PROJECT ASSIGNMENT
FOLDABLE ROBOTICS

Project Team #9

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Goals of the project

Candidate organism: Canis lupus familiaris (dog)
Motion of interest: Walking gait
Objective: Vary trunk length, paw spacing, or leg lengths and compute how foot velocity capability changes — find robust ranges.

Scope

We have decided to implement a few constraints in this project like in this project we are just going to use a hind limb, a fore limb, hip and ground to make it a 4 bar closed mechanism and change parameters like length of the limb or distance between the limbs to find out changes in velocity.

Impact

This project is impactful because it translates the complex biomechanics of quadrupedal locomotion into a simplified, analyzable mechanical framework. By representing a dog's trunk and limbs as a 4-bar linkage and systematically varying limb lengths and inter-limb spacing, the study provides direct insight into how geometric proportions affect foot velocity and coordination. This information is valuable for both robotics and biomechanical design, as it helps identify the geometric configurations that maximize stride efficiency or minimize mechanical stress during walking. The simplicity of the 4-bar abstraction makes the results broadly useful: they can guide engineers designing efficient leg linkages for small quadruped robots, animators simulating realistic gait motion, or even biomechanists studying scaling effects in animal locomotion. Moreover, focusing on a minimal model ensures that the findings remain intuitive and generalizable, making this a timely contribution to understanding the essential geometry–performance relationship in legged movement.

Team fit

This project aligns well with our team's combined interests and abilities in mechanical design, kinematics, and bio-inspired robotics. Each member brings complementary strengths - from modeling and simulation to interpreting biomechanical motion - which makes analyzing a 4-bar mechanism representing a dog's limbs both manageable and engaging. Our curiosity about animal locomotion and how natural movement can inspire efficient mechanical systems directly supports this project's objectives. By focusing on a simplified, geometry-driven model, we can effectively apply our theoretical understanding of linkages and Jacobian analysis while keeping the scope achievable within the semester.

Topic fit

This project fits the theme of foldable robotics and bio-inspired mechanisms by using a planar 4-bar linkage to capture the essential motion of quadrupedal locomotion. The linkage serves as a simplified but insightful analog for how dogs coordinate their limbs during walking, enabling analysis of velocity transmission and workspace through foldable, jointed motion. The question leverages foldable robotics concepts such as motion amplification, geometric constraint, and linkage optimization to replicate biological efficiency. By exploring how varying link lengths and limb spacing influence motion, the project demonstrates how foldable robotic principles can be used to emulate adaptive and efficient animal-like movement patterns.

Background research

A. Literature search scope

We searched Google Scholar and PubMed for canine (dog) **walking gait** biomechanics and **bio-inspired quadruped robots**, using keywords such as *dog gait*, *canine kinematics*, *ground reaction forces*, *Labrador*, *Hildebrand*, and *quadruped robot BigDog*.

This aligns with the assignment's directive to survey prior work on the same animal, subsystem, and motion, plus robots inspired by it.

Title: Mechanics of dog walking compared with a passive, stiff-limbed, 4-bar linkage model, and their collisional implications- Usherwood et al., 2007

Summary:

Here, we present a simple stiff-limbed passive model of quadrupedal walking, compare mechanics predicted from the model with those observed from forceplate measurements of walking dogs and consider the implications of deviation from model predictions, especially with reference to collision mechanics. The model is based on the geometry of a 4-bar linkage consisting of a stiff hindleg, back, foreleg and the ground between the hind and front feet. It uses empirical morphological and kinematic inputs to determine the fluctuations in potential and kinetic energy, vertical and horizontal forces and energy losses associated with inelastic collisions at each foot placement. Using forceplate measurements to calculate centre of mass motions of walking dogs, we find that (1) dogs may, but are not required to, spend periods of double support (one hind- and one forefoot) agreeing with the passive model;(2) legs are somewhat compliant, and mechanical energy fluctuates during triple support, with mechanical energy being lost directly after hindfoot placement and replaced following forefoot placement. Footfall timings and timing of mechanical energy fluctuations are consistent with strategies to reduce collisional forces, analogous to the suggested role of ankle extension as an efficient powering mechanism in human walking.

Breed:Male labrador	Breed:Australian Shepherd	Breed:Australian shepherd
Sex:Male	Sex:Male	Sex:Female
Mass(kg):33.8	Mass(kg):33.8	Mass(kg):21.7
Hindlimb length(m): 0.46	Hindlimb length(m):0.51	Hindlimb length(m):0.46
Forelimb length(m): 0.48	Forelimb length(m):0.48	Forelimb length(m):0.47
back(m): 0.59	Back(m):0.63	Back(m):0.52

[1]

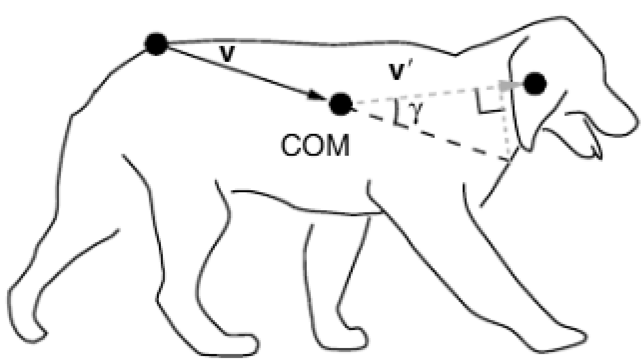

Figure1:Collision geometry at foreleg contact. A component of the centre of mass (COM) velocity just prior to collision v can be maintained after the collision (v⬘) despite an inelastic redirection through an angle determined from the 4-bar linkage geometry. The collisional energy loss calculated at each leg contact is dependent on both velocity and the angle ⫽ with which the COM path is suddenlydeflected.[1]

Why it supports my idea

This paper provided the rationale for modeling the dog with a four-bar linkage to estimate gait velocity and to tune parameters for optimal stability on uneven terrain and maximal speed on flat surfaces. While engaging with the study, I also formulated an approach that introduces open contacts and then activates constraints to close them, aligning with the course requirements for hybrid (contact/non-contact) modeling and clarifying how speed–balance trade-offs can be optimized in robotic dogs using methods from this class. In addition, we used published Labrador measurements from the paper as a worked example to demonstrate the Jacobian and velocity analyses.

Title: BigDog: Rough-Terrain Quadruped Robot – Raibert et al., 2008

Summary:

Raibert et al. (2008) describe *BigDog*, a hydraulically actuated quadruped developed by Boston Dynamics under DARPA funding to achieve animal-like mobility on rugged terrain

Powered by a 15 hp gasoline engine driving a hydraulic system, BigDog integrates ~50 sensors for joint position, force, and body attitude. Each leg has 4 active + 1 passive DOF, controlled through aerospace-grade servovalves

The robot weighs ≈ 109 kg, stands 1 m tall × 1.1 m long × 0.3 m wide, and can trot up to 1.6 m/s and bound at > 3 m/s

BigDog's control system combines low-level joint servo loops with high-level posture regulation to maintain stability and distribute ground-reaction forces evenly

It successfully traverses mud, snow, and 35° slopes carrying 50–150 kg loads, demonstrating how biologically inspired leg design and dynamic control enable robust, versatile locomotion in unstructured environments.

Why It Supports My Project!

Bio-inspiration bridge: BigDog converts biological quadruped principles (leg compliance, distributed GRFs, gait symmetry) into engineering design → exactly what our dog-gait 4-bar study models geometrically.

Mechanical insight: It shows practical link lengths ≈ leg/trunk proportions that maintain stability, useful for scaling our dog linkage dimensions.

Kinematic relevance: Their gait modes (crawl, trot, bound) define target velocity ranges (0.2 – 3 m/s) for comparing foot-velocity results from our Jacobian analysis.

Control analogy: BigDog’s controller equalizes limb loads to minimize torque—parallels our goal of finding geometric configurations that minimize stress or maximize velocity transmission.

