Knowledge Discovery & Data Mining
一 Data Exploration: Descriptive Statistics一

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Outline

- **Central Tendency**

  - Mean, Median, Mode,

  - Symmetric vs. Skewed Data

- **Dispersion of Data**

  - Range, Quantiles, Quartiles, Interquartile Range(IQR),

  - Variance, Standard Deviation

Population vs. Sample

- **A set of data points is a sample from a population:**

  - A **population** is the entire set of objects or events under study.

    - E.g., population can be hypothetical "all students" or all students in this class.

    - E.g., population can be all the houses in a region

  - A **sample** is a "representative" subset of the objects or events under study. Needed because it's impossible or intractable to obtain or compute with population data.

Basic Statistical Descriptions of Data

- Central Tendency

- Dispersion of the Data

Central Tendency

Suppose that we have some attribute X, like salary, which has been recorded for a set of objects. Let x1, x2, . . . , xN be the set of N observed values or observations for X. Here, these values may also be referred to as the data set (for X). If we were to plot the observations for salary, where would most of the values fall?

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Measuring the Central Tendency

- Mean (algebraic measure) (sample vs. population):
  $$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mu = \frac{\sum x}{N}$$

  Note: n is sample size and N is population size.

  - Weighted arithmetic mean:
  $$\overline{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

- Median:

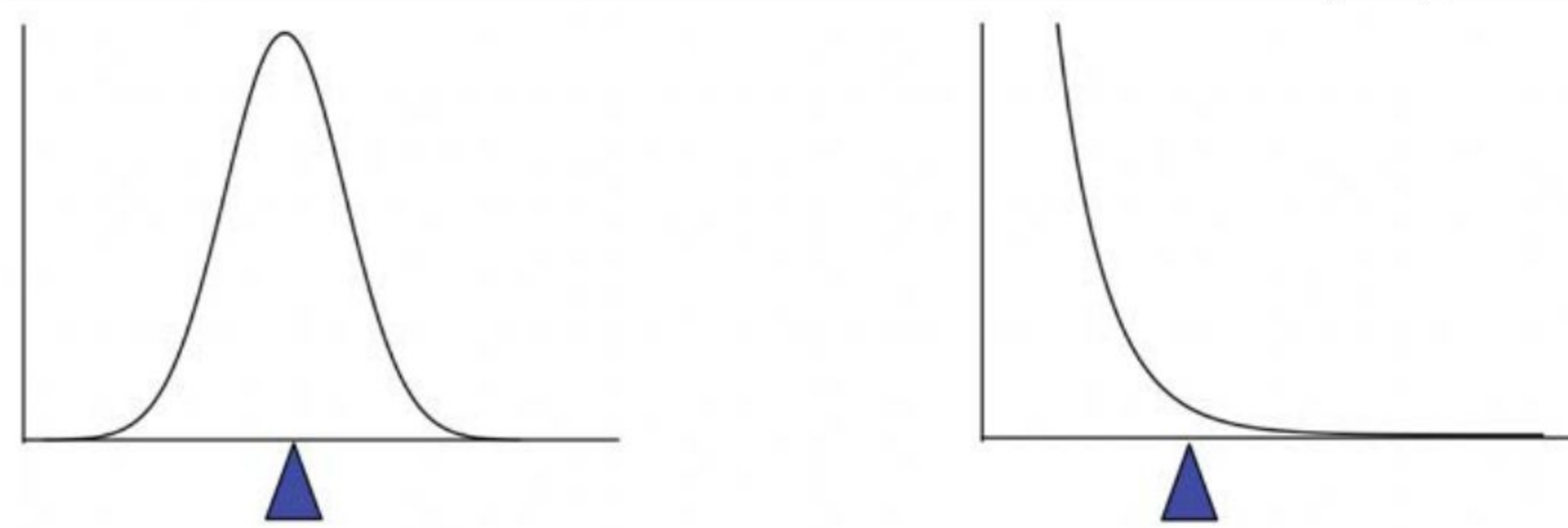 - Middle value if odd number of values, or average of the middle two values otherwise

- Mode

  - Value that occurs most frequently in the data

  - Unimodal, bimodal, trimodal

- The mean of a set of n observations of a variable is denoted $\bar{x}$ and is defined as:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- The mean describes what a "typical" sample value looks like, or where is the "center" of the distribution of the data.

- Key theme: there is always uncertainty involved when calculating a sample mean to estimate a population mean.

Sample Median

- The median of a set of n number of observations in a sample, ordered by value, of a variable is is defined by

$$\text{Median} = \begin{cases} x_{(n+1)/2} & \text{if } n \text{ is odd} \\ \dfrac{x_{n/2}+x_{(n+1)/2}}{2} & \text{if } n \text{ is even} \end{cases}$$

- Example (already in order):
  - Ages: 17, 19, 21, 22, 23, 23, 23, 38
  - Median = (22+23)/2 = 22.5
- The median also describes what a typical observation looks like, or where is the center of the distribution of the sample of observations.

