Ejercicio 1

3.68 Verifique que a) $\frac{d}{dz}(1+z^2)^{\frac{3}{2}} = \frac{3}{2}(1+z^2)^{\frac{1}{2}}(2z) = \boxed{3z(1+z^2)^{\frac{1}{2}}}$

b) $\frac{d}{dz}(z+2\sqrt{z})^{\frac{1}{3}} = \frac{1}{3}(z+2\sqrt{z})^{-\frac{2}{3}}(\sqrt{z}+1)$

$\frac{d}{dz}(z+2\sqrt{z})^{\frac{1}{3}} = \frac{1}{3}(z+2\sqrt{z})^{-\frac{2}{3}}\left(1+\left(\frac{1}{2}\right)(2z^{-\frac{1}{2}})\right) = \boxed{\frac{z^{-\frac{1}{2}}(z+2\sqrt{z})^{-\frac{2}{3}}(\sqrt{z}+1)}{3}}$

Ejercicio 2

3.56 Sea $w = iz^2 - 4z + 3i$ Encuentre a) Δw b) dw

c) $\Delta w - dw$ en el punto donde $z = 2i$

a) $\Delta w = i(z+\Delta z) - f(z) = \{i(z+\Delta z)^2 - 4(z+\Delta z) + 3i\} - \{iz^2 - 4z + 3i\}$

$i(z^2 + 2z\Delta z + \Delta z^2) - 4z - 4\Delta z + 3i - iz^2 + 4z - 3i$

$\Delta w = iz^2 + 2iz\Delta z + i\Delta z^2 - 4\Delta z = (i2z - 4)\Delta z + i\Delta z^2$

$\boxed{-8\Delta z + \Delta z^2} = (i2(2i) - 4)\Delta z + i\Delta z^2$

b) $\boxed{dw = -8\,dz}$

c) $\Delta w - dw = -8\Delta z + i\Delta z^2 + 8\,dz = \boxed{i\Delta z^2}$

Ejercicio 3

3.57 Supongq que $w = (2z+1)^3$ $z = -i$ $\Delta z = 1+i$

Encuentre a) Δw y b) dw

a) $\Delta w = f(z+\Delta z) - f(z) = (2(z+\Delta z)+1)^3 - (2z+1)^3$

$(2(-i+1+i)+1)^3 - (-2i+1)^3 = (3)^3 - 2i + n = 38 - 2i$

b) $dw = \Delta z$

$(2z+2\Delta z+1)^3 = 24(z)^3\Delta z + 24z\Delta z + 6\Delta z$

$(2z+2\Delta z+1)(2z+2\Delta z+1) = 4z^2 + 4z\Delta z + 2z + 4\Delta z + 4(\Delta z)^2 + 2\Delta z + 2z + 2\Delta z + 1$

$24(1+i) + 24(-i)(1+i) + 6(1+i)$

$24 + 24i - 24i + 24 + 6 + 6i = \boxed{54 + 6i}$

Ejercicio 9

3.63 Con las reglas de diferenciación encuentre la derivada de cada función siguiente:

a) $\frac{d}{dz}[(1+4i)z^2 - 3z - 2] = (2+8i)z - 3z$

b) $(2z+3i)(z-i) = 2z^2 - 2iz + 3iz - 3i^2 \quad d(2z^2+zi-3i^2) = \boxed{4z+i}$

c) $\left(\frac{z-1}{z+2i}\right) = \left(\frac{z-1}{z+2i}\right)\left(\frac{z-2i}{z-2i}\right) = \frac{z^2-4iz-iz+2i^2}{z^2+4} = \frac{z^2-5iz-2}{z^2+4}$

d) $(2iz+1)^2 = 4i^2z^2+1+4iz \quad = -4z^2+1+4iz \quad d(1-4z^2+4iz) = \boxed{-8z+4i}$

c) $\left(\frac{z-i}{z+2i}\right) = \frac{2z^2-5iz-2}{z^2+4} = \frac{(z^2+4)(4z-5i)-(2z^2-5iz-2)(2z)}{(z^2+4)^2}$

