INSTITUTO POLITÉCNICO NACIONAL
ESCUELA SUPERIOR DE COMPUTO

LISTA DE EJERCICIOS 1-12
SEMANA 11

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GRUPO: 4CV3

MATERIA: MATEMATICAS AVANZADAS PARA LA INGENIERIA

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INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

ESCOM

Libro: O'neil

Serie Compleja

$$\boxed{\sum_{n=-\infty}^{\infty} d_n e^{inw_0x}}$$

$$\boxed{d_n = \frac{1}{2L}\int_{-L}^{L} f(x)\, e^{-inw_0x}\,dx}$$

$$d_n = \frac{1}{T}\int_0^T f(x)\, e^{-i\frac{n\pi x}{L}}\,dx$$

Ejercicio 1

1. $f(x) = 2x$ para $0 \le x \le 3$ periodo 3

$$d_n = \frac{1}{3}\int_0^3 2x e^{-inw_0x}\,dx = \frac{1}{3}\left[-2x\left(\frac{3}{n\pi i}e^{-\frac{in\pi x}{3}}\right) + 2\int \frac{(3)e^{-\frac{in\pi x}{3}}}{n\pi i}\,dx\right]_0^3$$

$$\frac{1}{3}\left[+\frac{6xi}{n\pi}e^{-i\frac{n\pi x}{3}} + \frac{i6(3)e^{(3)}}{n^2\pi^2}^{-i\pi x n} - \frac{6}{\pi^2 n^2}\right]_0^3 = \cancel{-\frac{6\left(-i\pi n e^{-i\pi n} - e^{-i\pi n} + 1\right)}{\pi^2 n^2}}$$

$$\sum_{n=-\infty}^{\infty} \frac{-6}{\pi^2 n^2}\left(-i\pi n e^{-i\pi n} - e^{-i\pi n} + 1\right) e^{\left(\frac{in\pi x}{3}\right)}$$

$$e^{\left(\frac{in\pi x}{3}\right)}\left[-\frac{+i6}{\pi n^2}\left(-i\pi n\left(\cos(\pi n) + i\sin(\pi n)\right) - \cos(\pi n) + i\sin(\pi n) + 1\right)\right]$$

$$-\frac{6}{\pi^2 n^2}\left[-i\pi n(-1)^n - (-1)^n + 1\right] e^{i\left(\frac{n\pi x}{3}\right)}$$

$$\sum_{n=-\infty}^{\infty} \frac{-6}{\pi^2 n^2}\left[-i\pi n e^{-\frac{i\pi n x}{3}} - i e^{-\frac{+in\pi x}{3}} + e^{\frac{in\pi x}{3}}\right]$$

$$d_n = \frac{1}{3}\int_0^3 2x e^{-\frac{2in\pi x}{3}}\,dt = \frac{1}{3}\left[\frac{2x(3)e^{-\frac{2in\pi x}{3}}}{(-2in\pi)} + \int \frac{(2)3 e^{-\frac{2in\pi x}{3}}}{(2)in\pi x}\,dx\right]_0^3$$

$$d_n = \frac{1}{3}\left[\frac{3ixe^{-\frac{2in\pi x}{3}}}{n\pi} + \frac{9e^{-\frac{2in\pi x}{3}}}{(2)n\pi x}\right]_0^3 = \boxed{-\frac{3i}{n\pi}e^{\frac{2in\pi x}{3}}}$$

$$d_0 = \frac{1}{3}\int_0^3 2x\,dx = \boxed{3}$$

$$f(x) = \sum_{n=-\infty}^{\infty} d_n e^{2in\pi x}$$

$$f(x) = 3 + \sum_{n=-\infty}^{\infty}\frac{3i}{n\pi}e^{-\frac{2in\pi x}{3}} = 3 + \frac{3i}{\pi}\sum_{n=-\infty}^{\infty} e^{-\frac{2in\pi x}{3}} = \boxed{3 + \frac{3i}{\pi}\sum_{-\infty}^{\infty}\frac{1}{n}e^{\frac{2in\pi x}{3}}}$$

