2. $f(x) = x^2$ para $0 \leq x < 2$ periodo 2

$c_0 = \frac{1}{T}\int^{T} f(x)dx = \frac{1}{2}\int_0^2 x^2 dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 = \boxed{\frac{4}{3}}$

INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

LISTA DE EJERCICIOS 1-12

SEMANA 10

NOMBRE DEL ALUMNO: GARCÍA QUIROZ GUSTAVO IVAN

GRUPO: 4CV3

MATERIA: MATEMATICAS AVANZADAS PARA LA INGENIERIA

NOMBRE DEL PROFESOR: MARTINEZ NUÑO JESUS ALFREDO

Ejercicio 1

Section 13.2 PROBLEM

1. $f(x) = 4 \quad -3 \le x \le 3$

$$\boxed{a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx} \qquad \boxed{a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)}$$

$$\boxed{b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{\pi n x}{L}\right)dx}$$

$$a_0 = \frac{1}{3}\int_{-3}^{3} 4\,dx = \frac{1}{3}\left[4x\right]_{-3}^{3} = \frac{1}{3}\left[4(3) - 4(-3)\right] = \frac{24}{3} = \boxed{8}$$

$$a_n = \frac{1}{3}\int_{-3}^{3} 4\cos\left(\frac{n\pi x}{3}\right)dx = \frac{4}{3}\left[-\operatorname{sen}\left(\frac{n\pi x}{3}\right)\left[\frac{n\pi}{3}\right]\right]_{-3}^{3} = \frac{4}{3}\left[-\frac{n\pi}{3}\operatorname{sen}\left(\frac{n\pi x}{3}\right)\right]_{-3}^{3}$$

$$\frac{4}{3}\left[-\frac{n\pi}{3}\operatorname{sen}\left(\frac{3n\pi}{3}\right) + \frac{n\pi}{3}\operatorname{sen}\left(\frac{-3\pi n}{3}\right)\right] = \left[-\frac{2n\pi}{3}\operatorname{sen}(\pi n)\right]\frac{4}{3} = \boxed{0}$$

$$b_n = \frac{1}{3}\int_{-3}^{3} 4\sin\left(\frac{\pi n x}{3}\right)dx = \frac{4}{3}\left[\frac{\pi n}{3}\cos\left(\frac{\pi n x}{3}\right)\right]_{-3}^{3} = \frac{4}{3}\left[\frac{\pi n}{3}\cos(\pi n) - \frac{\pi n}{3}\cos(-\pi n)\right]$$

$$\frac{4}{3}[0] = \boxed{0}$$

$$\boxed{f(x) = \frac{1}{2}a_0 + \sum_{k=1}^{\infty}\left[a_k\cos\left(\frac{\pi k x}{L}\right) + b_k\sin\left(\frac{k\pi x}{L}\right)\right]}$$

$$f(x) = \left(\frac{1}{2}\right)8 = \boxed{4}$$

3. Ejercicio 2

$f(x) = \cosh(x\pi) \qquad -1 \le x \le 1 \qquad \boxed{b_n = 0}$

$$a_0 = \frac{1}{2}\int_{-1}^{1}\cosh(x\pi)\,dx = \left[\frac{\pi\sin(x\pi)}{2\pi^2}\right]_{-1}^{1} = \frac{\pi\sin(\pi) - \pi\sin(-\pi)}{2\pi^2} = \boxed{\frac{2\pi\sin(\pi)}{2\pi^2}} = \frac{\sin h(\pi)}{\pi}$$

$$a_n = \int_{-1}^{1}\cosh(x\pi)\cos\left(\frac{n\pi x}{1}\right)dx = 2\int_{0}^{1}\cosh(\pi x)\cos(n\pi x)\,dx = \boxed{\frac{2\sin h(\pi)}{\pi}\left(\frac{(-1)^n}{n^2+1}\right)}$$

$$\boxed{a_n = \frac{\sin(\pi)}{\pi}}$$

$$\boxed{f(x) = \frac{\sinh(\pi)}{\pi} + \frac{2\sin h(\pi)}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1}\cos(n\pi x)}$$

3.