Mean vs. Median

- The mean is sensitive to extreme values (outliers)

Mode

Mode: The mode for a set of data is the value that occurs most frequently compared to all neighboring values in the set. It is possible to have more than one mode.

- unimodal, bimodal, trimodal, multimodal…

Example. The data from 30, 36, 47, 50, 52, 52, 56, 60, 63, 70, 70, 110. are _____. The modes are:

Mode

Mode: The mode for a set of data is the value that occurs most frequently compared to all neighboring values in the set. It is possible to have more than one mode.

● unimodal, bimodal, trimodal, multimodal…

Example. The data from 30, 36, 47, 50, 52, 52, 56, 60, 63, 70, 70, 110. are **bimodal**. The modes are: **$52,000 and $70,000**.

Mode

For unimodal numeric data that are moderately skewed (asymmetrical), we have the following empirical formula:
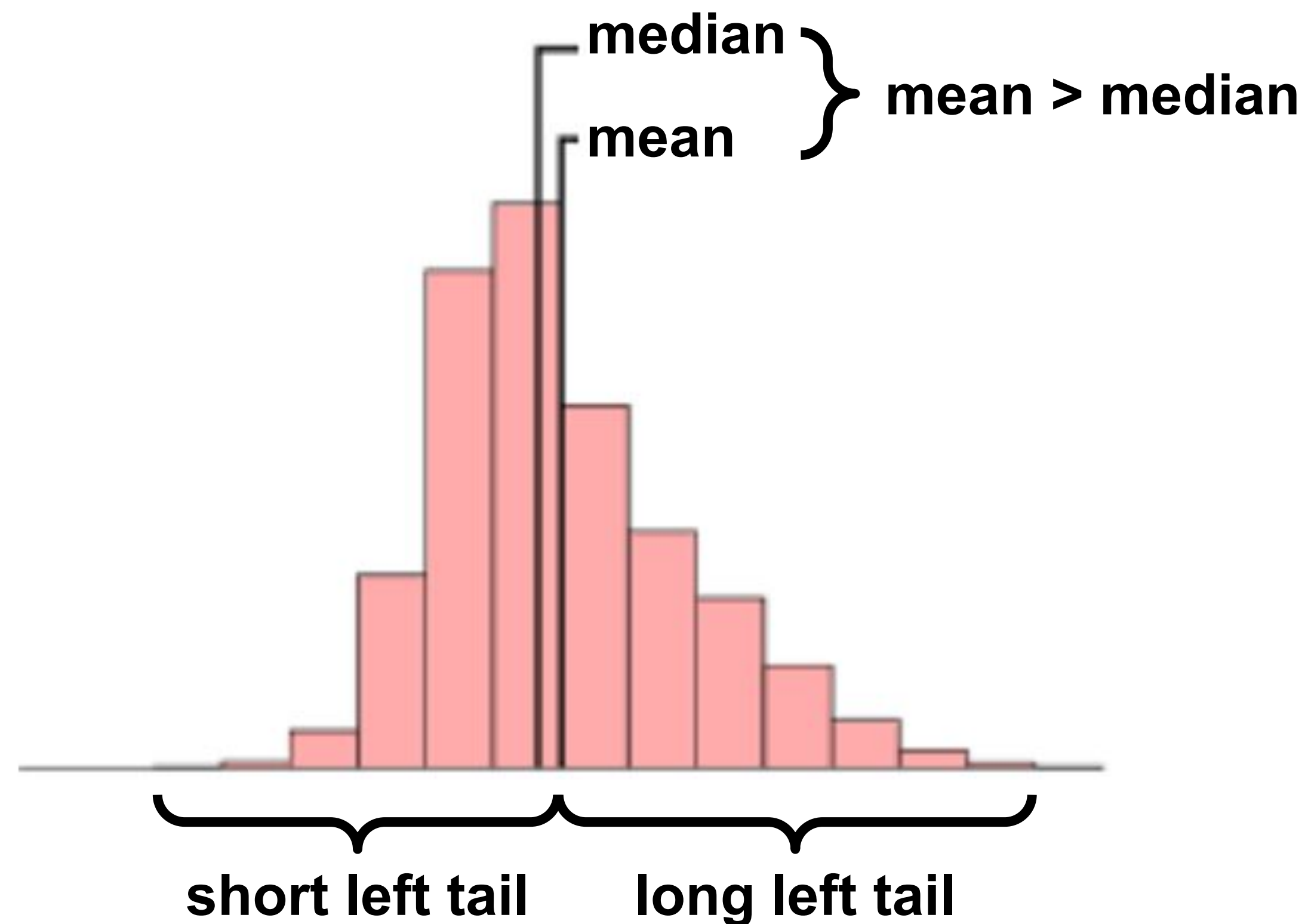
$$mean - mode \approx 3 \times (mean - median).$$

