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Analytical-2 CSA0669

20) Given an array of [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9] integers sort the following elements using insertion sort using brute-force approach strategy, analyse complexity of algorithm.

Q

sol:- Given

4 -2 5 3 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

1) -2 4 5 3 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

2) -2 3 4 5 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

3) -2 3 4 5 10 -5 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

4) -5 -2 3 4 5 10 ~~8~~ 2 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

5) -5 -2 2 3 4 5 10 8 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

6) -5 -2 2 3 4 5 8 10 -3 6 7 -4 1 9 -1 0 -6 -8 11 -9

7) -5 -3 -2 2 3 4 5 8 10 6 7 -4 1 9 -1 0 -6 -8 11 -9

8) -5 -3 -2 2 3 4 5 6 8 10 7 -4 1 9 -1 0 -6 -8 11 -9

9) -5 -3 -2 2 3 4 5 6 7 8 10 -4 1 9 -1 0 -6 -8 11 -9

10) -5 -4 -3 -2 2 3 4 5 6 7 8 10 1 9 -1 0 -6 -8 11 -9

11) -5 -4 -3 -2 1 2 3 4 5 6 7 8 10 9 -1 0 -6 -8 11 -9

12) -5 -4 -3 -2 1 2 3 4 5 6 7 8 9 10 -1 0 -6 -8 11 -9

13) -5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 9 10 0 -6 -8 11 -9

14) -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 -6 -8 11 -9

15) -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 -8 11 -9

16) -8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 -9

17) -9 -8 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11

Insertion sort time complexity:-

Best case - $O(n)$, if the list is already sorted, where n is number of elements in the list

Average case - $O(n^2)$, if the list is randomly ordered.

Worst case - $O(n^2)$, if the list is in reverse order.

Q19) Sort the following elements using insertion sort using brute Approach strategy, [38, 27, 43, 3, 9, 8, 2, 10, 15, 88, 52, 60, 5] and analyse complexity of the algorithm.

Sol:- Given [38, 27, 43, 3, 9, 8, 2, 10, 15, 88, 52, 60, 5]

1) 38 27 43 3 9 8 2 10 15 88 52 60 5

2) 27 38 43 3 9 8 2 10 15 88 52 60 5

3) 27 38 43 3 9 8 2 10 15 88 52 60 5

4) ~~27 38 39 43 8 2 10 15 88 52 60 5~~

5) ~~8 27 38 39 43 2 10 15 88 52 60 5~~

4) 3 27 38 43 9 8 2 10 15 88 52 60 5

5) 3 9 27 38 43 8 2 10 15 88 52 60 5

6) 3 8 9 27 38 43 2 10 15 88 52 60 5

7) 2 3 8 9 27 38 43 10 15 88 52 60 5

8) 2 3 8 9 10 27 38 43 15 88 52 60 5

9) 2 3 8 9 10 15 27 38 43 88 52 60 5

10) 2 3 8 9 10 15 27 38 43 52 88 60 5

11) 2 3 8 9 10 15 27 38 43 52 88 60 5
12) 2 3 8 9 10 15 27 38 43 52 60 88 5
13) 2 3 5 8 9 10 15 27 38 43 52 60 88

Insertion sort time complexity:-

Best case - O(n) if the list is already sorted, where n is number of elements in the list

Avg case - O(n²) if the list is randomly ordered.

worst case - O(n²) if the list is in reverse order.

Q18) Sort the array 64, 25, 12, 22, 11 using selection sort, what is the time complexity of selection sort in the best, worst and avg cases.

sol:- Given,

64, 25, 12, 22, 11,
↑ ↑
start Min

64, 25, 12, 22, 11
↑ ↑
start min

64, 25, 12, 22, 11
↑ ↑
Start Min

64, 25, 12, 22, 11
↑ ↑
Start Min.

11 | 25, 12, 22, 64
Sorted array | ↑ start ↑ min | unsorted array

11 12 | 25, 22, 64
↑ start ↑ Min

11 12 22 | 25, 64
Start Min

11 12 22 25 | 64
Start Min

11	12	22	25	64

Time Complexity:-

Best Case – $O(n^2)$

Avg Case – $O(n^2)$

worst case – $O(n^2)$

Q17) Sort the array 64, 34, 25, 12, 23, 11, 90 using bubble sort. what is the time complexity of best, worst and avg case.

