INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

LISTA DE EJERCICIOS 1-12

SEMANA 14

NOMBRE DEL ALUMNO: GARCÍA QUIROZ GUSTAVO IVAN

GRUPO: 4CV3

MATERIA: MATEMATICAS AVANZADAS PARA LA INGENIERIA

NOMBRE DEL PROFESOR: MARTINEZ NUÑO JESUS ALFREDO

Ejercicio 12

Parseval

Ejercicio 1

Ejercicio 1 Libro O'neil

17.

$$F^{-1}\left(\frac{1}{(1+iw)^2}\right) = H(t)e^{-t} \; H(t)e^{-t}$$

$$= \int_{-\infty}^{\infty} H(\tau)e^{-\tau} H(t-\tau)e^{-(t-\tau)}d\tau$$

$$= H(t)e^{-t}\int_0^t d\tau = \boxed{H(t)\,t\,e^{-t}}$$

Ejercicio 2

18 $F^{-1}\left\{\frac{\sin(3w)}{(2+iw)\,w}\right\} = \frac{1}{2}\left(H(t+3)-H(t-3)\right\} H(t)e^{-2t}$

$$= \frac{1}{2}\int_{-\infty}^{\infty} (H(t+3) - H(t-3))H(t-\tau)e^{-2(t-\tau)}d\tau$$

$$\frac{1}{2}e^{-2t}\left[H(t+3)\int_{-3}^{t} e^{2t}dt - H(t-3)\int_{3}^{t} e^{2\tau}d\tau\right]$$

$$= \frac{1}{4}(1-e^{-2(t+3)})H(t+3) - \frac{1}{4}(1-e^{-2(t-3)})H(t-3)$$

Ejercicio 3

Demostrar el siguiente versión de Parseval:

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(w)|^2 dw$$

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)\overline{F(w)}\,dw = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(w)|^2 dw$$

14.1 The Fourier Integral

2

$$\int_{-\infty}^{\infty} |f(x)|\,dx = \int_{-\infty}^{\infty} |x|dx = 2\int_0^{\pi} x\,dx = \boxed{\pi^2}$$

$$A_w = 0 \quad B_w = \frac{1}{\pi}\int_{-\infty}^{\infty} t\sin(wt)\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} t\sin(wt)dt$$

$$\frac{2}{\pi} = \left[\frac{\sin \pi w}{w^2} - \frac{\pi}{w}\cos(\pi w)\right.$$

$$\int_0^{\infty}\left[\frac{2\sin(\pi w)}{\pi w^2} - \frac{2\cos(\pi w)}{w}\right]\sin(wx)\,dw$$

Ejercicio 4

$$f(x) = \begin{cases} k & para\ -10 \le x \le 10 \\ 0 & para\ |x| > 10 \end{cases}$$

2. $\int_{-\infty}^{\infty} |f(w)| dx = \int_{-k}^{k} k\, dx = 20k$

$$A_N = \frac{1}{\pi} \int_{-10}^{10} k \cos(wt) dt = \frac{2k}{\pi w} \sin(10w)$$

$$\int_0^{\infty} \frac{2k}{\pi w} \sin(10w) \cos(wx) dw$$

Ejercicio 5

3. $$f(x) = \begin{cases} -1 & para & -\pi \le x \le 0 \\ 1 & para & 0 < x \le \pi \\ 0 & para & |x| > \pi \end{cases}$$

$A_w = 0$ Función impar

$$B_N = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(wt) dt = \frac{2}{\pi w}(1 - \cos(\pi w))$$

$$\int_0^{\infty} \frac{2}{\pi w}(1 - \cos(\pi w)) \sin(wx) dw$$

Ejercicio 6

4. $$f(x) \begin{cases} \sin(x) & para & -4 \le x \le 0 \\ \cos(x) & para & 0 \le x \le 4 \\ 0 & para & |x| > 4 \end{cases}$$

$$A_w = \frac{1}{\pi} \int_{-4}^{0} \sin(t) \cos(wt) dt + \frac{1}{\pi} \int_0^4 \cos(t) \cos(tw) dt$$

$$= \frac{1}{2\pi(w-1)} [1 + \sin(4(w-1)) - \cos(4(w-1))] -$$

$$\frac{1}{2\pi(w+1)} [1 - \sin(4(w+1)) - \cos(4(w+1))]$$

$$B_w = \frac{1}{2\pi(w-1)} (1 - \cos(4(w-1)) + \sin(4(w-1))) +$$

$$\frac{1}{2\pi(w+1)} [1 - \cos(4(w+1)) - \sin(4(w+1))]$$

$$\int_0^{\infty} (A_w \cos wx + B_w (\sin(wx))) dw$$

Ejercicio 7

5. $f(x) = \begin{cases} x^2 & \text{para } -100 \le x \le 100 \\ 0 & \text{para } x > 100 \end{cases}$

$\int_{-\infty}^{\infty} |f(x)|\, dx$ converge. La función es par entonces $B_w = 0$

$A_w = \frac{1}{\pi}\int_{-100}^{100} t^2 \cos(wt)\, dt = \frac{2}{\pi}\int_0^{\pi} t^2 \cos(wt)\, dt$

$A_w = \frac{2}{\pi}\left[\frac{t^2 \cos(wt)}{w} + \frac{2t\cos(wt)}{w^2} - \frac{2\sin(wt)}{w^3}\right]_0^{100}$

$A_w = \frac{20000\sin(100w)}{\pi w} - \frac{4\sin(100w)}{\pi w^3} + \frac{400\sin(100w)}{\pi w^2}$

La integral de Fourier es

$\int_0^{\infty}\left[\frac{400\cos(100w)}{\pi w^2} + \frac{20000w^2 - 4}{\pi w^3}\sin(100w)\right]\cos(wx)\, dw$

