INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

**LISTA DE EJERCICIOS 1-12**

**SEMANA 2**

NOMBRE DEL ALUMNO: GARCÍA QUIROZ GUSTAVO IVAN

GRUPO: 4CV3

**MATERIA:** MATEMATICAS AVANZADAS PARA LA INGENIERIA

**NOMBRE DEL PROFESOR:** MARTINEZ NUÑO JESUS ALFREDO

FECHA: 07/03/2023

Instituto Politécnico Nacional
Escuela Superior de Computo

Lista de Ejercicios 1-11
Semana 2

Nombre del alumno: Gustavo Ivan García Quiroz
Grupo: 3CV3

Nombre del profesor: Martinez Nuño Jose Alfredo
Materia: Matematicas avanzadas para la Ingenieria

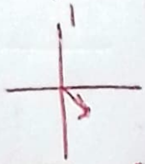
Fecha: 07/03/2023

Ejercicio 2

Exprese en forma polar cada numero complejo de los incisos siguientes

Libro: Schaum

a) $2-2i$



$r = \sqrt{4+4} = \boxed{2\sqrt{2}}$ $\quad \theta = tg^{-1}\left(\frac{2}{-2}\right) = \boxed{\frac{\pi}{-4}}$

$2-2i = 2\sqrt{2}\left(\cos\left(\frac{\pi}{-4}+2K\pi\right) + i\,sen\left(\frac{\pi}{-4}+2K\pi\right)\right)$

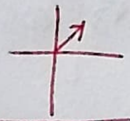
b) $-1+\sqrt{3}\,i$

$r = \sqrt{1+3} = \boxed{2}$ $\quad \theta = -tg^{-1}\left(\frac{\sqrt{3}}{1}\right) + 180°$

$\theta = \pi + \left(-\frac{\pi}{3}\right) = \left(\frac{2}{3}\pi\right)$

$-1+\sqrt{3}\,i = 2\left(\cos\left(\frac{2\pi}{3}+2K\pi\right) + i\,sen\left(\frac{2\pi}{3}+2K\pi\right)\right)$

c) $2\sqrt{2}+2\sqrt{2}i$



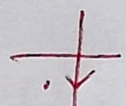
$r = \sqrt{4(2)+4(2)} = \sqrt{8+8} = \boxed{4}$

$\theta = tg^{-1}\left(\frac{2\sqrt{2}}{2\sqrt{2}}\right) = \boxed{\frac{\pi}{4}}$

$r = \sqrt{16} = 4$

$2\sqrt{2}+2\sqrt{2}i = 4\left(\cos\left(\frac{\pi}{4}+2K\pi\right) + i\,sen\left(\frac{\pi}{4}+2K\pi\right)\right)$

d) $-i$



$\boxed{r = 1}$ $\quad \theta = tg^{-1}\left(\frac{-1}{0}\right) = \boxed{\frac{3\pi}{2}}$

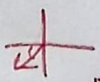
$-i = \left(\cos\left(\frac{3\pi}{2}+2K\pi\right) + i\,sen\left(\frac{3\pi}{2}+2K\pi\right)\right)$

e) $-4$

$\boxed{r = 4}$ $\quad \theta = tg^{-1}\left(\frac{0}{-4}\right) = \pi$

$-4 = 4\left(\cos(\pi+2K\pi) + i\,sen(\pi+2K\pi)\right)$

f) $-2\sqrt{3}-2i$



$r = \sqrt{4(3)+4} = \boxed{4}$ $\quad \theta = tg^{-1}\left(\frac{2}{2\sqrt{3}}\right) = \boxed{\frac{\pi}{6}}$

$-2\sqrt{3}-2i = 4\left(\cos\left(\frac{7\pi}{6}+2K\pi\right) + sen\left(\frac{7\pi}{6}+2K\pi\right)\right)$

g) $\sqrt{2}i$ $\boxed{r = \sqrt{2}}$ $\theta = tg^{-1}\left(\frac{\sqrt{2}}{0}\right) = \boxed{\frac{\pi}{2}}$