Libro Makarenko

Ejercicio 9

171. $\int_{|z|=1} \frac{\operatorname{tg} z \, dz}{z \, e^{\left(\frac{1}{z+2}\right)}} = \int_{|z|=1} \frac{\left(\frac{\operatorname{tg} z}{e^{\left(\frac{1}{z+2}\right)}}\right)}{z} dz$

$\int_{|z-z_0|=r} \frac{f(z)\,dz}{z-z_0} = 2\pi i \, f(z_0)$

$z_0 = 0$

$f(z) = \frac{\operatorname{tg} z}{e^{\frac{1}{z+2}}}$

$f(z_0) = \frac{\operatorname{tg}(0)}{e^{\left(\frac{1}{2}\right)}} = 0$

$\int_{|z|=1} \frac{\operatorname{tg} z \, dz}{z \, e^{\left(\frac{1}{z+2}\right)}} = \boxed{0}$

Ejercicio 10

172. $\int_{|z|=3} \frac{\cos(z+\pi i)\,dz}{z(e^z+2)} = \int_{|z|=3} \frac{\left(\frac{\cos(z+\pi i)}{e^z+2}\right)}{z}$

$\int_z \frac{f(z)\,dz}{z-z_0} = 2\pi i \, f(z_0)$

$f(z_0) = \frac{\cos(\pi i)}{e^0+2} = \frac{\cosh(\pi)}{3}$

$2\pi i \left(\frac{\cosh(\pi)}{3}\right) = \boxed{\frac{2}{3}\pi i \cosh \pi}$

Ejercicio 11

175. $\int_{|z|=1} \frac{\operatorname{sh}\left(\frac{\pi}{2}\right)(z+i)}{z^2-2z} dz = \int_{|z|=1} \frac{\operatorname{sh}\frac{\pi}{2}(z+i)}{z(z-2)} dz$

$z_0 = 0$ $\quad z_1 = 2$

$\int_{|z|=1} \frac{\operatorname{sh}\left(\frac{\pi}{2}\right)(z+i)}{z^2-2z} = \int_{|z|=1} \frac{\left(\frac{\operatorname{sh}\left(\frac{\pi}{2}\right)(z+i)}{z-2}\right)}{z}$

$f(z_0) = \frac{\operatorname{sh}\left(i\frac{\pi}{2}\right)}{0-2} = \frac{\operatorname{sen}\left(\frac{\pi}{2}\right)}{-2} = \boxed{\frac{-1}{i2}}$

$\int_{|z|=1} \frac{\operatorname{sh}\left(\frac{\pi}{2}\right)(z+i)}{z^2-2z} dz = \left(\frac{-1}{i2}\right) 2\pi i = \boxed{\frac{\pi i}{i}} = \boxed{\pi}$

$$\int_{|z|=2} \frac{sen(z)\, sen(z-1)}{z^3 - z}\, dz = \int_{|z|=2} \frac{sen(z)\, sen(z-1)}{z(z^2-1)}\, dz$$

$z_0 = 0 \quad z_1 = 1 \quad z_2 = -1$

$$\int_{|z|=2} \frac{sen(z)\, sen(z-1)}{z^3 - z}\, dz = \int_{C_0} \frac{\frac{sen(z)\, sen(z-1)}{z^2-1}}{z}\, dz + \int_{C_1} \frac{\frac{sen(z)\, sen(z-1)}{z(z+1)}}{(z-1)}\, dz$$

$$+ \int_{C_2} \frac{\frac{sen(z)\, sen(z-1)}{z(z-1)}}{(z+1)}\, dz$$

$$f(z_1) = \frac{sen(1)\, sen(0)}{(1)(2)} = 0$$

$$f(z_0) = \frac{\frac{sen(0)\, sen(-1)}{(0-1)}}{-1} = 0$$

$$f(z_3) = \frac{sen(-1)\, sen(-2)}{(-1)(-2)} = \frac{sen(-1)\, sen(-2)}{2}$$

$$\int_{|z|=2} \frac{sen(z)\, sen(z-1)\, dz}{z^3 - z} = \boxed{sen(-1)\, sen(-2)\, \pi i}$$