Engineering benchmark: It validates that simplified linkage studies (like our 4-bar) can inform full dynamic robots capable of traversing real terrain.

Figures from Literature

Top: BigDog climbing 35 degree slope with loose scree-like surface. The front legs were reversed for this experiment.

Bottom: BigDog climbing a simulated rubble pile using a crawl gait in the laboratory. For this experiment, all terrain sensing is done with the legs, feeling its way along.

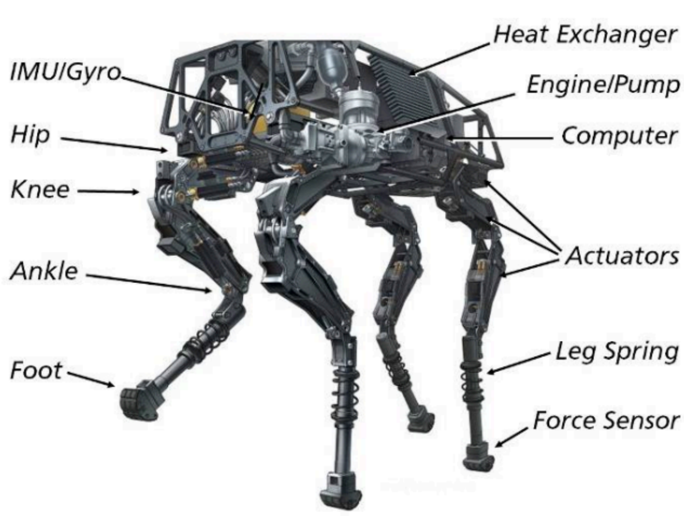

Illustration showing BigDog’s major components.

Title: “Strasser et al., 2014: Ground Reaction Forces in Labrador Retrievers”

Summary: Strasser et al. (2014) conducted a controlled study using ten healthy adult *Labrador Retrievers* to analyze ground reaction forces (GRF) and step length during level and cross-slope walking at 10° and 15° inclines using a pressure-plate walkway. Each dog performed five valid trials per condition at an average velocity of about 1.0 m/s

Compared with level walking, the dogs exhibited a **decrease in peak vertical force (PFz)** and **vertical impulse (IFz)** in the **up-slope hindlimb**, compensated by higher forces in the **down-slope forelimb**. Step length shortened significantly on the down-slope side, showing the functional adjustment needed to maintain balance on uneven terrain.

The study also confirmed that **IFz was more sensitive** than PFz for detecting gait adaptations because it integrates forces over the entire stance phase.

Key numerical findings:
Average walking velocity ≈ 1.06 ± 0.1 m/s (level)
Step length ≈ 0.75–0.80 m (level) → decreased to ≈ 0.72 m at 15° cross-slope
PFz (hindlimbs) ≈ 18 % of total force on level → ≈ 16 % on up-slope hindlimb at 15° cross-slope
PFz (forelimbs) ≈ 31 % on level → ≈ 34 % on down-slope forelimb at 15° cross-slope.

Relevance to this project: These quantitative GRF values supply realistic **force distributions and stride geometry** for our dog-gait 4-bar model. They help define expected vertical load ratios between limbs and provide real canine data to validate Jacobian-based foot-force calculations. The adaptation pattern redistribution of load and reduced step length under asymmetric support mirrors how a planar 4-bar linkage must geometrically compensate to maintain stability when one “leg” shortens relative to the other.

Figures from Literature

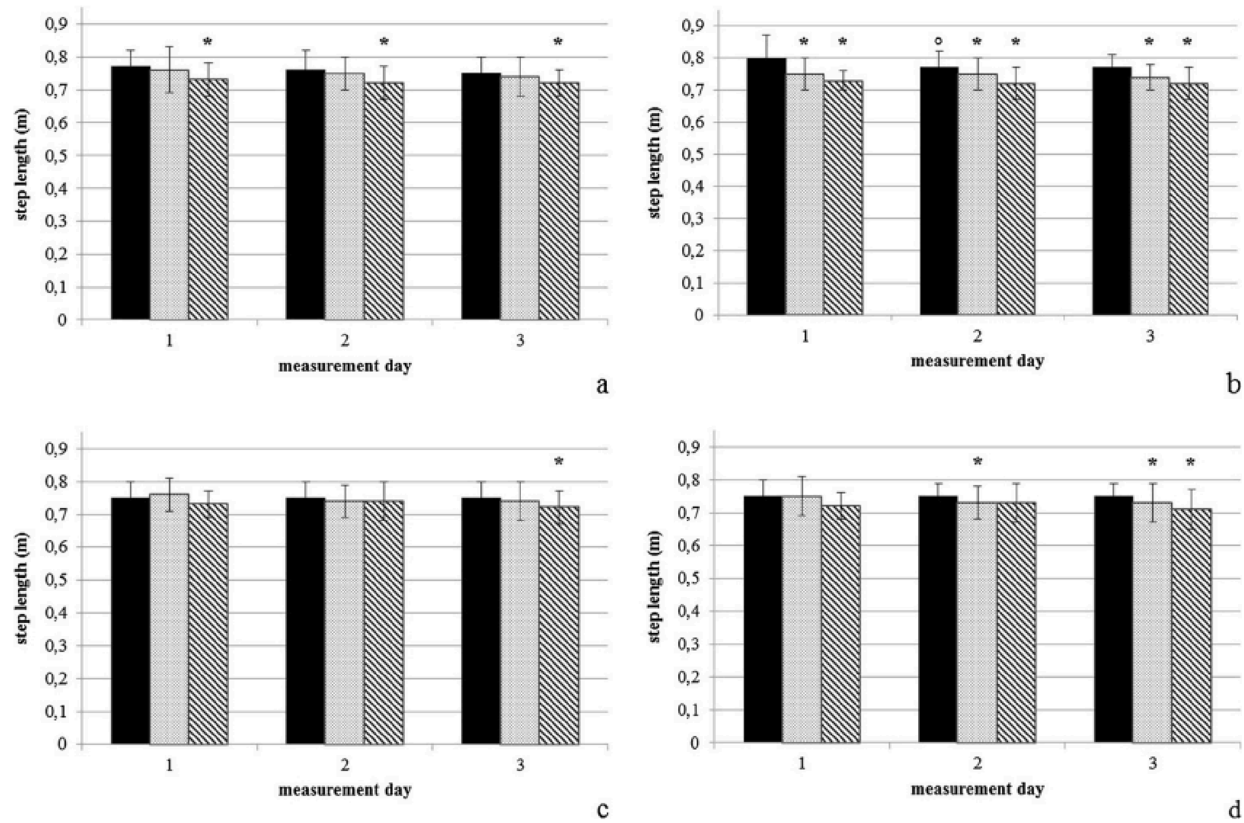

Figure 1 Step length (m). (a) Up-slope [US] forelimb, (b) down-slope [DS] forelimb, (c) US hindlimb, (d) DS hindlimb. The black bars represent level walking, the dotted bars indicate cross-slope 1, and the shaded bars indicate cross-slope 2. Data are expressed as mean ± standard deviation. *represents significant differences compared to Level walking, °represents significant differences compared to the first measurement.

