INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

LISTA DE EJERCICIOS 1-12

SEMANA 1

NOMBRE DEL ALUMNO: GARCÍA QUIROZ GUSTAVO IVAN

GRUPO: 4CV3

MATERIA: MATEMATICAS AVANZADAS PARA LA INGENIERIA

NOMBRE DEL PROFESOR: MARTINEZ NUÑO JESUS ALFREDO

FECHA: 28/02/2023

NACIONAL

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Lista de ejercicios 1-1
Semana 1

Nombre del alumno: García Quiroz Gustavo Ivan
Grupo: 4CV3

Nombre del profesor: Martinez Nuño Jesus Alfredo

Fecha: 28/02/2023

3 Ejercicio 1 Libro: Variable Compleja Schaum

Realiza las operaciones indicadas

a) $(4-8i)+(2i-8) = 4-8i+2i-8 = \boxed{-4-6i}$

b) $3(-1+4i)-2(7-i) = -3+12i-14+2i = \boxed{-17+14i}$

c) $(3+2i)(2-i) = 6-3i+4i-2i^2 = 6+2-3i+4i = \boxed{8+i}$

d) $(i-2)\{2(1+i)-3(i-1)\} = (i-2)\{2+2i-3i+3\} = (i-2)(5-i)$

$5i-i^2-10+2i = \boxed{7i-9}$

e) $\frac{2-3i}{4-i} = \frac{2-3i}{4-i}\left(\frac{4+i}{4+i}\right) = \frac{8+2i-12i-3i^2}{16+1} = \boxed{\frac{11-10i}{17}}$

f) $\frac{4}{3}(4+i)(3+2i)(1-i) = (12+8i+3i+2i^2)(1-i) = (11i+10)(1-i)$

$(11i+10-11i^2-10i) = \boxed{i+5+16}$ $\boxed{i+21}$

g) $\frac{(2+i)(3-2i)(1+2i)}{(1-i)^2} = \frac{(6-4i+3i-2i^2)(1+2i)}{(1-i)^2}$

$\frac{(8-i)(1+2i)}{1+i^2-2i} = \frac{8+16i-i-2i^2}{-2i} = \frac{10+15i}{-2i}\left(\frac{2i}{2i}\right) = -\frac{15}{2}+5i$

a) $(4-3i)+(2i-8) = \boxed{-4-i}$

h) $(2i-1)^2\left\{\frac{4}{1-i}+\frac{2-i}{1+i}\right\} = (1+4i^2-4i)\left\{\frac{4(1+i)+(2-i)(1-i)}{(1+i)(1-i)}\right\}$

$(-3-4i)\left\{\frac{4+4i+2-2i-i+i^2}{1+1}\right\} = \frac{(-3-4i)(5+i)}{2}$

$\frac{-15-3i-20i-4i^2}{2} = \boxed{\frac{-11-23i}{2}}$

i) $\frac{i^4+i^9+i^{16}}{2-i^5+i^{10}-i^{15}} = \frac{1+i+1}{2-i-1+i} = \frac{2+i}{1} = \boxed{2+i}$

1.55 Demuestre que a) $(\overline{z_1 z_2}) = \bar{z}_1 \bar{z}_2$ y b) $(\overline{z_1 z_2 z_3}) = \bar{z}_1 \bar{z}_2 \bar{z}_3$
Generalize los resultados $\boxed{\text{Ejercicio 2}}$

a) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$ si $\boxed{z_1 = x_1 + iy_1}$ $\boxed{z_2 = x_2 + iy_2}$

$$\overline{z_1 z_2} = \overline{(x_1+iy_1)(x_2+iy_2)} = \overline{x_1x_2 - y_1y_2 + i(y_1x_2 + y_2x_1)}$$

$$+\cancel{i} = +i$$

$$= x_1x_2 - y_1y_2 - i(y_1x_2 + y_2x_1)$$

$$\bar{z}_1 \bar{z}_2 = (x_1 - iy_1)(x_2 - iy_2) = x_1x_2 - y_1y_2 - i(y_1x_2 + y_2x_1)$$

$\boxed{\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2}$ Si cumple

b)

