INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

LISTA DE EJERCICIOS 1-12

SEMANA 5

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GRUPO: 4CV3

MATERIA: MATEMATICAS AVANZADAS PARA LA INGENIERIA

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FECHA: 21/03/2023

Libro: Zill Pág. 275

14. $\oint_C \frac{10}{(z+i)^4}\,dz \quad |z+i|=1$

$$\oint_C \frac{f(z)}{(z-z_0)^n}\,dz = 2\pi i \,\frac{f^{(n-1)}(z_0)}{(n-1)!}$$

$$\oint_C \frac{10\,dz}{(z+i)^4} = 2\pi i\left[\frac{f^{(3)}(z_0)}{3!}\right] = \frac{2\pi i(0)}{3!} = \boxed{0}$$

$f^{(3)}(z_0) = 0$

$f' \; 0 = f''(z)\, 0 = f^{(3)}(z) = 0$

15. $\oint_C \frac{2z+1}{z^2+z}\,dz \quad$ a) $|z| = \frac{1}{2}$ $\quad$ b) $|z| = 2$ $\quad$ c) ~~$|z|=4$~~ $|z-3i|=1$

a) $\oint_{|z|=\frac{1}{2}} \frac{2z+1}{z(z+1)}\,dz = \oint_C \frac{\frac{2z+1}{z+1}}{z}\,dz = 2\pi i\left[\frac{2(0)+1}{(0)+1}\right] = \frac{6\pi i}{3} = \boxed{2\pi i}$

$\boxed{z_1 = 0} \quad \boxed{z_2 = -1}$

b) $\oint_{|z|=4} \frac{2z+1}{z(z+1)}\,dz = \boxed{0}$

b) $\oint_{|z|=2} \frac{2z+1}{z(z+1)}\,dz = \oint_{C_1} \frac{\frac{2z+1}{z+1}}{z}\,dz + \oint_{C_2} \frac{\frac{2z+1}{z}}{z+1}\,dz$

$\oint_{|z|=2} \frac{2z+1}{z(z+1)}\,dz = 2\pi i\left(\frac{2(0)+1}{0+1}\right) + 2\pi i\left(\frac{2(-1)+1}{-1+0}\right) = 2\pi i + 2\pi i = \boxed{4\pi i}$

c) $\oint_{|z-3i|=1} \frac{2z+1}{z(z+1)}\,dz = \boxed{0}$

16. $\oint_C \frac{2z}{z^2+3}\,dz \quad$ a) $|z|=1$ $\quad$ b) $|z-2i|=1$ $\quad$ c) $|z|=4$

a) $\oint_{|z|=1} \frac{2z\,dz}{(z+\sqrt{3}i)(z-\sqrt{3}i)} = \boxed{0}$ $\quad$ b) $\oint_{|z-2i|=1} \frac{2z\,dz}{(z+\sqrt{3}i)(z-\sqrt{3}i)} = \oint_{C_1} \frac{\frac{2z}{z+\sqrt{3}i}}{z-\sqrt{3}i}\,dz = 2\pi i\left(\frac{2\sqrt{3}i}{2\sqrt{3}i}\right)$

b) $\oint_{|z-2i|=1} \frac{2z\,dz}{(z+\sqrt{3}i)(z-\sqrt{3}i)} = \boxed{2\pi i}$

c) $\oint_{|z|=4} \frac{2z\,dz}{(z+\sqrt{3}i)(z-\sqrt{3}i)} = \oint_{C_1} \frac{\frac{2z}{(z-\sqrt{3}i)}}{(z+\sqrt{3}i)}\,dz + \oint_{C_2} \frac{\frac{2z}{(z+\sqrt{3}i)}}{(z-\sqrt{3}i)}\,dz = 2\pi i\left(\frac{-2\sqrt{3}i}{-2\sqrt{3}i}\right) + 2\pi i\left(\frac{2\sqrt{3}i}{2\sqrt{3}i}\right)$

