INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

LISTA DE EJERCICIOS 1-12

SEMANA 9

NOMBRE DEL ALUMNO: GARCÍA QUIROZ GUSTAVO IVAN

GRUPO: 4CV3

MATERIA: MATEMATICAS AVANZADAS PARA LA INGENIERIA

NOMBRE DEL PROFESOR: MARTINEZ NUÑO JESUS ALFREDO

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Libro: Schaum

7.49 Demuestre que $\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$

Ejercicio 1

Ceros de la ecuación

$x^4 = -1$

$x = e^{i\left(\frac{\pi + 2K\pi}{4}\right)}$

$\boxed{x_0 = e^{i\frac{\pi}{4}} = \frac{1}{\sqrt{2}}(1+i)}$ $\quad x_1 = e^{i\frac{(3\pi)}{4}} = \boxed{\frac{1}{\sqrt{2}}(-1+i)}$

$\boxed{x_2 = e^{\frac{i5\pi}{4}} = \frac{1}{\sqrt{2}}(-1-i)}$ $\quad x_3 = e^{\frac{i7\pi}{4}} = \boxed{\frac{1}{\sqrt{2}}(1-i)}$

$$\int_0^\infty \frac{dx}{z^4+1} = \frac{1}{2}\int_{C_1} \frac{dx}{z^4+1} + \frac{1}{2}\int_{C_2} \frac{dx}{z^4+1} = \frac{2\pi i}{2}\left(\frac{1}{4(z_0)^3+0}\right) + \pi i\left(\frac{1}{4(z_1)^3+0}\right)$$

$$\int_0^\infty \frac{dz}{z^4+1} = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{dz}{z^4+1} = \pi i\left(\frac{2\sqrt{2}}{4(4i)(1+i)} + \frac{2\sqrt{2}}{4\left(\frac{-4i}{2}\right)(-1+i)}\right)$$

$$\frac{\pi i}{2}\left(\frac{2\sqrt{2}}{\frac{4i-2}{2}} + \frac{2\sqrt{2}}{4i+2}\right)$$

$$\left(\frac{\pi i}{2}\right)\left(\frac{\sqrt{2}(i+1) + \sqrt{2}(i-1)}{2(i-1)(i+1)}\right) = \left(\frac{\pi i}{2}\right)\left(\frac{2\sqrt{2}\,i}{-2(2)}\right)$$

$$\int_0^\infty \frac{dz}{z^4+1} = \boxed{\frac{\pi}{2\sqrt{2}}}$$

Ejercicio 2

7.50 Calcule $\int_0^\infty \frac{dx}{(x^2+1)(x^2+4)^2} = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)(x^2+4)^2} = \frac{\pi}{9(2)} + \frac{11\pi}{9(16)} = \boxed{\frac{5\pi}{288}}$

Ceros de la ecuación

$x = \sqrt{-1} = \pm i$ $\quad \boxed{x_0 = \pm i}$ $\quad \boxed{x_1 = \pm i2}$

$x = \sqrt{-4} = \pm i2$

$$\frac{1}{2}\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)(x^2+4)^2} = \frac{1}{2}\int_{C_1} \frac{\frac{1}{(x^2+4)^2}}{(x^2+1)}dx + \frac{1}{2}\int_{C_2} \frac{\frac{1}{(x^2+1)}}{(x^2+4)^2}dx$$

$$\frac{1}{2}\int_{C_1} \frac{\frac{1}{(x^2+4)^2(x+i)}}{(x-i)}dx = \frac{2\pi i}{2}\left(\frac{1}{(3)^2(2i)}\right) = \boxed{\frac{\pi}{9(2)}}$$

$$\frac{1}{2}\int_{C_2} \frac{\frac{1}{(x^2+1)(x+2i)^2}}{(x-2i)^2}dx = \frac{2\pi i}{2!}\left(\frac{(-1)[(x_1+2i)^2(2x_1) + (x_1^2+1)(2(x_1+2i))]}{((x_1^2+1)(x_1+2i)^2)^2}\right)$$

$$\rightarrow \frac{2\pi i}{1}\left(\frac{(-1)[(-16)(4i) + (-3)(8i)]}{((-3)(4i)^2)^2}\right) = \frac{-2\pi i}{1}\left[\frac{-(11i)}{(9)(16)^2}\right] = \boxed{\frac{11\pi}{(9)(16)}}$$