$\sqrt{2}i = \cos\left(\frac{\pi}{2}\right) + i\,sen\left(\frac{\pi}{2}\right)$

h) $\frac{\sqrt{3}}{2} - \frac{3i}{2}$

$r = \sqrt{\frac{3}{4} - \frac{9}{4}} = \sqrt{\frac{12}{4}} = \boxed{\sqrt{3}}$

$\theta = tg^{-1}\left(\frac{-3}{\sqrt{3}}\right) = \frac{\pi}{3}$

$\theta = 2\pi - \frac{\pi}{3} = \frac{7\pi - 2\pi}{3} = \boxed{\frac{5\pi}{3}}$

$\frac{\sqrt{3}}{2} - \frac{3i}{2} = \sqrt{3}\left[\cos\left(\frac{5\pi}{3} + 2\pi K\right) + i\,sen\left(\frac{5\pi}{3} + 2K\pi\right)\right]$

Libro Schaum

Ejercicio 3

1.82 Muestre que $2+1=\sqrt{5}\,e^{i\tan^{-1}\left(\frac{1}{2}\right)}$

$r=\sqrt{4+1}=\sqrt{5}$

$\theta=\tan^{-1}\left(\frac{1}{2}\right)$

$\sqrt{5}\left(\cos\left(\tan^{-1}\left(\frac{1}{2}\right)\right)+i\,\text{sen}\left(\tan^{-1}\left(\frac{1}{2}\right)\right)\right)=\boxed{\sqrt{5}\,e^{i\tan^{-1}\left(\frac{1}{2}\right)+\frac{\pi}{2}}}$

1.83 Exprese en forma polar

Ejercicio 4

a) $-3-4i$

$r=\sqrt{16+9}=\boxed{\sqrt{25}}$

$\theta=\tan^{-1}\left(\frac{4}{3}\right)=\boxed{53.13^\circ}+\boxed{180^\circ}=\boxed{233.13^\circ}$

$z=\sqrt{25}\left[\cos\left(233.13^\circ+2k\pi\right)+i\,\text{sen}\left(233.13^\circ+2k\pi\right)\right]$

a) $1-2i$

$r=\sqrt{4+1}=\sqrt{5}$

$\theta=\tan^{-1}\left(\frac{-2}{1}\right)=$

$z=\sqrt{5}\left[\cos\left[\tan^{-1}(-2)+2k\pi\right]+i\,\text{sen}\left(+\tan^{-1}(-2)+2k\pi\right)\right]$

1.97

Ejercicio 5

a) $z^4+81=0$

$z=\sqrt[4]{81}=\sqrt[4]{81}\left(\cos\frac{(\pi+2k\pi)}{4}\right)+i\,\text{sen}\,\frac{(\pi+2k\pi)}{4}$

$z_0=3\left[\cos\left(\frac{\pi}{4}\right)+i\,\text{sen}\left(\frac{\pi}{4}\right)\right]$ $\quad z_1=3\left(\cos\left(\frac{3\pi}{4}\right)+i\,\text{sen}\left(\frac{3\pi}{4}\right)\right)$

$z_2=3\left[\cos\left(\frac{5\pi}{4}\right)+i\,\text{sen}\left(\frac{5\pi}{4}\right)\right]$ $\quad z_3=3\left[\cos\left(\frac{7\pi}{4}\right)+i\,\text{sen}\left(\frac{7\pi}{4}\right)\right)$

b) $z^6=\sqrt{3}i-1$ $\quad z=\sqrt[6]{\sqrt{3}i-1}$

$z=\sqrt[6]{2}\left[\cos\left(\left(\frac{1}{6}\right)\left(\frac{2\pi}{3}+2k\pi\right)\right)+i\,\text{sen}\left(\frac{1}{6}\left(\frac{2\pi}{3}+2k\pi\right)\right)\right]$ $\quad e^{i\left(\frac{2\pi}{18}+\frac{\pi k}{3}\right)}$

$z_0=\sqrt[6]{2}\,e^{i\left(\frac{\pi}{9}\right)}$ $\quad z_1=\sqrt[6]{2}\,e^{i\left(\frac{4\pi}{9}\right)}$ $\quad z_2=e^{\frac{7\pi}{9}i}\sqrt[6]{2}$

$z_3=\sqrt[6]{2}\,e^{i\frac{10\pi}{9}}$ $\quad z_4=\sqrt[6]{2}\,e^{i\frac{13\pi}{9}}=\sqrt[6]{2}\,e^{i\frac{13\pi}{6}}$ $\quad z_5=\sqrt[6]{2}\,e^{i\frac{15\pi}{9}}$

