Libro: H Wei

INSTITUTO POLITÉCNICO NACIONAL
ESCUELA SUPERIOR DE COMPUTO

**LISTA DE EJERCICIOS 1-12**
**SEMANA 12**

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GRUPO: 4CV3

**MATERIA:** MATEMATICAS AVANZADAS PARA LA INGENIERIA

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7.38 Demostrar que

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots + = \frac{\pi^2}{6}$$

[Sugerencia: hacer $t = \pi$ en el resultado del problema 7.11].

$$\pi^2 = \frac{1}{3}\pi^2 + \sum_{n=1}^{\infty} \frac{4(-1)^n \cos(n\pi)}{n^2}$$

$$\pi^2 = \frac{1}{3}\pi^2 + \sum_{n=1}^{\infty} \frac{4(-1)^n (-1)^n}{n^2}$$

$$\left(\frac{3}{3}\right)\pi^2 - \frac{1}{3}\pi^2 = 4\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{2\pi^2}{3}$$

$$\frac{2\pi^2}{12} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\boxed{\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}}$$

Problema 7.35 Desarrollar $f(t) = e^{r\cos(t)} \cos(r\operatorname{sen} t)$ en serie de Fourier

[Sugerencia: Usar la serie de potencias para $e^z$ cuando $z = re^{it}$

$$f(t) = e^{r\cos(t)} \cos(r\operatorname{sen}(t))$$

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \frac{z^5}{5!} + \dots = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$z = re^{it} = r\cos(t) + ir\operatorname{sen}(t)$$

$$e^{r\cos(t) + ir\operatorname{sen}(t)} = e^{r\cos}[\cos(r\operatorname{sen}(t)) + i e^{r\cos(t)} \operatorname{sen}(r\operatorname{sen} t))$$

$$\sum_{n=0}^{\infty} \frac{r^n(\cos(t) + i\operatorname{sen}(t))^n}{n!}$$

$$\sum_{n=0}^{\infty} \frac{r^n \cos(nt)}{n!} + \frac{r^n \operatorname{sen}(tn)}{n!}$$

$$f(t) = e^{r\cos(t)} \cos(r\operatorname{sen}(t)) = \sum_{n=0}^{\infty} \frac{r^n}{n!} \cos(nt)$$

$$\boxed{f(t) = 1 + \sum_{n=1}^{\infty} \frac{r^n}{n!} \cos(nt)}$$

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Problema 1.33 Encontrar la serie de Fourier para la función $f(t) = |A \operatorname{sen} w_0 t|$ (Ver Figura 1.10)

$$a_0 = \frac{2A}{\frac{\pi}{2w_0}} \int_0^{\frac{\pi}{2w_0}} \operatorname{sen}(w_0 t)\, dt = \frac{-2A}{\frac{\pi}{2w_0}} \left[\frac{\cos w_0 t}{w_0}\right]_0^{\frac{\pi}{2w_0}} = \boxed{\frac{4A}{\pi}}$$

$$a_n = \frac{2A}{\frac{\pi}{2w_0}} \int_0^{\frac{\pi}{2w_0}} \operatorname{sen}(w_0 t) \cos(2w_0 n t)\, dt = \frac{2A}{\frac{\pi}{2w_0}} \int_0^{\frac{\pi}{2w_0}} [\operatorname{sen}(w_0 t + 2w_0 n t) + \operatorname{sen}(w_0 t - 2w_0 n t)]\, dt$$

$$a_n = \frac{-2A}{\frac{\pi}{2w_0}} \left[\frac{\cos(w_0 t + 2w_0 n t)}{w_0 + 2w_0 n} + \frac{\cos(w_0 t - 2w_0 n t)}{w_0 - 2w_0 n}\right]_0$$

$$a_n = -\frac{A}{\frac{\pi}{2w_0}} \left[\frac{\cos\left(\frac{\pi}{2} + n\pi\right)}{w_0 + 2w_0 n} + \frac{\cos\left(\frac{\pi}{2} - n\pi\right)}{(w_0 - 2w_0 n)} - \frac{1}{w_0 + 2w_0 n} - \frac{1}{w_0 - 2w_0 n}\right]$$

$$a_n = \frac{4A}{\pi}\left[\frac{1}{1 - 4n^2}\right]$$

$$\boxed{b_n = 0}$$

$$\boxed{f(x) = \frac{2A}{\pi} + \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{1}{1 - 4n^2} \cos(2 n w_0 t)}$$