SR. ROBERTO AGUILAR AGUILAR	GARCIA QUIROZ GUSTAVO IVAN
GERENTE GENERAL	CONTADOR GENERAL

NOMBRE DEL ALUMNO: GARCÍA QUIROZ GUSTAVO IVAN
GRUPO: 3CV3

7.53 Calcule $\int_0^{2\pi} \frac{\cos 3\theta \, d\theta}{5+4\cos\theta}$

Ejercicio 6

Calcule. Demuestre que $\int_0^{2\pi} \frac{\cos^2 3\theta}{5-4\cos 2\theta} d\theta = \frac{3\pi}{8}$

$\cos(2\theta) = \frac{z^2+z^{-2}}{2}$ $\boxed{z=e^{i\theta}}$ $d\theta = \frac{dz}{iz}$

$\int_0^{2\pi} \frac{\cos^2(3\theta)}{5-4\left(\frac{z^2+z^{-2}}{2}\right)}\left(\frac{dz}{iz}\right) = \frac{1}{i}\int_0^{2\pi} \frac{\cos^2(3\theta)\, dz}{5z-4\left(\frac{z^3+z^{-1}}{2}\right)} = \frac{z}{i}\int_0^{2\pi} \frac{\cos^2(3\theta)\,dz}{\left(\frac{4z^4+4}{2}\right)+5z^2}$

$\frac{z}{i}\int_0^{2\pi} \frac{\cos^2(3\theta)\cancel{dz}}{-2z^4+5z^2-\cancel{4z}} = \frac{1}{i}\int_0^{2\pi} \frac{z\,\cos^2(3\theta)\,dz}{-2z^4+5z^2-2} = \frac{1}{i}\int_0^{2\pi} \frac{z\left(\frac{z^3-z^{-3}}{2}\right)^2 dz}{-2z^4+5z^2-2}$

Ceros función

$z^2 = \frac{-5 \pm \sqrt{25-4(-2)(-2)}}{2(-2)} = \frac{5}{4} \pm \frac{\sqrt{25-16}}{4} = \frac{5\pm 3}{4}$

$\boxed{z = \pm\sqrt{\frac{5\pm 3}{4}}}$ $\boxed{\begin{matrix} z_2 = \frac{1}{\sqrt{2}} \\ z_3 = -\frac{1}{\sqrt{2}} \end{matrix}}$

$z_0 = \sqrt{\frac{5+3}{4}} = \frac{\sqrt{8}}{2} \simeq \sqrt{2} \simeq 1.41$ $z_2 = \sqrt{\frac{5-3}{4}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \simeq 0.70$

$z_1 = -\sqrt{\frac{5+3}{4}} = -\frac{\sqrt{8}}{2} \simeq \sqrt{2} \simeq -1.41$ $z_3 = -\sqrt{\frac{5-3}{4}} = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}} \simeq -0.70$

$\frac{1}{4i}\int_0^{2\pi} \frac{z\,(z^3-z^{-3})^2\, dz}{-2z^4+5z^2-2} = \boxed{\frac{1}{4i}}$

Ejercicio 3

7.51. Calcule $\int_0^{2\pi} \frac{\operatorname{sen}(3\theta)}{5-3\cos(\theta)}\,d\theta$

$\boxed{z=e^{i\theta} \quad d\theta=\frac{dz}{zi}}$ $\cos(\theta)=\frac{z+z^{-1}}{2}$ $\operatorname{sen}3\theta=\frac{z^3-z^{-3}}{2i}$

$$\int_0^{2\pi} \frac{\left(\frac{z^3-z^{-3}}{2i}\right)}{5-3\left(\frac{z+z^{-1}}{2}\right)}\left(\frac{dz}{zi}\right) = -\int_0^{2\pi} \frac{(z^3-z^{-3})\,dz}{10z-3z^2-3} = -\int_{|z|=1} \frac{(z^3-z^{-3})\,dz}{-3z^2+10z-3}$$

