1. Evalue las siguientes potencias de i

a) $i^8 = i^4 \cdot i^4 = \boxed{1}$ b) $i^{11} = i^8 \cdot i^2 \cdot i = 1(-1)i = \boxed{-i}$

c) $i^{42} = (i^4)^{10}(i^2) = \boxed{-1}$ d) $i^{105} = ((i^4)^{26})i = \boxed{i}$

2. Escribir los números dados en la forma a+bi

a) $2i^3 - 3i^2 + 5i = -2i + 3 + 5i = 5i - 2i + 3 = \boxed{3i+3}$

b) $3i^5 - i^4 + 7i^3 - 10i^2 - 9 = 3i - 1 - 7i + 10 - 9 = \boxed{-4i}$

c) $\frac{5}{i} + \frac{2}{i^3} - \frac{20}{i^{18}} = -5i + 2i + 20 = \boxed{-3i+20}$

d) $2i^6 + \left(\frac{2}{-i}\right)^3 + 5i^{-5} - 12i = -2 + \left(\frac{8}{-i^3}\right) + \left(\frac{5}{i^5}\right) - 12i$

$\cancel{\boxed{-10-17i}} = -2 - 8i - 5i - 12i$

$\boxed{-2-25i}$

En los problemas 3-20 escribe el número dado en a+bi

3. $(5-9i) + (2-4i) = \boxed{7-13i}$

4. $3(4-i) - 3(5+2i) = 12 - 3i - 15 - 6i = \boxed{-3-9i}$

5. $i(5+7i) = \boxed{5i-7}$

6. $i(4-i) + 4i(1+2i) = 4i + 1 + 4i + 8i^2 = \boxed{8i-7}$

7. 9. $3i + \frac{1}{2-i} = 3i + \frac{1}{2-i}\left(\frac{2+i}{2+i}\right) = 3i + \frac{2+i}{4+1} = \frac{2}{5} + \frac{i}{5} + \frac{15i}{5}$

$\boxed{\frac{2}{5} + \frac{16i}{5}}$

8. 10. $\frac{i}{1+i} = \left(\frac{i}{1-i}\right)\frac{1+i}{1+i} = \frac{i-1}{2} = \boxed{-\frac{1}{2} + \frac{i}{2}}$

9. 11. $\frac{2-4i}{3+5i} = \frac{2-4i}{3+5i}\left(\frac{3-5i}{3-5i}\right) = \frac{6-10i-12i+20i^2}{9+25}$

$\boxed{-\frac{8}{17} - \frac{11i}{17}} = \frac{-16}{34} - \frac{22}{34}i$

10. 12. $\frac{10-5i}{6+2i}$

$\frac{10-5i}{6+2i}\left(\frac{6-2i}{6-2i}\right) = \frac{60-20i-30i+10i^2}{36+4}$

$\frac{50}{40} \; \frac{-50i}{40} = \boxed{\frac{5}{4}(1-i)}$

$\frac{5}{4} - \frac{5}{4}i$

Encuentre los números reales tales que x

$$2x - 3yi + 4xi - 2y - 5 - 10i = x + y + 2 - (y - x + 3)i$$

$$x - 3y - 3yi + 4xi - 7 = y + xi - 3i + 10i$$

$$x - 3y - 2yi + 3xi = 7 + 7i$$

$$x - 3y = 7$$
$$3x - 2y = 7$$

$$\left(\begin{array}{cc|c} 1 & -3 & 7 \\ 3 & -2 & 7 \end{array}\right)$$

[illegible]

5.56 Pruebe que a) $\overline{z_1/z_2} = \overline{z_1}/\overline{z_2}$ y $|z_1/z_2| = |z_1|/|z_2|$

a) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$ Si $z_1 = x_1 + iy_1$
$z_2 = x_2 + iy_2$

$$\overline{\frac{z_1}{z_2}} = \overline{\frac{x_1+iy_1}{x_2+iy_2}\left(\frac{x_2-iy_2}{x_2+(-iy_2)}\right)} = \overline{\frac{x_1x_2 + iy_1x_2 - iy_2x_1 + y_1y_2}{x_2^2+y_2^2}}$$

$$\frac{x_1x_2+y_1y_2+i\,\cancel{(y_1x_2+y_2x_1)}}{x_2^2+y_2^2}$$

$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{x_1-iy_1}{x_2-iy_2}\left(\frac{x_2+iy_2}{x_2+iy_2}\right) = \frac{x_1x_2+y_1y_2+i(y_1x_2+y_2x_1)}{x_2^2+y_2^2}$$

$$\boxed{\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}}$$ Si se cumple

b) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ Si $z_2 \neq 0$

$$\left|\frac{z_1}{z_2}\right| = \left|\frac{x_1+iy_1}{x_2+iy_2}\left(\frac{x_2-iy_2}{x_2-iy_2}\right)\right| = \left|\frac{x_1x_2+y_1y_2 \quad i(-y_2x_1+y_1x_2)}{x_2^2+y_2^2}\right|$$