Libro: Makarenko

339. $$\oint_{|z|=2} \frac{e^z\,dz}{z^3(z+1)} = \oint_{C_1} \frac{\frac{e^z}{z^3}\,dz}{z+1} + \oint_{C_2} \frac{\frac{e^z}{z+1}\,dz}{z^3}$$

$$2\pi i\left[\frac{e^{-1}}{-1}\right] + 2\pi i\left[\frac{f^{(2)}(z_0)}{2!}\right]$$

$$f(z) = \frac{e^z}{z+1}$$

$$f'(z) = \frac{(z+1)e^z + (-e^z(1))}{(z+1)^2} = \frac{ze^z}{(z+1)^2}$$

$$f''(z) = \frac{(z+1)^4(e^z + (z+1)e^z + e^z) + (-(z+1)e^z + (-e^z))\,2(z+1)^3}{(z+1)^4}$$

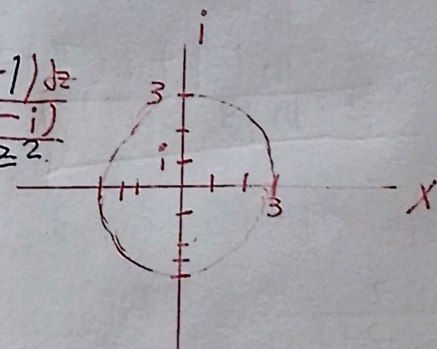
$$f''(0) = \frac{(1)(1) - (0)(2)}{1} = \boxed{1}$$

$$\oint_{|z|=2} \frac{e^z\,dz}{z^3(z+1)} = 2\pi i[-e^{-1}] + \frac{2\pi i}{2}[1] = \boxed{[1 - 2e^{-1}]\pi i}$$

Ejercicio 12

340. $$\oint_{|z-i|=3} \frac{e^{z^2}-1\,dz}{z^3 - iz^2}$$

$$\oint_{|z-i|=3} \frac{e^{z^2}-1\,dz}{z^2(z-i)} = \oint_{C_1} \frac{\frac{e^{z^2}-1}{z^2}\,dz}{z-i} + \oint_{C_2} \frac{\frac{(e^{z^2}-1)\,dz}{(z-i)}}{z^2}$$



$$2\pi i\left[\frac{e^{i^2}-1}{i^2}\right] + 2\pi i\left[\frac{f'(z_0)}{1!}\right]$$

$$f(z) = \frac{e^{z^2}-1}{z-i}$$

$$f'(z) = \frac{(z-i)(2z\,e^{z^2}) - (e^{z^2}-1)(1)}{(z-i)^2} = \frac{2z(z-i)e^{z^2} - e^{z^2} + 1}{(z-i)^2}$$

$$f'(0) = \frac{0 - 1 + 1}{(-i)^2} = \boxed{0}$$

$$\oint_{|z-i|=3} \frac{(e^{z^2}-1)\,dz}{z^2(z-i)} = -2\pi i e^{-1} + 2\pi i + 2\pi i(0) = \boxed{2\pi i(-e^{-1}+1)}$$

Ejercicio 8

341. $$\oint_{|z|=\frac{1}{2}} z^2 \, sen\left(\frac{1}{z}\right) dz = 2\pi i\left[z^2\left[z^{-1} - \frac{z^{-3}}{3!} + \frac{z^{-5}}{5!} - \frac{z^{-7}}{7!} + \frac{z}{4!} + \cdots\right]\right]$$

$$2\pi i\left[z - z^{-1}\left(\frac{1}{3!}\right) + \cdots\right] = \frac{2\pi i}{3!} = \boxed{\frac{\pi i}{3}}$$