Ejercicio 2

2.37 Encontrar la serie de Fourier para la función $f(t)$ definida por $f(t)=e^t$ en el intervalo $(-\pi,\pi)$ y $f(t+2\pi)=f(t)$ (Ver la figura 19)

$$f(t)=\sum_{n=-\infty}^{\infty} d_n e^{i\frac{n\pi x}{L}} = \sum_{n=-\infty}^{\infty} d_n e^{inx}$$

$$d_n=\sum_{n=-\infty}^{\infty} c_n \qquad d_n=\frac{1}{2L}\int_{-L}^{L} f(x)e^{-inx}\,dx$$

$$d_n=\frac{1}{2\pi}\int_{-\pi}^{\pi} e^t e^{-int}\,dt=\left(\frac{1}{2\pi}\right)\left[\frac{e^{\pi(1-in)}-e^{-\pi(1-in)}}{(1-in)}\right]$$

$$f(x)=\left(\frac{1}{2\pi}\right)\sum_{-\infty}^{\infty}\left[\frac{e^{\pi(1-in)}-e^{-\pi(1-in)}}{1-in}\right]e^{inx}$$

$$e^{inx}=\cos(nx)+i\,sen(nx)=e^{inx}$$
$$e^{i\pi n}=\cos(\pi n)+i\,sen(n\pi)=e^{in\pi}$$

$$f(x)=\left(\frac{1}{2\pi}\right)\sum_{n=-\infty}^{\infty}\left[\frac{e^{\pi}(\cos(n\pi)-i\,sen(n\pi))-e^{-\pi}(\cos(n\pi)+i\,sen(n\pi))}{(2\pi)(1-in)}\right]e^{inx}$$

$$\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{(1-in)}\left(\frac{e^{\pi}-e^{-\pi}}{(2)\pi}\right)e^{inx}=\sum_{n=-\infty}^{\infty}\frac{\sinh(\pi)}{\pi}\left[\frac{(-1)^n(1+in)}{1+n^2}e^{inx}\right]$$

$$\boxed{f(x)=\sum_{n=-\infty}^{\infty}\frac{\sinh(\pi)}{\pi}\left[\frac{(-1)^n(1+in)}{1+n^2}\right]\cos(nx)+i\,sen(nx)}$$

$$f(x)=\frac{\sinh(\pi)}{\pi}\left(\sum_{n=-\infty}^{-1}\frac{(-1)^n(1+in)}{1+n^2}(\cos(nx)+i\,sen(nx))+1+\sum_{n=1}^{\infty}\frac{(-1)^n(1+in)}{1+n^2}(\cos(nx)+i\,sen(nx))\right)$$

$$f(x)=\frac{\sinh(\pi)}{\pi}\left(\sum_{n=1}^{\infty}\frac{(-1)^n(1-in)}{1+n^2}(\cos(nx)-i\,sen(nx))+1+\sum_{n=1}^{\infty}\frac{(-1)^n(1+in)}{1+n^2}(\cos(nx)-i\,sen(nx))\right)$$

$$\boxed{f(x)=\frac{2\,sen\,h(\pi)}{\pi}\left(\frac{1}{2}+\frac{(-1)^n}{1+n^2}\sum_{n=1}^{\infty}(\cos(nx)-n\,sen(nx))\right)}$$

$$f(t) = \begin{cases} 1 & 0 \le t < 1 \\ -1 & -1 \le t < 0 \\ 0 & |t| > 1 \end{cases}$$

$$F(w) = \int_{-1}^{0} -e^{-iwt}\,dt + \int_{0}^{1} e^{-iwt}\,dt = \frac{2i}{w}(\cos(w) - 1)$$

$$\frac{e^{-iwt}}{(iw)}\Big|_{-1}^{0} + \left[\frac{e^{-iwt}}{-wi}\right]_{0}^{1} = \frac{e^{iw}}{iw} - \frac{1 - e^{iw}}{iw} + \frac{e^{-iw} - e^{0}}{(-wi)}$$