5. $f(x) = \begin{cases} -4 & \text{para } -\pi \leq x \leq 0 \\ 4 & \text{para } 0 < x \leq \pi \end{cases}$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-4)\,dx + \int_0^{\pi} 4\,dx\right] = \frac{1}{\pi}\left[[-4x]_{-\pi}^{0} + [4x]_0^{\pi}\right]$$

$$\frac{1}{\pi}[-4\pi + 4\pi] = \boxed{0}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos\left(\frac{n\pi x}{\pi}\right)dx = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-4)\cos\left(\frac{n\pi x}{\pi}\right)dx + \int_0^{\pi} 4\cos\left(\frac{n\pi x}{\pi}\right)dx\right]$$

$$\frac{1}{\pi}\left[\left[-4\left(\frac{1}{n}\right)\operatorname{sen}(nx)\right]_{-\pi}^{0} + \left[4\operatorname{sen}\left(\frac{n\pi x}{\pi}\right)\right]_0^{\pi}\right]$$

$$a_n = \frac{1}{\pi}[0+0] = \boxed{0}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{\pi n x}{L}\right)dx \quad a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-4)\,dx + \int_0^{\pi} 4\,dx\right]$$

$$\frac{1}{\pi}\left[[-4x]_{-\pi}^{0} + [4x]_0^{\pi}\right] = \frac{1}{\pi}[-4\pi + 4\pi] = 0$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -4\sin\left(\frac{\pi n x}{\pi}\right)dx + \int_0^{\pi} 4\sin\left(\frac{\pi n x}{\pi}\right)dx\right] = \frac{1}{\pi}\left[[4n\cos(nx)]_{-\pi}^{0} + [-4n\cos(nx)]_0^{\pi}\right]$$

$$\frac{1}{\pi}[4n\cos(0) - 4n\cos(\pi n) - 4n\cos(\pi n) + 4n\cos(0)]$$

$$\frac{1}{\pi}[4n - 4n(-1)^n - 4n(-1)^n + 4n] = \frac{8n - 8n(-1)^n}{\pi} = \frac{8n}{\pi}(1-(-1)^n)$$

$$f(x) = \sum_{n=1}^{\infty}\frac{8n}{\pi}(1-(-1)^n)\sin\left(\frac{\pi n x}{\pi}\right) = \sum_{n=1}^{\infty}\frac{8n}{\pi}(1-(-1)^n)\sin(nx)$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -4\sin(nx)\,dx + \int_0^{\pi} 4\sin(nx)\,dx\right] = \frac{4}{\pi}\left[+\left[\frac{1}{n}\cos(nx)\right]_{-\pi}^{0} + \left[-\frac{1}{n}\cos(nx)\right]_0^{\pi}\right]$$

$$\frac{4}{\pi}\left[\frac{1}{n}\cos(0) - \frac{1}{n}\cos(\pi n)\right] + \frac{4}{\pi}\left[-\frac{1}{n}\cos(\pi x) + \frac{1}{n}\cos(0)\right]$$

$$\frac{4}{\pi}\left[\frac{1}{n} - \frac{1}{n}(-1)^n - \frac{1}{n}(-1)^n + \frac{1}{n}\right] = \frac{4(2)}{\pi}\left[\frac{1}{n} - \frac{(-1)^n}{n}\right]$$

$$f(x) = \sum_{n=1}^{\infty}\frac{8}{\pi}\left[\frac{1-(-1)^n}{n}\right]\sin(nx) = \boxed{\frac{16}{\pi}\sum_{n=1}^{\infty}\frac{\sin((2n-1)x)}{2n-1}}$$

5. $f(x) = \begin{cases} -4 & \text{para } -\pi \le x \le 0 \\ 4 & \text{para } 0 < x \le \pi \end{cases}$

3. 8. $f(x) = \begin{cases} -x & -5 \le x \le 0 \\ 1+x^2 & 0 \le x \le 5 \end{cases}$

$$a_0 = \frac{1}{5}\left[\int_{-5}^{0} -x\,dx + \int_0^5 (1+x^2)\,dx\right] = \boxed{\frac{71}{3}}$$

$$a_n = \frac{1}{5}\left[\int_{-5}^{0} (-x)\cos\left(\frac{n\pi x}{5}\right)dx + \int_0^5 (1+x^2)\cos\left(\frac{n\pi x}{5}\right)dx\right]$$

$$a_n = \left[-\frac{x5}{n\pi}\sin\left(\frac{n\pi x}{5}\right) + \int \frac{5}{n\pi}\sin\left(\frac{n\pi}{5}x\right)\right]_{-5}^{0} +$$

$$\left[\frac{5(1+x^2)}{\pi n}\sin\left(\frac{n\pi x}{5}\right) - \frac{2(5)}{\pi n}\int x\sin\frac{\pi n x}{5}\,dx\right]_{-5}^{0}$$