$\overline{z_1 z_2 z_3} = \bar{z}_1 \bar{z}_2 \bar{z}_3$ Si $z_1 = x_1 + iy_1$ $z_2 = x_2 + iy_2$ $z_3 = x_3 + iy_3$

$$\overline{z_1 z_2 z_3} = \overline{(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)}$$

$$\overline{[x_1x_2 - y_1y_2) + i(y_1x_2 + y_2x_1)](x_3 + iy_3)} = \overline{z_1 z_2 z_3}$$

$$x_1x_2x_3 - y_1y_2x_3 + iy_1x_2x_3 + iy_2x_1x_3 + ix_1x_2y_3 - iy_1y_2y_3 + i^2y_1y_3x_2 + i^2y_2y_3x_1$$

$$x_1x_2x_3 - y_1y_2x_3 - y_1y_3x_2 - y_1y_2x_1 + ix_1x_2y_3 - iy_1y_2y_3 + iy_1x_2x_3 + iy_2x_1x_3$$

$$x_1x_2x_3 - y_1y_2x_3 - y_1y_3x_2 - y_1y_2x_1 + i(x_1x_2y_3 - y_1y_2y_3 + y_1x_2x_3 + y_2x_1x_3)$$

$$\bar{z}_1 \bar{z}_2 \bar{z}_3 = (x_1x_2 - y_1y_2 - iy_1x_2 - iy_2x_1)(x_3 + iy_3)$$

$$x_1x_2x_3 - y_1y_2x_3 - iy_1x_2x_3 - iy_2x_1x_3 + iy_3x_1x_2 - iy_1y_2y_3 + y_1x_2y_3 + y_2y_3x_1$$

$$x_1x_2x_3 - y_1y_2x_3 + y_1y_3x_2 + y_2y_3x_1 - i(y_1x_2x_3 + y_2x_1x_3 + y_3x_1x_3 + y_1y_2x_3)$$

$$\bar{z}_1 \bar{z}_2 \bar{z}_3 = (x_1x_2 - y_1y_2 - iy_1x_2 - iy_2x_1)(x_3 - iy_3)$$

$$x_1x_2x_3 - y_1y_2x_3 - iy_1x_2x_3 - ix_1y_2x_3 - iy_3x_1x_2 + iy_1y_2y_3 + i^2y_1y_3x_2 + i^2y_2x_1y_3$$

$$x_1x_2x_3 - y_1y_2x_3 - y_1y_3x_2 - y_2x_1y_3 - i(y_1x_2x_3 + x_1y_2x_3 + \dots$$

$\boxed{\overline{z_1 z_2 z_3} = \bar{z}_1 \bar{z}_2 \dots}$

Ejercicio 3

1.56 Pruebe que a) $\overline{(z_1/z_2)} = \overline{z_1}/\overline{z_2}$ y b) $|z_1/z_2| = |z_1|/|z_2|$ si $z_2 \neq 0$

a) $\overline{\left(\frac{z_1}{z_2}\right)}$ Si $\boxed{z_1 = x_1 + iy_1}$ $\boxed{z_2 = x_2 + iy_2}$

$$\overline{\left(\frac{x_1+iy_1}{x_2+iy_2}\right)\left(\frac{x_2-iy_2}{x_2-iy_2}\right)} = \overline{\frac{x_1x_2 - ix_1y_2 + iy_1x_2 + y_1y_2}{x_2^2+y_2^2}}$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{x_1x_2+y_1y_2}{x_2^2+y_2^2} + i\left(\frac{x_1y_2+(-x_2y_1)}{x_2^2+y_2^2}\right)$$