$\oint_{|z|=4} \frac{2z\,dz}{(z+\sqrt{3}i)(z-\sqrt{3}i)} = \boxed{4\pi i}$

Libro: Schaum

4.32 Evalúe $\int_{(0,1)}^{(2,5)} (3x+y)dx+(2y-x)dy$ a lo largo de

a) La curva $y=x^2+1$ $\quad dy=2x\,dx$

$z=0+i+(2+5i-i)t$

$z=i+2t+4it$

$x=2t$

$y=1+4t$

$$\int_i^{2+5i} (3x+x^2+1)dx+(2x^2+2-x)(2x\,dx)$$

$$\left[\frac{3x^2}{2}+\frac{x^3}{3}+x\right]_i^{2+5i}+\int_i^{2+5i}(4x^3+4x-2x^2)dx$$

$$\left[\frac{3x^2}{2}+\frac{x^3}{3}+x\right]_i^{2+5i}+\left[x^4+2x^2-\frac{2}{3}x^3\right]_i^{2+5i}$$

$$\left[\frac{3x^2}{2}+\frac{x^3}{3}+x+x^4+2x^2-\frac{2}{3}x^3\right]_i^{2+5i}$$

$$\left[\frac{7x^2}{2}-\frac{x^3}{3}+x+x^4+2x^2\right]_i^{2+5i}$$

$$\left[\frac{7(2+5i)^2}{2}-\frac{(2+5i)^3}{3}+2+5i+(2+5i)^4+2(2+5i)^2\right]$$

$$-\left[-\frac{7}{2}-\frac{(i)^3}{3}+i+1-2\right]$$

$$\frac{7}{2}(4+25i^2-20i)+\frac{58+85i}{3}+2+5i+304i-460+2(20i-21)$$

$$-\left[\frac{i}{3}+i-1-\frac{7}{2}\right]=\boxed{\frac{88}{3}}$$

c) la recta que une $(0,1)$ y $(2,5)$

$$\int_{(0,1)}^{(2,5)} (3x+y)\,dx + (2y-x)\,dy$$

$A = i$
$B = 2+5i$

$z = A + (B-A)t$
$z = i + (2+5i-i)t$
$x = 2t$
$y = (t4+1) \quad 0 \le t \le 1 \quad dx = 2\,dt \quad dy = 4\,dt$

$$\int_{0}^{1} (3(2t) + (4t+1))(2\,dt) + (2(4t+1) - (2t))(4\,dt)$$

$$2\int_0^1 (6t + 4t + 1)\,dt + 4\int_0^1 (8t + 2 - 2t)\,dt$$

$$2\int_0^1 (10t+1)\,dt + 4\int_0^1 (6t+2)\,dt$$

$$2(5t^2 + t)\Big]_0^1 + 4\left[3t^2 + 2t\right]_0^1$$

$$12 + 20 = \boxed{32}$$

d) Las rectas que van de $(0,1)$ a $(0,5)$ y de $(0,5)$ a $(2,5)$

$$\int_{(0,1)}^{(0,5)} (3x+y)\,dx + (2y-x)\,dy$$

$z = i + (5i - i)t \qquad x = 0 \quad y = 1(+4t) \quad 0 \le t \le 1$
$dx = 0 \quad dy = 4\,dt$

$$\int_0^1 (3x+y)(0) + (2(1+4t) - 0)(4\,dt) = 4\int_0^1 (2+8t)\,dt = 4\left[2t + \frac{8}{2}t^2\right]_0^1 = \boxed{24}$$

$z = 5i + (2+5i-5i)t$
$x = 2t \quad dx = 2\,dx \quad 0 \le t \le 1$
$y = 5i \quad dy = 0$

$$\int_0^1 (3(2t) + (5i))(2\,dt) + (2y-x)(0) = 2\int_0^1 (6t+5)\,dt = 2\left[3t^2 + 5t\right]_0^1 = 16$$

$$2 + 12 + 16 = 28 + 12$$

$$\int_{(0,1)}^{(0,5)} (3x+y)\,dx + (2y-x)\,dy + \int_{(0,5)}^{(2,5)} (3x+y)\,dx + (2y-x)\,dy = \boxed{40}$$

d) las rectas que van de (0,1) a (2,1) y de (2,1) a (2,5)