$$a_n = \left[\frac{x5}{\pi n}\sin\left(\frac{n\pi x}{5}\right) + \frac{25}{n^2\pi^2}\cos\left(\frac{n\pi x}{5}\right)\right]_{-5}^{0} +$$

$$\left[\frac{5(1+x^2)}{\pi n}\sin\left(\frac{n\pi x}{5}\right) - \frac{10}{\pi n}\left(-\frac{x5}{\pi n}\cos\left(\frac{\pi n x}{5}\right) + \int \frac{5}{\pi n}\cos\left(\frac{n\pi x}{5}\right)dx\right)\right]_0^5$$

$$\boxed{a_n = \frac{25}{n^2\pi^2}\left[11(-1)^n - 1\right]}$$

$$b_n = \frac{1}{5}\left[\int_{-5}^{0} x\sin\left(\frac{n\pi x}{5}\right)dx + \int_0^5 (1+x^2)\sin\left(\frac{n\pi x}{5}\right)dx\right]$$

$$b_n = \frac{1}{5}\left[\left[-\frac{x5}{n\pi}\cos\left(\frac{n\pi x}{5}\right) + \int \frac{5}{n\pi}\cos\left(\frac{n\pi x}{5}\right)dx\right]_{-5}^{0} +\right.$$

$$\left.\left[-\frac{(1+x^2)5}{n\pi}\cos\left(\frac{n\pi x}{5}\right) + \frac{2(5)}{n\pi}\int x\cos\left(\frac{n\pi x}{5}\right)dx\right]_0^5\right]$$

$$b_n = \left[-\frac{5x}{n\pi}\cos\left(\frac{n\pi x}{5}\right) + \frac{25}{n^2\pi^2}\sin\left(\frac{n\pi x}{5}\right)\right]_{-5}^{0} +$$

$$\left[-\frac{5(1+x^2)}{n\pi}\cos\left(\frac{n\pi x}{5}\right) + \frac{10}{n\pi}\left[\frac{x5}{n\pi}\sin\left(\frac{n\pi x}{5}\right) - \frac{5}{n\pi}\int \sin\left(\frac{n\pi x}{5}\right)dx\right]\right]_0^5$$

$$b_n = \frac{5}{n\pi}\left[1 - 21(-1)^n\right] + \frac{25}{n^3\pi^3}\left[(-1)^n - 10\right]$$

$$\boxed{f(x) = \frac{71}{6} + \sum_{n=1}^{\infty}\left[\frac{25}{n^2\pi^2}(11(-1)^n - 1)\cos\left(\frac{n\pi x}{5}\right) + \left\{\frac{5}{n\pi}\left[1 - 21(-1)^n\right] + \frac{25}{n^3\pi^3}\left[(-1)^n - 10\right]\right\}\sin\left(\frac{n\pi x}{5}\right)\right]}$$

5.

Ejercicio

8. 4. $f(x) = 1 - |x| \quad -2 \leq x \leq 2$

$\frac{a_0}{2} = (\frac{1}{2}) \frac{1}{L} \int_{-L}^{L} f(x) dx = \frac{1}{2(2)} \int_{-L}^{L} 1 - |x| = \boxed{0}$

$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin(\frac{\pi x n}{L}) dx = \frac{1}{2} \int_{-2}^{2} 1 - |x| \sin(\frac{\pi n}{2} x) dx$

$b_n = \frac{1}{2} \int_{-2}^{0} [1 - (-x)] \sin(\frac{\pi n}{2} x) dx + \frac{1}{2} \int_{0}^{2} [(1 - x] \sin(\frac{\pi n}{2} x)] dx$

a_n $\frac{1}{2} \left[-\frac{2(1+x)}{\pi n} \cos(\frac{\pi n x}{2}) \right]_{-2}^{0} + \frac{1}{2} \left[-\frac{2(1-x)}{\pi n} \cos(\frac{\pi n}{2} x) \right]_{0}^{2} = b_n$

$b_n = \sum_{n=1}^{\infty} \frac{1}{2} \left[\left[\frac{-2}{\pi n} (-1)^n - \frac{2}{\pi n} \right] + \frac{2(-1)^n - 2}{\pi n} \right] \sin(\frac{\pi n}{2} x) = \boxed{0}$

a_n $a_n = \frac{1}{2(2)} \int_{-2}^{2} (1 - |x|) dx \cos(\frac{\pi n}{2} x)$

$a_n = \frac{1}{2} \int_{-2}^{0} (1 - (-x)) \cos(\frac{\pi n}{2} x) dx + \int_{0}^{2} (1 - (x)) \cos(\frac{\pi n}{2} x)$