Sol:- 64, 34, 25, 12, 22, 11, 90

I

64, 34, 25, 12, 22, 11, 90
i j

34, 64, 25, 12, 22, 11, 90

34, 64, 25, 12, 22, 11, 90

i j

34, 25, 64, 12, 22, 11, 90

i j

34, 25, 12, 64, 22, 11, 90

i j

34, 25, 12, 22, 64, 11, 90

i j

34, 25, 12, 22, 11, 64, 90

i j

34, 25, 12, 22, 11, 64, 90

II

34, 25, 12, 22, 11, 64, 90

i j

25, 34, 12, 22, 11, 64, 90

i j

25, 12, 34, 22, 11, 64, 90

i j

25, 12, 22, 34, 11, 64, 90

i j

25, 12, 22, 11, 34, 64, 90

i j

25, 12, 22, 11, 34, 64, 90

i j

25, 12, 22, 11, 34, 64, 90

III

25, 12, 22, 11, 34, 64, 90

i j

12, 25, 22, 11, 34, 64, 90

i j

12, 22, 25, 11, 34, 64, 90
i j

12, 22, 11, 25, 34 64 90
i j

12, 22, 11, 24, 34, 64, 90
i j

12, 22, 11, 24, 34, 64, 90
i j

12, 22, 11, 24 34, 64, 90

IV

12, 22, 11, 24 34, 64, 90
i j

12, 22, 11, 24 34 64, 96
i j

12, 11, 22, 24 34, 64, 90
i j

12, 11, 22, 24 34, 64, 90
i j

12, 11, 22, 24, 34 64 90
i j

12, 11, 22, 24, 34 64, 90
i j

V

12, 11, 22, 24, 34 64 90
i j

11, 12, 22, 24, 34, 64, 90
i j

11, 12, 22, 24 34, 64 90
i j

11, 12, 22, 24 34 64, 90
i j

Time complexity:-

Best case - O(n)

worst case - O(n²)

Avg case - O(n²)

) solve
and
52

6) Solve the following elements using Merge Sort and divide and conquer strategy [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5]

Sol:- Given

0 1 2 3 4 5 6 7 8 9 10 11

38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5

$Mid = \frac{l+h}{2} = \frac{0+11}{2} = \frac{11}{2} = 5.5 \simeq 6$

0 1 2 3 4 5 6 | 7 8 9 10 11

38, 27, 43, 3, 9, 82, 10 | 15, 88, 52, 60, 5

$M = \frac{l+h}{2} = \frac{0+6}{2} = 3$ $M = \frac{l+h}{2} = \frac{7+11}{2} = \frac{18}{2} = 9$

0 1 2 3 | 4 5 6 | 7 8 9 | 10 11

38, 27, 43, 3 | 9, 82, 10 | 15, 88, 52 | 60, 5

$m = \frac{l+h}{2} = \frac{0+3}{2} = 1.5 \simeq 2$ $M = \frac{l+h}{2} = \frac{10}{2} = 5$ $M = \frac{7+9}{2} = 8$ $M = \frac{l+h}{2} = 0.5 \simeq 1$

0 1 2 4 5 7 8

38, 27, 43 | 3 | 9, 82 | 10 | 15, 88 | 52 | 60 | 5

$M = \frac{0+2}{2} = 1$

0 1

38, 27 | 43 | 3 | 9 | 82 | 10 | 15 | 88 | 52 | 60 | 5

M = 0

38 | 27 | 43 | 3 | 9 | 82 | 10 | 15 | 88 | 52 | 60 | 5

27 | 38   43   3   9 | 82   10   15 | 88   52   5 | 60

27 | 38 | 43   3   9 | 10 | 82   15 | 52 | 88   5 | 60

3 | 27 | 38 | 43   9 | 10 | 82   5 | 15 | 52 | 60 | 88

3 | 9 | 27 | 38 | 43 | 82   5 | 15 | 52 | 60 | 88

3 | 5 | 9 | 15 | 27 | 38 | 43 | 52 | 60 | 82 | 88

Time complexity

Best case $- O(n^2)$
Avg case $- O(n^2)$
Wrst Case $- O(n^2)$

Q 15) Find the index of the target value 10 using binary search from the following list of elements
[2, 4, 6, 8, 10, 12, 14, 16, 18, 20]