Libro = Makurenko

12. Calcular;

a) $\left(\frac{1+i\sqrt{3}}{1-i}\right)^{40}$

$1+i\sqrt{3} = 2\left(\cos\left(\frac{\pi}{3}\right)+i\,\text{sen}\left(\frac{\pi}{3}\right)\right) = 2e^{i\frac{\pi}{3}}$

$r = 2 \quad \theta = tg^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$1-i = \sqrt{2}\left(\cos\left(\frac{7}{4}\pi\right)+i\,\text{sen}\left(\frac{7\pi}{4}\right)\right) = \sqrt{2}\,e^{i\frac{7\pi}{4}}$

$r = \sqrt{2} \quad \theta = tg^{-1}\left(\frac{1}{-1}\right) = \frac{\pi}{4}$

$2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$\frac{1+i\sqrt{3}}{1-i}\left(\frac{(1+i)}{(1+i)}\right) = \frac{1+i\sqrt{3}+i-\sqrt{3}}{1+1} \quad \frac{1-\sqrt{3}}{2}+\frac{i(1+\sqrt{3})}{2}$

Ejercicio 6

1.48 Encuentre las raices cuadradas de

a) $z^2 = 5 - 12i \qquad z = \sqrt[2]{5-12i}$

$r = \sqrt{25+144} = \sqrt{169} = 13$

$\theta = \tan^{-1}\left(\frac{-12}{5}\right) =$

$z = \sqrt[2]{13}$

$\left[\cos\left(\frac{-\tan^{-1}\left(\frac{12}{5}\right) + 2k\pi}{2}\right) + i\,\text{sen}\left(\frac{-\tan^{-1}\left(\frac{12}{5}\right) + 2k\pi}{2}\right)\right]$

$z = \sqrt[2]{13}\; e^{i\left(\frac{\pm\tan^{-1}\left(\frac{12}{5}\right) + 2k\pi}{2}\right)}$

$$\boxed{z_1 = \sqrt[2]{13}\; e^{i\left(\frac{-\tan^{-1}\left(\frac{12}{5}\right)}{2}\right)}}$$

$$\boxed{z_2 = \sqrt[2]{13}\; e^{i\left(\frac{-\tan^{-1}\frac{12}{5} + 2k\pi}{2}\right)}}$$

b) $z^2 = 8 + 4\sqrt{5}\,i$

$r = \sqrt{64 + 16(5)} = \boxed{12}$

$\boxed{\theta = \tan^{-1}\left(\frac{\sqrt{5}}{2}\right)}$

$z = \sqrt[2]{12}\left(\cos\left(\frac{\tan^{-1}\left(\frac{\sqrt{5}}{2}\right) + 2k\pi}{2}\right) + \text{sen}\left(\frac{\tan^{-1}\left(\frac{\sqrt{5}}{2}\right) + 2k\pi}{2}\right)\right)$

$$\boxed{\begin{aligned} z_0 &= \sqrt[2]{12}\, e^{i\left(\frac{\tan^{-1}\frac{\sqrt{5}}{2}}{2}\right)} \\ z_1 &= \sqrt[2]{12}\, e^{i\left(\frac{\tan^{-1}\left(\frac{\sqrt{5}}{2}\right) + \pi}{2}\right)} \end{aligned}}$$

Ejercicio 7

199 Encuentra las raices cobicas de $-11-2i$

$$z = \sqrt[3]{-11-2i}$$

$$\boxed{\begin{aligned} r &= \sqrt{125} \\ \theta &= \pi + \tan^{-1}\left[\frac{2}{11}\right] \end{aligned}}$$

$$z = (\sqrt{125})^{\frac{1}{3}} e^{i\left(\frac{1}{3}\right)\left(2k\pi + \tan^{-1}\left(\frac{2}{11}\right)\right)}$$

$$\cancel{e^{i\left(\frac{\tan^{-1}\left(\frac{2}{11}\right)}{3}\right)}}$$

$$z_0 = \sqrt[6]{125}\, e^{i\left(\frac{\tan^{-1}\left(\frac{2}{11}\right) + 2\pi}{3}\right)}$$

$$z_1 = \sqrt[6]{125}\, e^{i\left(\frac{\tan^{-1}\left(\frac{2}{11}\right) + 4\pi}{3}\right)}$$

$$z_2 = \sqrt[6]{125}\, e^{i\left(\frac{\tan^{-1}\left(\frac{2}{11}\right) + 6\pi}{3}\right)}$$