Ceros de la función

$$z_{1,2}=\frac{-10\pm\sqrt{(10)^2-4(-3)(-3)}}{2(-3)} = \frac{-10\pm\sqrt{100-36}}{-6} = \frac{-10\pm 8}{-6}$$

$\boxed{z_1=\frac{1}{3}}$ $z_2=\frac{-18}{-6}=\frac{9}{3}=\boxed{3}$

$$=\int_{|z|=1} \frac{\frac{z^3-z^{-3}}{(z-3)}}{(z-\frac{1}{3})(1)}\,dz = 2\pi i\left(\frac{\left(\frac{1}{3}\right)^3+(3)^{3}}{\frac{1}{3}-3}\right) = \left(\frac{\frac{-1+(3)^6}{9(3)}}{\frac{1-9}{3}}\right)(2\pi i) = \frac{(2\pi i)(-1+3^6)}{(9)(-8)}$$

$$\int_{|z|=1} \frac{\frac{z^3-z^{-3}}{(z-\frac{1}{3})}}{(z-3)}\,dz = \int_{|z|=3} \frac{\frac{z^3-\frac{1}{z^3}}{(z-\frac{1}{3})}}{z-3}\,dz = 2\pi i\left(\frac{(3)^3-\frac{1}{(3)^3}}{3-\frac{1}{3}}\right) = \frac{(3)(2\pi i)((3)^6-1)}{(1+9)(3)^3}$$

$$-\int_0^{2\pi} \frac{(z^3-z^{-3})\,dz}{-3z^2+10z-3} = -\int_0^{2\pi} \frac{\frac{z^3-z^{-3}}{(z-3)}\,dz}{(z-\frac{1}{3})} - \int_0^{2\pi} \frac{\frac{z^3-z^{-3}}{(z-\frac{1}{3})}\,dz}{(z-3)} = \frac{(2\pi i)(3^6-1)}{(-8)(9)} - \frac{(2\pi i)(3^6-1)}{(-8)(9)}$$

$\boxed{0}$

Ejercicio 1

7.52. Calcule $\int_0^{2\pi} \frac{\cos(3\theta)}{5+4\cos(\theta)}\,d\theta = \int_0^{2\pi} \frac{\frac{z^3+z^{-3}}{2}}{5+4\left(\frac{z+z^{-1}}{2}\right)}\,d\theta$

$$\int_0^{2\pi} \frac{z^3+z^{-3}}{10z+4z^2+4}\left(\frac{dz}{i}\right) = \int_0^{2\pi} \frac{(z^3+z^{-3})\,dz}{(2z+4)(2z+1)}$$

$$z_{1,2}=\frac{-10\pm\sqrt{100-4(4)(4)}}{2(4)} = -\frac{5}{4}\pm\frac{\sqrt{36}}{8} = -\frac{5}{4}\pm\frac{6}{8} = -\frac{5}{4}\pm\frac{6}{8} = \frac{-5\pm 3}{4}$$

$\boxed{z_1=-2}$ $\boxed{z_0=-\frac{1}{2}}$

$$\frac{1}{2i}\int_0^{2\pi} \frac{(z^3+z^{-3})\,dz}{(z+2)(z+\frac{1}{2})} = \pi\left[\frac{z_0^3+\frac{1}{z_0^3}}{8z_0^{2}+10z_0} + \frac{z_1^3+z_1^{-3}}{8z_1+10}\right]$$

$$\pi\left[\frac{8+\frac{1}{8}}{8(2)+10} + \frac{\frac{1}{8}+8}{4+10}\right] = \pi\left[\frac{64+1}{(16+10)(8)} + \frac{1+64}{(8)(14)}\right]$$

$$\pi\left[\frac{65}{26(8)} + \frac{65}{8(14)}\right] = \boxed{\pi\frac{25}{28}}$$