$z = i + (2+i-i)t$

$x = 2t \quad dx = 2dt \quad 0 \leq t \leq 1$

$y = 1 \quad dy = 0$

$$\int_0^1 (3(2t)+1)\,2dt + 0 = 2\int_0^1 (6t+1)dt = 2[3t^2+t]_0^1 = \boxed{8}$$

$z = 2+i+(2+5i-2-i)t = 2+i+(4i)t$

$x = 2 \quad dx = 0$

$y = 1+4t \quad dy = 4dt$

$$4\int_0^1 (2(1+4t)-2)\,dt = 4\int_0^1 (2+8t-2)dt = 4\left[\frac{8t^2}{2}\right]_0^1 = \boxed{16}$$

$16+8 = \boxed{24}$

4.33 a) Evalúe $\oint_C (x+2y)dx + (y-2x)dy$ alrededor de la elipse C definida por $x = 4\cos\theta$, $y = 3\,\text{sen}\,\theta$ $0 \leq \theta < 2\pi$ si C se escribe en dirección contraria a las manecillas del reloj

$$\oint_C (4\cos\theta + 2(3\,\text{sen}\,\theta))(-4\,\text{sen}\,\theta\, d\theta) + (3\,\text{sen}\,\theta - 2(4\cos\theta))(3\cos\theta\, d\theta)$$

$dx = -4\,\text{sen}\,\theta\, d\theta \quad dy = 3\cos\theta\, d\theta$

$$-4\oint_C (4\cos\theta\,\text{sen}\,\theta + 6\,\text{sen}^2\theta)d\theta + 3\oint_C (3\,\text{sen}\,\theta\cos\theta - 8\cos^2\theta)\,d\theta$$

$$\oint_C (-16\,\text{sen}\,\theta\cos\theta - 24\,\text{sen}^2\theta + 9\,\text{sen}\,\theta\cos\theta - 24\cos^2\theta)d\theta$$

$$\oint_C -7\,\text{sen}\,\theta\cos\theta\, d\theta - 24\oint_C d\theta = -7\left[\frac{\text{sen}^2\theta}{2}\right]_0^{2\pi} - 24[\theta]_0^{2\pi}$$

$$-7[0] - 24(2\pi) = \boxed{-48\pi}$$

b) ¿Cuál es el resultado en el inciso a) si C se describen en el sentido de las manecillas del reloj?

$\boxed{R = 48\pi}$

4.34 Evalue $\int_c (x^2 - iy^2)dz$ a lo largo de

a) la parábola $y = 2x^2$ desde $(1,2)$ hasta $(2,8)$

$dy = 4x\,dx$

$$\int_c (16x^2 - iy^2)(dx + idy) = \int_c x^2dx + ix^2dy - iy^2dx + y^2dy$$

$$\int_{(1,2)}^{(2,8)} (x^2 - i(2x^2)^2)dx + \int(ix^2 + 4x^4)(4x\,dx)$$

$$\int_c (x^2 + 16x^5 + i(-4x^4 + x^2))dx = \left[\frac{x^3}{3} + \frac{16x^6}{6} + i\frac{x^3}{3} - i\frac{4x^5}{5}\right]_{(1,2)}^{(2,8)}$$

$$\frac{(2+8i)^3}{3} + \frac{16}{6}(2+8i)^6 + i\frac{(2+8i)^3}{3} - i\frac{4(2+8i)^5}{5} - \left[\frac{(1+2i)^3}{3} - \frac{16}{6}(1+2i)^6 + i\frac{(1+2i)^3}{3} - i\frac{4}{5}(1+2i)^5\right]$$

