CS535 Homework 1

Due: Sep. 6, 2019.

1. Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buys 49 Indian rupees, 1 Indian rupee buys 2 Japanese yen, and 1 Japanese yen buys 0.0107 U.S. dollars. Then, by converting currencies, a trader can start with 1 U.S. dollar and buy $49 \times 2 \times 0.0107 = 1.0486$ U.S. dollars, thus turning a profit of 4.86 percent.

   Suppose that we are given n currencies $c_1, c_2, \cdots, c_n$ and an $n \times n$ table R of exchange rates, such that one unit of currency c_i buys $R[i, j]$ units of currency c_j.

   (a) Give an efficient algorithm to determine whether or not there exists a sequence of currencies $c_{i_1}, c_{i_2}, \cdots, c_{i_k}$ such that

   $$R[i_1, i_2] \cdot R[i_2, i_3] \cdot \cdots \cdot R[i_{k-1}, i_k] \cdot R[i_k, i_1] > 1$$

   Analyze the running time of your algorithm.

   (b) Give an efficient algorithm to print out such a sequence if one exists. Analyze the running time of your algorithm.

2. Suppose that we are given n intervals $[x_i, y_i]$ for $1 \leq i \leq n$, which together cover an interval $[x_0, y_0]$. Each interval $[x_i, y_i]$ has a positive cost c_i. Give an efficient algorithm to a subset of intervals with minimum total cost which together still cover the interval $[x_0, y_0]$.

3. Consider a digraph $D = (V, A)$ with two distinct vertices s and t.

   (a) Let F denote the set of edges in A which appear in some shortest s-t path (in terms of the number of edges) in D. Give an algorithm to output F in $O(|V| + |A|)$ time.

   (b) Give an efficient algorithm to find an inclusion-wise maximal (not necessarily) edge-disjoint shortest s-t paths in D in $O(|V| + |A|)$ time.

4. Suppose that a digraph $D = (V, A)$ with edge length function ℓ has no negative circuit but has a 0-length circuit C. Let p be an arbitrary potential function, and ℓ^* be the edge length function obtained by reweighting ℓ with p. Prove that for any edge $a \in C$, $\ell^*(a) = 0$.

5. **[PhD Session only]** Suppose that V is a set of points lying within a horizontal strip of height $\sqrt{3}/2$ in a plane. Each point in v has a positive weight $w(v)$. A subset U of points in V is said to be *well-separated* if every pair of points in S are separated by an Euclidean distance greater than one. Give an efficient algorithm to produce a well-separated subset U of V with the largest total weight, prove its correctness, and analyze its running time.