Ejercicio 9

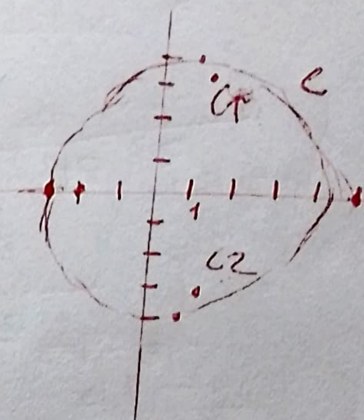
342. $$\oint_{|z|=\sqrt{3}} \frac{sen(\pi z)\, dz}{z^2 - z} = \oint_{|z|=\sqrt{3}\cong 1.73} \frac{sen(\pi z)\, dz}{z(z-1)}$$

$$\oint_{C1} \frac{\frac{sen(\pi z)}{z}}{(z-1)} dz + \oint_{C2} \frac{\frac{sen(\pi z)}{(z-1)}}{z} dz$$

$$2\pi i\left[sen(\pi) + \frac{sen(0)}{(-1)}\right] = \boxed{0}$$

Ejercicio 10

343. $$\oint_{|z+1|=4} \frac{z\, dz}{e^z + 3}$$



Ceros de la función

$$e^z + 3 = 0$$

$$\ln e^z = \ln|3|$$

$$-3 = 3e^{i(\pi + 2k\pi)}$$

$$z = \ln|-3|$$

$$z = \ln|3| + i(\pi + 2k\pi) \cong 1.09 + i(\pi + 2k\pi)$$

$$\boxed{z = \ln|3| \pm i(\pi)}$$

$$\oint_{|z+1|=4} \frac{z\, dz}{e^z + 3} = \oint_{C1} \frac{z\, dz}{e^z+3} + \oint_{C2} \frac{z\, dz}{e^z+3} = 2\pi i\left(\frac{\phi(z_0)}{\psi'(z_0)} + \frac{\phi(z_1)}{\psi'(z_1)}\right)$$

$$\oint_{|z+1|=4} \frac{z\, dz}{e^z+3} = 2\pi i\left[\frac{z_0}{e^{z_0}+0} + \frac{z_1}{e^{z_1}}\right] = 2\pi i\left[\frac{\ln|3| + i\pi}{e^{i\pi + \ln|3|}} + \frac{\ln|3| - i\pi}{e^{\ln|3| - i\pi}}\right]$$

$$2\pi i\left[\frac{\ln|3| + \pi}{e^{i\pi}\, e^{\ln|3|}} + \frac{\ln|3|(+i\pi)}{e^{\ln|3|}\, e^{-i\pi}}\right] = 2\pi i\left[\frac{\ln|3| + \pi}{-3} + \frac{\ln|3| - i\pi}{-3}\right]$$

$$\oint_{|z+1|=4} \frac{z}{e^z+3} dz = 2\pi i\left[\frac{2\ln|3|}{-3}\right] = \boxed{-\frac{4\pi i \ln|3|}{3}}$$

Ejercicio 7

1)b) $\oint_{|z-i|=1} \frac{e^z}{z^4+2z^2+1}\,dz = \int_C \frac{e^z}{(z^2+1)^2}\,dz = \int_C \frac{e^z}{(z+i)^2(z+(-i))^2}\,dz$

$$\oint_{C_1} \frac{e^z (z+i)^{-2}}{(z-i)^2}\,dz + \oint_{C_2} \frac{e^z (z-i)^{-2}}{(z+i)^2}\,dz$$

$$2\pi i\left[\frac{e^i(-2)(2i)^{-3} + e^i(2i)^{-2}}{1!} + \frac{e^{-i}(-2)(-2i)^{-3} + e^{-i}(-2i)^{-2}}{1!}\right]$$

$$f'(z) = e^z(-2)(z+i)^{-3} + e^z(z+i)^{-2}$$

$$f'(z) = e^z(-2)(z-(+i))^{-3} + e^z(z-i)^{-2}$$

$$2\pi i\left[\frac{(-2i)e^i}{8} + \left(-\frac{e^i}{4}\right) + \frac{e^{-i}\,i}{4} - \frac{e^{-i}}{4}\right]$$