$$\boxed{\frac{511}{3} - \frac{49}{5}i}$$

b) Las rectas de $(1,1)$ a $(1,8)$ y de $(1,8)$ a $(2,8)$

$$\int_{(1,1)}^{(1,8)} (x^2 - iy^2)dz = \int_c (x^2 - iy^2)dx + i(x^2 + y^2)dy$$

$z = 1+i + (1+8i - (1+i))t$ $\quad x = 1$ $\quad y = 1+7t$

$z = 1+i + (7i)$ $\quad dx = dx(0) = 0$ $\quad dy = 7\,dt$ $\quad 0 \le t \le 1$

$$7i\int_0^1 (1 + (1+7t)^2)dt = 7i\left[t + \frac{(1+7t)^3}{7(3)}\right]_0^1 = 7i\left[1 + \frac{(8)^3}{21} - \left[\frac{1}{21}\right]\right]$$

$$7i + \frac{i\,8^3}{3} - \frac{1}{3} = \frac{20}{3} + \frac{i\,8^3}{3}$$

$z = 1+8i + (2+8i - 1-8i)t = 1+8i + (1)t$ $\quad x = 1+t$ $\quad y = 8$ $\quad 0 \le t \le 1$

$dx = dt$ $\quad dy = 0$

$$\int_0^1 (1+t)^2 - i(8)^2)(dt)$$

$$\int_0^1 (1 + 2t + t^2 - i64)dt = \left[t + \frac{2t^2}{2} + \frac{t^3}{3} - i64t\right]_0^1$$

$$1 + 1 + \frac{1}{3} - i64 = \frac{7}{3} - i64$$

$$\frac{7}{3} - \frac{i64(3)}{(3)} + \frac{20}{3} + i\frac{8^3}{3} = \boxed{\frac{518}{3} - \frac{57}{3}i}$$

4.38 Evalúe $\int_{i}^{2-i}(3xy+iy^2)\,dz$

a) a largo de la curva $x=2t-2$ $\quad y=1+t-t^2$

$dx=2dt \quad dy=dt-2t\,dt=(1-2t)\,dt$

$1\le t\le 2$

$$\int_1^2(3xy+iy^2)(dx+idy)=\int_1^2[(3(2t-2)(1+t-t^2)+i(1+t-t^2)](2dt+i(1-2t)dt)$$

$$\int_1^2[(6t-6)(1+t-t^2)+i(1+t-t^2)][2dt+i(1-2t)dt]$$

$$\int_1^2[6t+6t^2-6t^3-6-6t+6t^2+i(1+t-t^2)][2dt+i(1-2t)dt]$$

$$\int_1^2[-6+6t^2+6t^2-6t^3+i(1+t-t^2)]$$

$$2\int_1^2([-6+6t^2+6t^2-6t^3]dt+i(1+t-t^2))dt$$

$$2\left[-6t+\frac{12t^3}{3}-\frac{6t^4}{4}+i\left(t+\frac{t^2}{2}-\frac{t^3}{3}\right)\right]_1^2$$

$$2\left[-12+4(8)-\frac{6(2)^4}{4}+i\left(2+2-\frac{8}{3}\right)\right]$$

$$-2\left[-6+4-\frac{3}{2}+i\left(1+\frac{1}{2}-\frac{1}{3}\right)\right]$$

$$2\left(-12+32-24+i\left(4-\frac{8}{3}\right)-\left(-2-\frac{3}{2}+i\left(\frac{7}{6}\right)\right)\right)$$

$$2\left(4+2+\frac{3}{2}+i\left(4-\frac{8}{3}-\frac{7}{6}\right)\right)=15+i\frac{1}{3}$$

$\frac{24}{-16}$
8

$$i\int_1^2(-6+12t^2-6t^3+i(1+t-t^2))(1-2t)\,dt$$

$$+12t-24t^3-12t^4+i(-2t-2t^2+2t^3))dt$$

$$i\int_1^2(-6+12t^2-6t^3+i(1+t-t^2)+12t-24t^3-12t^4+i(1-t-3t^2+2t^3))dt$$

$$i\int_1^2(-6+12t-30t^3+12t^2-12t^4+i(1-t-3t^2+2t^3)$$

$$i\left[-6t+6t^2-\frac{30t^4}{4}+4t^3-\frac{12t^5}{5}+i\left(1-\frac{t^2}{2}-t^3+\frac{t^4}{2}\right)\right]_1^2$$