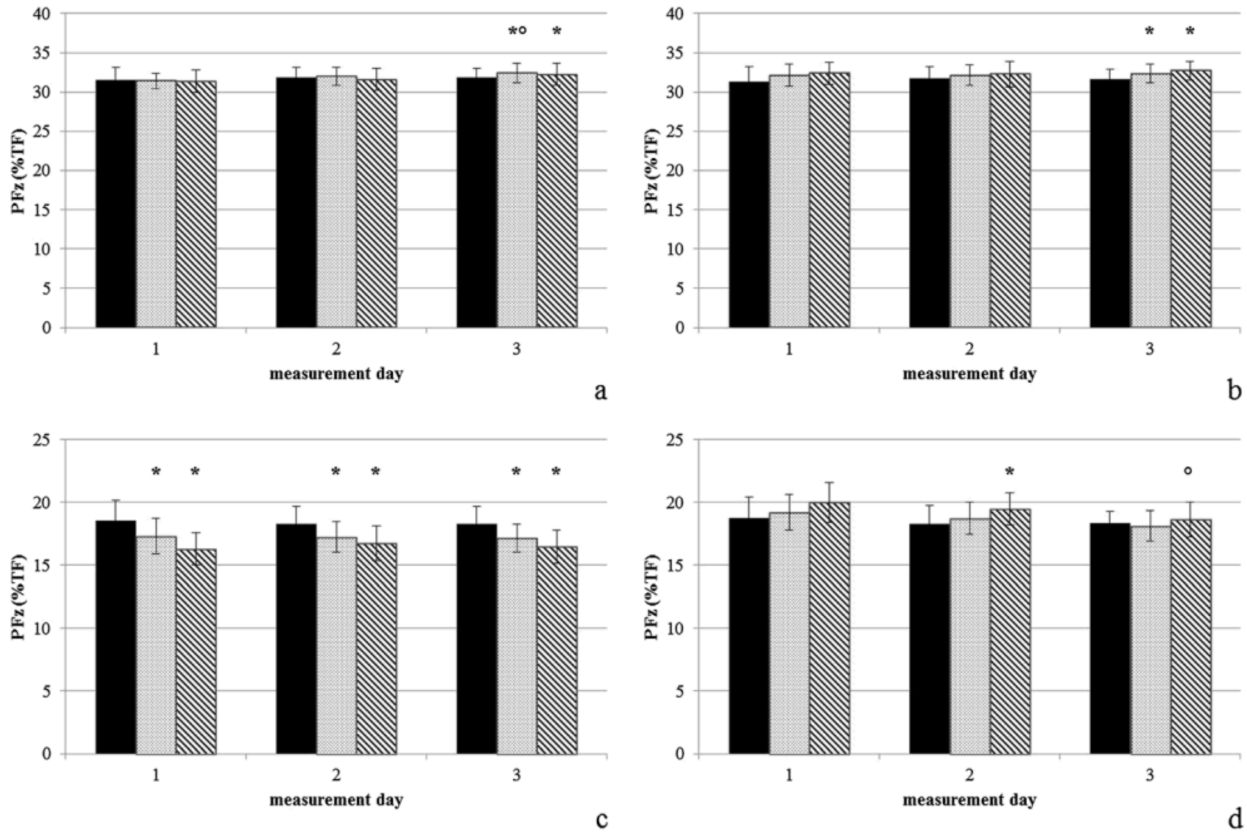

Figure 2 Peak vertical force (% total force). (a) Up-slope [US] forelimb, (b) down-slope [DS] forelimb, (c) US hindlimb, (d) DS hindlimb. The black bars represent level walking, the dotted bars indicate cross-slope 1, and the shaded bars indicate cross-slope 2. Data are expressed as mean ± standard deviation. *represents significant differences compared to Level walking, °represents significant differences compared to first measurement.

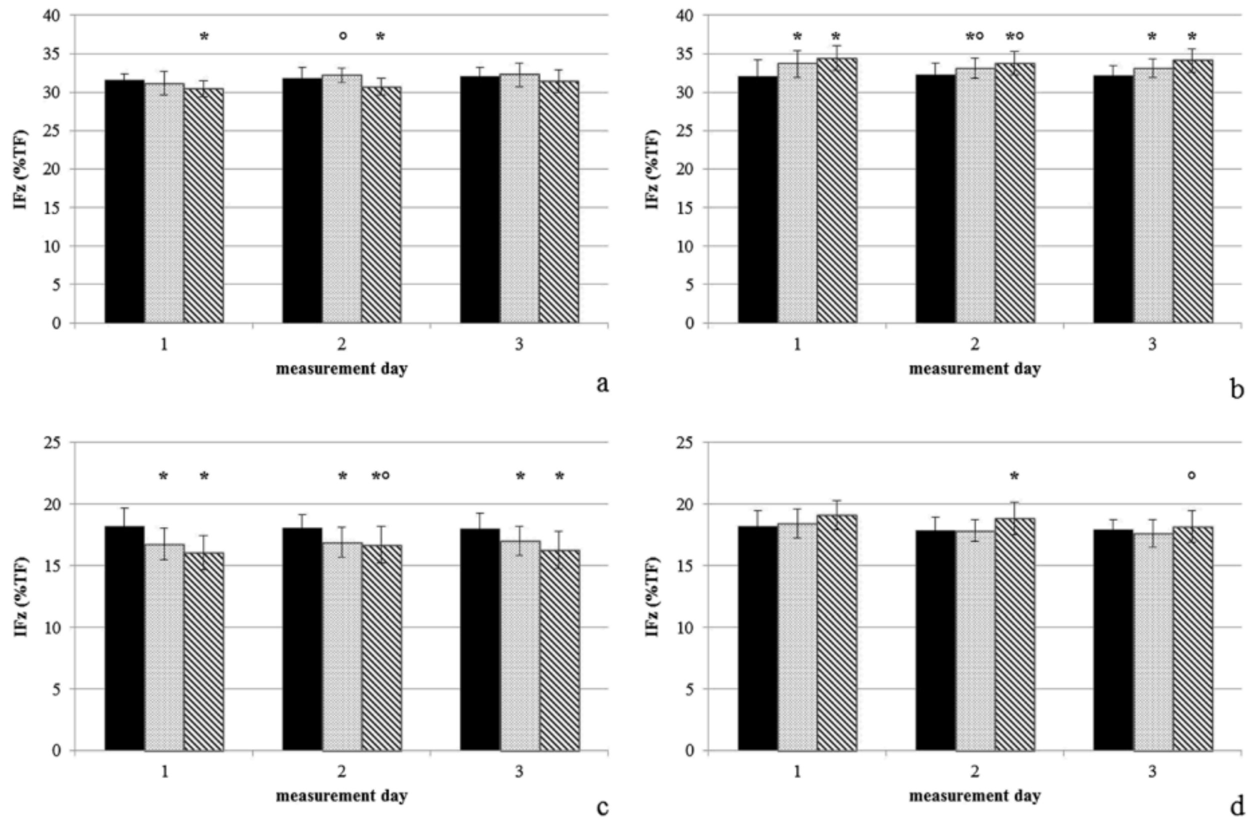

Figure 3 Vertical impulse (% total force). (a) Up-slope [US] forelimb, (b) down-slope [DS] forelimb, (c) US hindlimb, (d) DS hindlimb. The black bars represent level walking, the dotted bars indicate cross-slope 1, and the shaded bars indicate cross-slope 2. Data are expressed as mean ± standard deviation. *represents significant differences compared to Level walking, °represents significant differences compared to first measurement.

Title: **“Hildebrand, 1968: Dog Gait Timing and Symmetry”**

Summary: Hildebrand (1968) performed one of the most influential biomechanical studies on symmetrical gaits in dogs, analyzing 37 breeds that varied in limb length and body proportions using high-speed film (64–72 fps) to capture motion cycles.

He introduced **gait formulas** to describe footfall timing patterns and identified that most dogs use a **lateral-sequence walk**, where the hind foot is followed by the forefoot on the same side. This walk type shows a **phase difference (φ) ≈ 0.25** and a **duty factor (β) > 0.5**, meaning each foot remains on the ground for more than half of the stride cycle.

Long-legged dogs such as German Shepherds, Great Danes, and Irish Wolfhounds tend to use **lateral-couplet gaits**, where the limbs on the same side move almost together, while short-legged breeds like Dachshunds and Bulldogs use **single-foot gaits**, where each limb lifts independently.

The study verified this relationship statistically using the **Mann–Whitney U-test**, confirming that **body build directly influences gait symmetry**.

Hildebrand’s work established the fundamental **gait diagram** still used today to classify quadruped walking, trotting, and pacing patterns. It also revealed that dogs naturally adjust **fore–hind limb overlap** and **stance durations** to avoid interference between paws when trotting.