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Mean, median, and skewness

- The following distribution is called right-skewed since the mean is greater than the median. Note: skewness often "follows the longer tail"
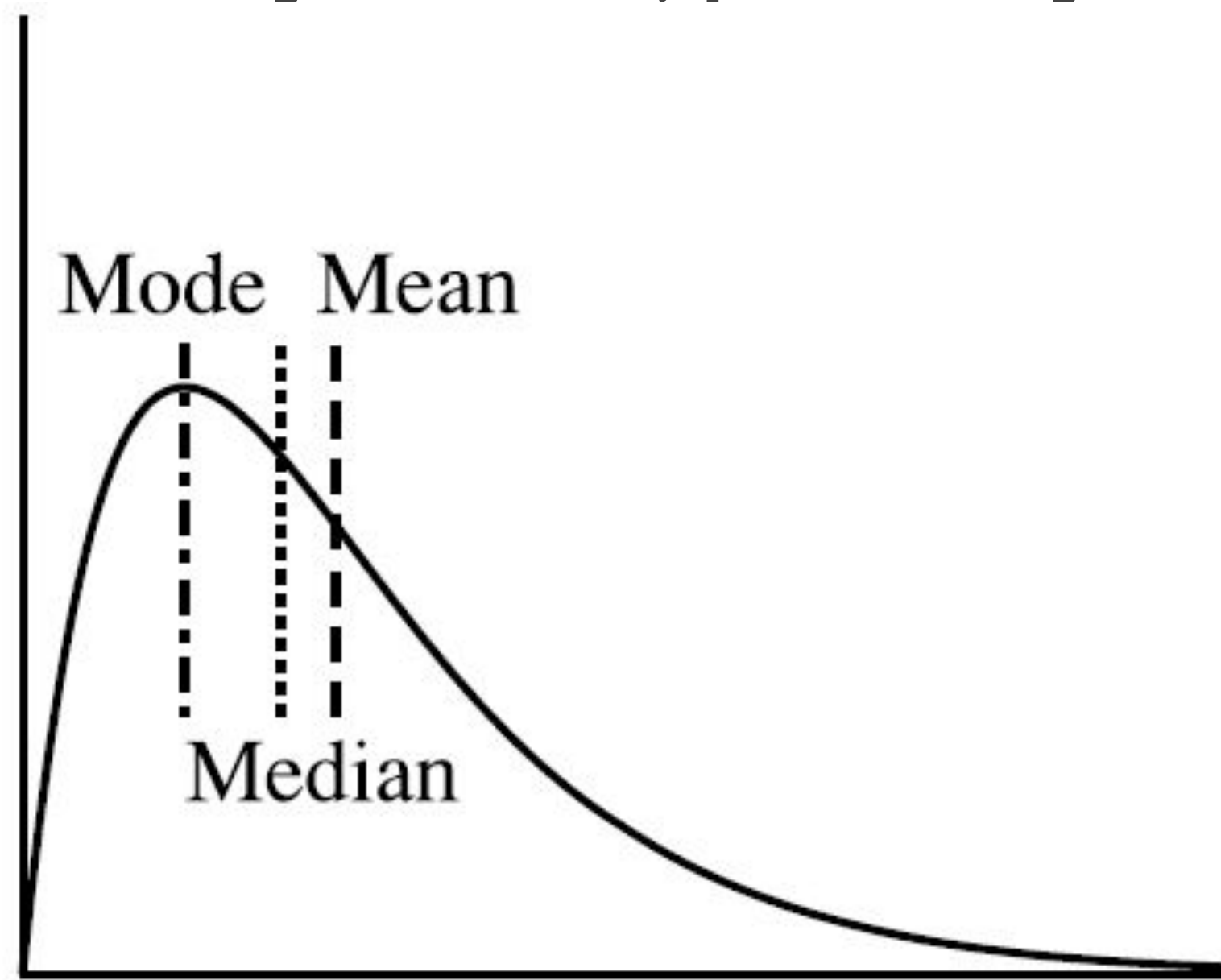
Symmetric vs. Skewed Data

