is in $O(n \log n)$
Prove a rigorous proof for your conclusion.

Sol:- Given,

$$f(n) = n \log n + n \in O(n \log n)$$

$$f(n) = n \log n + n \text{ and } g(n) = n \log n$$

Upper bound:

$$n \log n + n \leq c_2 n \log n$$

$$n \log n + n \leq n \log n + n \log n = 2n \log n$$

$$c_2 = 2$$

Lower bound:

$$n \log n + n \geq c_1 n \log n$$

$$n \log n + n \geq n \log n$$

$$c_1 = 1$$

Conclusion:-

$$n \log n \leq n \log n + n \leq 2n \log n$$

for all $n \geq n_0$

$h(n) = n \log n + n$ is in $\Theta(n \log n)$

- 8) Let $f(n) = n^3 - 2n^2 + n$ and $g(n) = n^2$ show that whether $f(n) \in \Omega(g(n))$ is true or false and justify your answer?

Sol:-

Given

$$f(n) = n^3 - 2n^2 + n; g(n) = n^2$$

$$f(n) \geq c \cdot g(n)$$

$$n^3 - 2n^2 + n \geq c(n^2)$$

$$n^3 - (2+c)n^2 + n \geq 0$$

choosing constants:-

$$n^3 \geq (2+c)n^2$$

$$n \geq 2+c$$

$$\text{choosing } c = 1$$

$$n^3 - 3n^2 + n \geq 0$$

Conclusion:-

$$f(n) = n^3 - 2n^2 + n \in \Omega(g(n)) //$$

Big theta Notation: Determine whether $h(n) = \ln^2 n + 3$
is $\Theta(n^2)$ or not.

Sol:- Given,

$$h(n) = 4n^2 + 3n \text{ is } \Theta(n^2)$$

$$h(n) \geq c \cdot n^2$$

$$h(n) = 4n^2 + 3n = n^2 \left(4 + \frac{3}{n}\right)$$

$h(n)$ and n^2 comparison:-

$$h(n) = n^2 \left(4 + \frac{3}{n}\right) \geq c \cdot n^2$$

$$\text{we need } n^2 \left(4 + \frac{3}{n}\right) \geq c \cdot n^2$$

$$n^2 \left(4 + \frac{3}{n}\right) \geq c \cdot n^2$$

$$4 + \frac{3}{n} \geq c$$

$$h(n) \geq c \cdot n^2 \text{ for all } n$$

$$h(n) \neq \Theta(n^2)$$

6) Big Omega Notation: Prove that $g(n) = n^3 + 2n^2 + 4n$ is
 $\omega(n^3)$.

Sol:- Given,

$$g(n) = n^3 + 2n^2 + 4n \text{ is } \omega(n^3)$$

$$c \cdot n^3 \geq h(n)$$

$$g(n) = n^3 + 2n^2 + 4n = n^2(n+2) + 4n$$

$$g(n) = n^2(n+2) + 4n \geq c \cdot n^3$$

$$n^2(n+2) + 4n \geq c \cdot n^3$$

$$n^2(n+2) + 4n \geq c \cdot n^3$$

$$n^2(n+2) + 4n - c \cdot n^3 \geq 0$$

$$n^2(n+2) + 4n - c \cdot n^3 \geq 0$$

we need:-

$$n^2(n+2) + 4n - c \cdot n^3 \geq 0$$

$$0 \cdot n^2(n+2) + 4n - c \cdot n^3 \geq 0$$

$\Rightarrow g(n) \geq c \cdot n^3$ for all n

$$g(n) \neq \Omega(n^3)$$

$\therefore g(n) = n^3 + 2n^2 + 4n$ is not $\Omega(n^3)$

5) Big O notation : $f(n) = n^2 + 3n + 5$ is $O(n^2)$

Sol:- $f(n) \leq c \cdot n^2$

$$f(n) = n^2 + 3n + 5 = n^2 + 3n + 5$$

$$f(n) = n^2 + 3n + 5 \leq c \cdot n^2$$

$$n^2 + 3n + 5 \leq c \cdot n^2$$

$$3n + 5 \leq c \cdot n^2$$

$$f(n) = n^2 + 3n + 5 \in O(n^2)$$

$$\therefore f(n) = n^2 + 3n + 5 \in O(n^2)$$

$$T(n) = \begin{cases} 2T(n-1) & \text{if } n > 0 \\ 1 & \text{otherwise} \end{cases}$$

$$\text{if } T(n) = 2T(n-1)$$

$$T(n-1) = 2[T(n-2)] = 2^2 T(n-2)$$

$$T(n) = 2^2 [2T(n-3)] = 2^3 T(n-3)$$

$$T(n) = 2^k T(n-k)$$

$$n-k=0, n=k$$

$$T(0)=1$$

$$T(n) = O(2^n) \quad //$$

$$\text{(2)} \quad T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + 1 & \text{if } n > 1 \\ 1 & \text{otherwise} \end{cases}$$

$$\text{sol: } T(n) = aT(n/6) + f(n)$$

$$\text{if } f(n) = O(n^{\log_6 a - \epsilon})$$

$$\text{then } T(n) = O(n^{\log_6 a})$$

$$\text{if } f(n) = \Theta(n^{\log_6 a} (\log k_n))$$

$$\text{then } T(n) = O(n^{\log_6 a} (\log k + \log n))$$

$$\text{if } f(n) = \Omega(n^{\log_6 a + \epsilon})$$

$$\text{then } T(n) = \Theta(f(n))$$

$$T(n) = 2T(n/2) + 1$$

$$\begin{matrix} a=2 \\ b=2 \end{matrix} \quad k=1, p=1$$

$$\log \frac{b}{a} = \log \frac{2}{2} = 1$$

$$\log \frac{b}{a} = k$$

$$p \geq -1 \Theta(n^k \log^{p+1} n)$$

$$\Theta(n^1 \log^2 n) \Rightarrow \Theta(n \log n) //$$

1) If $t_1(n) \in O(g_1(n))$ and $t_2(n) \in O(g_2(n))$, then $t_1(n) + t_2(n) \in O(\max(g_1(n), g_2(n)))$, prove that assertions.

Sol:- Given,

$$t_1(n) \leq c_1 \cdot g_1(n)$$

$$t_2(n) \leq c_2 \cdot g_2(n)$$

Consider $t_1(n) + t_2(n)$

$$t_1(n) + t_2(n) \leq c_1 \cdot g_1(n) + c_2 \cdot g_2(n)$$

Find an upper bound for $t_1(n) + t_2(n)$,

$$\max\{g_1(n), g_2(n)\} \geq g_1(n) \text{ and}$$

$$\max\{g_1(n), g_2(n)\} \geq g_2(n)$$

Therefore,

$$t_1(n) + t_2(n) \leq c_1$$

$$\max\{g_1(n), g_2(n)\} + c_2$$

$\max \{g_1(n), g_2(n)\}.$

Let $C = C_1 + C_2$ then

$t(n) + t_2(n) \leq C.$

$\max \{g_1(n), g_2(n)\}.$

Thus, $t(n) + t_2(n) \in O(\max \{g_1(n), g_2(n)\})$
 $|t(n) + t_2(n)| \leq C.$

$\max \{g_1(n), g_2(n)\}$

Thus, the statement is proven.