$$\sqrt{\left(\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}\right)^2+\left(\frac{y_1x_2-y_2x_1}{x_2^2+y_2^2}\right)^2}$$

$$\sqrt{\frac{(x_1x_2+y_1y_2)^2+(y_1x_2-y_2x_1)^2}{(x_2^2+x_2^2)^2}} = \frac{\sqrt{x_1^2x_2^2+y_1^2y_2^2+2x_1x_2y_1y_2+y_1^2x_2^2+y_2^2x_1^2-2y_1y_2x_1x_2}}{x_2^2+y_2^2}$$

$$\left|\frac{z_1}{z_2}\right| = \sqrt{\frac{x_1^2x_2^2+y_1^2y_2^2+y_1^2x_2^2+y_2^2x_1^2}{(x_2^2+y_2^2)^2}}$$

$$\frac{|z_1|}{|z_2|} = \frac{\sqrt{x_1^2+y_1^2}}{\sqrt{x_2^2+y_2^2}}\left(\frac{\sqrt{x_2^2+y_2^2}}{\sqrt{x_2^2+y_2^2}}\right) = \sqrt{\frac{x_1^2x_2^2+y_1^2y_2^2+x_1^2y_2^2+y_1^2x_2^2}{(x_2^2+y_2^2)^2}}$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$ Si se cumple

1.55 Demuestre que a) $\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$ y b) $\overline{(z_1 z_2 z_3)} = \bar{z}_1 \bar{z}_2 \bar{z}_3$
Generalice estos resultados

a) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$ Si $z_1 = x_1 + iy_1$ $z_2 = x_2 + iy_2$

$$\overline{z_1 z_2} = \overline{(x_1 + iy_1)(x_2 + iy_2)} = \overline{x_1x_2 - y_1y_2 + i(y_2x_1 + y_1x_2)}$$
$$x_1x_2 - y_1y_2 - iy_2x_1 - iy_1x_2$$
$$\bar{z}_1 \bar{z}_2 = (x_1 - iy_1)(x_2 - iy_2) = y_1x_2 - y_1y_2 - iy_1x_2 - iy_2 - iy_1x_2$$

$\boxed{\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2}$ Si se cumple

b) $\overline{z_1 z_2 z_3} = \bar{z}_1 \bar{z}_2 \bar{z}_3$ Si $z_1 = x_1 + iy_1$ $z_2 = x_2 + iy_2$
$z_3 = x_3 + iy_3$

$$\overline{z_1 z_2 z_3} = \overline{(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3)} = \overline{z_1 z_2 z_3}$$
$$\overline{z_1 z_2 z_3} = \overline{(x_1x_2 + (-y_1y_2) + i(y_2x_1 + y_1x_2))(x_3 + iy_3)}$$
$$\overline{z_1 z_2 z_3} = \overline{x_1x_2x_3 - y_1y_2x_3 + iy_2x_1x_3 + iy_1x_2x_3 + ix_1x_2y_3 - y_1y_2y_3 i + i^2 y_2y_3x_1 + i^2 y_1y_3x_2}$$
$$\overline{z_1 z_2 z_3} = x_1x_2x_3 - y_1y_2x_3 + y_2y_3x_1 + y_1y_3x_2 - i(y_2x_1x_3 + y_1x_2x_3 + x_1x_2y_3 - y_1y_2y_3)$$

$$\bar{z}_1 \bar{z}_2 \bar{z}_3 = \overline{(x_1 + iy_1)}\,\overline{(x_2 + iy_2)}\,\overline{(x_3 + iy_3)}$$
$$\bar{z}_1 \bar{z}_2 \bar{z}_3 = (x_1x_2 - y_1y_2 - i(y_2x_1 + y_1x_2))(x_3 - iy_3)$$
$$\bar{z}_1 \bar{z}_2 \bar{z}_3 = x_1x_2x_3 - y_1y_2x_3 - i(y_2x_1x_3) - iy_1x_2x_3 - iy_3x_1x_2 + iy_1y_2y_3 + i^2 y_2y_3x_1 + i^2 y_3y_1x_2$$
$$\bar{z}_1 \bar{z}_2 \bar{z}_3 = x_1x_2x_3 - y_1y_2x_3 - y_2y_3x_1 - y_3y_1x_2 - i(y_2x_1x_3 + y_1x_2x_3 + y_3x_1x_2 - y_1y_2y_3)$$
$$\bar{z}_1 \bar{z}_2 \bar{z}_3 = x_1x_2x_3 - y_1y_2x_3 - y_2y_3x_1 - y_3y_1x_2 - i(y_1x_1x_3 + y_1x_2x_3 + y_3x_1x_2 - y_1y_2y_3)$$

$\boxed{\overline{z_1 z_2 z_3} = \bar{z}_1 \bar{z}_2 \bar{z}_3}$ Si se cumple.