- In a unimodal frequency curve with perfect symmetric data distribution, the mean, median, and mode are all at the same center value.
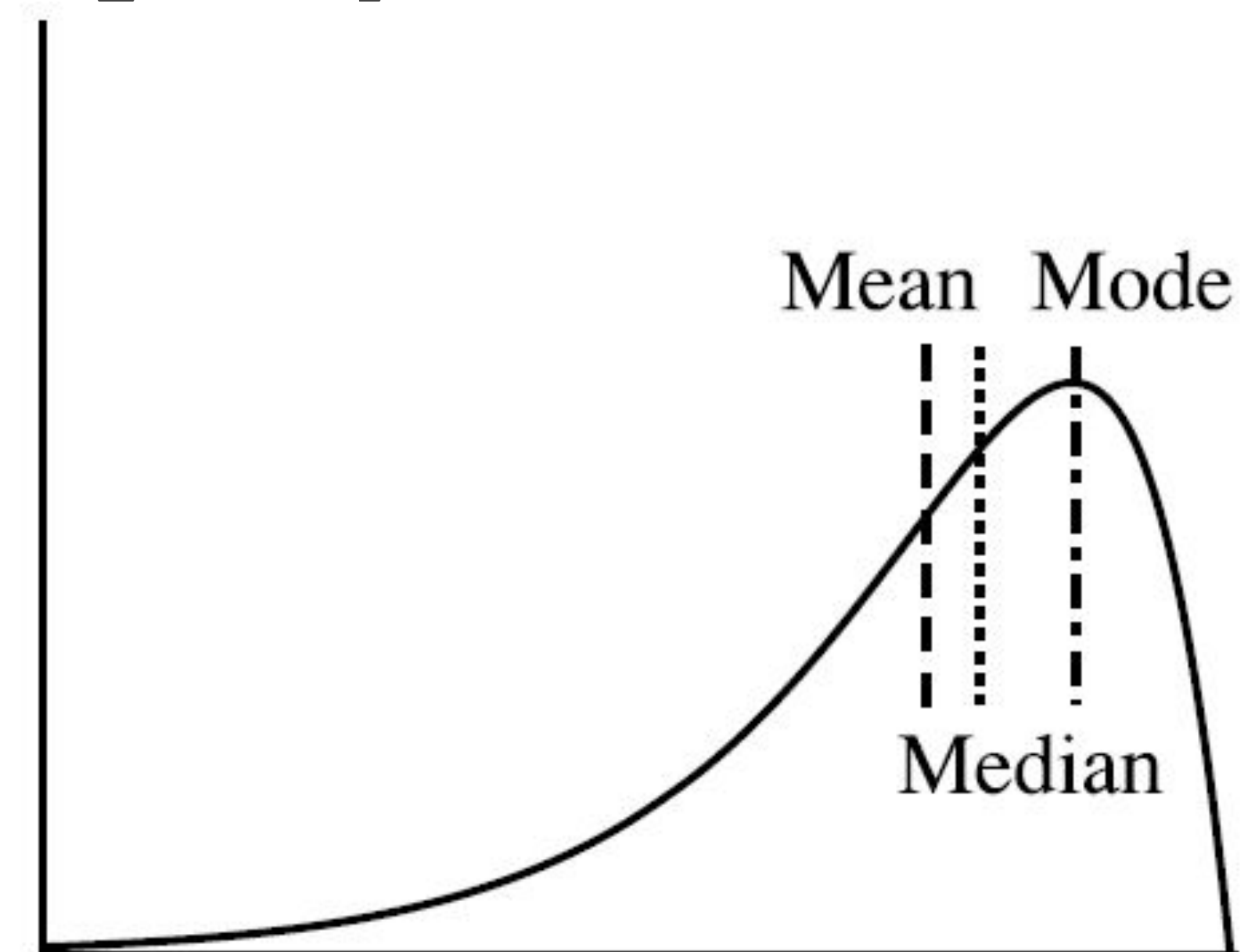- Data in most real applications are not symmetric. positively skewed, or negatively skewed.