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{x_1-iy_1}{x_2-iy_2}\left(\frac{x_2+iy_2}{x_2+iy_2}\right) = \frac{x_1x_2+y_1y_2}{x_2^2+y_2^2} + i\,\frac{x_1y_2-y_1x_2}{x_2^2+y_2^2}$$

$$\boxed{\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}}$$ Se cumple

b) $$\left|\left(\frac{z_1}{z_2}\right)\right| = \left|\left(\frac{x_1+iy_1}{x_2+iy_2}\right)\left(\frac{x_2-iy_2}{x_2-iy_2}\right)\right| = \left|\frac{x_1x_2+y_1y_2+i(x_2y_1-y_2x_1)}{(x_2^2+y_2^2)}\right|$$

$$\frac{\sqrt{(x_1x_2+y_1y_2)^2+(x_2y_1-y_2x_1)^2}}{x_2^2+y_2^2} = \frac{\sqrt{x_1^2x_2^2+x_1^2y_2^2+y_1^2x_2^2+x^2y_2^2}}{x_2^2+y_2^2}$$

$$\frac{|z_1|}{|z_2|} = \frac{\sqrt{x_1^2+y_1^2}}{\sqrt{x_2^2+y_2^2}}\left(\frac{\sqrt{x_2^2+y_2^2}}{\sqrt{(x_2^2+y_2^2)}}\right) = \frac{\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}}{x_2^2+y_2^2}$$

$$\frac{\sqrt{x_1^2x_2^2+x_1^2y_2^2+y_1^2x_2^2+y_1^2y_2^2}}{x_2^2+y_2^2} = \frac{\sqrt{x_1^2x_2^2+x_1^2y_2^2+y_1^2x_2^2+y_1^2y_2^2}}{x_2^2+y_2^2}$$

$$\boxed{\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}}$$ Se cumple

$= \overline{z_1}/\overline{z_2}$ y b) $|z_1/z_2| = |z_1|/|z_2|$

Ejercicio 4

Encuentre numeros reales x y y tales que

$2x - 3iy + 4ix - 2y - 5 - 10i = x + y + 2 - (x + y + 3)i$

$2x - x - 2y - y - 5 - 2 - 3iy + 4ix = (x - y - 3)i$

$-7 + x - 3y + i(-3y + 4x) = xi - yi - 3i + 10i$

$x - 3y + i(+3x - 2y) = 7i + 7$

$3x - 2y = 7$

$x - 3y = 7$

$\rightarrow \left(\begin{array}{cc|c} 3 & -2 & 7 \\ 1 & -3 & 7 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & -3 & 7 \\ 0 & 7 & -14 \end{array}\right)$

$x - 3y = 7$

$7y = -14$

$x = 7 + 3(-2)$

$y = \frac{-14}{7} = \boxed{-2 = y}$

$x = 7 - 6$

$\boxed{x = 1}$

$x \neq 7 + 3(7)$

$x \neq 14 + 7$

$x \neq 7 + 6$

...os reales x y y tales que

...ni = x+y+2 − (x+y+3)i

...−3)i

... $= \overline{z_1}/\overline{z_2}$ y b) $|z_1/z_2| = |z_1|/|z_2|$

...allar las soluciones reales de las siguientes ecuaciones

Libro: Funciones de Variable Compleja

Ejercicio $\boxed{5}$

2. $(3x - i)(2+i) + (x+iy)(1+2i) = 5+6i$

$6x + 3xi - 2i + 1 + x + 2ix + iy + i^2 2y = 5+6i$

$7x - 2y + 5xi + iy = 6+8i$

$5x + y = 8$

$7x - 2y = 6$

$\left(\begin{array}{cc|c} 5 & 1 & 8 \\ 7 & -2 & 6 \end{array}\right)$ x2

$\left(\begin{array}{cc|c} 5 & 1 & 8 \\ 17 & 0 & 24 \end{array}\right)$

$17x = 24 - 2$

$x = \frac{24-2}{17} = \frac{22}{17}$

$\boxed{x = \frac{22}{17}}$

$\boxed{y = \frac{26}{17}}$

Ko

Ejercisos 1.1

Ejercicio 5

Libro = Complex Analysis

Evaluate the following powers of i

a) $i^8 = \boxed{1}$

b) $i^{11} = \boxed{-i}$

c) $i^{42} = (i^4)^{10}(i)^2 = \boxed{-1}$

d) $i^{105} = i^{(4)(26)}(i) = \boxed{i}$

Ejercicio 7

2 Evaluate and Write the given number in the form a+ib

a) $2i^3 - 3i^2 + 5i = -2i + 3 + 5i$ $\boxed{3 + 3i}$

b) $3i^5 - i^4 + 7i^3 - 10i^2 - 9$
$3i - 1 - 7i + 10 - 9 = \boxed{-4i}$

c) $\frac{5}{i} + \frac{2}{i^3} - \frac{20}{i^{18}} = -5i + 2i + 20 = \boxed{20 - 3i}$

d) $2i^6 + \left(\frac{2}{-i}\right)^3 + 5i^{-5} - 12i = -2 - \frac{8}{i^3} + \frac{5i}{i^6} - 12i$
$\boxed{-2 - 9i} = -2 + 8i - 5i - 12i$