$\frac{1}{2} \left[\left[\frac{2(1+x)}{\pi n} \sin(\frac{\pi n}{2} x) + \frac{4}{\pi^2 n^2} \cos(\frac{\pi n}{2} x) \right]_{-2} + \left[\frac{4}{\pi^2 n^2} \cos(\frac{\pi n}{2} x) \right]_{0}^{2} \right]$

$a_n = \frac{1}{2} \left[\frac{4(1 - (-1)^n)}{\pi^2 n^2} + \frac{4(-(-1)^n + 1)}{\pi^2 n^2} \right] = \sum_{n=1}^{\infty} \frac{4 \cos(\frac{\pi n x}{2}) (-(-1)^n + 1)}{\pi^2 n^2}$

a $\boxed{f(x) = \sum_{n=1}^{\infty} \frac{4(-(-1)^n + 1)}{\pi^2 n^2} \cos(\frac{\pi n}{2} x)}$

b_n

Ejercicio 7

7. $f(x) = x^2 - x + 3 \qquad -2 \leq x \leq 2$

$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx = \frac{1}{2}\int_{-2}^{2}(x^2 - x + 3)\,dx = \frac{1}{2}\left[\frac{x^3}{3} - \frac{x^2}{2} + 3x\right]_{-2}^{2}$

$\frac{1}{2}\left[\frac{8}{3} - 2 + 6 - \left[\frac{-8}{3} - 2 - 6\right]\right] = \frac{1}{2}\left[\frac{16}{3} + 12\right] = \frac{8}{3} + 6 = \boxed{\frac{26}{3}}$

$a_n = \frac{1}{2}\int_{-2}^{2}(x^2 - x + 3)\cos\left(\frac{n\pi x}{2}\right)dx$

$a_n = \frac{1}{2}\left[\int_{-2}^{2} x^2\cos\left(\frac{n\pi x}{2}\right)dx - \int_{-2}^{2} x\cos\left(\frac{n\pi x}{2}\right)dx + 3\int_{-2}^{2}\cos\left(\frac{n\pi x}{2}\right)dx\right]$

Integral 1

$\int_{-2}^{2} x^2\cos\left(\frac{n\pi x}{2}\right)dx$

$u = x^2 \qquad dv = \cos\left(\frac{n\pi x}{2}\right)dx$

$du = 2x\,dx \qquad v = \frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)$

$\frac{x^2\,2}{n\pi}\sin\left(\frac{n\pi x}{2}\right) - \int 2x\left(\frac{2}{n\pi}\right)\sin\left(\frac{n\pi x}{2}\right)dx = \frac{2x^2}{n\pi}\sin\left(\frac{n\pi x}{2}\right) - 4\int\frac{x}{n\pi}\sin\left(\frac{n\pi x}{2}\right)dx$

$u = x \qquad dv = \sin\left(\frac{n\pi x}{2}\right)dx$

$du = dx \qquad v = -\frac{2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)$

$a_n = \left[\frac{2x^2}{n\pi}\sin\left(\frac{n\pi x}{2}\right) + \frac{4}{n\pi}\left[\frac{x\,2}{n\pi}\cos\frac{n\pi x}{2} - \int\frac{2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)dx\right]\right]_{-2}^{2}$

$a_n = \left[\frac{2x^2}{n\pi}\sin\left(\frac{\pi x}{2}n\right) + \frac{8x}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right) - \frac{16}{n^3\pi^3}\sin\left(\frac{n\pi x}{2}\right)\right]_{-2}^{2}$

$a_n = \frac{2^3}{n\pi}\sin(\pi x) + \frac{2^4}{n^2\pi^2}\cos(n\pi) - \frac{16}{n^3\pi^3}\sin(n\pi) - \left[\frac{2^3}{n\pi}\sin(-\pi x) + \frac{(2)(-2)}{n^2\pi^2}\cos(-n\pi) - \frac{16}{n^3\pi^3}\sin(-n\pi)\right]$

$a_n = \frac{2^4}{n^2\pi^2}(-1)^n + \frac{2^3}{n\pi}\sin(\pi n) + \left(-\frac{16}{n^2\pi^2}\cos(n\pi) - 0\right) = \frac{16(-1)^n}{n^2\pi^2} + \frac{16(-1)^n}{n^2\pi^2} = \boxed{\frac{32(-1)^n}{2n^2\pi^2}}$