Median, mean and mode of symmetric, positively and negatively skewed data
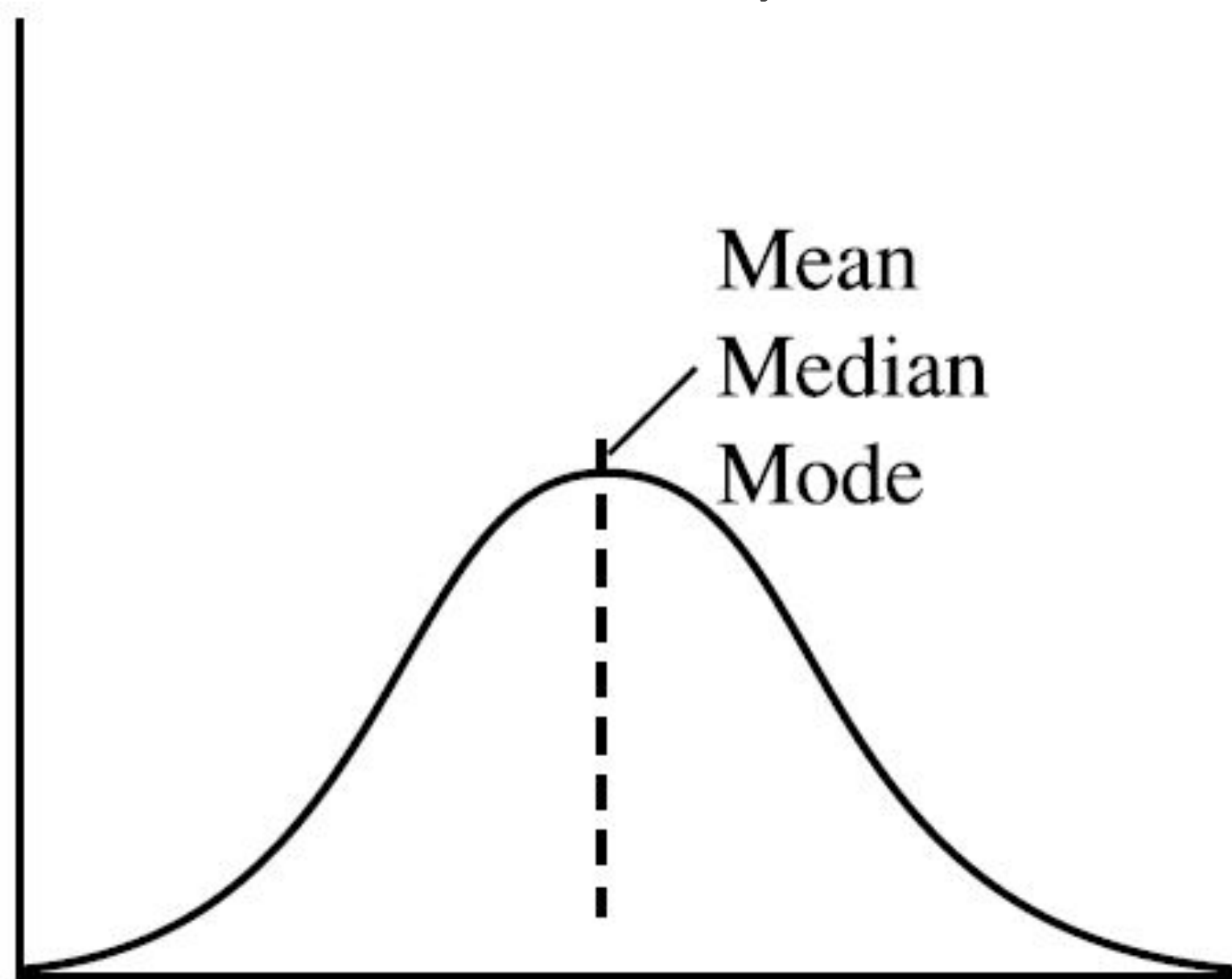
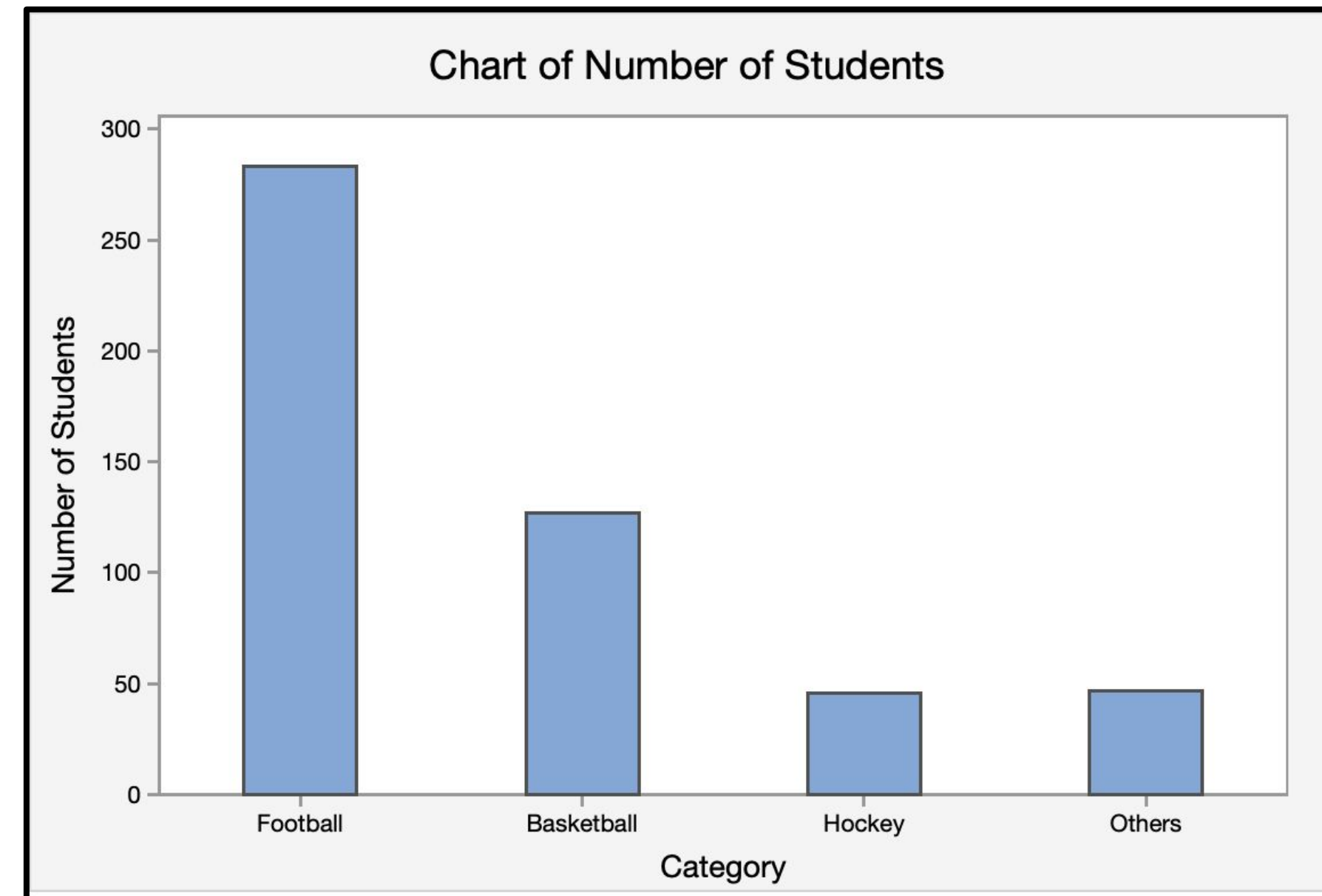(a) Symmetric data   (b) Positively skewed data   (c) Negatively skewed data

Question

Is income positively or negatively skewed?

