INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

LISTA DE EJERCICIOS 1-12

SEMANA 3

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GRUPO: 4CV3

MATERIA: MATEMATICAS AVANZADAS PARA LA INGENIERIA

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Ejercicio 1   Complex Analisys

1.100 Libro Schaum

Resuelva las ecuaciones siguientes. Encuentre todas las raices

a) $5z^2+2z+10=0$

$$z_{1,2}=\frac{-2\pm\sqrt{4-4(5)(10)}}{2(5)}=-\frac{1}{5}\pm\frac{\sqrt{196}}{10}=\frac{-1}{5}\pm\frac{i14}{10}$$

$$\boxed{z_{1,2}=-\frac{1}{5}\pm i\frac{7}{5}}$$

b) $z^2+(i-2)z+(3-i)=0$

$$z_{1,2}=\frac{-i+2\pm\sqrt{(i-2)^2-4(3-i)(1)}}{2(1)}=\frac{-i+2\pm\sqrt{3-4i-12+4i}}{2}$$

$$\frac{-i+2\pm i3}{2}=z_1,z_2$$

$\boxed{z_1=1+i}$ $\boxed{z_2=1-2i}$

1.101 Resuelva   Ejercicio 2

$$z^5-2z^4-z^3+6z-4=0$$

División Sintética

1	-2	-1	0	6	-4	1
	1	-1	-2	-2	4	
1	-1	-2	-2	4	0	1
	1	0	-2	-4		
1	0	-2	-4	0	0	
1	0	-2	-4	0	0	2
	2	4	4			
1	2	2	0	0	0	

$$(z-1)^2(z-2)(z^2+2z+2)$$

$$z_{1,2}=\frac{-2\pm\sqrt{4-4(2)}}{(2)(1)}=-1\pm i$$

$\boxed{z_1=1}$ $\boxed{z_2=1}$ $\boxed{z_3=2}$ $\boxed{z_4=-1+i}$ $\boxed{z_5=-1-i}$

1,102 Encuentre todas la raíces de $z^4+z^2+1=0$

Ejercicio 3

$$z_{1,2} = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$z_{1,2} = \pm\sqrt{\frac{-1 \pm i\sqrt{3}}{2}} \qquad z_{3,4} = \pm\sqrt{\frac{-1 \pm i\sqrt{3}}{2}}$$

$$(z^2)^2 = \left(\frac{-1 \pm i\sqrt{3}}{2}\right)^2$$

$$z^4 = \frac{(-1 \pm i\sqrt{3})^2}{4}$$

$$z_{1,2} = \frac{(-1 \pm i\sqrt{3})^{\frac{1}{2}}}{\sqrt{2}} = (\sqrt{4})^{\frac{1}{2}}\left(\cos\left(\frac{2\pi}{6}\right) + i\,\mathrm{sen}\left(\frac{2\pi}{6}\right)\right) = \pm\sqrt{\frac{-1 \pm i\sqrt{3}}{2}}$$

$$z_{1,2} = \frac{\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}2} \pm \frac{i\sqrt{3}}{2}\right) = \left(\frac{1}{2}\right)(1 \pm i\sqrt{3})$$

$$z_{3,4} = \sqrt{\frac{-1 - i\sqrt{3}}{2}} = \frac{1}{\sqrt{2}}\left[\sqrt{2}\left(\cos\left(\frac{4\pi}{3}\right) + i\,\mathrm{sen}\left(\frac{4\pi}{3}\right)\right)\right]$$

$$\boxed{z_{3,4} = (-1 \pm i\sqrt{3})^{\frac{1}{2}}}$$

$$\boxed{z_{3,4} = \frac{1}{2}(-1 \pm i\sqrt{3})}$$

$$\boxed{z_{1,2} = \frac{1}{2}(1 \pm i\sqrt{3})}$$

Ejercicio 4

1,104 $4 = x + y$

$xy = 8$

$\boxed{x = 4 - y}$

$(4-y)\,y = 8$

$-y^2 + 4y - 8 = 0$

$y^2 + 8 - 4y = 0$

$$y_{1,2} = \frac{4 \pm \sqrt{16 - 4(8)(1)}}{2(1)} = \frac{4 \pm i4}{2} = \frac{4 \pm i4}{2} = \boxed{2 \pm i2}$$