Ejercicio 8

7. $f(x) = \begin{cases} \sin(x) & \text{para } -3\pi \le x \le \pi \\ 0 & \text{para } x < -3\pi \text{ y para } x > \pi \end{cases}$

$\int_{-\infty}^{\infty} |f(x)|\, dx =$ converge

$A_w = \frac{1}{\pi}\int_{-3\pi}^{\pi} \sin t \cos(wt)\, dt = \int_{-3\pi}^{\pi}\left(\frac{1}{\pi}\right)\frac{\sin((w+1)x) - \sin((w-1)x)}{2}\, dx$

$\left[\frac{\cos((w-1)x)}{2(w-1)} - \frac{\cos((w+1)x)}{2(w+1)}\right]_{-3\pi}^{\pi} = \boxed{\frac{4\cos(\pi w)[\cos^2(\pi w) - 1]}{\pi(w^2-1)}}$

$B_w = \frac{1}{\pi}\int_{-3\pi}^{\pi} \sin(t)\sin(wt)\, dt = \int_{-3\pi}^{\pi}\left(\frac{1}{\pi}\right)\frac{\cos((w+1)x) - \cos((w-1)x)}{2}\, dx$

$\left[\frac{-\cos(x)\sin(wx) - w\sin(x)\cos(wx)}{w^2-1}\right]_{-3\pi}^{\pi} = \boxed{\frac{-4\sin(\pi w)\cos^2(\pi w)}{\pi(w^2-1)}}$

$\boxed{\int_0^{\infty} [A_w \cos(wx) + B_w \sin(wx)]\, dx}$

Ejercicio 9

8. $f(x) = \begin{cases} \frac{1}{2} & \text{para} \quad -5 \leq x < 1 \\ 1 & \text{para} \quad 1 \leq x \leq 5 \\ 0 & \text{para} \quad |x| > 5 \end{cases}$

$$A_W = \frac{1}{W} \int_{-5}^{1} \frac{1}{2} \cos(Wt)\, dt + \frac{1}{W} \int_{1}^{5} \cos(Wt)\, dt$$

$$A_W = \frac{\sin(W)}{\pi W} \left(24 \cos^4(W) - 18 \cos^2(W) + 1 \right)$$

$$B_W = \frac{1}{W} \int_{-5}^{1} \frac{1}{2} \sin(Wt)\, dt + \int_{1}^{5} \sin(Wt)\, dt$$

$$B_W = -\frac{2\cos(W)}{\pi W} \left(4\cos^4(W) - 5\cos^2(W) + 1 \right)$$

$$\boxed{\int_0^\infty \left(A_W \cos(Wx) + B_W \operatorname{sen}(Wx) \right) dW}$$

Ejercicio 10

9. $f(x) = \begin{cases} 1 & \text{para } -\pi \leq x < 0 \\ 2 & \text{para } 0 \leq x \leq \pi \end{cases}$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} dx + \int_{0}^{\pi} 2dx\right] = \frac{1}{\pi}\left[(x)\Big|_{-\pi}^{0} + (2x)\Big|_{0}^{\pi}\right] = \boxed{3}$$

$$\frac{a_0}{2} = \boxed{\frac{3}{2}}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin\left(\frac{\pi n x}{\pi}\right) = \frac{1}{\pi}\left[\int_{-\pi}^{0} \sin\left(\frac{\pi n x}{\pi}\right)dx + \int_{0}^{\pi} 2\sin(nx)dx\right]$$

$$\frac{1}{\pi}\left[\left[-\frac{1}{n}\cos(\pi x)\right]_{-\pi}^{0} + 2\left[-\frac{\cos(nx)}{n}\right]_{0}^{\pi}\right]$$

$$\frac{1}{\pi}\left[-\frac{1}{n}\cos(0) + \frac{1}{n}\cos(\pi x) - \frac{2}{n}\cos(\pi n) + \frac{2\cos(0)}{n}\right]$$

~~Ejercicio 11~~ Libro: Hwei

Teorema de Parseval

1.44 $f(x) =$

$$f(t) = \begin{cases} -1, & -\frac{T}{2} < t < 0 \\ 1, & 0 < t < \frac{T}{2} \end{cases}$$

$\boxed{T = 2L}$ $\boxed{w_0 = \frac{2\pi}{T}}$

$$\frac{4}{\pi}\sum_{n=\text{impar}}^{\infty} \frac{1}{n}\operatorname{sen}(nw_0 t)$$

$$\frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{2n-1}\operatorname{sen}\left((2n-1)\frac{w_0}{1} t\right)$$

$$\frac{1}{T}\int_0^T f^2(x)dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

$$\frac{1}{\pi}\int_0^{\pi} 1dx = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{4}{\pi(2n-1)}\right)^2$$

$$\frac{1}{\pi}(\pi) = \frac{1}{2}\sum_{n=1}^{\infty}\left[\frac{16}{\pi^2(2n-1)^2}\right]$$

$$\pi^2 = 8\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

Ejercicio 12

1.38 Teorema de Parseval

$f(t) = t \quad -\pi \leq t \leq \pi$

$$\boxed{\text{Respuesta } f(x) = \frac{1}{3}\pi^2 + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(nt)}$$

Demostrar que

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{\pi^2}{6}.$$

$$f(t) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} 4\,\frac{(-1)^n(-1)^n}{n^2} = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2}$$

$$\boxed{\frac{1}{T}\int_0^T f^2(x)\,dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)} \qquad T = 2L$$

$$\frac{1}{2\pi}\int_0^{2\pi} t^2\,dt = \pi^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2}$$

$$\frac{\pi^2}{3} = 4\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\boxed{\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}}$$

FIN