CORO2021

CLACO Lab2

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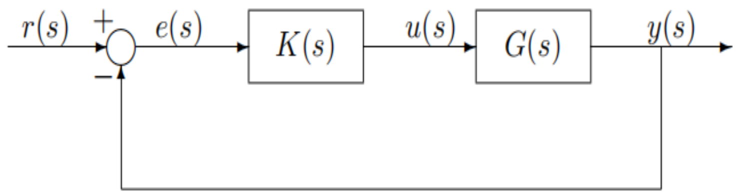
The aim of this study is to synthetize different PID type controllers, $K(s)$, with the code called ASTA for the system

$$G(s) = \frac{1}{(1+s)^2(1+10s)} \tag{1}$$

in the following closed loop architecture



1 Proportional controller

Tune a proportional control $K(s) = P$ such that the maximum of the complementary sensitivity function is $2.3db$. In order to compare the performances of the different closed loop that will be designed, note the characteristics of the step response $(M_p, t_p, t_{r5\%}, t_{r2\%}, \epsilon_r)$ and of the frequency response (M_r, ω_r). Note also the stability margins. (Note that $M_r \neq 2.3db$ and that Mp is far from 23%. What is the value of P which gives $M_r = 2.3db$?)

After several experiments, we found to get the maximum of the complementary sensitivity function is $2.3db$, the $P = 6.57$ and when $P = 5.36$, it gives $M_r = 2.3db$. The step response, Nichols plot and Bode plot is shown in figure 1, figure 2 and figure 3 respectively.