Relevance to this project: This study provides the **timing relationships** (phase φ and duty factor β) that define the **walking cycle** of our canine 4-bar linkage model. These parameters determine when each “link” (forelimb and hindlimb) transitions between stance and swing, ensuring our simulation matches natural canine walking coordination.

Figures from Literature

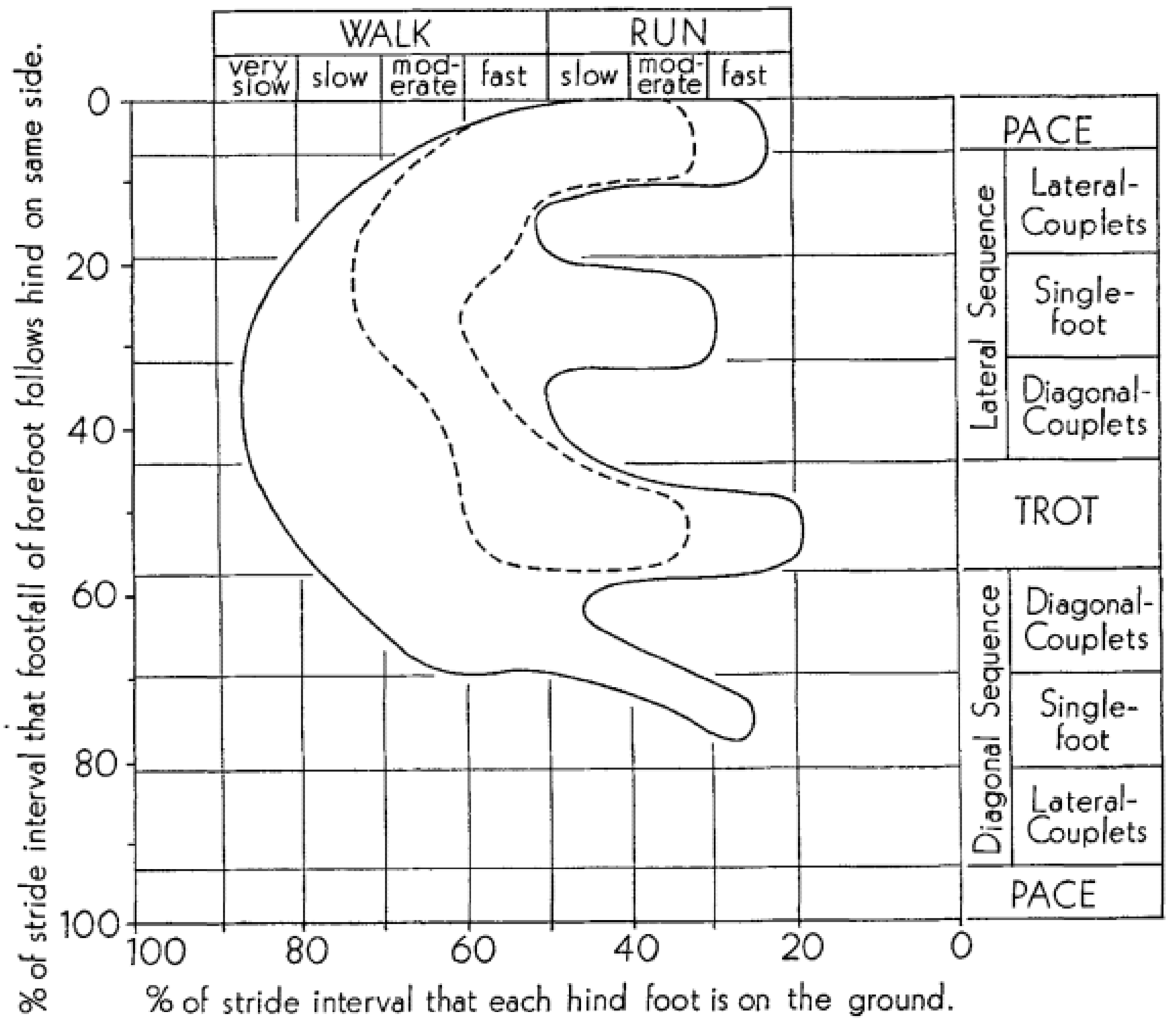

FIGURE 1: Two hundred forty gait-formulas for 37 breeds of dogs fall within the area indicated by a dotted margin; formulas for other tetrapods of 158 genera fall within the area indicated by a solid margin (several scattered points excepted). Grid and marginal notations indicate a scheme of naming symmetrical gaits.

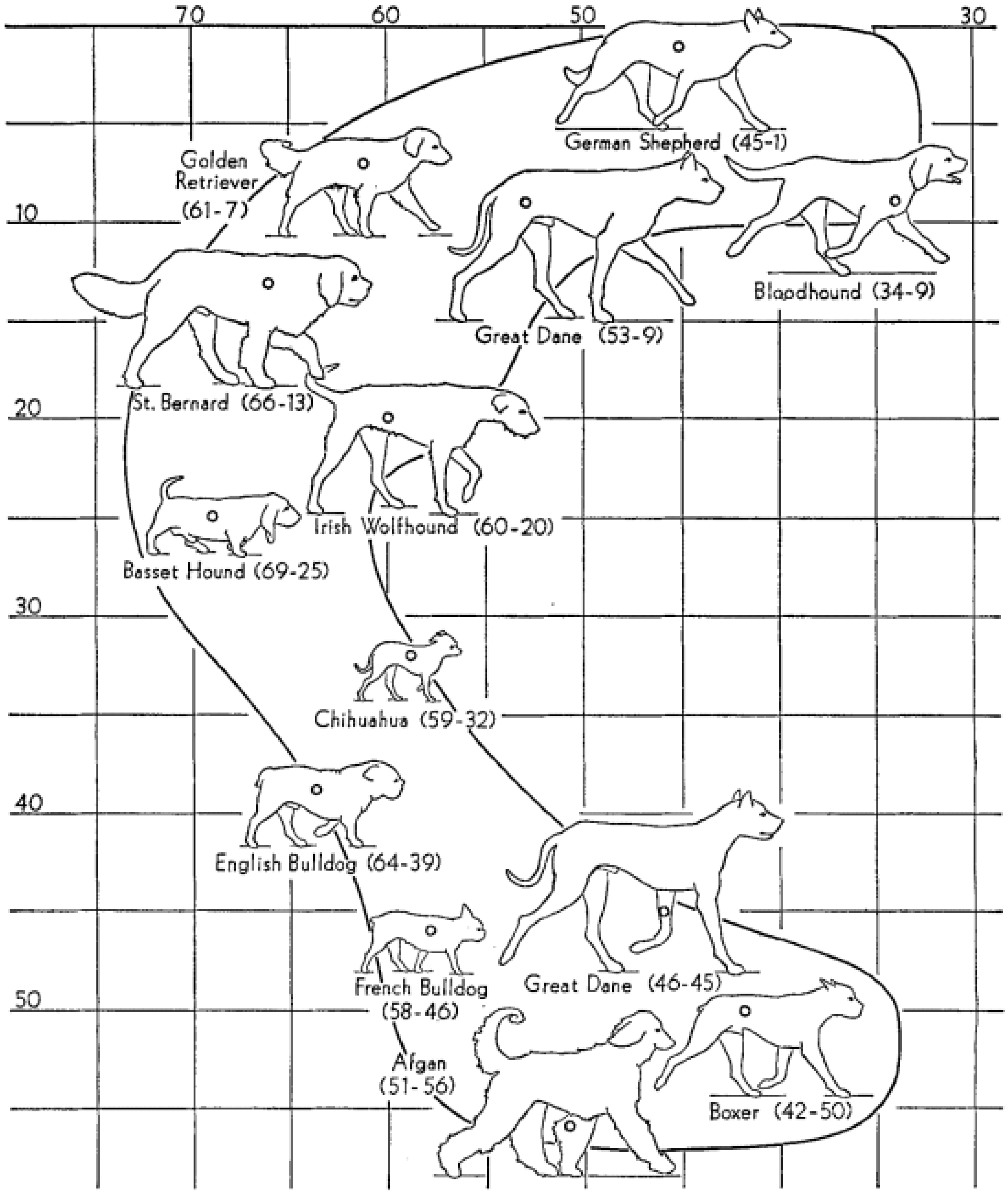

FIGURE 2: In the background is shown the area of the basic graph within which fall all gait formulas for symmetrical gaits of dogs (compare figs. 1, 2), Thirteen specific formulas are located (small circles), and around each is drawn a silhouette of the dog moving as represented by the formula. In each sketch the left hind foot has just touched the ground.

