INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

LISTA DE EJERCICIOS 1-12

SEMANA 6

NOMBRE DEL ALUMNO: GARCÍA QUIROZ GUSTAVO IVAN

GRUPO: 4CV3

MATERIA: MATEMATICAS AVANZADAS PARA LA INGENIERIA

NOMBRE DEL PROFESOR: MARTINEZ NUÑO JESUS ALFREDO

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Libro: Makarenko

García Quiroz Gustavo Iván

Ejercicio 1

144. $\int_C z\bar{z}\,dz$ $\quad c: |z|=1$. El recorrido se efectua en sentido antihorario

$z = e^{i\theta}$ $\quad dz = ie^{i\theta}d\theta$

$\bar{z} = e^{-i\theta}$ $\quad 0 \leq \theta \leq 2\pi$

$i\int_0^{2\pi} e^{i\theta - i\theta} d\theta = [\ \theta\ ]_0^{2\pi} = i[2\pi - 0] = i\,2\pi$

$\int_C z\bar{z}\,dz = \int_0^{2\pi}(e^{i\theta}e^{-i\theta})(ie^{i\theta}d\theta) = i\int_0^{2\pi} e^{i\theta}d\theta = [e^{i\theta}]_0^{2\pi} = e^{i2\pi} - 1$

$\boxed{\int_C z\bar{z}\,dz = 0}$

$e^{i2\pi} = \cos(2\pi) + i\,\mathrm{sen}(2\pi) = 1$

Ejercicio 2

145. $\int_1^i ze^z\,dz = \int_1^i ze^z dz$

$u = z$ $\quad dv = e^z dz$

$du = dz$ $\quad v = e^z$

$\left[ze^z - \int e^z dz\right]_1^i = \left[ze^z - e^z\right]_1^i = ie^i - e^i - [e(1-1)]$

$\boxed{\int_1^i ze^z dz = e^i(i-1)}$

Ejercicio 3

147. $\int_{1+i}^{-1-i} (2z+1)\,dz$ integral de recta

$z = A + (B-A)t = 1+i+(-2-2i)t$

$z = 1+i+(-1-i-1-i)t$

$z = 1-2t+i(1-2t)$ $\quad \boxed{x=y}$

$x = 1-2t$

$y = 1-2t$ $\quad 0 \leq t \leq 1$

$dx = dy = \boxed{-2dt}$

$\int_{1+i}^{-1-i} (2(x+iy)+1)(dx+idy) = \int_C (2x+2iy+1)(dx+idy)$

$\int_C (2x+2iy+1)dx + i\int_C (2x+2iy+1)dy$

$$\int_c (2x+2iy+1)\,dx = \int_0^1 (2(1-2t)+2i(1-2t)+1)(-2dt)$$

$$-4\int_0^1 (1-2t)\,dt - 4i\int_0^1 (1-2t)\,dt - \int_0^1 2\,dt$$

$$-4[t-t^2]_0^1 - 4i[t-t^2]_0^1 - 2[t]_0^1$$

$$-4[1-1]-4i[1-1]-2[1] = \boxed{-2}$$

$$i\int_c (2x+2y+1)\,dy = i[-2]$$

$$\int_{1+i}^{-1-i} (2z+1)\,dz = -2-2i = \boxed{-2(1-i)}$$

Ejercicio 4
148

$$\int_0^{1+i} z^3\,dz = \int_0^{1+i} [(x^3+(2xy^2)-xy)+i(3x^2y-y^2)](dx+idy)$$

Integral de recta

$$z = A+(B-A)t$$

$$z = 0+(1+i)t = (t)+i(t)$$

$$x+iy = t+it$$

$$x = t \qquad \boxed{x=y}$$

$$y = t \qquad \boxed{dx=dy=dx} \qquad 0\le t\le 1$$

$$\int_0^1 (t^3+(2t)^3-t^2+i(3t^3-t^2))(dt+i\,dt)$$

Parte 1

$$\int_0^1 [(t^3(+2t)^3-t^2)+i(3t^3-t^2)]\,dt = \int_0^1 ((3t^3-t^2)+i(3t^3-t^2))\,dt$$