$$\frac{1 - e^{iw} - e^{iw} + 1}{iw} = \frac{2(1 - e^{iw})}{iw} = \boxed{\frac{2(1-e^{iw})}{i^2 w}}$$

$$|F(w)| = \frac{2i(-1 + e^{iw})}{w} = |F(w)|$$

$$\boxed{F(w) = \frac{2i(\cos(w) - 1)}{w}}$$

$$2.\ f(t) = \begin{cases} \sin(t) & \text{para } -K \le t \le K \\ 0 & \text{para } |t| > K \end{cases}$$

$$H(t+K) - H(t-K)$$

$F[f(t)\sin(w_0 t)] = \frac{1}{2}\left(F(w - w_0) - F(w + w_0)\right)$

$$F(w) = \frac{1}{2}\left[\frac{2\sin(K(w+1))}{w+1} - \frac{2\sin(K(w-1))}{(w-1)}\right]$$

$$\boxed{F(w) = \left[\frac{\sin K(w+1)}{w+1} - \frac{\sin(K(w-1))}{w-1}\right] i}$$

$K = \sqrt{7}$

$t$

$nx$

Gustavo Ivan Garcia Quiroz

3. $f(t) = 5\,[H(t-3) - H(t-11)]$

$F(w) = \frac{1}{-iw} 5 \int_3^{11} e^{-iwt} (-iw\,dt) = \frac{5}{-iw} (e^{-iw11} - e^{-iw3})$

$= \frac{5}{-iw} (e^{-iw11} - e^{-iw3}) = \frac{5}{-iw} (e^{-iw4} - e^{iw4})\, e^{-iw7} = \frac{10}{w} \frac{e^{iw4} - e^{-iw4}}{2i} e^{-iw7}$

$\boxed{F(w) = \frac{10}{w} \operatorname{sen}(4w)\, e^{-iw7}}$

4. $f(t) = 5e^{-3(t-5)^2} = 5e^{-3(t^2-10t+25)}$

Por corrimiento en el tiempo

$F\{f(t-t_0)\}(w) = F(w)\, e^{-iwt_0} = \int_{-\infty}^{\infty} 5e^{-3(t-5)^2} e^{-iwt} dt$

$F(w) = 5\sqrt{\frac{\pi}{3}}\, e^{-w^2/12}\, e^{-5iw} = \boxed{5\sqrt{\frac{\pi}{3}}\, e^{-\frac{w^2}{12}}}$

5. $f(t) = H(t-K)\, e^{-\frac{t}{4}}$

$F(w) = \int_0^{\infty} e^{-\frac{t}{4}} e^{-iwt} dt = \int_K^{\infty} e^{(-iw-\frac{1}{4})t} dt = \left[ \frac{e^{(-iw-\frac{1}{4})t}}{(-iw-\frac{1}{4})} \right]_K^{\infty}$

$\lim_{t \to \infty} \frac{1}{e^{t(iw+\frac{t}{4})}(-iw-\frac{1}{4})} = \boxed{0}$

$F(w) = \frac{e^{-K(iw+\frac{1}{4})}}{\frac{-4iw-1}{4}} = \boxed{\frac{4e^{(iw+\frac{1}{4})K}}{1+4iw}}$

6. $f(t) = H(t-K)\, t^2$

$F(w) =$

Libro: II.

Gustavo ... Quiroz

4. $f(x) = 1-x$ para $0 \le x < 6$ periodo 6

$c_0 = \frac{1}{6}\int_0^6 (1-x)\,dx = \frac{1}{6}\left[x - \frac{x^2}{2}\right]_0^6 = \boxed{-3}$

$c_n = \frac{1}{6}\int_0^6 (1-x)\,e^{-\frac{2\pi i n x}{6}}\,dx = \frac{1}{6}\left[\frac{-3(1-x)}{\pi i n} + \frac{3}{\pi i n}\int e^{-\frac{\pi i n x}{3}}\,dx\right]_0^6$

