CS 535 Homework 2

Due: Sep. 23, 2019.

1. Let $G = (U, W, E)$ be a bipartite graph with maximal vertex degree k. Show that E can be decomposed into k matchings.

2. Let $G = (U, W, E)$ be a bipartite graph and M be a maximum matching of G. Suppose that M does not cover U. Show that the set $U \cap R_M$ is a Hall set, i.e., $|N(U \cap R_M)| < |U \cap R_M|$.

3. Let $G = (U, W, E)$ be a bipartite graph. Let $\mathcal{I}$ denote the collection of subsets of U which can be covered by some matching of G. Suppose that $S, T \in \mathcal{I}$ and $|S| < |T|$. Show that there exists $v \in T \setminus S$ such that $S \cup \{v\} \in \mathcal{I}$.

4. Let $G = (U, W, E)$ be a bipartite graph. Show that $\alpha'(G) = \min_{S \subseteq U} (|U| - |S| + |N(S)|)$.

5. **[PhD Session only]** Suppose that V is a set of points lying within a horizontal strip of height $\sqrt{3}/2$ in a plane. A subset U of points in V is said to be *well-separated* if every pair of points in S are separated by an Euclidean distance greater than one. Give an efficient algorithm to partition V into a smallest number of well-separated subset, prove its correctness, and analyze its running time.