$\boxed{\frac{5iz^2+20z-20i}{(z^2+4)^2}} = \frac{4z^3-5iz^2+16z-20i-(4z^3-10iz^2-4z)}{(z^2+4)^2}$

e) $(iz-1)^{-3} = -3(iz-1)^{-4}(i) = -3i(iz-1)^{-4}$

Ejercicio 5

3.64 Encuentre las derivadas de las funciones siguientes en el punto indicado.

a) $\left(\frac{(z+2i)(i-z)}{2z-1}\right) \quad z=1$

$zi - z^2 - 2 - 4i = \frac{(2z-1)(i-2z)-(zi-z^2-2-4i)(2)}{(2z-1)^2}$

$\frac{2z-1-2z^2-4+3i}{(2z-1)^2} = \frac{2zi-4z^2-i+2z-2zi+2z^2+4+4i}{(2z-1)^2}$

$\frac{2(1)-2(1)^2-4+3i}{(2(1)-1)^2} = \boxed{-4+3i}$

b) $(z+(z^2+1)^2)^2 \neq \quad z=1+i$

$2(z+(z^2+1)^2)(1+2(z^2+1)(2z)) = 2(z+(z^2+1)^2)(1+4z(z^2+1))$

$2(1+i) = (2(1+i)+2((1+i)^2+1)^2)(1+4(1+i)^3+4(1+i))$

$(2+2i+2(2i+1)^2)(1+4(-2+2i)+4+4i)$

$(2+2i+2(-4+4i+1))(1-8+8i+4+4i) = (-7+6i)(-3+12i) = 21-72i-36i-72 = \boxed{-108-78i}$

$(-1)^{3i}$ Excersise 9.2

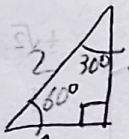
$3i \ln|-1| = 3i \ln|e^{i\pi+2K\pi}| = 3i^2(\pi+2K\pi) = \boxed{3\pi+6K\pi i}$

Ejercicio 10

Ejercicio 11

2. $3^{\frac{2}{\pi}} = 3^{\frac{2i}{\pi}}$

$\frac{2}{\pi}\ln|3| = \frac{2}{\pi}\ln|9e^{i(\pi+2K\pi)}| = \ln|9|\left(\frac{-2(\pi+2K\pi)}{\pi}\right) = \boxed{\ln|9|(-2+2k)}$



$\tan(30°) = \frac{1}{\sqrt{3}}$

Ejercicio 12

1. $(1+\sqrt{3}i)^i = \ln|2e^{i(\frac{\pi}{6}+2K\pi)}|^i = i\ln|2|\ln|e^{i(\frac{\pi+12K\pi}{6})}| = \ln|2|\frac{(\pi+12K\pi)}{6}$

$(1+\sqrt{3}i)^i = \boxed{\ln|2|\left(\frac{\pi+12\pi K}{6}\right)}$

Ejercicio 13

5. $(-i)^i = i\ln|-i| = i\ln|e^{i(\frac{3\pi}{2}+2K\pi)}| = \boxed{-\frac{3\pi}{2}+2K\pi}$

$$\frac{z^2-(z^2-1)^{\frac{1}{2}}\left(i\sqrt{z^2-1}+1\right)}{z(z^2-1)^{\frac{1}{2}}\left(i\sqrt{z^2-1}+1\right)}$$

$$\frac{z^2-iz^2+1-\sqrt{z^2-1}}{z\left(i(z^2-1)+\sqrt{z^2-1}\right)}$$

$$\frac{iz^2+z^2-1-\sqrt{z^2-1}}{zi(z^2-1+i\sqrt{z^2-1})}$$

Ejercicio 9

3.65 Comprueba que a) $\frac{d}{dz}\sec(z)=\sec(z)\tan(z)$

$\sec z=\frac{1}{\cos z}$

$\frac{d}{dz}\left(\frac{1}{\cos z}\right)=\frac{\cos z(0)-1(-\sin(z))}{\cos^2 z}=\left(\frac{1}{\cos z}\right)\left(\frac{\sin z}{\cos z}\right)=\sec(z)\tan(z)$

b) $\frac{d\cot z}{dz}=-\csc^2 z$ $\quad \cot z=\frac{1}{\tan(z)}$

$\frac{d}{dz}\left(\frac{1}{\tan(z)}\right)=\frac{0-\sec^2 z}{\tan^2 z}=-\left(\frac{\sec z}{\tan z}\right)^2=-\csc z$