$$i\left(-12+24-120+4(2)^3-\frac{12(2)^5}{5}+i(2-2-8+8)\right.$$

$$\left.-\left(-6+6-\frac{30}{4}+4-\frac{12}{5}+i\left(1-\frac{1}{2}-1+\frac{1}{2}\right)\right)\right)$$

$$i\left(12+24-120+32-\frac{12(16)}{5}+i(0)+\frac{15}{2}+4-\frac{12}{5}+i(0)\right)$$

$$44-120-\frac{192}{5}+4+\frac{75-24}{10}=-72+\frac{51-384}{10}=-72+\frac{-233}{10}=\boxed{\frac{953}{10}}$$

$$\int_i^{2-i}(3xy+iy^2)\,dz=15+i\frac{1}{3}-\frac{953}{10}=\boxed{\frac{1}{3}i+\frac{79}{30}i}$$

4. 3b Hallar $\oint_c (z^2+3z)\,dz$ a lo largo de

a) la circunferencia $|z|=2$ desde $(2,0)$ hasta $(0,2)$ en sentido contrario a las manecillas del reloj

$$\oint_c (x+i^2y+ixy+3x+3y)(dx+idy) = \oint_c (4x+2y+ixy)(dx+idy)$$

$$z=2e^{i\theta} \qquad 0\le\theta\le\frac{\pi}{2} \qquad dz=2ie^{i\theta}d\theta$$

$$\int_0^{\frac{3\pi}{2}}\left[(2e^{i\theta})^2+3(2e^{i\theta})\right](2ie^{i\theta}d\theta) = 2i\int_0^{\frac{\pi}{2}} (4e^{i\theta 3}+6e^{i2\theta})\,d\theta$$

$$2\int_0^{\frac{\pi}{2}}(4ie^{i\theta 3}+6ie^{i2\theta})\,d\theta = 2\left[\frac{4}{3}e^{i3\theta}+3e^{i\theta}\right]_0^{\frac{\pi}{2}}$$

$$2\left[\frac{4}{3}e^{i\frac{3\pi}{2}}+3e^{i\pi}\right]-2\left[\frac{4}{3}+3\right]$$

$$\frac{8}{3}\left(\cos\left(\frac{3\pi}{2}\right)+i\,\mathrm{sen}\left(\frac{3\pi}{2}\right)\right)+6(\cos(\pi)+i\,\mathrm{sen}(\pi))-\frac{26}{3}$$

$$-\frac{8}{3}i-6-\frac{26}{3}=\boxed{-\frac{8}{3}i-\frac{44}{3}}$$

$$\begin{array}{r} -26 \\ -18 \\ \hline -44 \end{array}$$

b) la recta desde $(2,0)$ hasta $(0,2)$

$$z=2+(2i-2)t$$
$$x=2-2t \qquad dx=-2dt$$
$$y=2t \qquad dy=2dt \qquad 0\le t\le 1$$

$$\int_0^1 (4(2-2t)+2(2t)+i(2-2t)(2t))(-2dt+i2dt)$$

$$\int_0^1 (8-8t+4t+i(4t-4t^2))(-2dt+i2dt)$$

$$-2\int_0^1(8-4t+i(4t-4t^2))\,dt = -2\left[8t-\frac{4t^2}{2}+i\left(2t^2-\frac{4t^3}{3}\right)\right]_0^1$$

$$-2\left[8-2+i\left(2-\frac{4}{3}\right)\right]=\boxed{-12+i\,\frac{-4}{3}}$$

$$2i\int_0^1(8-4t+i(4t-4t^2))\,dt = 2i\left[8t-2t+i2t^2-\frac{4}{3}t^3 i\right]_0^1$$

$$2i\left[6+i\left(2-\frac{4}{3}\right)\right]_0^1=\boxed{12i-\frac{4}{3}}$$

$$\oint_c (z^2+3z)\,dz = -12-i\frac{4}{3}+12i-\frac{4}{3}\ \boxed{\frac{44}{3}-\frac{8}{3}i}$$