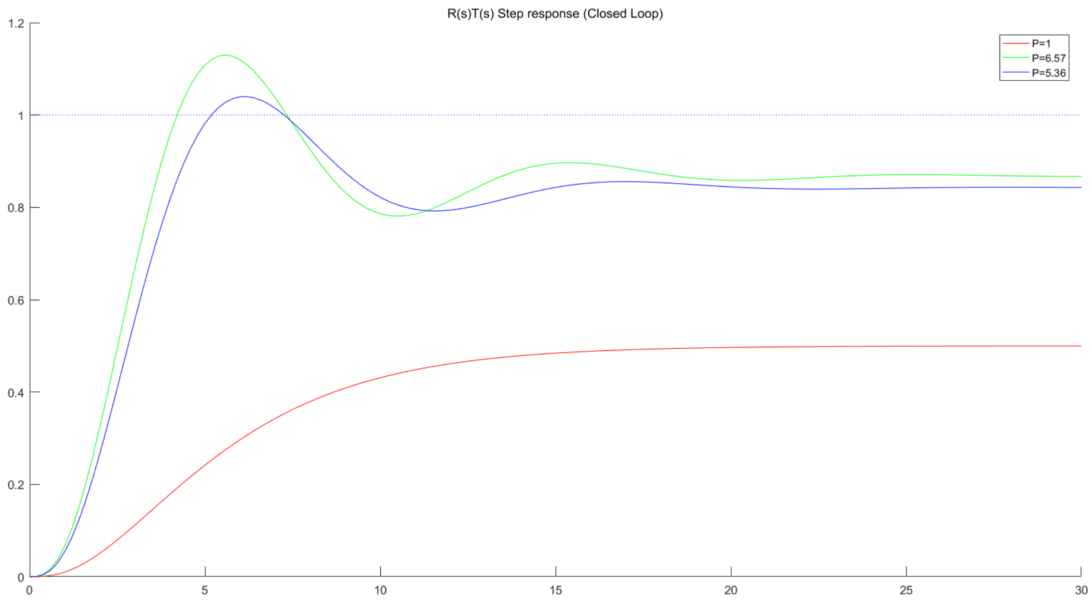


Figure 1: Step response with P controller

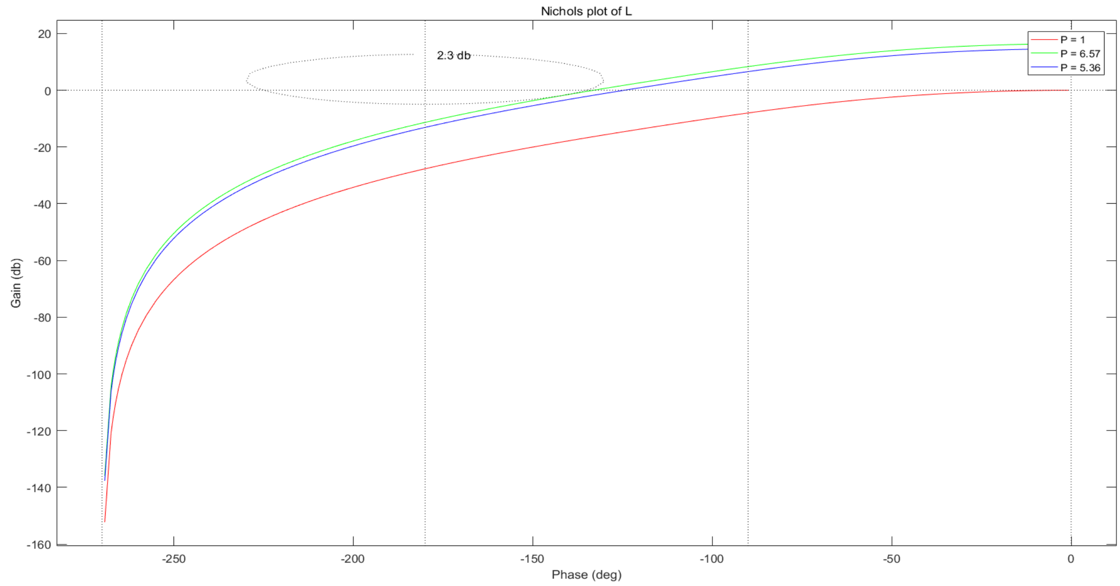


Figure 2: Nichols plot with P controller

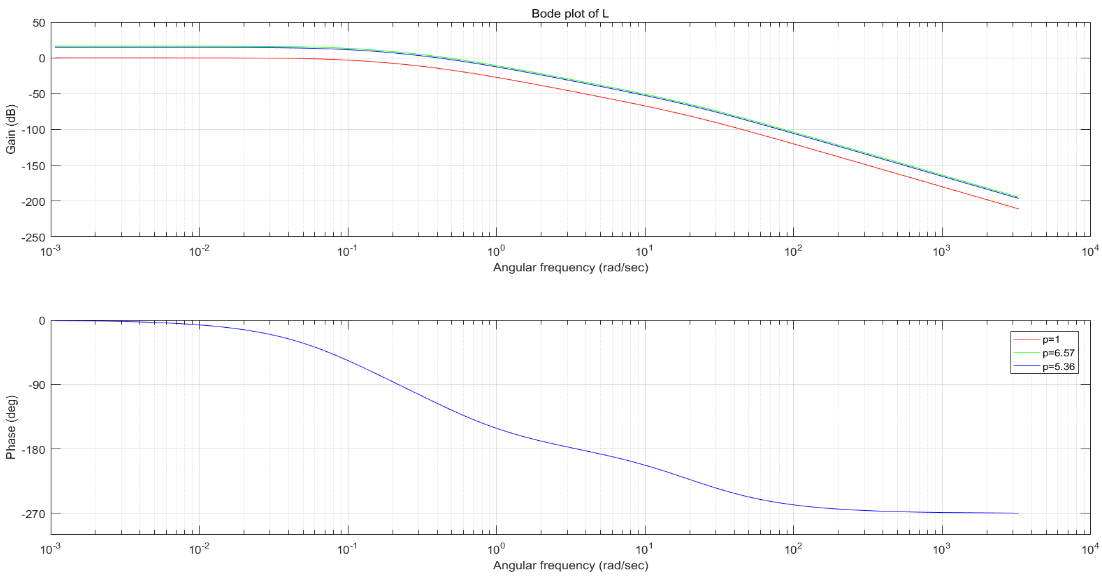


Figure 3: Bode plot with P controller

The statistic are recorded in Table 1-1.

Table 1-1 statistic 1

P	$\lvert T(jw)\rvert_{max}$(db)	$\lvert T(0)\rvert_{max}$(db)	M_r(db)	w_r(rd/s)	w_c(rd/s)
6.57	2.30	-1.23	3.53	0.57	0.92
5.36	0.81	-1.48	2.3	0.51	0.81

P	t_p(s)	$t_s(5\%)$(s)	$t_s(2\%)$(s)	$M_p(\%)$	$\epsilon_r(\%)$
6.57	5.57	12.26	16.94	30.19	13.21
5.36	6.13	12.56	13.95	23.41	15.72

2 Proportional and Integral controller

Tune a PI controller($K(s) = P(1 + \frac{1}{sT_i})$which gives the same maximum of the complementary sensitivity function ($2.3db$).

Compare the performances of the closed loop with the performances of the previous loop.

As the formula $T_i = \frac{10}{w_r} = 17.54$ with $w_r = 0.57$. We tune the value of $P = 5.6$. The step response, Nichols plot and Bode plot is shown in figure 4, figure 5 and figure 6 respectively. We can notice this system is stable in closed step response which means the error approaches 0. Compared with the previous system with proportional controller, the proportional integral controller can make the system stable. This system is tangent to the 2.3 M-circle in Nichols plot.

We can choose the value$\frac{1}{w_r} < T_i < \frac{10}{w_r}$. Finally we choose$T_i = \frac{5}{w_i} = 8.77$ to get a better PI controller. Then tune $P = 4.48$ to keep the system is tangent to 2.3 M-circle in the Nichols plot. The step response, Nichols plot and Bode plot is shown in figure 4, figure 5 and figure 6 respectively.