Regarding Categorical Variables...

- **For categorical variables, neither mean or median make sense. Why?**

- The mode might be a better way to find the most "representative" value.

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Measures of Spread: Range

- The spread of a sample of observations measures how well the mean or median describes the sample.

- One way to measure spread of a sample of observations is via the **range**.

Range = Maximum Value - Minimum Value

Suppose that the data for attribute X are sorted in ascending numeric order. Imagine that we can pick certain data points so as to split the data distribution into contiguous sets of equal size, These data points are called **quantiles**.

The 2-quantile is the data point dividing the lower and upper halves of the data distribution.
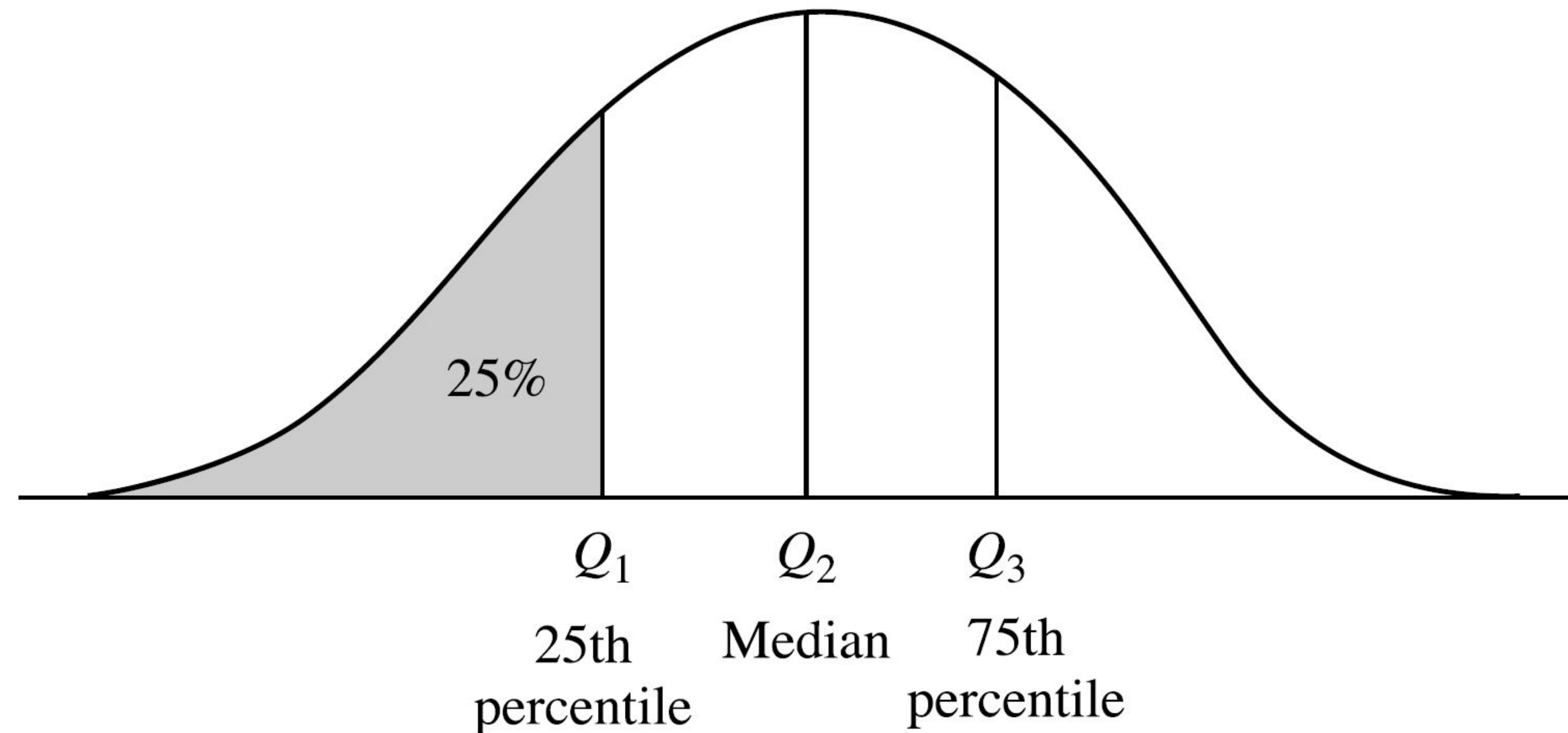
**What is the 2-quantile also called?**

median

The 4-quantiles are the three data points that split the data distribution into four equal parts; each part represents one-fourth of the data distribution. They are more commonly referred to as **quartiles**

The quantiles plotted are quartiles. The three quartiles divide the distribution into four equal-size contiguous subsets. The second quartile corresponds to the median.