c) las rectas de $(2,0)$ a $(2,2)$ y de $(2,2)$ a $(0,2)$

$$\oint_c (z^2+3z)\,dz=\boxed{-\frac{44}{3}-\frac{8}{3}i}$$

b) a lo largo de la recta que une $z=i$ y $z=2-i$

$$\int_i^{2-i}(3xy+iy^2)\,dz=\int_i^{2-i}(3xy+iy^2)(dx+idy)$$

$$\int_C (3xy+iy^2)dx+i(3xy+iy^2)dy$$

$$z=i+(2-i-i)t=i+2t-2it$$

$$x=2t \qquad dx=2dt$$

$$y=1-2t \qquad dy=-2dt \qquad 0\leq t\leq 1$$

$$\int_0^1 (3(2t)(1-2t)+i(1-2t)^2)(2dt)+i(6t-12t^2+i(1-2t)^2)(-2dt)$$

$$2\int_0^1(6t-12t^2+i(1-4t+4t^2))dt-i2\int_0^1(6t-12t^2+i(1-4t+4t^2))dt$$

$$2\left(3t^2-4t^3+i\left(t-\frac{4t^2}{2}+\frac{4t^3}{3}\right)\right)\Big|_0^1-i2\left(3t^2-4t^3+i\left(t-2t^2+\frac{4t^3}{3}\right)\right)\Big|_0^1$$

$$(2(-1+i(-1+\tfrac{4}{3}))-(i2)(-1+i(-1+\tfrac{4}{3}))$$

$$-2+i\frac{2}{3}+i2-\frac{2i}{3}=\boxed{-2+i2}$$

c) la recta de $(1,1)$ a $(2,8)$

$z = 1+i+(2+8i-1-i)t = 1+i+(1+7i)t$

$z=(1+t)+i(1+7t)$

$x = 1+t \qquad dx = dt$

$y = 1+7t \qquad dy = 7dt \qquad 0 \leq t \leq 1$

$$\int_C (x^2 - iy^2)\,dx + i(x^2+y^2)\,dy$$

$$\int_0^1 [(1+t)^2 - i(1+7t)^2](dt) + i(1+t+1+7t)(7dt)$$

$$\int_0^1 [(1+t^2+2t) - i(1+49t^2+14t) + i(8t+2)(7)]dt$$

$$\int_0^1 (1+t^2+2t)dt + i\int_0^1 (-1-49t^2-14t+56t+14)dt$$

$$i\int_0^1 (14-1-49t^2+42t)dt = i\left[13t - \frac{49t^3}{3} + 21t^2\right]_0^1 = i\left[13+21-\frac{49}{3}\right]$$

$$\left(34-\frac{49}{3}\right)i$$

$$\int_0^1 (1+t^2+2t)dt = \left[t+\frac{t^3}{3}+t^2\right]_0^1 = 2+\frac{1}{3} = \frac{7}{3}$$

$$\frac{7}{3} + \left(34 - \frac{49}{3}\right)i = \frac{7}{3} + \left(\frac{102-49}{3}\right)i$$

$$= \boxed{\frac{7}{3} + \frac{53}{3}i}$$

Evalúe $\oint_C |z|^2 dz$ al rededor del cuadrado con vértices en $(0,0), (1,0), (1,1), (0,1)$

$(0,0)$ a $(1,0)$

$z = x+iy$

$|z|^2 = x^2+y^2$

$dz = dx + i dy$

$z = \begin{cases} y = 0 \\ x = t \end{cases}$ $dy = 0$, $dx = dt$, $0 \le t \le 1$

$$\oint_C |z|^2 dz = \oint_C (x^2+y^2)(dx+idy)$$

$$\oint_C (x^2+y^2)dx + i\oint_C (x^2+y^2)dy$$

Integral 1

$$\int_{(0,0)}^{(1,0)} t^2 dt = \left[\frac{t^3}{3}\right]_0^1 = \boxed{\frac{1}{3}}$$