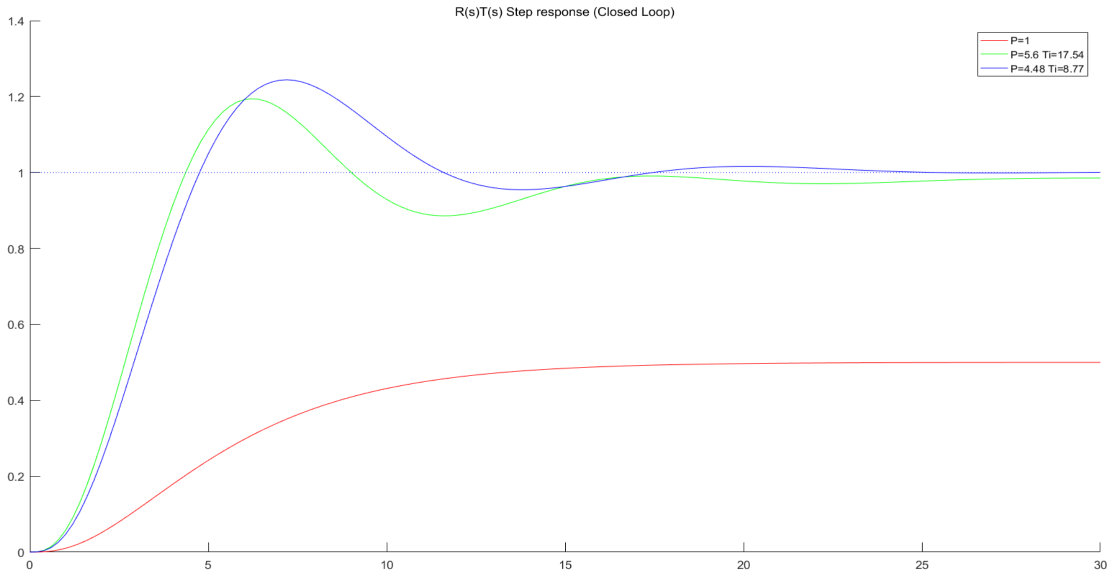


Figure 4: Step response with PI controller

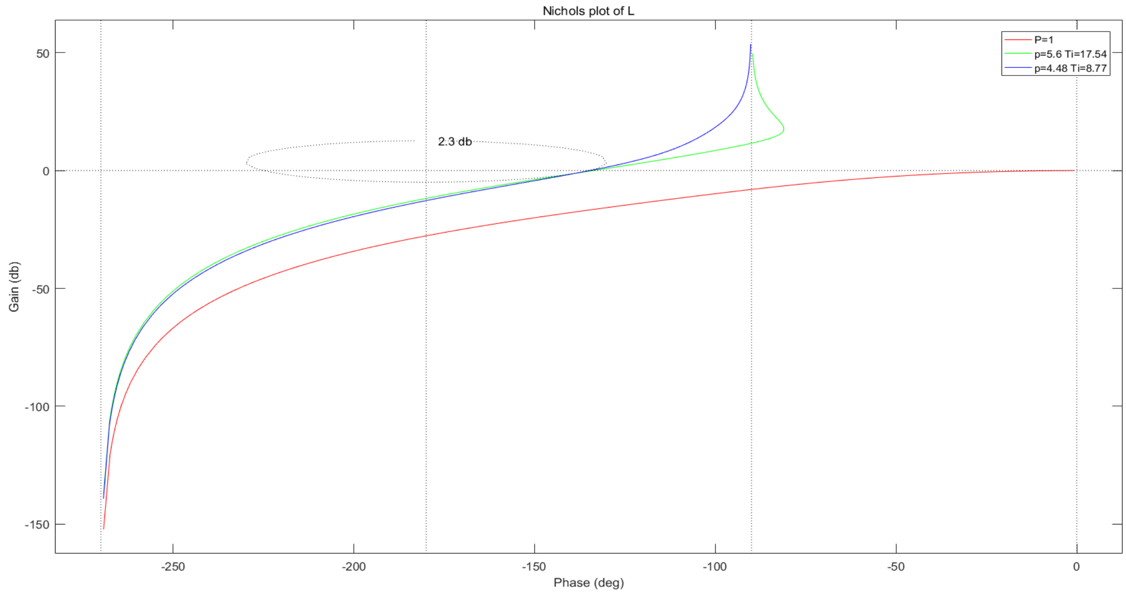


Figure 5: Nichols plot with PI controller

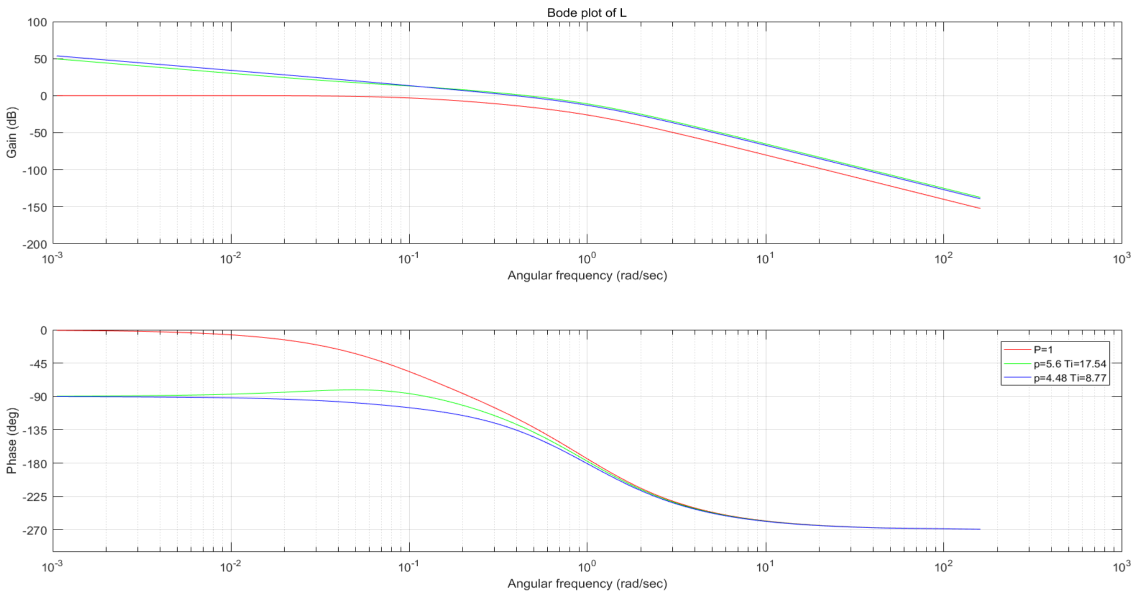


Figure 6: Bode plot with PI controller

The statistic are recorded in Table 2-1.

Table 2-1 statistic 2

P T_i	$\|T(jw)\|_{max}$(db)	$\|T(0)\|_{max}$(db)	M_r(db)	w_r(rd/s)	w_c(rd/s)
5.6 17.54	2.30	0	2.3	0.51	0.79
4.48 8.77	2.30	0	2.3	0.41	0.70

P T_i	t_p(s)	$t_s(5\%)$(s)	$t_s(2\%)$(s)	$M_p(\%)$	$\epsilon_r(\%)$
5.6 17.54	6.23	14.48	25.82	19.4	0
4.48 8.77	7.19	10.67	16.12	24.20	0

3 Lag controller

Tune a lag compensator ($K(s) = P(\frac{1+sT}{1+sTb})$ which gives the same maximum of the complementary sensitivity function (2.3db) but a steady state error equals to 5% for the step response.

Compare the performances of the closed loop with the performance of the previous proportional loops.

3.1 Lag controller design

We use an approximation of integration action.
We determine gain first to meet the e_∞ requirement.