$b_n = \frac{1}{2}\int_{2}^{2} f(x)\,dx$

Integral 2

$-\int_{-2}^{2} x\cos\left(\frac{n\pi x}{2}\right)dx = \frac{2x\sin\left(\frac{n\pi x}{2}\right)}{n\pi} - \int\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)dx$

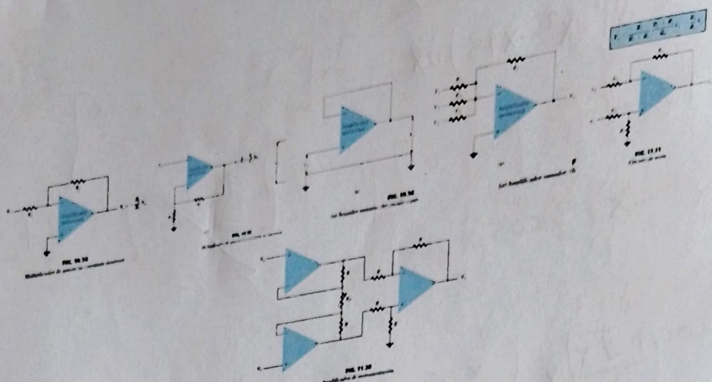
$u = x \qquad dv = \cos\left(\frac{n\pi x}{2}\right)dx$

$du = dx \qquad v = \frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)$

$\int_{-2}^{2} x\cos\left(\frac{n\pi x}{2}\right)dx = \left[\frac{2x\sin\left(\frac{n\pi x}{2}\right)}{n\pi} + \frac{4}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right)\right]_{-2}^{2} = \frac{4\sin(n\pi)}{n\pi} + \frac{4}{n^2\pi^2}\cos(n\pi) + \frac{4\sin(n\pi)}{n\pi} + \left(\frac{-4}{n^2\pi^2}\cos(n\pi)\right) \boxed{0}$

Integral 1

$3\int_{-2}^{2}\cos\frac{n\pi x}{2}dx = \left[\frac{2}{n\pi}\sin\frac{n\pi x}{2}\right]_{-2}^{2} 3 = \boxed{0}$ $\qquad \boxed{a_n = \frac{16(-1)^n}{n^2\pi^2}}$



$$b_n = \frac{2}{2}\int_{-2}^{2}(x^2-x+3)\sin\left(\frac{n\pi x}{2}\right)dx$$

$$\int_{-2}^{2}x^2\sin\left(\frac{n\pi x}{2}\right)dx = \left[\frac{-2x^2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)+\frac{4}{n\pi}\int x\cos\left(\frac{n\pi x}{2}\right)dx\right]_{-2}^{2}$$

$u = x^2 \qquad dv = \sin\left(\frac{n\pi x}{2}\right)dx$

$du = 2x\,dx \qquad v = \frac{-2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)$

$$\frac{4}{n\pi}\int x\cos\frac{n\pi x}{2} = \left[\frac{2x}{n\pi}\sin\left(\frac{n\pi x}{2}\right)-\int\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)dx\right]\frac{4}{n\pi}$$

$u = x \qquad dv = \cos\left(\frac{n\pi x}{2}\right)dx$

$du = dx \qquad v = \frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)$

$$\left[\frac{2x}{n\pi}\sin\left(\frac{n\pi x}{2}\right)+\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right)\right]\frac{4}{n\pi} = \frac{4}{n\pi}\int x\cos\left(\frac{n\pi x}{2}\right)dx$$

$$\left[\frac{-2x^2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)+\frac{4}{n\pi}\left[\frac{2x}{n\pi}\sin\left(\frac{n\pi x}{2}\right)+\frac{4}{\pi^2 n^2}\cos\left(\frac{n\pi x}{2}\right)\right]\right]_{-2}^{2}$$

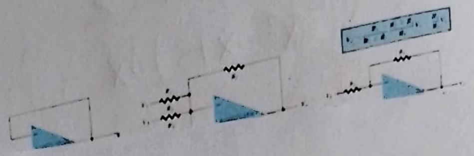
$$\left[\frac{(-1)2^3}{n\pi}(-1)^n+\frac{2^4}{n^2\pi^2}\sin(\pi n)+\frac{16}{n^3\pi^3}\cos(\pi n)\right]-\left[\frac{2^3}{n\pi}(-1)^n+\frac{16}{n^3\pi^3}\cos(n\pi)\right]$$