$z = 1+(1+i-1)t = 1+it$

$dz = dt$

$x = 1$, $y = t$, $dx = 0$, $dy = dt$, $0 \le t \le 1$

$(1,0)$ a $(1,1)$

Integral 2

$$i\int_{(1,0)}^{(1,1)} (1+t^2)(dt) = i\left[t+\frac{t^3}{3}\right]_0^1 = \boxed{i\,\frac{4}{3}}$$

Integral 3

$(1,1)$ a $(0,1)$

$z = 1+i+(i-1-i)t$

$x = 1-t$, $dx = -dt$

$y = 1$, $dy = 0$, $0 \le t \le 1$

$$-\int_{(0,1)}^{(0,1)} ((1-t)^2+1)dt = -\int_0^1 (1-2t+t^2+1)dt = -\left[2t - t^2 + \frac{t^3}{3}\right]_0^1$$

$$-\left[1+\frac{1}{3}\right] = \boxed{-\frac{4}{3}}$$

$z = i+(-i)t$, $0 \le t \le 1$

$x = 0$, $dx = 0$

$y = 1-t$, $dy = dt$

$(0,1)$ a $(0,0)$

Integral 4

$$-i\int_{(0,1)}^{(0,0)} (1-t)^2 dt = -i\int_0^1 (1-2t+t^2)dt = -i\left[t-t^2+\frac{t^3}{3}\right]_0^1 = \boxed{\frac{-i}{3}}$$

$$\oint_C |z|^2 dz = \frac{1}{3} + \frac{4i}{3} - \frac{4}{3} - \frac{i}{3} = \boxed{-1+i}$$

4.43 Evalue $\oint_C \frac{dz}{z-2}$ alrededor de

a) la circunferencia $|z-2|=4$

$$\oint_{|z-2|=4} \frac{dz}{z-2} = 2\pi i(1) = \boxed{2\pi i}$$

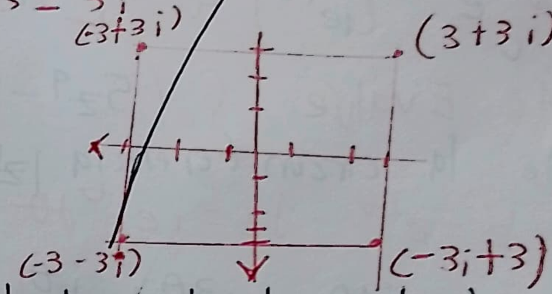
$\boxed{z=2}$

b) la circunferencia $|z-1|=5$

$$\oint_{|z-1|=5} \left(\frac{dz}{z-2}\right) = \boxed{2\pi i}$$

c) el cuadrado con vértices en $3\pm3i$, $-3\pm3i$

$$\oint_{|z|=3} \left(\frac{dz}{z-2}\right) = \boxed{2\pi i}$$



4.41 Evalue $\int_C (z^2+1)^2\,dz$ a lo largo de la cicloide $x=a(\theta-\text{sen}\,\theta)$, $y=a(1-\cos\theta)$ desde el punto en el que $\theta=0$ hasta el punto en el que $\theta=2\pi$.

$$\boxed{z=a(\theta-\text{sen}\,\theta)+i(a(1-\cos\theta))}$$

$$\int_0^{2\pi}(z^4+2z^2+1)\,dz = \left[\frac{z^5}{5}+\frac{2z^3}{3}+z\right]_0^{2\pi}$$

$z^5(0)=(a(0)+ia(0))^5=0$ $\qquad$ $z^5(2\pi)=(a(2\pi)+i(0))^5=(2\pi a)^5$

$z^3(0)=0$ $\qquad$ $z^3(2\pi)=(2\pi a)^3$

$z(0)=0$ $\qquad$ $z(2\pi)=(2\pi a)$

$$\left[\frac{(2\pi a)^5}{5}+\frac{2(2\pi a)^3}{3}+2\pi a\right] = \boxed{\frac{96\pi^2 a^2}{15}+\frac{80\pi^3 a^3}{15}+\frac{30\pi a}{15}}$$