Measures of Spread: IQR

The distance between the first and third quartiles is a simple measure of spread that gives the range covered by the middle half of the data. This distance is called the **interquartile range (IQR)**.

$$IQR = Q_3 - Q_1$$

25%

Q_1 Q_2 Q_3

25th percentile   Median   75th percentile
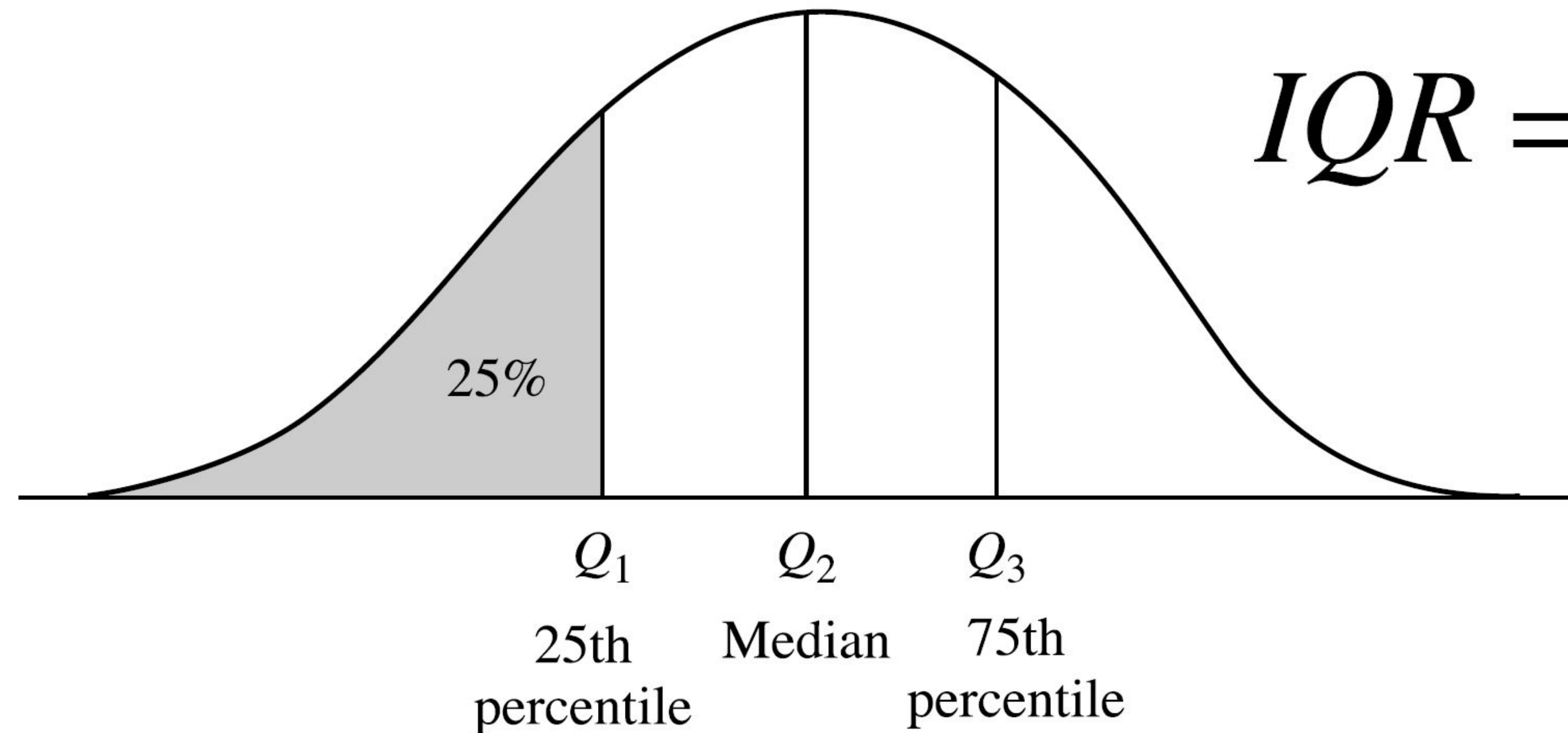
Measures of Spread: IQR

Example. Let's calculate the Q1, Q2, Q3 and IQR of the salary data, The data are already sorted in ascending order: 30, 36, 47, 50, 52, 52, 56, 60, 63, 70, 70, 110. Then:

- Q1 = **($47,000 + $50,000)/2 = $48,500**

- Q2 = **($52,000 + $56,000)/2 = $54,000**

- Q3 = **($63,000 + $70,000)/2 = $66,500**

- IQR = **$66,500 − $48,500 = $18,000**

$$IQR = Q_3 - Q_1$$

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Measures of Spread: Variance

- The (sample) variance, measures how much on average the sample values deviate from the mean:

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} |x_i - \bar{x}|^2$$

- Note: the term $|x_i - \bar{x}|$ measures the amount by which each x_i deviates from the mean $\bar{x}$. Squaring these deviations means that **variance is sensitive to extreme values (outliers)**.