~~3.58~~ Ejercicio 6

3.66 Demuestre que a) $\frac{d}{dz}(z^2+1)^{\frac{1}{2}} = \frac{z}{(z^2+1)^{\frac{1}{2}}}$

$(\frac{1}{2})(z^2+1))^{-\frac{1}{2}} \frac{d(z^2+1)}{dz} = (\frac{2z}{2})(z^2+1)^{-\frac{1}{2}} = \boxed{\frac{z}{(z^2+1)^{\frac{1}{2}}}}$

b) $\frac{d}{dz}\ln(z^2+2z+2) = \frac{2z+2}{z^2+2z+2}$

$\frac{d(z^2+2z+2)}{z^2+2z+2} = \boxed{\frac{2z+2}{z^2+2z+2}}$

Ejercicio 7

3.67 Encuentre las derivadas de cada una de las funciones siguientes e indique las restricciones que puede haber

a) $3\operatorname{sen}^2(\frac{z}{2}) = \frac{6}{2}\operatorname{sen}(\frac{z}{2})\cos(\frac{z}{2}) = 3\operatorname{sen}(\frac{z}{2})\cos(\frac{z}{2})$

b) $\tan^3(z^2-3z+1i) = 3\tan^2(z^2-3z+1i)\sec^2(z^2-3z+1i)(2z-3)$

c) $\ln(\sec z + \tan z) = \boxed{\frac{\sec^2(z)+\sec\tan z}{\sec(z)+\tan(z)}}$

d) $\csc((z^2+1)^{\frac{1}{2}}) = -\csc(z^2+1)^{\frac{1}{2}}\cot(z^2+1)^{\frac{1}{2}}(2z)$

e) $(z^2-1)\cos(z+2i) = \cos(z+2i)(2z) + (z^2-1)(-\operatorname{sen}(z+2i))$

Ejercicio 8

3.69 Compruebe que a) $\frac{d}{dz}(\tan^{-1}z) = \frac{1}{z^2+1}$

$\tan^{-1}z = -\frac{i}{2}\ln(\frac{i-z}{i+z})$

$\frac{d}{dz}(-\frac{1}{2}\ln(\frac{i-z}{i+z})) = \frac{1}{dz}(-\frac{i}{2})\ln(\frac{1+iz}{1-iz}) = -\frac{i}{2}\left[\frac{\frac{(1+iz)(i)-(1+iz)(-i)}{(1-iz)^2}}{\frac{1+iz}{1-iz}}\right]$

$(-\frac{i}{2})\left(\frac{i+z+i-z}{(1-iz)^2}\right)\left(\frac{1-iz}{1+iz}\right) = \frac{(-i)(2i)}{2(1-iz)(1+iz)}$

$\boxed{\frac{1}{z^2+1}}$

b) $\frac{d}{dz}(\sec^{-1}z) = \frac{1}{z\sqrt{z^2-1}}$

$\frac{d}{dz}(-i\ln(i\sqrt{z^2-1}+1)) = -i\frac{z(i(\frac{1}{2})(z^2-1)^{-\frac{1}{2}}(2z)) - (i\sqrt{z^2-1}+1)(1)}{z^2}$

$\frac{i\sqrt{z^2-1}+1}{z}$

$\left(\frac{z^2 i(z^2-1)^{-\frac{1}{2}} - (i\sqrt{z^2-1}+1)}{z(i\sqrt{z^2-1}+1)}\right) = -\frac{1}{z}\left|\frac{z^2}{z(z^2-1)^{\frac{1}{2}}(i\sqrt{z^2-1}+1)}\right.$