Ejercicio 6

Ejercicio 8

1.90 Demuestre que

a) $\operatorname{sen} 3\theta = 3\operatorname{sen}\theta - 4\operatorname{sen}^3\theta$ y b) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Formula de Moivre

$$\boxed{[\cos\theta + i\operatorname{sen}\theta]^n = \cos(n\theta) + i\operatorname{sen}(n\theta)}$$

$$[\cos\theta + i\operatorname{sen}\theta]^3 = \cos^3\theta + 3i\cos^2\theta\operatorname{sen}\theta + 3i^2\operatorname{sen}^2\theta\cos\theta + i^3\operatorname{sen}^3\theta$$

$$i\operatorname{sen}(3\theta) + \cos(3\theta) = \cos^3\theta - 3\operatorname{sen}^2\theta\cos\theta + 3i\cos^2\theta\operatorname{sen}\theta - i\operatorname{sen}^3\theta$$

$$\operatorname{sen}(3\theta) = 3\cos^2(\theta)\operatorname{sen}(\theta) - \operatorname{sen}^3(\theta)$$

$$\cos^2\theta + \operatorname{sen}^2\theta = 1$$

$$\cos^2\theta = 1 - \operatorname{sen}^2\theta$$

$$\boxed{\operatorname{sen}(3\theta) = 3\operatorname{sen}(\theta) - 4\operatorname{sen}^3(\theta)} = 3(1-\operatorname{sen}^2\theta)\operatorname{sen}\theta - \operatorname{sen}^3(\theta)$$

$$\cos(3\theta) = \cos^3\theta - 3\cos\theta(1-\cos^2\theta)$$

$$\boxed{\cos(3\theta) = 4\cos^3\theta - 3\cos\theta}$$

Libro: Makarenko    Ejercicio 9

12. Demostrar

a) $\left(\frac{1+i\sqrt{3}}{1-i}\right)^{40}$

$$\left[\left(\frac{1+i\sqrt{3}}{1-i}\right)\left(\frac{1+i}{1+i}\right)\right]^{40} = \left[\frac{1+i\sqrt{3}+i-\sqrt{3}}{2}\right]^{40} = \left[\frac{1}{2}\left(1-\sqrt{3}+i(1+\sqrt{3})\right)\right]^{40}$$

$$r = \sqrt{(1-\sqrt{3})^2+(1+\sqrt{3})^2} = \sqrt{1+3-2\sqrt{3}+1+3+2\sqrt{3}}$$

$$r = \sqrt{8} = \boxed{2\sqrt{2}}$$

$$\theta = \boxed{\tan^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right) = \theta}$$

$$\left[\frac{1}{2}\left(2\sqrt{2}\right)\right]^{40}\left(\cos\left(\tan^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right)(40)\right) + i\,\mathrm{sen}\left(40\tan^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right)\right)\right)$$

$$2^{20}\left[-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right] = \boxed{-2^{19}(1-i\sqrt{3})}$$

b) $(-2-2i)^7 = (2\sqrt{2})^7\left[\cos\left(\frac{35\pi}{4}\right)+i\,\mathrm{sen}\left(\frac{35\pi}{4}\right)\right]$

$$r = \sqrt{8} = 2\sqrt{2}$$

$$\theta = tg^{-1}\left(\frac{-2}{2}\right) = \frac{5\pi}{4}$$

c) $(2-2i)^7 = (2\sqrt{2})^7\left[\cos\left(\frac{-7\pi}{4}\right)+i\,\mathrm{sen}\left(-\frac{7\pi}{4}\right)\right]$

$$r = \sqrt{8} = \boxed{2\sqrt{2}}$$

$$\theta = \tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$$

$$(2)^{21}\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right) = 2^{?}(1+i)$$

$$(\sqrt{3}-i3)^6 = (2\sqrt{3})^6\left[\cos\left(\frac{-\pi}{3}\right)+i\,\mathrm{sen}\left(\frac{-\pi}{3}\right)\right]$$