$$\left[\frac{3t^4}{4}-\frac{t^3}{3}+i\left(\frac{3t^4}{4}-\frac{t^3}{3}\right)\right]_0^1 = \frac{3}{4}-\frac{1}{3}+i\left(\frac{3}{4}-\frac{1}{3}\right)-[0]$$

$$+\frac{5}{12}+i\frac{5}{12} = -\frac{1}{12}+i\frac{5}{12}$$

Parte 2

$$i\int_0^1 [(3t^3-t^2)+i(3t^3-t^2)]\,dt = i\left[\frac{5}{12}+i\frac{5}{12}\right] = i\frac{5}{12}+i^2\frac{5}{12} - \frac{i}{12}$$

$$\int_c z^3\,dz = \frac{5}{12}+i\frac{5}{12}+i\frac{5}{12}-\frac{5}{12}+\boxed{i\frac{10}{12}} = -\frac{1}{12}-\frac{5}{12}+i\frac{5}{12}-\frac{i}{12}$$

$$\boxed{-\frac{6}{12}+\frac{4i}{12}}$$

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Ejercicio 5

149 $\int_{1}^{i} (3z^4 - 2z^3)\,dz = \left[\frac{3z^5}{5} - \frac{2z^4}{4}\right]_1^{i} = i\frac{3}{5} - \frac{2}{4} - \left[\frac{3}{5} - \frac{1}{2}\right]$

$\boxed{i\frac{3}{5} - \frac{3}{5}}$

$\frac{3}{5}i[1+i]$

Ejercicio 6

150. $\int_C e^z\,dz$ C: a) Es el arco de la parábola $y=x^2$ que conecta los puntos $z_1=0$ y $z_2=1+i$

b) Es el segmento de la recta que conecta esos 2 puntos

a) $z = x+iy \qquad dz = dx + i\,dy$

$y = x^2$

$z = x + ix^2 \qquad dz = dx + i2x\,dx$

$\int_C e^z\,dz = \int_0^{1+i} e^{x+ix^2}\,dx(1+i2x) = [e^{x+ix^2}]_0^{1+i}$

$e^{1+i+i(1+i)^2} - e^0 = e^{1+i-2} - 1 = \boxed{e^{-1}\cos(1) + e^{-1}\operatorname{sen}(1) - 1}$

b) $\int_0^{1+i} e^z\,dz = (e^z)_0^{1+i} = e^{1+i} - [e^0] = \boxed{e\cos(1) + ie\operatorname{sen}(1) - 1}$

Ejercicio 7

151. $\int_C \cos z\,dz$, C: Es el segmento de recta que conecta $z_1 = \frac{\pi}{2}$ y $z_2 = \pi + i$.

$\int_{\frac{\pi}{2}}^{\pi+i} \cos z\,dz = [\operatorname{sen} z]_{\frac{\pi}{2}}^{\pi+i} = \operatorname{sen}(\pi+i) - \operatorname{sen}\left(\frac{\pi}{2}\right)$

$-\operatorname{sen}\left(\frac{\pi}{2}\right) = -1$

$\operatorname{sen}(\pi+i) = \frac{e^{i(\pi+i)} - e^{-i(\pi+i)}}{2i} = \frac{e^{i\pi-1} - e^{-i\pi+1}}{2}(-i)$

$e^{i\pi} = \cos(\pi) + i\operatorname{sen}(\pi) = -1$

$\operatorname{sen}(\pi+i) = \frac{-e^{-1} + e^{1}}{2}(-i) = -i\operatorname{senh}(1)$

$\int_{\frac{\pi}{2}}^{\pi+i} \cos z\,dz = +(-1 - i\operatorname{senh}(1)) = \boxed{-(1 + i\operatorname{senh}(1))}$