$\boxed{x_{1,2} = 4 - 2 \pm 2}$

En los problemas siguientes hallar conjuntos de puntos en el plano de la variable compleja z que se determinan por las condiciones dadas:

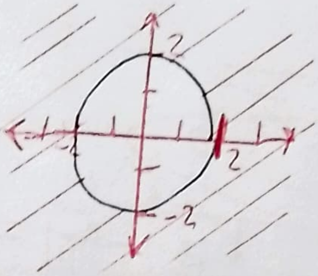
Libro: Makarenko Ejercicio 5

19. a) $|z| \geq 2$

$z = x + iy \qquad z = x + iy$

$a = h + ik \qquad a = 0$

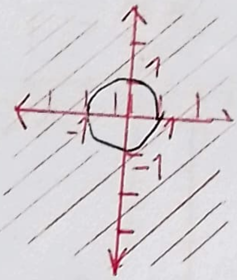
Centro (h,k) $\boxed{\begin{matrix} r = 2 \\ C(h,k) = 0 \end{matrix}}$



b) $\dfrac{1}{|z|} \geq 1 \qquad z \neq 0$

$|z| \leq 1$

$\boxed{\begin{matrix} r = 1 \\ C(h,k) = 0 \end{matrix}}$

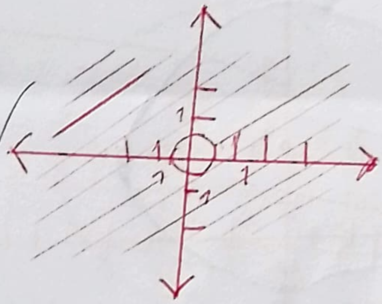


c) $\left|\dfrac{1}{z}\right| \leq 2$

$|1| \leq 2|z|$

$\dfrac{1}{2} \leq |z| \qquad \boxed{|z| \geq \dfrac{1}{2}}$

$\boxed{\begin{matrix} r = \frac{1}{2} \\ C(h,k) = 0 \end{matrix}}$



Ejercicio 6

22. $\left|\dfrac{z-1}{z+1}\right| \leq 1$

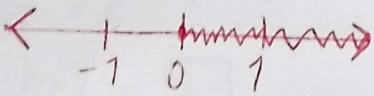
$\dfrac{z-1}{z+1} \leq 1 \qquad \dfrac{z-1}{z+1} \geq -1$

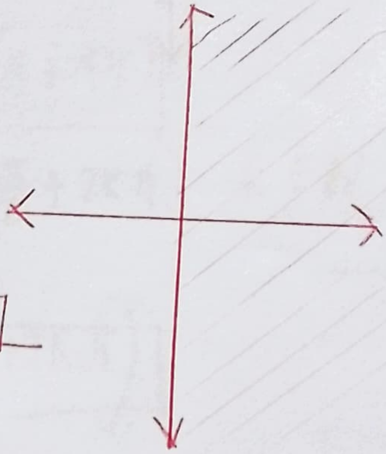
$\dfrac{z-1}{z+1} - 1 \leq 0 \qquad \dfrac{z-1}{z+1} + 1 \geq 0$