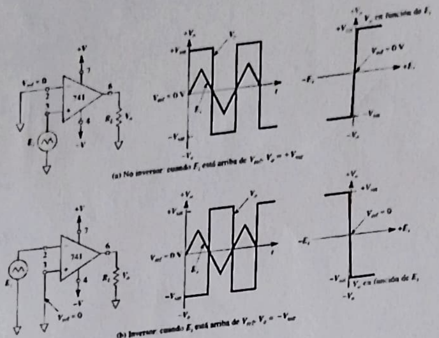
$$\frac{16\cos(n\pi)}{n^3\pi^3}+\frac{(-1)^{n+1}(2)^3}{n\pi}-\frac{(-1)^{n+1}(2)^3}{n\pi}-\frac{16}{n^3\pi^3}\cos(n\pi) \neq \boxed{b_n \neq -\frac{16}{n^3\pi^3}(-1)^n \neq 0}$$

$$-\int_{-2}^{2}x\sin\left(\frac{n\pi x}{2}\right)dx = \frac{-2x}{n\pi}\cos\left(\frac{n\pi x}{2}\right)+\int\left(\frac{2}{n\pi}\right)\cos\left(\frac{n\pi x}{2}\right)dx$$

$$\left[-\frac{2x}{n\pi}\cos\frac{n\pi x}{2}+\frac{4}{n^2\pi^2}\sin\left(\frac{n\pi x}{2}\right)\right]_{-2}^{2}$$

$$-\left[\frac{-4}{n\pi}\cos(n\pi)+\left(\frac{4}{n\pi}\cos(n\pi)\right)+0\right]+0 = \boxed{\frac{8}{n\pi}(-1)^n}$$





(a) No inversor: cuando E_i está arriba de V_{ref}, $V_o = +V_{sat}$

(b) Inversor: cuando E_i está arriba de V_{ref}, $V_o = -V_{sat}$

$$3\int_{-2}^{2} \sin\left(\frac{n\pi x}{2}\right) dx = 3\left[\frac{-2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\right]_{-2}^{2} = -3\left[\frac{-2}{n\pi}(-1)^n + \frac{2}{n\pi}(-1)^n\right] = \boxed{0}$$

$$b_n = 0 + \frac{8}{n\pi}(-1)^n + 0 = \boxed{\frac{8(-1)^n}{n\pi}}$$

$$f(x) = \frac{13}{3} + \sum_{n=1}^{\infty}\left[\frac{16(-1)^n}{n^2\pi^2}\cos(n\pi) + \frac{8(-1)^n}{n\pi}\sin(n\pi)\right]$$

$$f(x) = \frac{13}{3} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)$$

$\frac{13}{3}$ $\frac{13}{3}$ $f(x) + \frac{13}{3} + \sum_{n=1}^{\infty} (-1)^n [\cos]$

$$\boxed{f(x) = \frac{13}{3} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{16}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right) + \frac{8}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\right]}$$

Ejercicio 2

$f(x) = -x \qquad -1 \leq x \leq 1$

$$a_0 = \int_{-1}^{1} -x \, dx = \left[\frac{-x^2}{2} \right]_{-1}^{1} = \left[-\frac{1}{2} + \frac{1}{2} \right] = \boxed{0}$$

$$b_n = \int_{-1}^{1} (-x) \sin(\pi n x) \, dx = \left[\frac{+x}{\pi n} \cos(\pi n x) + \int \frac{(-1)}{\pi n} \cos(\pi n x) \, dx \right]_{-1}^{1}$$

$\boxed{du = -dx}$ $\boxed{v = \frac{-\cos(\pi n)}{\pi n}}$

$$b_n = \left[\frac{x}{\pi n} \cos(\pi n x) + \frac{(-1)\sin(\pi n x)}{\pi^2 n^2} \right]_{-1}^{1} = \frac{2}{\pi n} (-1)^n = \cancel{\boxed{\frac{2}{\pi n} (-1)^n}}$$

$$a_n = \int_{-1}^{1} (-x) \cos(\pi n x) \, dx = \left[\frac{-x}{\pi n} \sin(\pi n x) + \int \frac{\sin(\pi n x)}{\pi n} \, dx \right]_{-1}^{1}$$

$$\left[\frac{-x}{\pi n} \sin(\pi n x) + \left(- \frac{\cos(\pi n x)}{\pi n} \right) \right]_{-1}^{1}$$

$$- \frac{(-1)^n}{\pi n} + \frac{(-1)^n}{\pi n} = \boxed{0}$$

$$\boxed{f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(\pi n x)}$$