$$u(s) = P(\frac{1+sT}{1+sTb})e(s)$$

$$b > 1, T < bT \Rightarrow \frac{1}{bT} < \frac{1}{T}$$

$$e_\infty = 0.05$$

$$\lim_{t\to\infty} se(s) = \lim_{s\to 0} s\frac{1}{1+K(s)G(s)}r(s) = \lim_{s\to 0} s\frac{1}{1+K(s)G(s)}\frac{1}{s}$$

$$= \frac{1}{1+K(0)G(0)} = \frac{1}{1+PG(0)} = \frac{1}{1+P} = 0.05$$

$\Rightarrow P = 19$

Condition 1:

$$P = 19$$

$$T = \frac{10}{w_r} = \frac{10}{0.57} = 17.54$$

Tune b such that $|T(jw)|_{max} = 2.3db$:

$$b = 3.21$$

Condition 2:

$$P = 19$$

$$T = \frac{5}{w_r} = \frac{5}{0.57} = 8.77$$

Tune b such that $|T(jw)|_{max} = 2.3db$:

$$b = 3.745$$

3.2 Results analysing

Firstly, we compare the step response in time domain and frequency domain of 2 conditions above (different T and b).

Step response plot of closed loop, Nichols plot of L and Bode plot of L, and Bode plot of T are shown in figure 7, figure 8, figure 9 and figure 10. In the plot, red line is Condition 1 with $P = 19, b = 3.21, T = 17.54$ and green line is Condition 2 with $P = 19, b = 3.745, T = 8.77$.