$\dfrac{-2}{z+1} \leq 0 \qquad \dfrac{2z}{z+1} \geq 0$

$z \leq 2 \qquad \boxed{z \geq 0} \quad \boxed{z > -1}$

$\boxed{z < -1}$



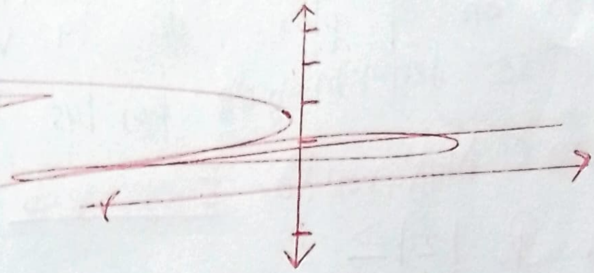


$\boxed{z \geq 0}$

b) $0 \leq \operatorname{Im} z \leq 1$

$z = x + iy$

$\boxed{\begin{matrix} y \geq 0 \\ y \leq 1 \end{matrix}}$



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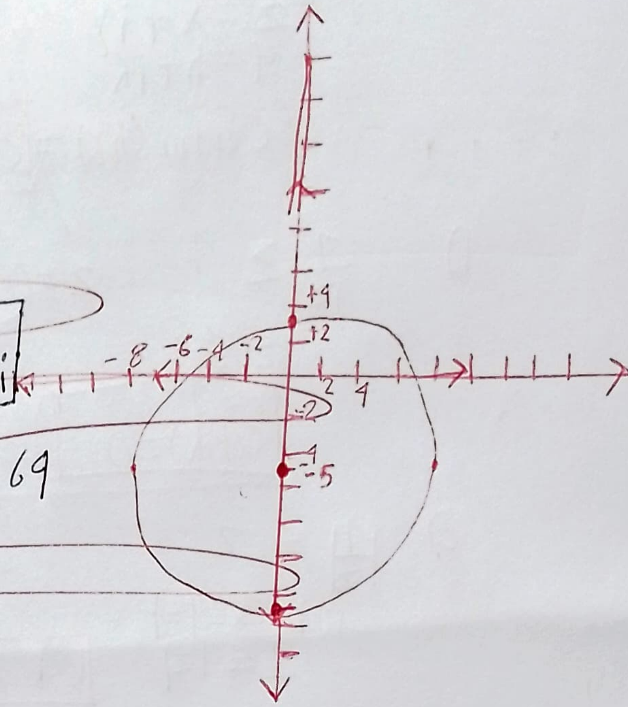
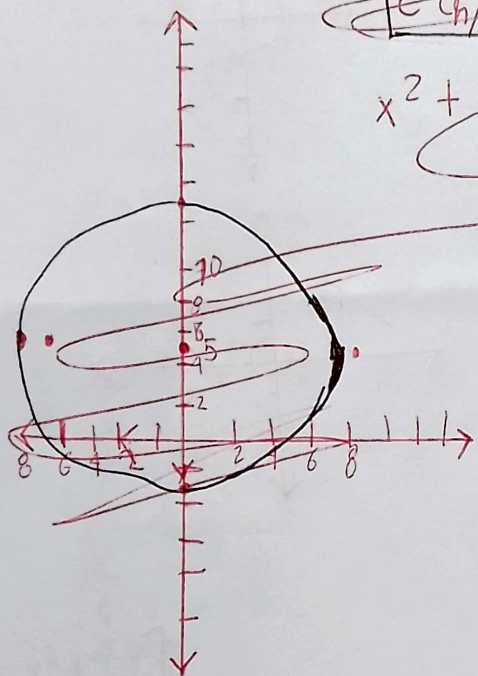
$|z - 5i| = 8$:

$\sqrt{(x-0)^2 + (y-5i)^2} = 8$

$\boxed{\begin{matrix} r = 8 \\ C(h,k) = -5i \end{matrix}}$

$x^2 + (y-5i)^2 = 69$



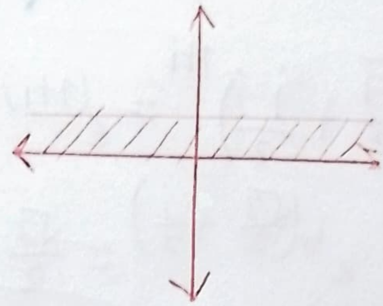


b) $0 \le \operatorname{Im} z \le 1$

$z = x+iy$

$\boxed{\begin{matrix}(y)\ge 0 \\ (y)\le 1\end{matrix}}$

$0 \le y \le 1$

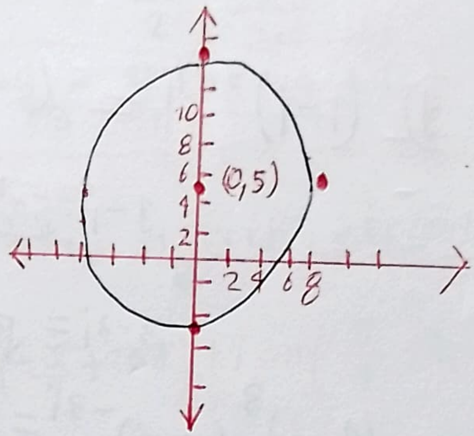


Ejercicio 7

20 a) $|z-5i| = 8$

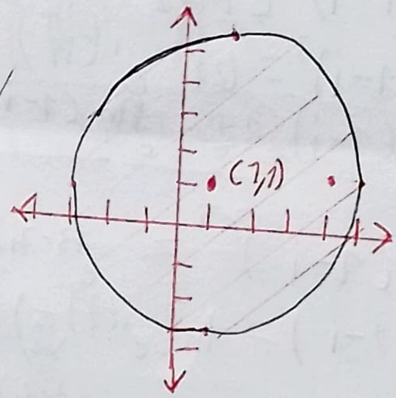
$\sqrt{(x)^2+(y-5)^2} = 8$

$x^2+(y-5)^2 = 64$



b) $|z-1-i| \le 4$

$(x-1)^2+(y-1)^2 \le 4$



Ejercicio 8 LIBRo : MaKyren Ko

67. a) i^i

$e^{i\ln|i|} = e^{i\cdot i(\frac{\pi}{2}+2k\pi)} = \boxed{e^{-(\frac{\pi}{2}+2k\pi)}}$

$\ln|i| = \ln(e^{i\theta}) = \ln|e^{i\frac{\pi}{2}}| = i(\frac{\pi}{2}+2k\pi)$ $k \in N$

b) $i^{\frac{1}{i}}$

$e^{\frac{1}{i}\ln|i|} = e^{-i(i(\frac{\pi}{2}+2k\pi))} = \boxed{e^{(\frac{\pi}{2}+2k\pi)}}$

c) $1^i = e^{i\ln(1)} = \boxed{1}$

d) $(-1)^{\sqrt{2}} = e^{\sqrt{2}\ln(-1)} = e^{\sqrt{2}} e^{\ln(-1)} = e^{\sqrt{2}} e^{\pi+2k\pi} = \boxed{e^{\sqrt{2}\pi i(\pi+2k)}}$

$e^{\ln(-1)} = e^{\ln i^2} = i^2 = (1)^2 e^{i(\frac{\pi}{2}+2k\pi)[2]} = e^{i\pi+4k\pi}$

f) $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{1+i} = e^{(1+i)\ln\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)} = e^{(1+i)\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)} = e^{[i+1]}e^{i\frac{\pi}{6}}$

$e^{\ln\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)} = \frac{\sqrt{3}}{2}+\frac{i}{2} = \frac{1}{2}\left[2e^{\left(\frac{\pi}{6}\right)i}\right] = e^{i\left(\frac{\pi}{6}+2k\pi\right)}$

$\boxed{e^{1+i\left(1+\frac{\pi}{6}\right)}}$

g) $(1-i)^{3-3i} = e^{(3-3i)\ln(1-i)} = \sqrt{2}e^{(3-3i)}e^{-i\frac{\pi}{4}} = \boxed{\sqrt{2}e^{3-3i\left(3+\frac{\pi}{4}\right)}}$

$1-i = \sqrt{2}e^{i\left(-\frac{\pi}{4}\right)}$

$3-3i = \sqrt{18}\; e^{i\left(-\frac{\pi}{4}\right)}$

$(1-i)^{3}(1-i)^{-3i} = (2)^{\frac{3}{2}}\left(e^{i\cdot -\frac{\pi}{4}\cdot\frac{3\pi}{4}}\right) =$