$$r = \sqrt{3+9} = \sqrt{12} = 2\sqrt{3}$$

$$\theta = \tan^{-1}\left(\frac{-3}{\sqrt{3}}\right) = -\frac{\pi}{3}$$

$$= (2\sqrt{3})^6\left(\cos\left(\frac{-\pi}{18}\right)+i\,\mathrm{sen}\left(-\frac{\pi}{18}\right)\right)$$

$$(2\sqrt{3})^6\left(\cos(-2\pi)+i\,\mathrm{sen}(-2\pi)\right) = \boxed{(2\sqrt{3})^6}$$

$$2^8\left[\cos\left(\frac{16}{16}\left(-\frac{\pi}{4}\right)\right)+i\,\mathrm{sen}\left(\frac{16}{16}\left(-\frac{\pi}{4}\right)\right)\right]$$

d) $\left(\frac{1-i}{1+i}\right)^8 = \frac{1}{2^8}\left[\ \right]^8 = \frac{(1-i)^{16}}{2^8}$

$$\left[\left(\frac{1-i}{1+i}\right)\left(\frac{1-i}{1-i}\right)\right]^8 = \left[\frac{(1-i)^2}{2}\right]^8$$

$$r = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{-1}{1}\right) = \boxed{-\frac{\pi}{4}}$$

Rubo - Makarenko

~~Ejercicio 10~~

3) $\sqrt[4]{-1} = (1)^{\frac{1}{4}}$

a) $\sqrt{i}$ $\left(\cos\left(\frac{\pi}{4}\right) + i\,sen\left(\frac{\pi}{4}\right)\right) = \left(\cos\left(\frac{\pi}{4} + \frac{k\pi}{2}\right) + i\,sen\left(\frac{\pi + 2k\pi}{4}\right)\right)$

$$\boxed{\begin{aligned} z_0 &= e^{i\frac{\pi}{4}} \\ z_1 &= e^{i\left(\frac{3\pi}{4}\right)} \\ z_2 &= e^{i\left(\frac{5\pi}{4}\right)} \\ z_3 &= e^{i\left(\frac{7\pi}{4}\right)} \end{aligned}}$$

b) $\sqrt{i} = (1)^{\frac{1}{2}}\left[\cos\left(\frac{\pi/2 + 2k\pi}{2}\right) + i\,sen\left(\frac{\frac{\pi}{2} + 2k\pi}{2}\right)\right] = e^{i\left(\frac{\pi}{4} + k\pi\right)}$

$\boxed{z_0 = e^{i\frac{\pi}{4}}}$ $\boxed{z_1 = e^{i\frac{5\pi}{4}}}$ $z_2 = e^{i\frac{9\pi}{4}}$ $z_4 = e^{i\left(\frac{12\pi + \pi}{4}\right)}$

c) $\sqrt[3]{i} = \left[e^{i\left[\left(\frac{1}{3}\right)\left(\frac{\pi}{2} + 2k\pi\right)\right]}\right] = e^{i\left(\frac{\pi + 2k\pi}{6}\right)}$

$\boxed{z_0 = e^{i\left(\frac{\pi}{6}\right)}}$ $\boxed{z_1 = e^{i\left(\frac{3\pi}{6}\right)}}$ $\boxed{z_2 = e^{i\frac{5\pi}{6}}}$

d) $\sqrt[4]{-i} = (1)^{\frac{1}{4}}\left[\cos\left(\frac{1}{4}\left(\frac{3\pi}{2} + 2k\pi\right)\right) + i\,sen\left(\frac{1}{4}\left(\frac{3\pi}{2} + 2k\pi\right)\right)\right] = e^{i\left(\frac{3\pi + 4k\pi}{8}\right)}$

$\boxed{z_0 = e^{i\frac{3\pi}{8}}}$ $\boxed{z_1 = e^{i\left(\frac{7\pi}{8}\right)}}$ $\boxed{z_2 = e^{i\left(\frac{15\pi}{8}\right)}}$ $\boxed{z_3 = e^{i\left(\frac{11\pi}{8}\right)}}$

~~Ejercicio 11~~

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$\sqrt[4]{1} = e^{i\left(\frac{1}{4}\right)(2k\pi)} = e^{i\frac{k\pi}{2}}$