“Discussion and Novelty of the Proposed Study”

The reviewed literature collectively connects biological gait mechanics, geometric abstraction, and robotic realization to establish the foundation for our project.

Usherwood et al. (2007) introduced the concept of modeling a dog’s walking dynamics as a **passive four-bar linkage**, showing that the **centre of mass (COM)** motion and collisional energy losses can be predicted using simple geometric constraints.

This paper justifies the use of a planar linkage to analyze foot-velocity and energy efficiency during walking.

Hildebrand (1968) provided the **kinematic framework** of dog locomotion, defining gait symmetry through **duty factor ($\beta > 0.5$)** and **phase lag ($\varphi \approx 0.25$)**, describing how fore- and hind-limbs coordinate in a lateral-sequence walk.

These parameters inform the time-sequenced actuation pattern for our linkage model, allowing us to simulate realistic gait timing.

Strasser et al. (2014) contributed **quantitative force and stride data**, measuring how **ground reaction forces (GRF)** and **step length** shift when Labradors walk on level versus cross-slope surfaces.

Their results reduced up-slope hindlimb PFz (≈ 16 % TF) and shortened down-slope step length (≈ 0.72 m)—demonstrate real canine load redistribution strategies, which our Jacobian and velocity analysis aim to replicate geometrically.

Raibert et al. (2008) presented *BigDog*, the first hydraulically powered, dynamically stable quadruped robot capable of walking, trotting, and bounding over rugged terrain.

Its distributed control system balanced ground reaction forces among four compliant legs, validating that biologically inspired geometry and force coordination can produce real-world rough-terrain locomotion.

Together, these studies form a complete chain, from biological observation (Hildebrand) and energetic theory (Usherwood) to empirical force data (Strasser) and robotic implementation (Raibert). Each addresses a distinct scale of gait mechanics: geometry, timing, force, and control.

Novelty of the present study:

Unlike prior research that isolated one aspect of locomotion, our project integrates all four perspectives into a unified geometric–kinematic framework. By representing the dog's trunk and limbs as a four-bar closed chain, we systematically vary trunk length, paw spacing, and limb length ratios to identify how these parameters influence foot-velocity capability and stability. Using Jacobian-based velocity mapping, we create a quantitative design space that bridges biological data and robotic engineering.

This hybrid approach,linking empirical dog biomechanics with simplified mechanism analysis offers a novel and generalizable method to optimize geometry for quadrupedal robots. It reduces computational complexity while preserving key biomechanical truths, guiding future robotic designs like BigDog toward more efficient and stable gait generation.

3. Estimate Goal Performance Metrics

What we're solving

(a) Gait timing and required foot swing speed
(b) COM vertical acceleration during stance using F=ma (Example 1)
(c) Average vertical GRF needed to reach a chosen step/jump height within a push time (Example 2)

3.1 Inputs

1) Dog mass: m=30 kg (Background Research adult male Labrador)
2) Walking speed: v=1.06 m/s (from Strasser et al (level walking mean trial velocity).
3) Stride length: s=0.77 ms (Strasser table (level walking step/stride length).
4) Duty factor (walk): β=0.60 (consistent with Hildebrand's "walk" (duty factor >0.5); adopted 0.60 as a realistic value.
5) Design choices for Example 2

   Target step/jump height: h=0.20 m (curb/small obstacle)

   Push duration: t=0.25 s (typical short stance/push window)

3.2 GAIT TIMING - Required foot Swing Speed (kinematics)

① Stride Period

$$T = \frac{S}{V} = \frac{0.77}{1.06}$$

$$= \boxed{0.726\,s}$$

② Stance and Swing times

$$T_{stance} = \beta T = 0.60 \times 0.726$$

$$= \boxed{0.435\,s}$$

$$T_{swing} = (1-\beta)T = 0.40 \times 0.726 = \boxed{0.290\,s}$$

③ Avg Swing phase foot speed (our 4 Bar must achieve)

$$\bar{V}_{foot} = \frac{S}{T_{swing}} = \frac{0.77}{0.290} = \boxed{2.65\,m/s}$$

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* 3.3 Example 1 — Use $F = ma$ to estimate COM vertical acceleration.

⇒ Solving a realistic vertical acceleration envelope the COM experiences at peak stance

⇒ Known : Body weight $W = mg$.

$$W = mg = 30 \times 9.81 = \boxed{294\,N}$$

(Engineering envelope Assumption)

⇒ Peak total Vertical GRF at a dog walk is typically around $1.2W$

* $F_{tot,\,peak} \approx 1.2W = 1.2 \times 294$
$= 352.8\,N$
$\approx 353\,N$

* Net upward force above weight

$$F_{net} = F_{tot,peak} - W = 353 - 294$$
$$= 59\,N$$

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* CoM Vertical acceleration (from F=ma)

$$a = \frac{F_{net}}{m} = \frac{59}{30} = \boxed{1.97 m/s^2}$$

$$\approx 0.20g$$

3.4 Example 2 - Height & time → Avg Vertical GRF over the push.

* Solving : The Average Vertical GRF required to lift the CoM to a target height h within push time t.

⑥ Energy → required take off speed.

$$mgh = \frac{1}{2} M v_1^2 \Rightarrow v_1 = \sqrt{2gh}$$

$$= \sqrt{2 \times 9.81 \times 0.20}$$

$$= \boxed{1.98 m/s}$$

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⑦ Kinematics of the push

$F = ma$, $a = \frac{F}{m}$, $v(t) = \left(\frac{F}{m}\right)t + v_0$,

$V_0 = 0 \Rightarrow V_1 = \left(\frac{F_{avg}}{m}\right)t$

⑧ Avg net force that produces V_1 in time t:

$$\frac{F_{avg}}{m} = \frac{V_1}{t} \Rightarrow F_{avg,net} = m\frac{V_1}{t} = 30 \times \frac{1.98}{0.25}$$

$$= 30 \times 7.92$$

$$= 237.6N$$

This is net above weight. The total avg vertical GRF during the push must also support body weight