Ejercicio 6

7.54 Demuestre que si $m \angle 0$ $\int_0^{\infty} \frac{\cos mx}{(x^2+1)^2} dx = \frac{\pi e^{-m}(1+m)}{4}$

ceros de la función

$x^2 = \pm\sqrt{-1} = \boxed{\pm i}$

$$\left(\frac{1}{2}\right)\int_{-\infty}^{+\infty} \frac{\cos(mx)\, dx}{((x+i)(x-i))^2} = \left(\frac{1}{2}\right)\int_{-\infty}^{+\infty} \frac{\cos(mx)\, dx}{(x+i)^2(x-i)^2} = \left(\frac{1}{2}\right)\oint_c \frac{e^{i(mz)}\, dz}{(z-i)^2(z+i)^2}$$

$$\frac{2\pi i}{2}\left[\frac{e^{i(mz)}}{(x^2+1)^2}\right]'$$

$$\left(\frac{1}{2}\right)\oint_0 \frac{\frac{e^{i(mz)}\, dz}{(z+i)^2}}{(z-i)^2} + \left(\frac{1}{2}\right)\oint_c \frac{\frac{e^{i(mz)}}{(z-i)^2}}{(z+i)^2} = \frac{1}{2} Re\left(\oint_c \frac{e^{i(mz)}\, dz}{(z^2+1)^2}\right)$$

$$\frac{2\pi i}{2} Re\left(\frac{e^{i(mz_0)}\, dz}{2(z_0^2+1)(2z_0)}\right) = \frac{1}{2}(2\pi i)\left(\frac{e^{i^2 m}}{2(-1+1)(2i)}\right)$$

$$\frac{(z+i)^2 e^{i(mz)}(im) - e^{imz}\, 2(z+i)}{(z+i)^4}$$

$$Re\left\{\left(\frac{1}{2}\right)(2\pi i)\left[\frac{(2i)^2 e^{i^2 m}(im) - e^{i^2 m}\, 2(2i)}{(2i)^4}\right]\right\}$$

$$Re\left\{\frac{2\pi i}{2}\left[\frac{(2)^2\left[-e^{-m}(im) - e^{-m} i\right]}{(2)^4}\right]\right\}$$

$$Re\left\{\left(\frac{\pi 2}{2}\right)\left[\frac{(-i^2)\left[e^{-m} m + e^{-m}\right]}{2^2}\right]\right\} = \boxed{\frac{\pi e^{-m}(1+m)}{4}}$$

7.54 Calcule $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4x+5)^2} = \oint_c \frac{dx}{(x-(-2-i))^2 \over (x-(2+i))^2}$

Ejercicio 8

Ceros

$$x_{1,2} = \frac{-4 \pm \sqrt{16-4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{-4}}{2}$$

$$x_1 = \frac{-4 \pm i2}{2} = \boxed{-2 \pm i}$$

$$x_2 = -2 \pm \frac{i2}{2} = \boxed{-2-i}$$

$$\oint_c \frac{\frac{1}{(z-(-2-i))^2} dz}{(z-(-2+i))^2} = \frac{2\pi i}{1!}\left[-\frac{2(z_1+2+i)}{(z_1+2+i)^4}\right]$$

$$2\pi i\left[-\frac{2(-2+i+2+i)}{(-2+i+2+i)^4}\right] = 2\pi i\left[-\frac{2i}{(1)}\right] = \boxed{4\pi}$$

Ejercicio 9

7.58

$$\int_0^{\infty} \frac{dx}{x^4+x^2+1} = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{dx}{x^4+x^2+1} = \frac{1}{2}\int_c \frac{dz}{z^4+z^2+1}$$