$\boxed{z_0 = 1}$ $\boxed{z_1 = e^{i\frac{\pi}{2}}}$ $\boxed{z_2 = e^{i\pi}}$ $\boxed{z_3 = e^{i\frac{3\pi}{2}}}$

18) $\sqrt[3]{-1+i} = (\sqrt{2})^{\frac{1}{3}} e^{i\left(\frac{1}{3}\right)\left(\frac{3\pi}{4} + 2k\pi\right)} = \sqrt[6]{2}\, e^{i\left(\frac{\pi}{4} + \frac{2k\pi}{3}\right)}$

$z_0) \sqrt[6]{2}\, e^{i\left(\frac{\pi}{4}\right)}$ $\boxed{z_0 = e^{i\frac{\pi}{4}} \sqrt[6]{2}}$ $\boxed{z_1 = \sqrt[6]{2}\, e^{i\left(\frac{\pi}{4} + \frac{2\pi}{3}\right)}}$ $\boxed{z_2 = \sqrt[6]{2}\, e^{i\left(\frac{\pi}{4} + \frac{4\pi}{3}\right)}}$

$\boxed{z_3 = \sqrt[6]{2}\, e^{i\left(\frac{\pi}{4} + \frac{8\pi}{3}\right)}}$

c) $\sqrt{2 - 2\sqrt{3}\,i} = \sqrt[4]{16}\left[\cos\left(\left(\frac{1}{2}\right)\left(\frac{\pi}{3} + 2k\pi\right)\right) + sen\left[\left(\frac{1}{2}\right)\left(\frac{\pi}{3} + 2k\pi\right)\right]\right]$

$\sqrt{2 - 2\sqrt{3}\,i} = 2\, e^{i\left(\frac{\pi + 6k\pi}{6}\right)}$

$\boxed{z_0 = 2e^{i\frac{\pi}{6}}}$ $\boxed{z_1 = 2e^{i\left(\frac{7\pi}{6}\right)}}$

Libro: Makarenko — Ejercicio 9

Libro: Schaums

1.54 Suponga que $z_1=1-i$ $z_2=-2+4i$ y $z_3=\sqrt{3}-2i$. Evalúe los incisos siguientes

a) $z_1^2+2z_1-3=(1-i)^2+2(1-i)-3=1-2i+i^2+2-2i-3$
$3-3-4i+i^2=\boxed{-1-4i}$

b) $|2z_2-3z_1|^2=|2(-2+4i)-3(1-i)|^2=|-4+8i-3+3i|^2$
$(\sqrt{(-7)^2+(11)^2})^2=\boxed{170}$

c) $|z_3-\overline{z_3}|^5=|(\sqrt{3}-2i)-(\sqrt{3}+2i)|^5=|-2i-2i|^5=|-4i|^5$
$(\sqrt{16})^5=\boxed{1024}$

d) $|z_1\overline{z_2}+z_2\overline{z_1}|=|(1-i)(-2-4i)+(-2+4i)(1+i)|$
$|-2-4i+2i+4i^2+4i+4i^2-2-2i|=|-8-4|=\sqrt{(12)^2}=\boxed{12}$

e) $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|=\left|\frac{(1-i)+(-2+4i)+1}{(1-i)-(-2+4i)+i}\right|=\left|\frac{1-2+1-i+4i}{1+2-i-4i+i}\right|=\left|\frac{3i}{3-4i}\right|$
$\left|\frac{3i}{3-4i}\left(\frac{3+4i}{3+4i}\right)\right|=\left|\frac{9i-12}{9-16i^2}\right|=\left|\frac{9i-12}{25}\right|=\sqrt{\left(\frac{9}{25}\right)^2+\left(\frac{12}{25}\right)^2}=\boxed{\frac{3}{5}}$

f) $\frac{1}{2}\left(\frac{z_3}{\overline{z_3}}+\frac{\overline{z_3}}{z_3}\right)=\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}-2i}{\sqrt{3}+2i}+\frac{\sqrt{3}+2i}{\sqrt{3}-2i}\right)=\left(\frac{1}{2}\right)\left(\frac{(\sqrt{3}-2i)^2+(\sqrt{3}+2i)^2}{3+4}\right)$
$\left(\frac{1}{2}\right)\left(\frac{3-4\sqrt{3}i+4i^2+3+4\sqrt{3}i+4i^2}{5+2}\right)=\frac{6+8i^2}{10+4}=\frac{6-8}{10+4}=\frac{-1}{5+2}=\boxed{-\frac{1}{7}}$