$c_n = \left[\frac{(i\pi n x - i\pi n + 3)\,e^{-i\frac{n\pi x}{3}}}{2\pi^2 n^2} + C\right]_0^6 = \boxed{\frac{5\pi n + 3i + i\pi n - 3}{2\pi^2 n^2}}$

$f(x) = -3 + \sum_{n=-\infty}^{\infty} \frac{-5\pi n - 3i - i\pi n + 3}{2\pi^2 n^2}$

$\boxed{f(x) = -3 + \frac{1}{2\pi^2}\sum_{n=-\infty}^{\infty} \frac{-5\pi n - 3i - i\pi n + 3}{n^2}}$

5. $f(x) = \begin{cases} -1 & \text{para } 0 \le x < 2 \\ 2 & \text{para } 2 \le x < 4 \end{cases}$ f tiene periodo 4

$c_0 = \frac{1}{4}\left[\int_0^2 (-1)\,dx + \int_2^4 2\,dx\right] = \frac{1}{4}\left[[-x]_0^2 + [2x]_2^4\right] = \boxed{\frac{1}{2}}$

$c_n = \frac{1}{4}\left[\int_0^4 f(x)\,e^{-\frac{2in\pi x}{4}}\,dx\right] = \frac{1}{4}\left[\int_0^2 (-1)\,e^{-\frac{in\pi x}{2}} + \int_2^4 2e^{-\frac{in\pi x}{2}}\,dx\right]$

$\frac{1}{4}\left[\left[e^{-\frac{in\pi x}{2}}\left(\frac{(-1)2}{i\pi n}\right)\right]_0^2 + \left[\frac{2(-2)}{\pi n i}\,e^{-\frac{in\pi x}{2}}\right]_2^4\right]$

$c_n = \frac{-2e^{-2i\pi n} + 3e^{-i\pi n} - 1}{2i\pi n} = \boxed{\frac{-3 + 3(-1)^n}{2i\pi n}}$

$\boxed{f(x) = \frac{1}{2} + \frac{3}{\pi}\sum_{n=-\infty}^{\infty} e^{\frac{(2n-1)\pi i x}{2}}}$

6. $f(x) = e^{-x}$ para $0 \leq x < 5$ periodo 5

$$C_0 = \frac{1}{5}\int_0^5 e^{-x}dx = \frac{1}{5}\left[\frac{e^{-x}}{-1}\right]_0^5 = \boxed{\frac{-e^{-5}+1}{5}}$$

$$C_n = \frac{1}{5}\int_0^5 e^{-x} e^{\frac{-2i\pi x n}{5}} dx = \frac{1}{5}\int_0^5 e^{\frac{-5x-2i\pi x n}{5}} dx = \frac{1}{5}\left[\left(\frac{5}{-5-2i\pi n}\right)e^{\frac{-5x-2i\pi n x}{5}}\right]$$

$$C_n = \frac{1}{5}\left(\frac{5}{2in\pi+5} - \frac{5}{e^{2i\pi n+5}(2i\pi n+5)}\right)$$

$$C_n = \frac{1}{2ni\pi+5} - \frac{\cancel{5}}{e^{2i\pi n+5}(2i\pi n+5)} = \boxed{\frac{1-e^{-5}}{2i\pi n+5}}$$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{-2i\pi n x}{T}} = \boxed{\frac{-e^{-5}+1}{5} + 1-e^{-5}\sum_{n=-\infty}^{\infty}\frac{1}{2i\pi n+5}e^{\frac{2\pi i n x}{5}}}$$

7. $f(x) = \begin{cases} x & \text{para } 0 \leq x \leq 1 \\ 2-x & \text{para } 1 \leq x < 2 \end{cases}$, f tiene periodo 2

$$C_0 = \frac{1}{2}\left[\int_0^1 x\,dx + \int_1^2 (2-x)dx\right] = \frac{1}{2}\left[\left[\frac{x^2}{2}\right]_0^1 + \left[2x-\frac{x^2}{2}\right]_1^2\right]$$

$$C_0 = \frac{1}{2}\left(\frac{1}{2} + [4-2] - \left[2-\frac{1}{2}\right]\right) = \boxed{0}$$