Sol:-

0	1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18	20

$B = \frac{l+h}{2} = \frac{0+9}{2} = \frac{9}{2} = 4.5 \simeq 4$

0	1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18	20

P_1 | P_2

key = 10

l	h	Mid	condition
0	9	4	A[mid] > key high = mid − 1 A[Mid] = = key return result

$\therefore B = \frac{l+h}{2} = \frac{0+9}{2} = 4.5 \simeq 4$

key value = 10

$\boxed{\text{Index } 4 = 10}$ //

4) Find the no. of items to perform swapping for selection sort. Also estimate the time complexity for the order of notation set S (12, 7, 5, -2, 18, 6, 13, 4)

Sol:- Given, S (12, 7, 5, -2, 18, 6, 13, 4)

12, 7, 5, -2, 18, 6, 13, 4

start Min Min Min Min

-2 ¦ 7, 5, 12, 18, 6, 13, 4

start Min Min Min

-2, 4 ¦ 5, 12, 18, 6, 13, 7

start Min

-2, 4, 5 ¦ 12, 18, 6, 13, 7

start Min Min

-2, 4, 5, 6 ¦ 18, 12, 13, 7

start Min Min Min

-2, 4, 5, 6, 7 ¦ 12, 13, 18

start Min

-2, 4, 5, 6, 7, 12 ¦ 13, 18

start Min

-2, 4, 5, 6, 7, 12, 13, 18

Time complexity:-

Best case:- $O(n^2)$

Avg. case:- $O(n^2)$

Worst case:- $O(n^2)$

Q3) Apply Merge sort and order the list of 8 elements given

Data d = (45, 67, 12, 5, 22, 30, 50, 20) Set up a recurrence relation for the number of key comparison made by merge sort.

Sol:- Given

d = (45, 67, -12, 5, 22, 30, 50, 20)

0	1	2	3	4	5	6	7
45	67	-12	5	22	30	50	20

$m = \frac{l+h}{2} = \frac{0+7}{2} = \frac{7}{2} = 3.5 \simeq 4$

45, 67, -12, 5, 22 | 30, 50, 20

$m = \frac{l+h}{2} = \frac{0+4}{2} = \frac{4}{2} = 2$ | $m = \frac{l+h}{2} = \frac{5+7}{2} = \frac{12}{2} = 6$

45, 67, -12 | 5, 22 | 30, 50 | 20

$m = \frac{l+h}{2} = \frac{0+2}{2} = 1$

45, 67 | -12 | 5 | 22 | 30 | 50 | 20

45 | 67 | -12 | 5 | 22 | 30 | 50 | 20

[45 | 67] [-12] [5 | 22] [30 | 50] [20]

[-12 | 45 | 67] [5 | 22] [20 | 30 | 50]

[-12 | 5 | 22 | 45 | 67] [20 | 30 | 50]

[-12 | 5 | 20 | 22 | 30 | 45 | 50 | 67]

Recurrence relation:-

$T(n) = 2T\left(\frac{n}{2}\right) + c(n)$

$a = 2.$

$b = 2 \quad k = 1 \quad P = 1$

$\log_2^2 = 1$

$\log_a^b = k$

$P > -1 \quad \Theta(n^k \log_n^{P+1})$

$\Theta(n^1 \log_n^{1+1})$

$= \Theta(n \log^2 n) = \Theta(n \log n)$ //

Q 12) Demonstrate Binary Search method to search key = 23 from the array arr[ ] = [2, 5, 8, 12, 16, 23, 38, 56, 72, 91]

Sol:- Given

$$\begin{array}{ccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array}$$

arr[ ] = [2, 5, 8, 12, 16, 23, 38, 56, 72, 91]

key = 23

$B = \frac{0 + \cancel{8}9}{2} = \frac{\cancel{1}9}{2} = 4.5 \approx 5$

$$\begin{array}{cccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2, & 5, & 8, & 12, & 16, & \boxed{23} & 38, & 56, & 72, & 91 \end{array}$$

Min

l	h	Mid	Condition
0	9	5	A[Mid] = key return result

∵ A[Mid] = key

∴ Index of 23 = 5 //

Q11) Given an array of $\{4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 10$
$9, -1, 0, -6, -8, 11, -9]$ integers, find max and Min prod
that can be obtained by Multiplying 2 integers
from array.

