

1) To implement the median of median algorithm ensures that you handle the worst-case time complexity efficiently while finding the  $k$ th smallest element in an unsorted array.

(i)  $arr = [12, 3, 5, 7, 19], k = 2$

Given  
 $arr = [12, 3, 5, 7, 19], k = 2$

Arrange the array in ascending order =  $\overset{\text{Index}}{\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 3 & 5 & 7 & 12 & 19 \end{bmatrix}}$   
 $k = 1 \quad 2 \quad 3 \quad 4 \quad 5$

$$\text{median} = \frac{\text{low} + \text{high}}{2} = \frac{0 + 4}{2} = 2 //$$

$$\text{median} = 7$$

As given  $k = 2$ , the value of  $(k = 2) = 5 //$

(ii)  $arr = [12, 3, 5, 7, 4, 19, 26], k = 3$

Given,  
 $arr = [12, 3, 5, 7, 4, 19, 26], k = 3$

Arrange the array in ascending order  $\overset{\text{Index}}{= \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 7 & 12 & 19 & 26 \end{bmatrix}}$   
 $k = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$

$$\text{median} = \frac{\text{low} + \text{high}}{2} = \frac{0 + 6}{2} = 3 //$$

$$\text{median} = 7$$

As given,  $k = 2$ ,

the value of  $(k = 3)$

$$= 5 //$$

(ii)  $arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$   $k = 6$

sol: Given,

$arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$   $k = 6$

Arrange the order in ascending order, it is already arranged

$$= \begin{matrix} \text{Index} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ k & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{matrix} [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$$

$$\text{Median} = \frac{\text{low} + \text{high}}{2} = \frac{0 + 9}{2} = 4.5 \approx 5$$

$\text{Median} = 6$

As, given  $k = 6$ , The value of  $(k = 6) = 6$

2) To implement a function median of median(arr, k) that takes an unsorted array arr and an integer k, and returns the kth smallest element in the array.

(i)  $arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$   $k = 6$

sol: Given,

$arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$   $k = 6$

Arrange it in ascending order, but it is already

$$\text{arranged} = \begin{matrix} \text{Index} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ k & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{matrix} [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$$

$$\text{Median} = \frac{0 + 9}{2} = 4.5 \approx 5$$

$\text{Median} = 6$

As given  $k = 6$ , The value of  $(k = 6) = 6$

## Closest pair of points

1) Given an array of points where  $\text{points}[i] = [x_i, y_i]$  represents a point

(i)  $\text{arr} = [23, 17, 31, 44, 55, 21, 20, 18, 19, 27]$   $k=5$

Given,

$\text{arr} = [23, 17, 31, 44, 55, 21, 20, 18, 19, 27]$

Arrange the order in ascending order

$$\begin{array}{cccccccccc} \text{Index} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ & [17, 18, 19, 20, 21, 23, 27, 31, 44, 55] \end{array}$$
  
$$\begin{array}{cccccccccc} k & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array}$$

$$\text{Median} = \frac{\text{low} + \text{high}}{2} = \frac{0 + 9}{2} = 4.5 \approx 5 \checkmark$$

$4.5 \approx 4$

As given  $k=5$ , the value of  $(k=5) = 21 //$

## Closest pair of points

1) Given an array of points where  $\text{points}[i] = [x_i, y_i]$  represents a point on the X-Y plane and an integer  $k$ , return the  $k$ -closest pair to the origin  $(0,0)$ .

(i)  $\text{points} = [[1,3], [-3,2], [5,8], [0,1]]$ ,  $k=2$

Given,

$\text{Points} = [[1,3], [-3,2], [5,8], [0,1]]$

Distance =  $\sqrt{x^2 + y^2}$

$$(1,3) = 1^2 + 3^2 = 10 //$$

$$[-3,2] = (-2)^2 + 2^2 = 8 //$$

$$\begin{aligned}
 [5, 8] &= 5^2 + 8^2 \\
 &= 25 + 64 \\
 &= 89
 \end{aligned}$$

$$\begin{aligned}
 [0, 1] &= 0^2 + 1^2 \\
 &= 1
 \end{aligned}$$

$$\text{Distance} = [10, 8, 89, 1]$$

Arrange the points in that order close to the origin by considering distances  
 $= [[0, 1], [-2, 2], [1, 3], [5, 8]]$

As the value  $k=2$ ,

Consider first 2 points, so the closest pair  
 $= [[0, 1], [-2, 2]]$

(ii) points =  $[[1, 3], [-2, 2]]$ ,  $k=1$

Given,

$$\text{points} = [[1, 3], [-2, 2]]$$

$$\text{Distance} = x^2 + y^2$$

$$\begin{aligned}
 [1, 3] &= 1^2 + 3^2 \\
 &= 10 //
 \end{aligned}$$

$$\begin{aligned}
 [-2, 2] &= (-2)^2 + 2^2 \\
 &= 4 + 4 = 8 //
 \end{aligned}$$

$$\text{Distance} = [10, 8]$$

Arrange the points in such a order that are close to the origin by considering distance  
 $= [[-2, 2], [1, 3]]$

As the value,  $k=1$

Consider first points, so the closest pair  
 $= [-2, 2]$



(ii) points =  $[[3, 3], [5, -1], [-2, 4]]$   $k=2$

Given

points =  $[[3, 3], [5, -1], [-2, 4]]$

Distance =  $x^2 + y^2$

$$\begin{aligned} [3, 3] &= 3^2 + 3^2 \\ &= 9 + 9 \\ &= 18 \end{aligned}$$

$$\begin{aligned} [5, -1] &= 5^2 + (-1)^2 \\ &= 25 + 1 \\ &= 26 \end{aligned}$$

$$\begin{aligned} [-2, 4] &= (-2)^2 + (4)^2 \\ &= 4 + 16 \\ &= 20 \end{aligned}$$

Distance =  $[18, 26, 20]$

As the arrangement of points should be done in such a way that are close to origin considering distances,

As the value  $k=2$ , Take two points into consideration,  $[3, 3], [-2, 4]$

2) Given four lists A, B, C, D of integer values, write a program to compute how many tuple  $(i, j, k, l)$  there are such that  $A[i] + B[j] + C[k] + D[l]$  is zero.

1)  $A = [1, 2], B = [-2, -1], C = [-1, 2], D = [0, 2]$   
from collections import defaultdict

def fourlists(A, B, C, D):

AB = ~~fourlists~~ defaultdict(int)

for a in A:

for b in B:

```
AB-sum-counts[a+b] += 1
```

```
count = 0
```

```
for c in C:
```

```
    for d in D:
```

```
        complement = -(c+d)
```

```
        if complement in AB-sum-counts:
```

```
            count += AB-sum-counts[complement]
```

```
    return count
```

```
A = [1, 2]
```

```
B = [-3, -1]
```

```
C = [-1, 2]
```

```
D = [0, 2]
```

```
print(four-sum-count(A, B, C, D))
```

(ii) 

```
A=[0], B=[0], C=[0], D=[0]
```

```
from collections import defaultdict
```

```
def four-sum-count(A, B, C, D):
```

```
    AB-sum-counts = defaultdict(int)
```

```
    for a in A:
```

```
        for b in B:
```

```
            AB-sum-counts[a+b] += 1
```

```
    count = 0
```

```
    for c in C:
```

```
        for d in D:
```

```
            complement = -(c+d)
```

```
            if complement in AB-sum-counts:
```

```
                count += AB-sum-counts[complement]
```

```
    return count
```

```
A = [0]
```

```
B = [0]
```

```
C = [0]
```

```
D = [0]
```

```
print(four-sum-count(A, B, C, D))
```