Ceros

$$x^2 = \frac{-1 \pm \sqrt{1-4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$x_1 = \sqrt{\frac{-1+i\sqrt{3}}{2}} = \frac{\sqrt{4}e^{i\frac{-\pi}{6}}}{\sqrt{2}} = \frac{\sqrt{2}e^{i\frac{\pi}{6}}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} + \left(\frac{i}{\sqrt{2}}\right) = \frac{\sqrt{3}}{\sqrt{2}} - \frac{i}{\sqrt{2}} = \frac{\sqrt{3}}{2} - \frac{i}{2}$$

$$x_c = -\sqrt{\frac{-1+i\sqrt{3}}{2}} = -\sqrt{2}e^{i\frac{\pi}{6}} = \left(-\frac{\sqrt{3}}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \boxed{-\frac{\sqrt{3}}{2} + \frac{i}{\sqrt{2}}}$$

$$x_2 = \sqrt{\frac{-1-i\sqrt{3}}{2}} = \frac{\sqrt{2}e^{i\frac{\pi}{6}}}{\sqrt{2}} = \boxed{\frac{\sqrt{3}}{2} + \frac{i}{2}}$$

$$x_3 = -\sqrt{\frac{-1-i\sqrt{3}}{2}} = -\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = -\frac{\sqrt{3}}{2} - \frac{i}{2}$$

$$\frac{1}{2}\oint_c \frac{dz}{z^4+z^2+1} = \frac{1}{2}\left[2\pi i\left[\left(\frac{1}{4\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^3+2\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)}\right) + \frac{1}{4\left(\frac{-\sqrt{3}}{2}+\frac{i}{2}\right)^3+2\left(\frac{-\sqrt{3}}{2}+\frac{i}{2}\right)}\right]\right]$$

$$\pi i\left[\frac{1}{\frac{4}{2^3}(\sqrt{3}+i)^3+i+\sqrt{3}} + \frac{1}{\frac{1}{2}(\sqrt{3}+i)^3+i-\sqrt{3}}\right] = \frac{\pi\sqrt{3}}{\sqrt{36}}$$

$$\boxed{\frac{\pi\sqrt{3}}{6}}$$

8 $\displaystyle\int_0^{2\pi} \frac{2\,\text{sen}(\theta)\,d\theta}{2+\text{sen}^2(\theta)} = \oint_{|z|=1} \frac{2\left(\frac{z-z^{-1}}{2i}\right)}{2+\left(\frac{z-z^{-1}}{2i}\right)^2}\left(\frac{dz}{iz}\right)$

$\text{sen}(\theta) = \dfrac{z-z^{-1}}{2i}$ $\boxed{z=e^{i\theta}}$

$$\frac{1}{i}\oint_{|z|=1} \frac{z-z^{-1}}{2-\frac{z^2-2+z^{-2}}{4}}\left(\frac{dz}{iz}\right) = \frac{4}{i}\oint_{|z|=1}\left(\frac{z-z^{-1}}{8-(z^2-2+z^{-2})}\right)\left(\frac{dz}{iz}\right)$$

$$= 4\oint_{|z|=1}\left(\frac{z-z^{-1}}{-8+(z^2+z^{-2}-2)}\right)\frac{dz}{z} = 4\oint \frac{z-z^{-1}}{z^3-10z+z^{-1}}\,dz$$

$$= 4\oint_{|z|=1} \frac{z^2-1}{z^4-10z^2+1}\,dz = 8\pi i\left(\frac{z_1^2-1}{4z_1^3-20z_1} + \frac{z_2^2-1}{4z_2^3-20z_2}\right)$$

Ceros de la función

$$z^2 = \frac{10 \pm \sqrt{100-4}}{2} = 5 \pm 2\sqrt{6}$$

$$\boxed{z_{1,2} = \pm\sqrt{5-2\sqrt{6}}}$$

$$\boxed{z_{3,4} = \pm\sqrt{5+2\sqrt{6}}}$$

$$8\pi i\left(\frac{5-2\sqrt{6}-1}{4(5-2\sqrt{6}-5)\sqrt{5-2\sqrt{6}}} - \frac{5-2\sqrt{6}-1}{4(5-2\sqrt{6}-5)\sqrt{5-2\sqrt{6}}}\right) = \boxed{0}$$