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Ejercicio 8

157 $$\int_0^i (z-i)e^{-z}\,dz = \left[(z-i)(-e^{-z}) + \int e^{-z}\,dz\right]_0^i$$

$$u = z-i \qquad dv = e^{-z}dz$$
$$du = dz \qquad v = -e^{-z}$$

$$\left[(z-i)(-e^{-z}) - e^{-z}\right]_0^i = -(i-i)e^{-z} - e^{-i} - [i-1]$$
$$-e^{-i} + 1 - i = \boxed{1-\cos(1) + i(-1+\operatorname{sen}(1))}$$

Ejercicio 9

159. $\int_1^i \frac{\ln z}{z}\,dz$ por el segmento de recta que conecta los puntos $z_1 = 1$ y $z_2 = i$

$$\int_1^i \frac{\ln z}{z}\,dz = \left[\frac{(\ln z)^2}{2}\right]_1^i = \frac{(\ln(i))^2}{2} - \left[\frac{\ln(1)^2}{2}\right]$$
$$\frac{\ln(i)^2}{2} = \left(\ln\left(e^{i\frac{\pi}{2}}\right)\right)^2\left(\frac{1}{2}\right) = \left(\frac{\pi i}{2}\right)^2\left(\frac{1}{2}\right) = \boxed{-\frac{\pi^2}{8}}$$

Ejercicio 10

160. $$\int_0^{1+i} \operatorname{sen}(z)\cos(z)\,dz = \left[\frac{(\operatorname{sen} z)^2}{2}\right]_0^{1+i}$$

$$\left[\frac{(\operatorname{sen}(z))^2}{2}\right]_0^{1+i} = \frac{(\operatorname{sen}(1+i))^2}{2} - \frac{(0)^2}{2} = \frac{(1-\cos^2(1+i))^2}{2}$$
$$\left(\frac{1}{2} - \cos^2(1+i) + \frac{\cos^4(1+i)}{2}\right)$$
$$\boxed{\frac{1-\cos^2(1+i)}{2}}$$

161. Ejercicio 11 $$\int_{-1}^{i} \frac{1+\operatorname{tg} z}{\cos^2 z}\,dz = \int_{-1}^{i} \sec^2(z)(1+\operatorname{tg} z)\,dz = \left[\frac{(1+\operatorname{tg} z)^2}{2}\right]_{-1}^{i}$$

$$\left(\frac{1}{2}\right)(1+\operatorname{tg}(i))^2 - \left[\left(\frac{1}{2}\right)(1+\operatorname{tg}(1))^2\right]$$
$$\left(\frac{1}{2}\right)(1+2\operatorname{tg}(i)+\operatorname{tg}^2(i)) - \left(\frac{1}{2}\right)(1+2\operatorname{tg}(1)+\operatorname{tg}^2(1))$$
$$\boxed{-\left(\operatorname{tg}(1)+\operatorname{tg}^2(1)\left(\frac{1}{2}\right)+\operatorname{tgh}(1)\right)+i\operatorname{tg}(1)}$$

16)

$$\int_{-1}^{i} \frac{\cos z}{\sqrt{\operatorname{sen} z}}\, dz = \left[(2)(\operatorname{sen} z)^{\frac{1}{2}} \right]_{-1}^{i} = 2(\operatorname{sen}(i))^{\frac{1}{2}} - 2(\operatorname{sen} -1)^{\frac{1}{2}}$$

$$\operatorname{sen}(z) = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\operatorname{sen}(i) = \left(\frac{e^{-1} - e^{1}}{2}\right)(-i) = i \operatorname{senh}(1)$$

$$\sqrt{\operatorname{sen}(-1)} = i\sqrt{\operatorname{sen}(1)}$$

$$\boxed{2i\left(\sqrt{\operatorname{senh} 1} - \sqrt{\operatorname{sen}(1)}\right)} = \int_{-1}^{i} \frac{\cos(z)}{\sqrt{\operatorname{sen}(z)}}\, dz$$