$$C_n = \frac{1}{2}\left[\int_0^1 x e^{-i\pi n x}dx + \int_1^2 (x-2)e^{-i\pi n x}dx\right]$$

$$\frac{1}{2}\left[\frac{x}{-i\pi n}e^{-i\pi n x} + \frac{1}{i\pi n}\int e^{-i\pi n x}\right]_0^1 + \frac{1}{2}\left[\frac{x-2}{-i\pi n x}e^{-i\pi n x} + \int e^{-i\pi n x}dx\right]_1^2$$

$$\frac{+i\pi n e^{-i\pi n} + (-1)^n - 1}{2\pi^2 n^2} + \frac{(-ie^{-i\pi n})}{\pi n} + \frac{-1+e^{-i\pi n}}{\pi^2 n^2}$$

$$\frac{i\pi n e^{-i\pi n} + 3(-1)^n - 3}{2\pi^2 n^2} + \frac{i(-1)^{n+1}}{\pi n}$$

$$\boxed{f(x) = \sum_{n=-\infty}^{\infty} \frac{i\pi e^{-i\pi n} + 3(-1)^n - 3}{2\pi^2 n^2} + \frac{i(-1)^{n-2}}{\pi n}}$$

2. $f(x) = x^2$ para $0 \le x < 2$ periodo 2

$$c_0 = \frac{1}{T}\int_0^T f(x)\,dx = \frac{1}{2}\int_0^2 x^2\,dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 = \boxed{\frac{4}{3}}$$

$$c_n = \frac{1}{T}\int_0^T f(t)\, e^{-\frac{2in\pi x}{T}}\,dt = \frac{1}{2}\int_0^2 x^2 e^{-\frac{2in\pi x}{2}}\,dx$$

$$c_n = \frac{1}{2}\left[-\frac{i}{\pi^3 n^3}\left(-\pi^2 n^2 e^{-i\pi n x} x^2 - 2(-i\pi n e^{-i\pi n x} x - e^{-i\pi n x})\right)\right]_0^2$$

$$\frac{1}{2}\left[-\frac{i}{\pi^3 n^3}\left[(-\pi^2 n^2 (4) - 2(-i\pi n\, 2 - 1))\right] - \frac{1}{2}\right] = \frac{2}{n^2\pi^2} - \frac{2i}{n\pi}$$

$$\boxed{f(x) = \frac{4}{3} + \sum_{n=-\infty}^{\infty}\left[\frac{2}{n^2\pi^2} - \frac{2i}{n\pi}\right] e^{n\pi i x}}$$

3. $f(x) = \begin{cases} 0 & \text{para } 0 \le x < 1 \\ 1 & \text{para } 1 \le x < 4 \end{cases}$, $f$ tiene periodo 4

$$c_0 = \frac{1}{4}\int_0^4 f(x)\,dx = \frac{1}{4}\left[\int_0^1 0\,dx + \int_1^4 dx\right] = \frac{1}{4}[x]_1^4 = \boxed{\frac{3}{4}}$$

$$c_n = \frac{1}{4}\int_0^4 f(t)\, e^{-\frac{2in\pi x}{T}}\,dt = \frac{1}{4}\left[\int_0^1 0\, e^{-\frac{2in\pi x}{4}}\,dx + \int_1^4 e^{-\frac{2in\pi x}{4}}\,dt\right]$$

$$\frac{1}{4}\left[\frac{-2e^{-\frac{in\pi x}{2}}}{in\pi}\right]_1^4 = \frac{1}{4}\left[-\frac{2(e^{-2i\pi n})}{in\pi} + \frac{2e^{-\frac{2in\pi}{2}}}{in\pi}\right] = \frac{i}{2n\pi}\left[e^{-2i\pi n} - e^{-\frac{in\pi}{2}}\right]$$

$$\boxed{f(x) = \frac{3}{4} - \frac{1}{2\pi}\sum_{n=\infty}^{\infty}\frac{1}{n}\left(\sin\left(\frac{n\pi}{2}\right) + \left[\cos\left(\frac{n\pi}{2}\right) - 1\right] i\right) e^{\frac{n\pi i x}{2}}}$$