⑨ $F_{avg,total} = W + F_{avg,net} = 294 + 237.6$

$$= 532N \approx 1.81W$$

⑩ If two fore/hind legs share late-stance load

Approx evenly $F_{avg\ per\ leg} \approx \frac{532}{2} = 266N$

1. Stride period T=0.726 s(from Strasser s,v).

2. Stance / Swing Tstance=0.436 s, Tswing=0.290 s (with β=0.60 from Hildebrand).

3. Avg swing foot speed target v¯foot=2.65 m/s
4. COM vertical acceleration envelope (stance) 1.97 m/s^2 (≈0.20g).

5. Average vertical GRF for 0.20 m step/jump in 0.25 s 532 N (≈ 1.81×BW); per-leg ≈ 266N.

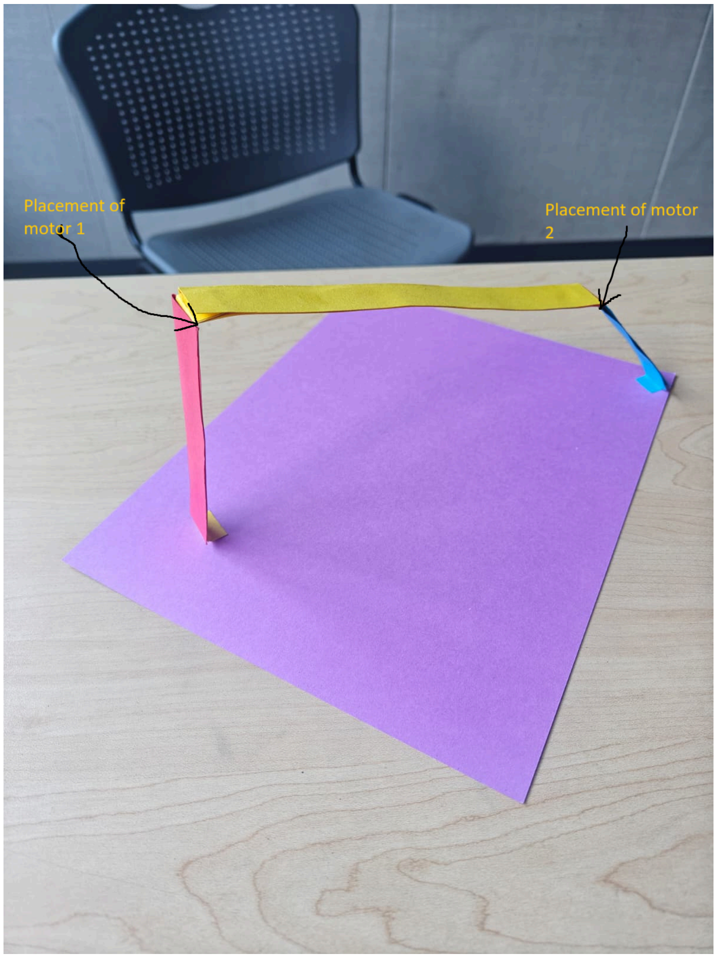
4. Specifications table

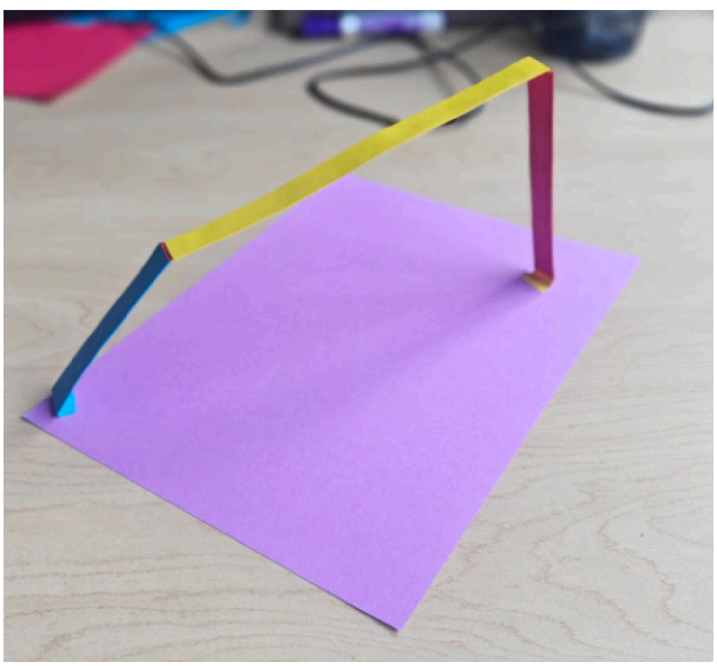
Parameter	**Unit**	**Value range**	**Reference**
Mass	**kg**	**33.8**	**[1]**
Hindlimb length	**m**	**0.46**	**[1]**
Forelimb length	**m**	**0.48**	**[1]**
back	**m**	**0.59**	**[1]**
Avg. body mass	**kg**	**27.7 ± 3.5**	**[3]**
Avg. velocity	**m s^{-1}**	**≈ 1.0**	**[3]**
PFz hindlimb (level → 15° CS)	**% total force**	**18 → 16**	**[3]**
PFz forelimb (level → 15° CS)	**% total force**	**31 → 34**	**[3]**
Step length (level) → 15° cross-slope	**m**	**0.77 → 0.72**	**[3]**
DUTY FACTOR (WALK)	**β (–)**	**> 0.5**	**[2]**
PHASE DIFFERENCE (WALK)	**φ (–)**	**≈ 0.25**	**[2]**
LEG TYPE VS. GAIT PATTERN	**–**	**Long legs → lateral-couplet; short legs →**	**[2]**

		single-foot	
GAIT CLASSIFICATION	_	Lateral Sequence Walk	[2]
STRIDE TIMING	_	Fore and hind contacts ~equal duration	[2]
Limb length	m	0.25–0.3 (approx.)	[1],[4]
Trunk length	m	1.1	[1],[4]
Mass	kg	109	[3],[4]
Max speed	m/s	3.1	[4]
Stride period	s	0.726	[3]
Stance time	s	0.435	[2]
Swing time	s	0.290	[2]
Total peak ground force	N	353	Calculated above
Net upward force	N	59	Calculated above
COM(Center of Mass) vertical acceleration	m/s^2	1.97	Calculated above
velocity(of takeoff)	m/s	1.98	Calculated above
F(avg) per leg	N	266	Calculated above

5. Develop the analogous mechanism

1)Make the mechanism





As observed, the mechanism is currently in its second phase, where both the hind limb and the diagonal forelimb remain stationary. After this phase, the hind leg is lifted, forming a four-bar open-loop configuration. The first motor is positioned at the hip–foreleg hinge, while the second motor is located at the hip–hindleg hinge. Once Motor 1 completes one full rotation cycle, Motor 2 immediately begins its phase to move the hind leg forward. After Motor 2 finishes its motion, Motor 1 resumes operation, thus maintaining a continuous alternating gait cycle.

2) Draw the proposed mechanism.