Ejercicio 7

381 $\displaystyle\int_{-\infty}^{\infty} \frac{x\cos(x)\,dx}{x^2-2x+10}$ Ceros de la Funcion

$x_{1,2} = \dfrac{2 \pm \sqrt{4-40}}{2(1)} = \boxed{1 \pm 3i}$ $\quad \text{Re}\left(\oint_C \dfrac{z e^{iz}\,dz}{z^2-2z+10}\right) = \text{Re}\left(\dfrac{2\pi i\,(1+3i)e^{i(1+3i)}}{2(1+3i)-2}\right)$

$$I = \frac{2\pi i\,(1+3i)\,e^{i(1+3i)}}{6i} = \frac{\pi}{3}(1+3i)\,e^{i}e^{-3} = \frac{\pi e^{-3}}{3}(1+3i)(\cos(1)+i\,\text{sen}$$

$$I = \frac{\pi}{3}\left(\cos(1) - 3\,\text{sen}(1) + i(\text{sen}(1) + 3\cos(1))\right)e^{-3}$$

$$\boxed{I = \frac{\pi}{3}e^{-3}\left(\cos(1) - 3\,\text{sen}(1)\right)}$$

Ejercicio 11

1. $\int_0^{2\pi} \frac{1}{6+\sin(\theta)} (d\theta) = \int_{|z|=1} \left(\frac{1}{6+\frac{1}{2i}\left(z-\frac{1}{z}\right)} \right) \frac{dz}{iz}$

$\int_{|z|=1} \left(\frac{dz}{6iz + \frac{1}{2}(z^2-1)} \right) = \int_c \frac{2\,dz}{12iz+z^2-1} = \int_c \frac{2dz}{z^2+12iz-1}$

$2 \oint_{|z|=1} \frac{dz}{(z^2+12iz-1)}$

$z_{1,2} = \frac{-12i \pm \sqrt{(12i)^2-4(1)(-1)}}{2(1)} = \frac{-12i \pm 2i\sqrt{35}}{2} = i(-6 \pm \sqrt{35})$

$2\oint_c \frac{dz}{(z^2+12iz-1)} = 2\oint_{|z|=1} \frac{\frac{dz}{(z+i(6+\sqrt{35})}}{(z-i(-6+\sqrt{35}))}$

$4\pi i \left[\frac{1}{(i(-6+\sqrt{35}) + i(6+\sqrt{35}))} \right] = \boxed{\frac{2\pi}{\sqrt{35}}}$

2 Ejercicio 12

$I = \int_{-\infty}^{\infty} \frac{1}{1+x^4} (dx) = \oint_{c_1} \frac{dz}{1+z^4} + \oint_{c_2} \frac{dz}{1+z^4} = 2\pi i \left(\frac{1}{4z_0^3} + \frac{1}{4z_1^3} \right)$

$z^4 = -1 = e^{i(\pi+2k\pi)}$

$z_0 = e^{i\left(\frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}}(1+i) \quad z_1 = e^{i\frac{3\pi}{4}} = \frac{1}{\sqrt{2}}(-1+i)$

$z_2 = e^{i\frac{5\pi}{4}} = \frac{1}{\sqrt{2}}(-1-i) \quad z_3 = e^{i\frac{7\pi}{4}} = \frac{1}{\sqrt{2}}(1-i)$

$I = \frac{\pi i}{2}(z_0^{-3} + z_1^{-3}) = \frac{\pi i}{2}\left(e^{-i\frac{3\pi}{4}} + e^{-9i\frac{\pi}{4}}\right)$

$I = \frac{\pi i}{2}\left(\frac{1}{\sqrt{2}}(-1-i) + \frac{1}{\sqrt{2}}(1-i)\right) = \boxed{\frac{\pi}{\sqrt{2}}}$