Sol:- Given,

arr $[4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9]$

Maximum no's:-

Two Max no's $= 11, 10 = 11 \times 10 = 110$.
(largest)

Two smallest no's $= -9 \times -8 = 72$

product $= 110$ is the highest (Maximum)

Minimum no's:-

Two Min no's $= 11 \times -9 = -99$

Two Min no's $11 \times -8 = -88$

product $= -99$ is the (Minimum).

Q10) Solve the following recurrence relations and find the order of growth for solution.

$T(n) = 4T(n/2) + n^2, T(1) = 1$

Sol:- $a = 4$
$b = 2$ $\quad k = 2 \quad p = 1$

$\log_b^a$

$\log_2^4 \Rightarrow \log_2^{2^2} \Rightarrow 2\log_2^2 = 2(1) = 2$

$\log_b^a = k$

$P \geq -1 \quad \Theta(n^k \log^{P+1} n)$

$\Theta(n^2 \log^2 n) \Rightarrow \Theta(n^2 \log n)$

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9) Determine whether $h(n) = n\log n + n$ is in $\Theta(n\log n)$ prove a rigorous proof for your conclusion.

Sol:- Given,

$h(n) = n\log n + n \in \Theta(n\log n)$

$f(n) = n\log n + n$ and $g(n) = n\log n$

upper bound:-

$n\log n + n \leq c_2 n\log n$

$n\log n + n \leq n\log n + n\log n = 2n\log n$

$c_2 = 2$

lower bound:-

$n\log n + n \geq c_1 n\log n$

$n\log n + n \geq n\log n$

$c_1 = 1$

Conclusion:-

$n\log n \leq n\log n + n \leq 2n\log n$

for all $n \geq n_0$

$h(n) = n\log n + n$ is in $\theta(n\log n)$

8) Let $f(n) = n^3 - 2n^2 + n$ and $g(n) - n^2$ show that whether $f(n) = \Omega(g(n))$ is true or false and justify your answer?