- What does a variance of 1,008 mean? Or 0.0001?

Measures of Spread: Standard Deviation

- The (sample) standard deviation, denoted s, is the square root of the variance

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} |x_i - \bar{x}|^2}$$

Measuring the Dispersion of Data

- Quartiles, outliers and boxplots

  - Quartiles: Q1 (25th percentile), Q3 (75th percentile)

  - Inter-quartile range: IQR = Q3 –Q1

  - Five number summary: **min, Q1, median,Q3, max**

  - **Outlier: usually, a value higher/lower than 1.5 x IQR of Q3 or Q1**

- Variance and standard deviation

  - Standard deviation s (or σ)

    is the square root of variance

Population Variance	**Sample Variance**
$$\sigma^2 = \dfrac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$	$$s^2 = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$
σ^2 = population variance	s^2 = sample variance
x_i = value of ith element	x_i = value of ith element
μ = population mean	$\bar{x}$ = sample mean
N = population size	n = sample size

Box Plot
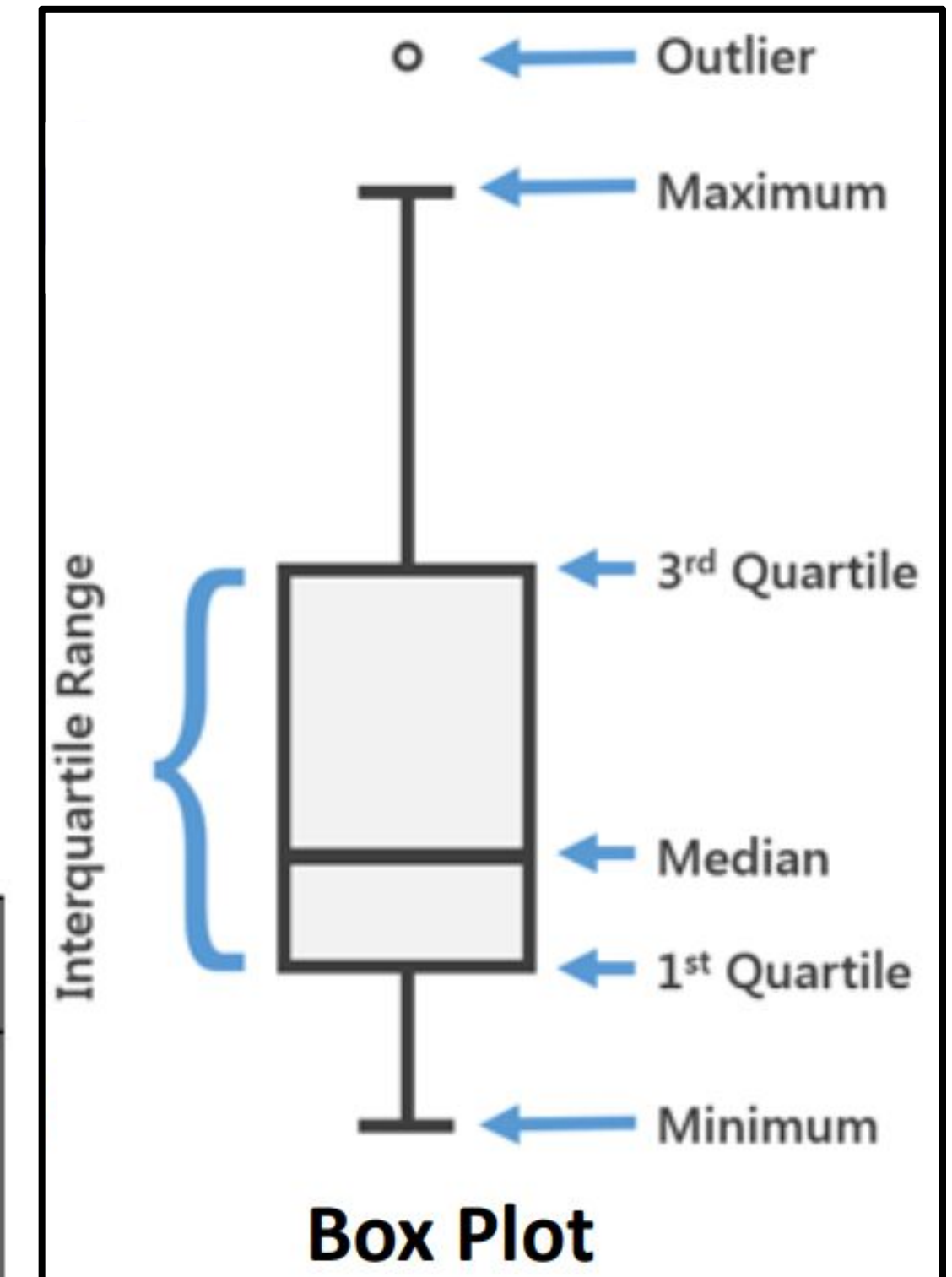
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