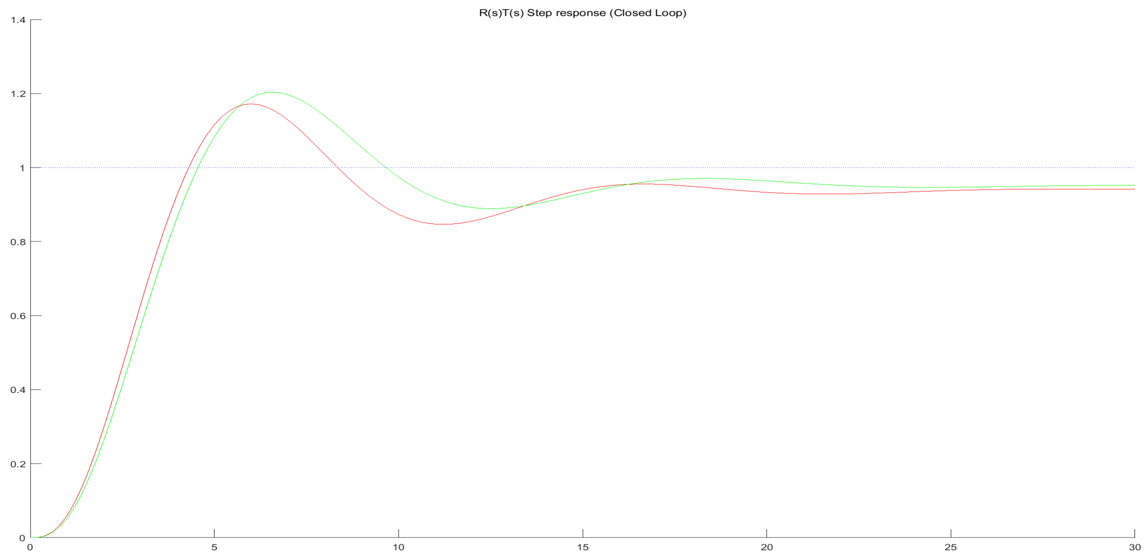


Figure 7: Step response plot of closed loop

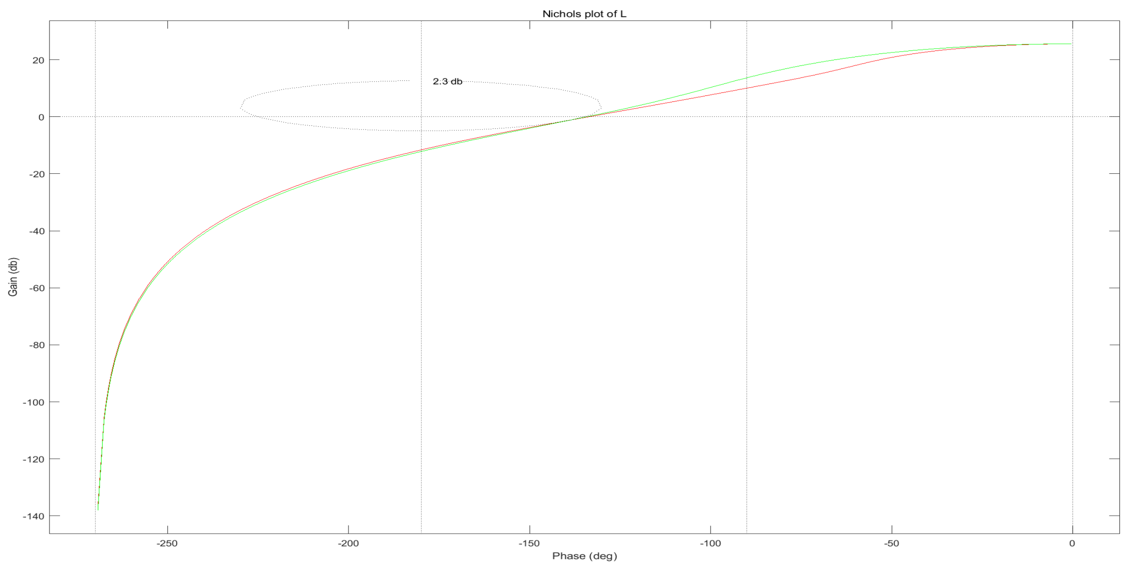


Figure 8: Nichols plot of L

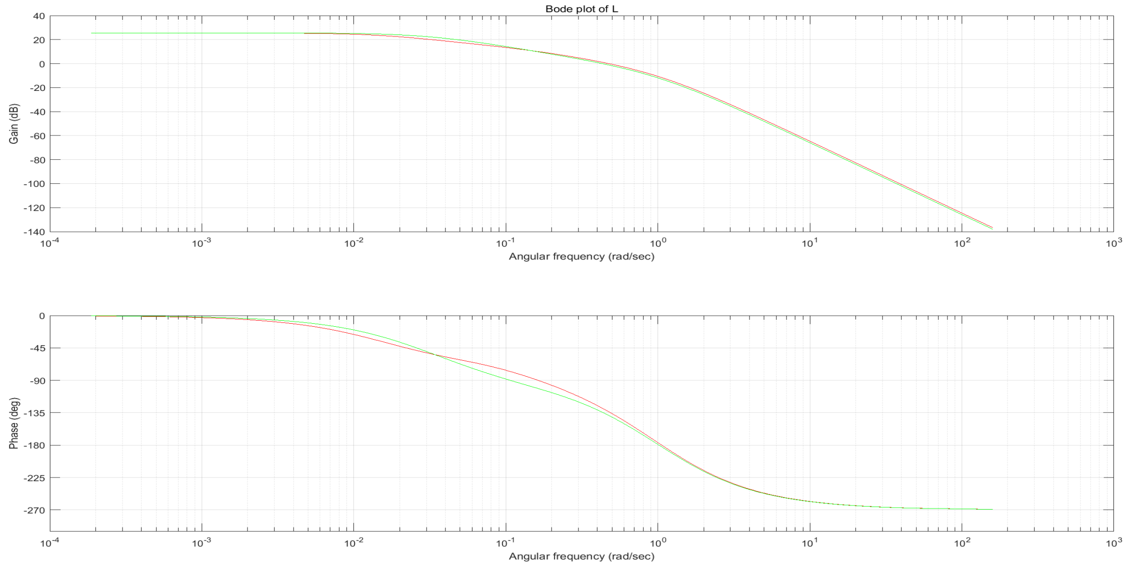


Figure 9: Bode plot of L

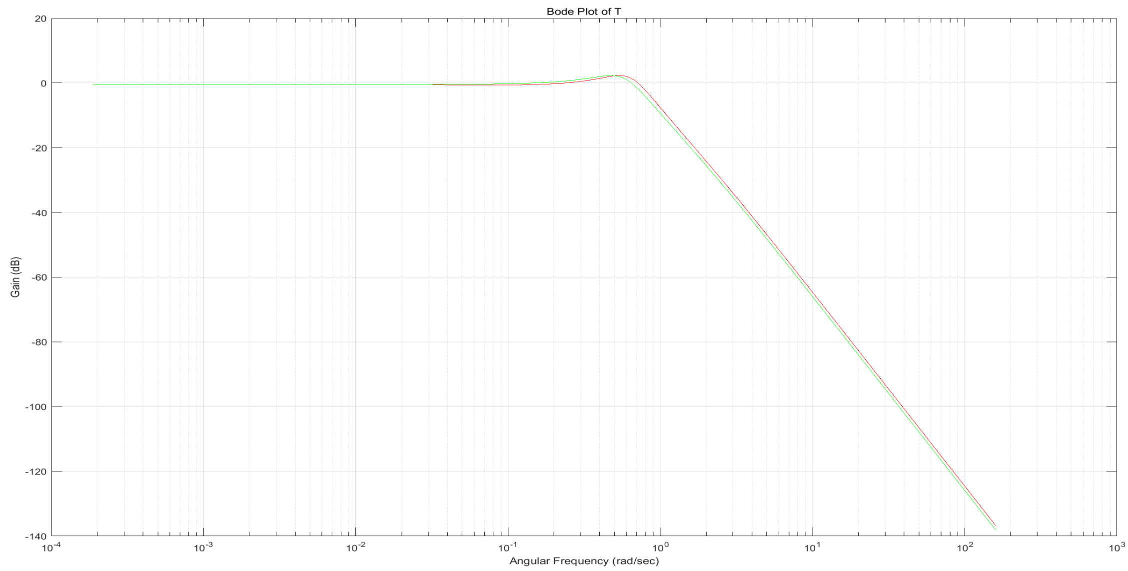


Figure 10: Bode plot of T

The statistic are recorded in Table 3-1.

Table 3-1 statistic 3

P, b, T	$\|T(jw)\|_{max}$(db)	$\|T(0)\|_{max}$(db)	M_r(db)	w_r(rd/s)	w_c(rd/s)
19, 3.21, 17.54	2.303	-0.446	2.748	0.559	0.844
19, 3.745, 8.77	2.309	-0.446	2.754	0.468	0.769

P, b, T	t_p(s)	t_s(5%)(s)	t_s(2%)(s)	M_p(%)	ϵ_r(%)
19, 3.21, 17.54	5.987	13.592	22.961	23.389	5
19, 3.745, 8.77	6.573	13.743	19.057	26.700	5

$w_r$is the resonant frequency. Usually we choose$\frac{1}{w_r} < T < \frac{10}{w_r}$. With the same $|T(jw)|_{max} = 2.3db$, noticed that t_p of condition 1 is smaller than condition 2, $t_s(2\%)$ of condition 1 is greater than condition 2. System has larger overshoot and settles faster with larger T, and responses faster with smaller T. Larger T means more integrate effect, it can make system slower but less static error. It is more reasonable to choose $T = 5/w_r = 8.77$.

Secondly, we compare step response of time domain and frequency domain of lag compensator and proportional and integral controller.

Step response plot of closed loop, Nichols plot of L and Bode plot of L, and Bode plot of T are shown in figure 11, figure 12, figure 13 , figure 14 and figure 15. In the plot, red line is Proportional and integral controller with $P = 4.48, Ti = \frac{5}{w_r} = 8.77$ and green line is Lag controller with $P = 19, b = 3.745, T = 8.77$.