Sol:- Given,

$f(n) = n^3 - 2n^2 + n$; $g(n) - n^2$

$f(n) \geq c.g(n)$

$n^3 - 2n^2 + n \geq c(n^2)$

$n^3 - (2+c)n^2 + n \geq 0$

Choosing constants;-

$n^3 \geq (2+c)n^2$

$n \geq 2+c$

choosing $c = 1$

$n^3 - 3n^2 + n \geq 0$

Conclusion:-

$f(n) = n^3 - 2n^2 + n \in \Omega(g(n))$ //

Big theta Notation: Determine whether $h(n) = 4n^2 + 3n$ is $\theta(n^2)$ or not

Sol:- Given,

$h(n) = 4n^2 + 3n$ is $\theta(n^2)$

$h(n) \geq c \cdot n^2$

$h(n) = 4n^2 + 3n = n^2(4 + \frac{3}{n})$

$h(n)$ and n^2 comparison:-

$h(n) = n^2(4 + \frac{3}{n}) \geq c \cdot n^2$

we need $n^2(4 + \frac{3}{n}) \geq c \cdot n^2$

$n^2(4 + \frac{3}{n}) \geq c \cdot n^2$

$4 + \frac{3}{n} \geq c$

$h(n) \geq c \cdot n^2$ for all n

$h(n) \neq \theta(n^2)$

6) Big Omega Notation: Prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

Sol:- Given,

$g(n) = n^3 + 2n^2 + 4n$ is $\omega(n^3)$

$c \cdot n^3 \geq h(n)$

$g(n) = n^3 + 2n^2 + 4n = n^2(n+2) + 4n$

$g(n) = n^2(n+2) + 4n \geq c \cdot n^3$

$n^2(n+2)+4n \geq c \cdot n^3$

$n^2(n+2)+4n \geq c \cdot n^3$

$n^2(n+2)+4n - c \cdot n^3 \geq 0$

$n^2(n+2)+4n - c \cdot n^3 \geq 0$

we need:-

$n^2(n+2)+4n - c \cdot n^3 \geq 0$

$0 \cdot n^2(n+2)+4n - c \cdot n^3 \geq 0$

~~Ans~~ $g(n) \geq c \cdot n^3$ for all n

$g(n) \neq \Omega(n^3)$

$\therefore$ $g(n) = n^3+2n^2+4n$ is not $\Omega(n^3)$

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5) Big O notation: $f(n) = n^2+3n+5$ is $O(n^2)$

Sol:- $f(n) \leq c \cdot n^2$

$f(n) = n^2+3n+5 = n^2+3n+5$

$f(n) = n^2+3n+5 \leq c \cdot n^2$

$n^2+3n+5 \leq c \cdot n^2$

$n^2+3n+5 \leq c \cdot n^2$

$3n+5 \leq c \cdot n^2$

$f(n) = n^2+3n+5 \in O(n^2)$

$\therefore$ $f(n) = n^2+3n+5 \in O(n^2)$

$$T(n) = \begin{cases} 2T(n-1) & \text{if } n>0 \\ 1 & \text{otherwise} \end{cases}$$

$T(n) = 2T(n-1)$

$T(n-1) = 2[2T(n-2)] = 2^2 T(n-2)$

$T(n) = 2^2[2T(n-3)] = 2^3 T(n-3)$

$T(n) = 2^k T(n-k)$

$n-k=0, n=k$

$T(0)=1$

$\boxed{T(n) = O(2^n)}$ //

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2) (i) $T(n) = \begin{cases} 2T(\frac{n}{2})+1 & \text{if } n>1 \\ 1 & \text{otherwise} \end{cases}$

Sol:- $T(n) = aT(n/b) + f(n)$

if $f(n) = O(n^{\log_b a - \epsilon})$

then $T(n) = \Theta(n^{\log_b a})$

if $f(n) = \Theta(n^{\log_b a} \log^k n)$

then $T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$

if $f(n) = \Omega(n^{\log_b a + \epsilon})$

then $T(n) = \Theta(f(n))$

$T(n) = 2T(n/2) + 1$

$a = 2 \quad k = 1, \; P = 1$
$b = 2$

$\log_a^b = \log_2^2 = 1$

$\log_a^b = k$

$P \geq -1 \; \Theta(n^k \log^{p+1} n)$

$\Theta(n^1 \log^2 n) \Rightarrow \Theta(n \log n)$ //

1) If $t_1(n) \in O(g_1(n))$ and $t_2(n) \in O(g_2(n))$, then $t_1(n) + t_2(n) \in O(\max(g_1(n)), g_2(n))$, prove that assertions.

Sol:- Given,

$t_1(n) \leq c_1 \cdot g_1(n)$

$t_2(n) \leq c_2 \cdot g_2(n)$

Consider $t_1(n) + t_2(n)$

$t_1(n) + t_2(n) \leq c_1 \cdot g_1(n) + c_2 \cdot g_2(n)$

Find an upper bound for $t_1(n) + t_2(n)$,.

$\max\{g_1(n), g^2(n)\} \geq g_1(n)$ and

$\max\{g_2(n), g_2(n)\} \geq g_2(n)$

Therefore,

$t_1(n) + t_2(n) \leq c_1$
$\max\{g_1(n), g_2(n)\} + c_2$

$\max\{g_1(n), g_2(n)\}$.

Let $c = c_1 + c_2$ then

$t_1(n) + t_2(n) \leq c$.

$\max\{g_1(n), g_2(n)\}$.

Thus, $t_1(n) + t_2(n) \in O(\max\{g_1(n), g_2(n)\})$

$|t_1(n) + t_2(n)| \leq c$.

$\max\{g_1(n), g_2(n)\}$

Thus, the statement is proven.