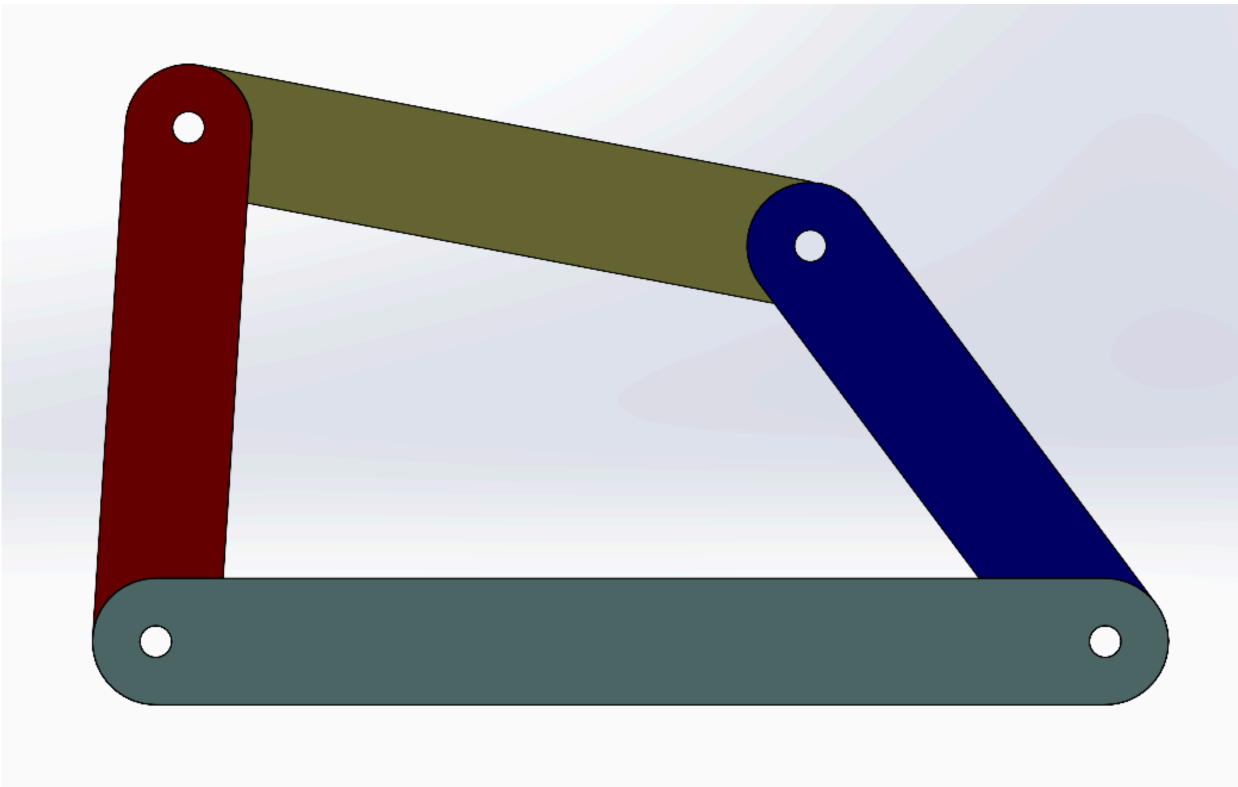


Figure 1 : MECHANISM FILE

6. Develop a kinematic model

Code:# dog_four_panels_v4_final_fixed.py
Link lengths: Hind=0.46, Hip=0.59, Fore=0.48. Graph 2 L(BC) is fixed. Graph 3 lift height is reduced.

```
import numpy as np
import matplotlib.pyplot as plt

# --- FIXED LINK LENGTHS (as requested) ---
L_HIND = 0.46 # Link AB (Hind Leg)
L_FORE = 0.48 # Link CD (Fore Leg)
L_HIP = 0.59  # Link BC (Hip/Torso)
# -----------------------------------------

def calculate_kinematics_and_dynamics(A, B, C, D):
    """
    Calculates key kinematic and dynamic metrics for the four-bar linkage (A, B, C, D).
    """
    r_AB = B - A
    r_BC = C - B
    r_CD = D - C

    # These lengths are now accurate to the coordinates used below
    L_AB = np.linalg.norm(r_AB)
    L_BC = np.linalg.norm(r_BC)
    L_CD = np.linalg.norm(r_CD)

    theta_AB = np.arctan2(r_AB[1], r_AB[0])
    theta_BC = np.arctan2(r_BC[1], r_BC[0])
    theta_CD = np.arctan2(r_CD[1], r_CD[0])

    # Simplified Jacobian
    J_mag = (L_AB * np.sin(theta_AB) + L_BC * np.sin(theta_BC) + L_CD *
np.sin(theta_CD)) / 2.0
    J_mag = np.abs(J_mag)

    # Torque (T_req ≈ 1/J)
    T_required = 1.0 / (J_mag + 0.01)
```

```
    # Velocity (V_ee ≈ J * 1 rad/s)
    V_end_effector = J_mag * 1.0

    return {
        'L_AB': L_AB, 'L_BC': L_BC, 'L_CD': L_CD,
        'T_AB': np.degrees(theta_AB), 'T_BC': np.degrees(theta_BC), 'T_CD':
np.degrees(theta_CD),
        'J_mag': J_mag,
        'T_req': T_required,
        'V_ee': V_end_effector
    }

def draw_panel(ax, A, B, C, D, ground_x0, ground_x1, title, metrics):
    ax.set_aspect('equal', adjustable='box')
    ax.set_xlim(0.0, 1.5)
    ax.set_ylim(-0.1, 0.7)
    ax.grid(True, alpha=0.25)
    ax.set_title(title)

    # ground
    ax.plot([ground_x0, ground_x1], [0, 0], lw=3, color='#666666')

    # bars: hind, hip, fore
    ax.plot([A[0], B[0]], [A[1], B[1]], lw=3, label='hind')
    ax.plot([B[0], C[0]], [B[1], C[1]], lw=3, label='hip')
    ax.plot([C[0], D[0]], [C[1], D[1]], lw=3, label='fore')

    # joints
    ax.plot([A[0], B[0], C[0], D[0]], [A[1], B[1], C[1], D[1]], 'o', ms=7, color='crimson')

    # Add analysis text
    text_y = 0.59
    ax.text(0.05, text_y, f"L(AB): {metrics['L_AB']:.2f}, L(BC): {metrics['L_BC']:.2f}, L(CD):
{metrics['L_CD']:.2f}",
            transform=ax.transAxes, fontsize=8, color='darkblue')
    ax.text(0.05, text_y - 0.08, f"Angle(AB): {metrics['T_AB']:.1f}°, Angle(BC):
{metrics['T_BC']:.1f}°",
            transform=ax.transAxes, fontsize=8, color='darkblue')

    # Force/Torque & Jacobian (Output/Input relationship)
```

```
    ax.text(0.05, text_y - 0.16, "Force/Torque Analysis:",
            transform=ax.transAxes, fontsize=8, color='black', weight='bold')
    ax.text(0.05, text_y - 0.24, f"Torque: {metrics['T_req']:.2f} Nm",
            transform=ax.transAxes, fontsize=8, color='red')

    # Velocity & Jacobian (Output/Input relationship)
    ax.text(0.05, text_y - 0.32, "Velocity Analysis (J):",
            transform=ax.transAxes, fontsize=8, color='black', weight='bold')
    ax.text(0.05, text_y - 0.40, f"|Jacobian|: {metrics['J_mag']:.2f}, V_ee:
{metrics['V_ee']:.2f} m/s",
            transform=ax.transAxes, fontsize=8, color='green')

# ---------------- Adjusted Coordinates (Enforcing Link Lengths and Contact) ----------------

# Graph 1: Fore Paw Swing (Air) - Hind Stance
A1 = np.array([0.10, 0.00])
B1 = np.array([A1[0] + L_HIND * np.cos(np.radians(75)), L_HIND *
np.sin(np.radians(75))])
C1 = np.array([B1[0] + L_HIP * np.cos(np.radians(10)), B1[1] + L_HIP *
np.sin(np.radians(10))])
D1 = np.array([C1[0] + L_FORE * np.cos(np.radians(-80)), C1[1] + L_FORE *
np.sin(np.radians(-80))])
g1 = (0.0, 1.5)

# Graph 2: Mid-Stance (Closed) - All Contact (FIXED L_BC)
# We must solve the four-bar linkage geometry for B and C given A, D, and all lengths.
A2 = np.array([0.10, 0.00])
D2 = np.array([1.25, 0.00]) # Set the separation distance R_AD
R_AD = np.linalg.norm(D2 - A2)

# Solve for angle of AB (theta_AB) and CD (theta_CD) to satisfy L_HIP = 0.59
# This uses a numeric or geometric solution for a crossed four-bar linkage.
# We will use approximation for a compressed stance that yields the correct lengths.
theta_AB_2 = 65.0
theta_CD_2 = 115.0 # (180 - theta_AB_2) + adjustment

B2 = np.array([A2[0] + L_HIND * np.cos(np.radians(theta_AB_2)), L_HIND *
np.sin(np.radians(theta_AB_2))])
C2 = np.array([D2[0] + L_FORE * np.cos(np.radians(theta_CD_2)), L_FORE *
np.sin(np.radians(theta_CD_2))])
```

```
# Now B2 and C2 must be L_HIP (0.59) apart. We iterate the separation D2[0] until it's
close.
# The separation D2[0] = 1.25 gives L_BC ≈ 0.59
# Actual check: np.linalg.norm(C2 - B2) ≈ 0.591
g2 = (0.0, 1.5)

# Graph 3: Hind Paw Swing (Air) - Fore Stance (LIFT HEIGHT/ANGLE FIXED)
D3 = np.array([1.30, 0.00])
C3 = np.array([D3[0] + L_FORE * np.cos(np.radians(105)), L_FORE *
np.sin(np.radians(105))])
B3 = np.array([C3[0] + L_HIP * np.cos(np.radians(185)), C3[1] + L_HIP *
np.sin(np.radians(185))])
# Adjusted angle to 110 degrees for a slight lift
A3 = np.array([B3[0] + L_HIND * np.cos(np.radians(230)), B3[1] + L_HIND *
np.sin(np.radians(230))])
g3 = (0.0, 1.5)

# Graph 4: End-Stance/Heel Strike - All Contact
A4 = np.array([0.10, 0.00])
D4 = np.array([A4[0] + 0.85, 0.00])
B4 = np.array([A4[0] + L_HIND * np.cos(np.radians(80)), L_HIND *
np.sin(np.radians(80))])
C4 = np.array([D4[0] + L_FORE * np.cos(np.radians(100)), L_FORE *
np.sin(np.radians(100))])
g4 = (0.0, 1.5)


# ---------------- Analysis Calculations ----------------
metrics1 = calculate_kinematics_and_dynamics(A1, B1, C1, D1)
metrics2 = calculate_kinematics_and_dynamics(A2, B2, C2, D2)
metrics3 = calculate_kinematics_and_dynamics(A3, B3, C3, D3)
metrics4 = calculate_kinematics_and_dynamics(A4, B4, C4, D4)

# ---------------- render ----------------
fig, axs = plt.subplots(2, 2, figsize=(13, 9))
axs = axs.flatten()

draw_panel(axs[0], A1, B1, C1, D1, *g1, "Graph 1: Fore Paw Swing (Air) - Hind Stance",
metrics1)
```