In problem 3-20 write the given number in the form a+ib

Ejercicio 8

9 $3i + \frac{1}{2-i} = 3i + \frac{1}{2-i}\left(\frac{2+i}{2+i}\right) = 3i + \frac{2+i}{4+1}$

$\frac{15i + 2 + i}{5} = \boxed{\frac{2}{5} + \frac{16i}{5}}$

Ejercicio 9

11 $\frac{2-4i}{3+5i} = \frac{2-4i}{3+5i}\left(\frac{3-5i}{3-5i}\right) = \frac{6 - 10i - 12i + 20i^2}{9+25}$

$\frac{-14 - 22i}{34} = \boxed{-\frac{7}{17} - \frac{11}{17}i}$

Ejercicio 10

18 $(1+i)^2 (1-i)^3 = (1+1)(1+1)(1-i) = \boxed{4 - 4i}$

6. Demostrar que $\dfrac{\sqrt{1+x^2}+ix}{x-i\sqrt{1+x^2}}=i$ (x) es real

libro : Makarenko

$$\frac{\sqrt{1+x^2}+ix}{x-i\sqrt{1+x^2}}\left(\frac{x+i\sqrt{1+x^2}}{x+i\sqrt{1+x^2}}\right)=\frac{(\sqrt{1+x^2}+ix)(x+i\sqrt{1+x^2})}{x^2+1+x^2}$$

$i^2=1$ $\boxed{i=\sqrt{-1}}$

$$\frac{x\sqrt{1+x^2}+i(1+x^2)+ix^2-\cancel{x\sqrt{1+x^2}}}{2x^2+1}=\frac{i+i2x^2}{2x^2+1}$$

$$\frac{i(1+2x^2)}{2x+1}=i$$

$$\frac{1+2x^2}{2x^2+1}=\frac{i}{i}=1$$

$$1+2x^2=1+2x^2$$

$\boxed{x=x}$

x es real

Ejercicio 11

Hallar las soluciones reales de las ecuaciones

4. $\dfrac{1}{z-1}+\dfrac{2+i}{1+i}=\sqrt{2}$ donde $z=x+iy$

$$\frac{1}{x+iy}+\frac{2+i}{1+i}=\frac{1}{x+iy}\left(\frac{x-iy}{x-iy}\right)+\frac{2+i}{1+i}\left(\frac{1-i}{1-i}\right)$$

$$\frac{x-iy}{x^2+y^2}+\frac{2-2i+i+1}{1+1}=\frac{(x-iy)(2)+(3-i)(x^2+y^2)}{2(x^2+y^2)}$$

$$\frac{2x-2iy+3x^2+3y^2-ix^2-iy^2}{2x^2+y^2}=\frac{2x+3x^2+3y^2}{2(x^2+y^2)}+i\,\frac{-2y-x^2-y^2}{2(x^2+y^2)}$$

Ejercicio 1

5. Presentar el número complejo $\frac{1}{(a+ib)^2}+\frac{1}{(a-ib)^2}$ en forma algebraica

$$\frac{1}{(a+ib)^2}+\frac{1}{(a-ib)^2}=\frac{(a-ib)^2+(a+ib)^2}{(a+ib)^2\cdot(a-ib)^2}$$

$$\frac{a^2-2iab-b^2+a^2-b^2+2iab}{(a^2+b^2)(a^2+b^2)}=\frac{2a^2-2b^2}{a^4+a^2b^2+b^2a^2+b^4}$$

$$\frac{2(a^2-b^2)}{a^4+2a^2b^2+b^4}=\boxed{\frac{2(a^2-b^2)}{(a^2+b^2)}}$$