4.39 Evalúe $\oint_c \bar{z}^2 dz$ al rededor de las circunferencias a) $|z|=1$ $dz = ie^{i\theta} d\theta$

$z = e^{i\theta}$ $\bar{z} = e^{-i\theta}$

$\oint_c e^{-2i\theta}(ie^{i\theta})\,d\theta = \int_0^{2\pi} i e^{-i\theta} d\theta = [-e^{-i\theta}]_0^{2\pi}$

$[-e^{-i2\pi} + e^{-i(0)}] = [-(\cos(-2\pi) + i\,sen(2\pi)) + \cos(-0) + i\,sen(0)] = -1+1 = \boxed{0}$

b) $|z-1| = 1$

$\oint_c e^{-2i\theta}(ie^{i\theta})\,d\theta = [-e^{-i\theta}]_0^{2\pi} = [e^{-i\theta} - e^{-i2\pi}] = \boxed{0}$

4.40 Evalúe $\oint_c (5z^4 - z^3 + z)\,dz$ al rededor

a) de la circunferencia $|z| = 1$

$z = e^{i\theta}$ $dz = ie^{i\theta} d\theta$ $0 \le \theta \le 2\pi$

$\int_0^{2\pi} (5e^{i4\theta} - z^{i3\theta} + e^{i\theta})(ie^{i\theta} d\theta) = \int_0^{2\pi} (5ie^{i5\theta} - iz^{i4\theta} + ie^{i2\theta}) d\theta$

$\left[e^{i5\theta} - \frac{e^{i4\theta}}{4} + \frac{e^{i2\theta}}{2}\right]_0^{2\pi} = \left[e^{i10\pi} - \frac{e^{i8\pi}}{4} + \frac{e^{i4\pi}}{2} - 1\right] + \frac{1}{4} - \frac{1}{2}$

$-1 + \frac{1}{4} - \frac{1}{2} + 1 - \frac{1}{4} + \frac{1}{2} = \boxed{0}$

b) del cuadrado con vértices en $(0,0), (1,0), (1,1)$, y $(0,1)$

$\oint_c (5z^4 - z^3 + z)\,dz = \int_{x=0}^{1} (5z^4 - z^3 + z)\,dz + \int_{y=0}^{1} (5z^4 - z^3 + z)\,dz$

$+ \int_{x=1}^{0} (5z^4 - z^3 + z)\,dz + \int_{y=1}^{0} (5z^4 - z^3 + z)\,dz = \boxed{0}$

c) de la curva que consta de las parábolas $y = x^2$ desde $(0,0)$ hasta $(1,1)$ y $y^2 = x$ desde $(1,1)$ hasta $(0,0)$

$y = x^2$ $\quad y^2 = (x)$

$dy = 2x\,dx$ $\quad dx = 2y\,dy$

$\oint_0^1 (5(x+ix^2)^4 - (x+ix^2)^3 + (x+ix^2))(dx + i2x\,dx) +$

$\int_1^0 (5(x^2+iy)^4 - (y^2+ix)^3 + (y^2+iy))(2y\,dy + i\,dy) = \boxed{0}$

$(2+3i)$ A $+ t(B-A)$ $= (0,2)$

$(2,3)$

$3i + (2)t$

$(0,3)$ $3i$

$(2,4)$ $2+4i$

$z = 3i + (2+4i - 3i)t$

$z = 3i + (2+i)t$

$(2)t + i(3+t)$

x

$(x, y) = (2t) + i(t+3)$

$\boxed{\begin{array}{l} x = 2t \\ y = t+3 \end{array}}$ $0 \le t \le 1$

$\int_{(0,3)}^{(2,4)} (2y + x^2)\,dx + (3x - y)\,dy$

$\boxed{dx = 2\,dt}$ $\boxed{dy = dt}$

$\int_0^1 2(t+3) + (2t)^2 \cdot 2\,dt$

$(2t + 6 + 4t^2)\,2\,dt$

$(4t + 12 + 8t^2)\,dt$

$(6t - (t+3))\,dt$

$\int_0^1 4t + 12 + 8t^2 \ldots$

$(4t - t - 3)\,dt$

$(3t - 3)\,dt$

$3t - 3 + 4 \ldots$

$7t + 4 + \ldots$