$(1-i)^{3} = (2)^{\frac{3}{2}}e^{i\left(-\frac{\pi}{4}\right)}$

$(1-i)^{-3i} = e^{-i\ln(1-i)^{3}} = e^{-i}(2)^{\frac{3}{2}}e^{i\frac{\pi}{4}} = (2)^{\frac{3}{2}}e^{\frac{-3\pi}{4}}$

g) $(1-i)^{3-3i} = e^{(3-3i)\ln(1-i)}$

$\ln(1-i) = \ln\left(e^{i\left(\frac{\pi}{4}\right)}\sqrt{2}\right) = \ln(\sqrt{2}) + \ln\left(e^{-i\frac{\pi}{4}}\right) = \boxed{\ln(\sqrt{2}) - \frac{\pi}{4}i}$

$e^{(3-3i)\left(\ln\sqrt{2}-\frac{\pi}{4}i\right)} = \boxed{e^{3\ln\sqrt{2}-\frac{\pi}{4}i-i3\left(\ln\sqrt{2}-\frac{\pi}{4}i\right)}}$

1\. $z^2 + iz - 2 = 0$ Excersise 1.6 ~~Ejercicio 4~~ Libro: Complex Analisys

$$z_{1,2} = \frac{-i \pm \sqrt{i^2 - 4(1)(-2)}}{2(1)} = \frac{-i \pm \sqrt{7}}{2}$$

$$\boxed{z_1 = \frac{-i}{2} + \frac{\sqrt{7}}{2}} \quad \boxed{z_2 = -\frac{\sqrt{7}}{2} - \frac{i}{2}}$$

2\. $iz^2 - z + i = 0$ ~~Ejercicio 10~~

$$z_{1,2} = \frac{1 \pm \sqrt{1 - 4(i)(i)}}{2(i)} = \frac{1 \pm \sqrt{5}}{2i} = \frac{-i}{2} \frac{\pm\sqrt{5}}{2i}$$

$$\boxed{z_1 = -\frac{i}{2} - \frac{\sqrt{5}}{2i}} \quad \boxed{z_2 = -\frac{i}{2} + \frac{\sqrt{5}}{2i}}$$

Expresar los numeros complejos dados en la forma exponencial de $\boxed{z = re^{i\theta}}$

~~Ejercicio 11~~

10\. $$\frac{2}{1+i} = \frac{2(1-i)}{2} = 1-i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4} + 2k\pi\right) + i\,\text{sen}\left(-\frac{\pi}{4} + 2k\pi\right)\right)$$

$r = \sqrt{2}$

$\theta = \tan^{-1}\left(-\frac{1}{1}\right) = -\frac{\pi}{4}$

$$\boxed{\sqrt{2}\, e^{\left(-\frac{\pi}{4} + 2k\pi\right)i}}$$

~~Ejercicio 12~~

11\. $$(3-i)^2 = (\sqrt{10})^2 \left[\cos\left(2\tan^{-1}\left(-\frac{1}{3}\right)\right) + i\,\text{sen}\left(2\tan^{-1} -\frac{1}{3}\right)\right]$$

$r = \sqrt{10}$

$\theta = \tan^{-1}\left(\frac{-1}{3}\right) = \theta$

$$10\left[\cos\left(2\tan^{-1} \frac{-1}{3}\right) i\,\text{sen}\left[2\left(\tan^{-1} - \frac{1}{3}\right)\right]\right] = \boxed{10e^{+i\left(\tan^{-1}\left(-\frac{1}{3}\right)\right)2}} = \boxed{10e^{i\left(2\tan^{-1}\left(-\frac{1}{3}\right) + 2k\pi\right)}}$$

12\. $$(1+i)^{20} = (\sqrt{2})^{20} e^{i20\left(\frac{\pi}{4}\right)} = \boxed{2^{10} e^{i(\pi 5 + 2k\pi)}}$$

$r = \sqrt{2}$

$\theta = \tan^{-1}(1) = \frac{\pi}{4}$

9\. $$-4-4i = \sqrt{32}\, e^{i\left(\frac{5\pi}{4} + 2k\pi\right)}$$

$\theta = \tan(1) + \pi = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

8\. $$-2\pi i = \sqrt{\ } = \boxed{2\pi e^{i\left(3\frac{\pi}{2} + 2k\pi\right)}}$$

7\. $$-10 = \boxed{10 e^{i(\pi + 2k\pi)}}$$