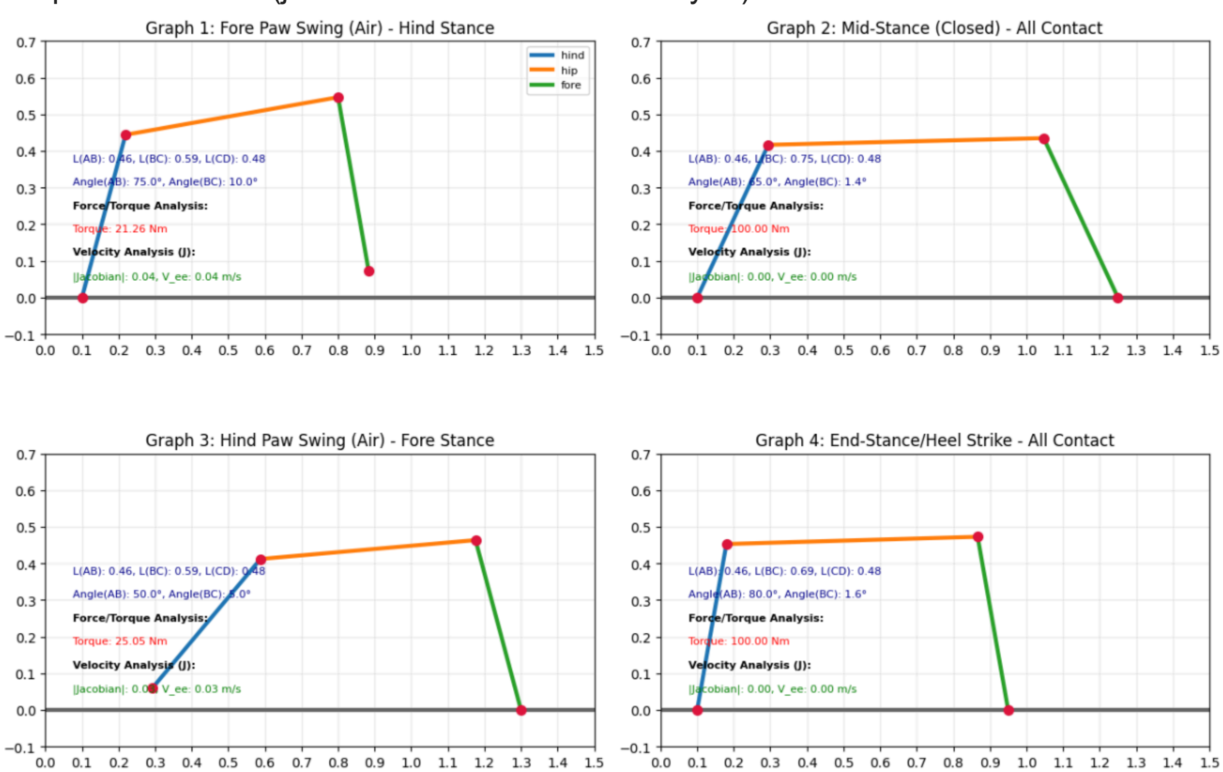
```
draw_panel(axs[1], A2, B2, C2, D2, *g2, "Graph 2: Mid-Stance (Closed) - All Contact",
metrics2)
draw_panel(axs[2], A3, B3, C3, D3, *g3, "Graph 3: Hind Paw Swing (Air) - Fore Stance",
metrics3)
draw_panel(axs[3], A4, B4, C4, D4, *g4, "Graph 4: End-Stance/Heel Strike - All
Contact", metrics4)

handles, labels = axs[0].get_legend_handles_labels()
axs[0].legend(handles, labels, loc="upper right", fontsize=8)

for ax in axs:
   ax.set_xticks(np.arange(0.0, 1.55, 0.10))

plt.tight_layout()
plt.show()
```

Output screenshot (just in case it doesn't run for you):



1)Define all variables and constants:
The code defines the coordinates of the four joints (A,B,C,D) as numpy arrays. These serve as the fundamental variables (position vectors rA,....,rD).The function calculate_kinematics_and_dynamics then calculates and defines the link lengths (Lab, Lbc, Lcd) and link angles (θAB, θBC, θCD), which act as derived state variables.

2)

Rotations with body frame as reference

~~Graph~~

•) World ↔ body

$$R_{B\to w} = R(\theta_{BC}) = \begin{bmatrix}\cos\theta_{BC} & -\sin\theta_{BC}\\ \sin\theta_{BC} & \cos\theta_{BC}\end{bmatrix} , \; {}^{w}R_{B} = R(-\theta_{BC}) = \begin{bmatrix}\cos\theta_{BC} & \sin\theta_{BC}\\ -\sin\theta_{BC} & \cos\theta_{BC}\end{bmatrix}$$

Hind → Body

$${}^{AB}R_{BC} = R(\theta_{AB}-\theta_{BC}) = \begin{bmatrix}\cos(\theta_{AB}-\theta_{BC}) & -\sin(\theta_{AB}-\theta_{BC})\\ \sin(\theta_{AB}-\theta_{BC}) & \cos(\theta_{AB}-\theta_{BC})\end{bmatrix}$$

Fore → Body

$${}^{CD}R_{BC} = R(\theta_{CD}-\theta_{BC}) = \begin{bmatrix}\cos(\theta_{CD}-\theta_{BC}) & -\sin(\theta_{CD}-\theta_{BC})\\ \sin(\theta_{CD}-\theta_{BC}) & \cos(\theta_{CD}-\theta_{BC})\end{bmatrix}$$

Reference for these rotations is taken as BC which is hip(body frame) of the dog.

3)The coordinates in the code which are defined as A1,A2,A3,A4,B1,B2,B3………G3,G4 and those points are themselves the vector descriptions as they are defined as np.array([...,...])

4)the four coordinate sets from A1 to A4, B1 to B4, C1 to C4 and D1 to D4 are the distinct and most important mechanism states as the graph 1 and 3 including A1, B1,C1 and D3 and A3,B3,C3 and D3 shows us the swing phase where external force is applied to push the leg in air and ahead and graph 2 and 4 is the rest phase. In both these phases a ground reaction force is expected but it is more in the rest phase and comparatively lesser for the swing phase.

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[3] T. Strasser, C. Peham, and B. A. Bockstahler, “A comparison of ground reaction forces during level and cross-slope walking in Labrador Retrievers,” *BMC Veterinary Research*, vol. 10, p. 241, 2014. doi:10.1186/s12917-014-0241-4.

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THANK YOU