$$\frac{\pi i}{2}\left[-ie^i - e^i + e^{-i}i - e^{-i}\right]$$

$-e^i = -[\cos(1) + i\,\mathrm{sen}(1)] = -\cos(1) - i\,\mathrm{sen}(1)$

$-ie^i = -[i\cos(1) - \mathrm{sen}(1)] = -i\cos(1) + \mathrm{sen}(1)$

$+ie^{-i} = [i\cos(1) - \mathrm{sen}(-1)] = i\cos(1) - \mathrm{sen}(-1)$

$-e^{-i} = -[\cos(1) + i\,\mathrm{sen}(-1)] = -\cos(1) - i\,\mathrm{sen}(-1)$

$$\oint \frac{e^z}{z^4+2z^2+1}\,dz = \boxed{\left[\cos(1) + \mathrm{sen}(1) + i(\mathrm{sen}(1) - \cos(1))\right]\frac{\pi}{2}}$$

$\oint_C \frac{\mathrm{senh}\,3z\,dz}{(z - \frac{\pi}{4}i)^3} = \boxed{\frac{\sqrt{2}\pi}{2}}$

Ejercicio 4
7.40 Demuestre que $\oint_C \frac{\cosh z}{z^3}\,dz = \pi i$ si C con vertices en $\pm 2 \pm 2i$

$$\oint_C \frac{\cosh z\,dz}{z^3} = 2\pi i\left[\frac{f^{(2)}(0)}{2!}\right] = 2\pi i\left[\frac{e^{(0)}+e^{-(0)}}{2(2)}\right] = \boxed{\pi i}$$

$f'(z) = \operatorname{senh} z$

$f''(z) = \cosh z = \boxed{\frac{e^{(z)}+e^{-(z)}}{2}}$

Ejercicio 5
7.42 Demuestre que el residuo que. Evalúe $\oint_C \frac{e^z\,dz}{\cosh z}$ a lo largo de la circonferencia C definida por $|z|=5$.

$$2\int_C \frac{e^z}{e^z+e^{-z}}\,dz = 2\int_C \frac{e^{2z}}{e^{2z}+1}\,dz = 2\int_{C_1} \frac{e^{2z}\,dz}{e^{2z}+1} + 2\int_{C_2} \frac{e^{2z}\,dz}{e^{2z}+1} + 2\int_{C_3} \frac{e^{2z}\,dz}{e^{2z}+1} + 2\int_{C_4} \frac{e^{2z}\,dz}{e^{2z}+1} + 2\int_{C_5} \frac{e^{2z}\,dz}{e^{2z}+1} + 2\int_{C_6} \frac{e^{2z}\,dz}{e^{2z}+1}$$

Ceros de la función

$-1 = e^{i(\pi+2K\pi)}$

$\ln e^{2z} = \ln(-1)$

$2z = i(\pi+2K\pi)$

$z = \frac{i(\pi+2K\pi)}{2}$

$$(2\pi i)2\left[\frac{e^{\pi}}{2e^{\pi}} + \frac{e^{3\pi}}{2e^{3\pi}} + \frac{e^{-3\pi}}{2e^{-3\pi}} + \frac{e^{-\pi}}{2e^{-\pi}}\right] = 2\pi i(2)[1+1] = \boxed{8\pi i}$$

Ejercicio 6
7.44. Evalúe $\oint_C e^{-\frac{1}{z}} \operatorname{sen}\left(\frac{1}{z}\right) dz$ donde C es la circonferencia $|z|=1$.

$$\oint_C e^{-\frac{1}{z}} \operatorname{sen}\left(\frac{1}{z}\right) dz = \oint_C \left(1 - \frac{1}{z} + \frac{1}{z^2 2!} - \frac{1}{z^3 3!} + \cdots\right)\left(\frac{1}{z} - \frac{1}{z^3 3!} + \frac{1}{z^5 5!}\right)$$

$$z\left(\frac{1}{z} - \frac{1}{z^2}\left(\frac{1}{z^3 3!} - \frac{1}{z^4 3!} + \cdots\right)\right)\left(\frac{1}{z} - \frac{1}{z^3 3!} + \frac{1}{z^5 5!} + \cdots\right)$$

$$z^2\left(\frac{1}{z^2} - \frac{1}{z^3} + \frac{1}{z^4 2!} - \frac{1}{z^5 3!} + \cdots\right) z\left(\frac{1}{z^2} - \frac{1}{z^4 3!} + \frac{1}{z^6 5!} + \cdots\right)$$

$$\oint_C e^{-\frac{1}{z}} \operatorname{sen}\left(\frac{1}{z}\right) dz = 2\pi i(1) = \boxed{2\pi i}$$