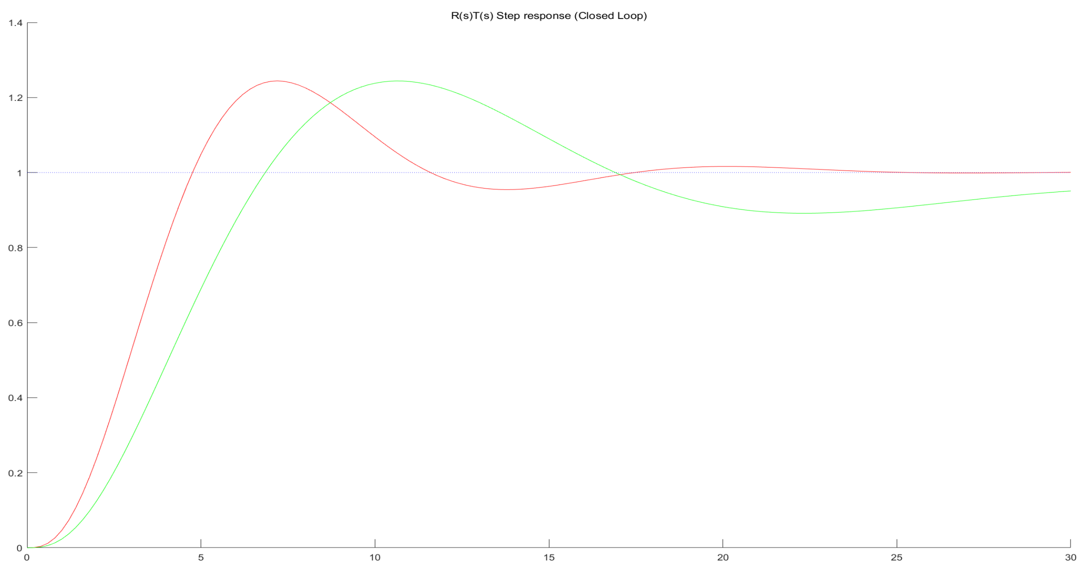


Figure 11: Step response plot of closed loop

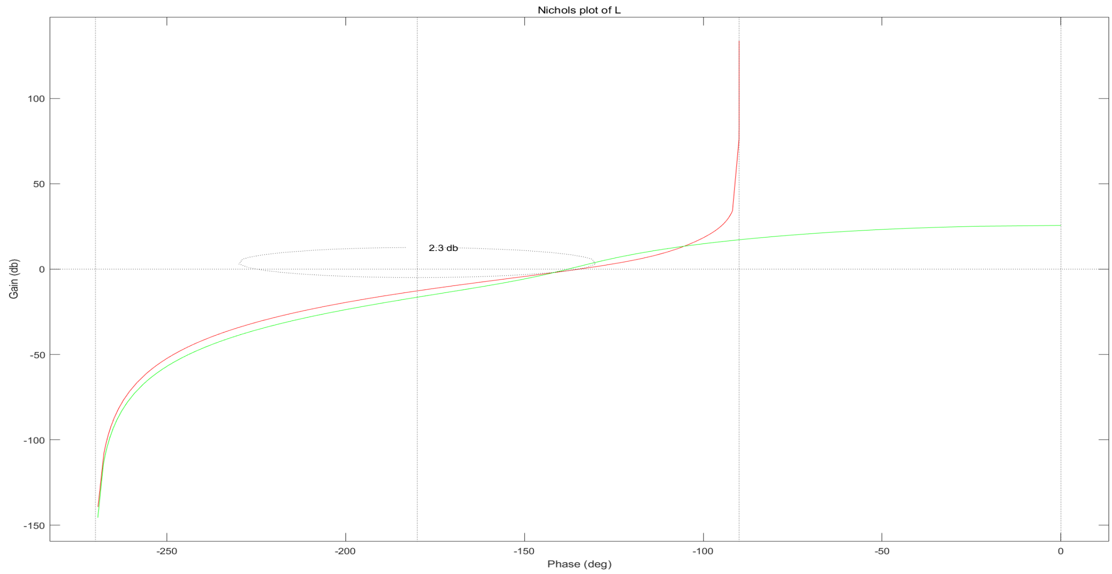


Figure 12: Nichols plot of L

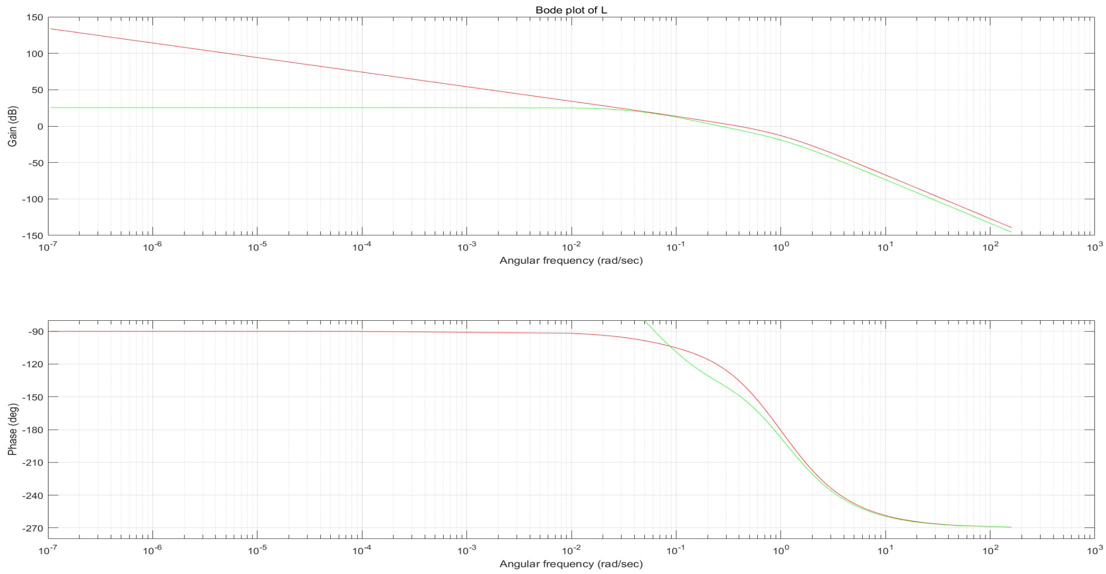


Figure 13: Bode plot of L

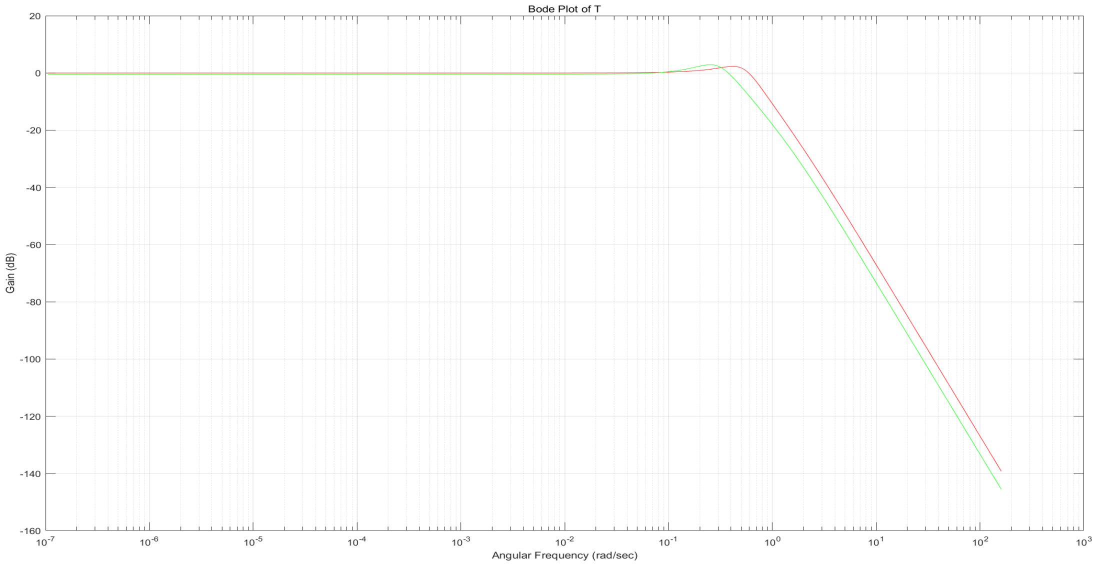


Figure 14: Bode plot of T

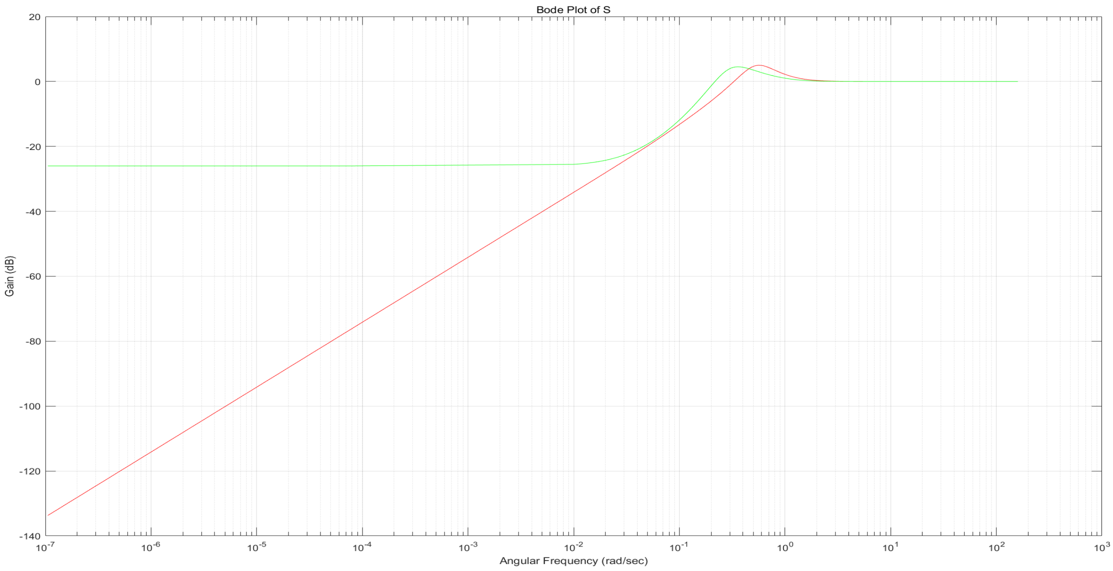


Figure 15: Bode plot of S

The statistic are recorded in Table 3-2.

Table 3-2 statistic 4

Controller	$\lvert T(jw)\rvert_{max}$(db)	$\lvert T(0)\rvert_{max}$(db)	M_r(db)	w_r(rd/s)	w_c(rd/s)
PI controller	2.3	0	2.3	0.41	0.70
Lag compensator	2.309	-0.446	2.754	0.468	0.769

Controller	t_p(s)	$t_s(5\%)$(s)	$t_s(2\%)$(s)	$M_p(\%)$	$\epsilon_r(\%)$
PI controller	7.19	10.67	16.12	24.20	0
Lag compensator	6.573	13.743	19.057	26.700	5

The aim of introduce Lag compensator is that PI controller with pure integration will cause phase lag in low frequency (which has unstable effect) and undesirable saturation (in presence of actuator limitation). In a lag compensator, pole is closer to the origin than zero.

So we want to push the negative phase to a very low frequency——lag compensator limits phase lag in low frequencies. According to the Nichols plot, PI controller faces the risk of unstability, but using lag compensator can move the curve to be away from the M-circle.

In addition, integral action has infinite increase of gain in zero frequency (according to Bode plot of S), but using a lag compensator can reduce this gain in the system without affect the phase too much to make the system more stable. The gain still increases in high frequencies.