g) $(\overline{z_2+z_3})(z_1-z_3)=(\overline{-2+4i+\sqrt{3}-2i})((1-i)-(\sqrt{3}-2i))$
$(\sqrt{3}-2+2i)(1-\sqrt{3}+i)=\sqrt{3}-2+2i-3+2\sqrt{3}-2\sqrt{3}i+2i-2\sqrt{3}i+2i$
$\boxed{3\sqrt{3}-7+\sqrt{3}i}$

h) $|z_1^2+\overline{z_2}^{-2}|^2+|\overline{z_3}^{-2}-z_2^{2}|^2=|(1-i)^2+\left(\frac{1}{-2+4i}\right)^2|^2+|\left(\frac{1}{\sqrt{3}-2i}\right)^2-(-2+4i)^2|^2$
$|-2i\ -\frac{7}{100}|^2+|\frac{1+4\sqrt{3}i}{49}-3+4i|^2=\sqrt{\left(\frac{2-3\sqrt{10}}{7}\right)}+\left(\frac{2}{7}\right)+\sqrt{\left(\frac{4\sqrt{3}}{7}\right)^2}$

i) $Re\{2z_1^3+3z_2^2-5z_3^2\}=$
$\sqrt{(35)^2+(52+20\sqrt{3})^2}=$

Ejercicio 12

18 $z = \sqrt[5]{\sqrt{2}\left[\cos\left(\frac{\pi}{6}\right) + i\,\text{sen}\left(\frac{\pi}{6}\right)\right]} = (2)^{\frac{1}{10}}\left[\cos\left(\frac{\pi}{6} + 2k\pi\right)\left(\frac{1}{5}\right) + i\,\text{sen}\left(\frac{\pi}{6} + 2k\pi\right)\left(\frac{1}{5}\right)\right]$

$z = \sqrt[10]{2}\, e^{i\left(\frac{\pi + 12k\pi}{30}\right)}$

$z_0 = \sqrt[10]{2}\, e^{i\left(\frac{\pi}{30}\right)}$

$z_1 = \sqrt[10]{2}\, e^{i\frac{13\pi}{30}} = \sqrt[10]{2}\, e^{i\frac{13\pi}{30}}$

$z_3 = \sqrt[10]{2}\, e^{i\frac{25\pi}{30}}$

$z_4 = \sqrt[10]{2}\, e^{i\frac{37\pi}{30}}$

$z_5 = \sqrt[10]{2}\, e^{i\frac{49\pi}{30}}$

h) $|z_1^2 + \overline{z_2}^2|^2 + |\overline{z_3}^2 - \overline{\overline{z_2}}^2|^2$

$|(1-i)^2 + (-2-4i)^2|^2 + |(\sqrt{3}+2i)^2 - (-2+4i)^2|^2$

$|1-2i+i^2+4+16i^2+8i|^2 + |3+4i^2+4\sqrt{3}i - (4+16i^2-8i)|^2$

$|5+17i^2+6i|^2 + |-1-12i^2+4\sqrt{3}i+8i|^2$

$|-12+6i|^2 + |11+i(4\sqrt{3}+8)|^2$

$(\sqrt{(-12)^2+(6)^2})^2 + (\sqrt{(11)^2+(4\sqrt{3}+8)^2})^2 =$

i) $Re\{2z_1^3 + 3z_2^2 - 5z_3^2\}$

$2z_1^3 + 3z_2^2 - 5z_3^2 = 2(1-i)^3 + 3(-2+4i)^2 - 5(\sqrt{3}-2i)^2$

$2(1-2i-1)(1-i) + 3(4+16i^2-16i) - 5(3+4i^2-4\sqrt{3}i)$

$2(-2-2i) + 12+48i^2-48i - 15-20i^2+20\sqrt{3}i$

$-4-4i-48i+12-48-15+20+20\sqrt{3}i$

$-35+i(20\sqrt{3}-52)$

$\boxed{Re\{-35\}}$