7.47 Evalúe $\oint_C \frac{2+3\,\text{sen}(\pi z)}{z(z-1)^2}\,dz$ donde es el cuadrado con vértices $3+3i$, $3-3i$, $-3+3i$, $-3-3i$.

$$\oint_C \frac{2+3\,\text{sen}(\pi z)}{z(z-1)^2}\,dz = \int_{C_1} \frac{\frac{2+3\,\text{sen}(\pi z)}{z}}{(z-1)^2}\,dz + \int_{C_2} \frac{\frac{2+3\,\text{sen}(\pi z)}{(z-1)^2}}{z}\,dz$$

$$2\pi i\left[\frac{3\pi\cos(\pi)-2}{1!}\right] + 2\pi i\left[\frac{2+0}{1}\right] = \boxed{-6\pi^2 i}$$

$$f'(z) = \frac{z(3\pi\cos(\pi z)) - (2+3\,\text{sen}(\pi z))}{z^2}$$

Ejercicio 2

7.48 Evalúe $\frac{1}{2\pi i}\oint_C \frac{e^{zt}}{z(z^2+1)}\,dz$, $t>0$ a lo largo del cuadrado con vértices en $2+2i$, $-2+2i$, $-2+2i$, $-2-2i$, $2-2i$.

$$\oint_{C_1} \frac{\frac{e^{zt}}{z}}{z^2+1}\,dz + \oint_{C_2} \frac{\frac{e^{zt}}{z^2+1}}{z}\,dz = \oint_{C_1} \frac{\frac{e^{zt}}{(z-i)z}}{(z+i)} + \int_{C_2} \frac{\frac{e^{zt}}{(z+i)z}}{(z-i)}\,dz$$

$$\int_{C_3} \frac{\frac{e^{zt}}{(z^2+1)}}{z}\,dz$$

$z = \sqrt{-1}$

$\boxed{z = \pm i}$

$$\frac{2\pi i}{2\pi i}\left[\frac{e^{-it}}{(-2i)(-i)} + \frac{e^{it}}{(2i)(i)} + \frac{1}{1}\right] = \left[-\frac{e^{-it}}{2} - \frac{e^{it}}{2} + 1\right]$$

$$\frac{-[\cos(t)+i\,\text{sen}(-t)] - [\cos(t)+i\,\text{sen}(t)]}{2} + 1]$$

$$-\frac{2\,\text{sen}(t)}{2} + 1 = \boxed{-\text{sen}(t) + 1}$$

Ejercicio

7.45 Sea C el cuadrado limitado por $x = \pm 2$, $y \pm 2$. Evalúe $\oint_C \frac{\operatorname{senh} 3z \, dz}{\left(z - \frac{\pi}{4} i\right)^3}$

$$2\pi i \left[\frac{\operatorname{senh}\left(\frac{3\pi}{4}\right)}{2!} \right] = \pi i \frac{e^{i\frac{3\pi}{4}} - e^{-i\frac{3\pi}{4}}}{2i} \Rightarrow = \frac{\pi}{2} \left[\cos\left(\frac{3\pi}{4}\right) + i \operatorname{sen}\left(\frac{3\pi}{4}\right)\right]$$

$$+ \left[-\cos\left(\frac{3\pi}{4}\right) + i \operatorname{sen}\left(\frac{3\pi}{4}\right)\right]$$

$$\oint_C \frac{\operatorname{senh} 3z \, dz}{\left(z - \frac{\pi}{4} i\right)^3} = \boxed{\frac{\sqrt{2}\pi}{2}}$$