Foundations of Computing

Lecture 20

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April 3, 2025

Outline

Lecture 19 Review

- Polynomial Time Computation
- The Complexity Class $\mathcal{P}$

$$\mathcal{P} = \bigcup_k TIME(n^k)$$

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  - RELPRIME
  - Pretty much everything you studied in algorithms class
- But, some problems have resisted our efforts to find efficient algorithms
- Today we will study one important class of such problems

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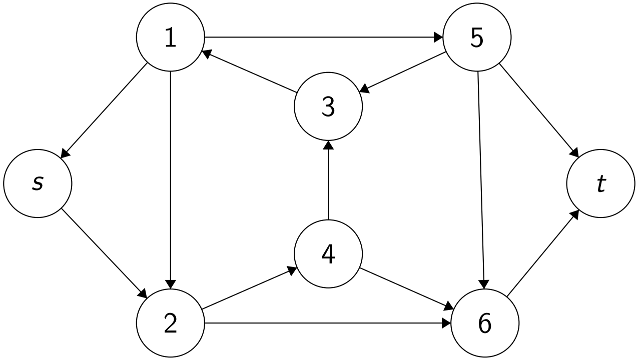

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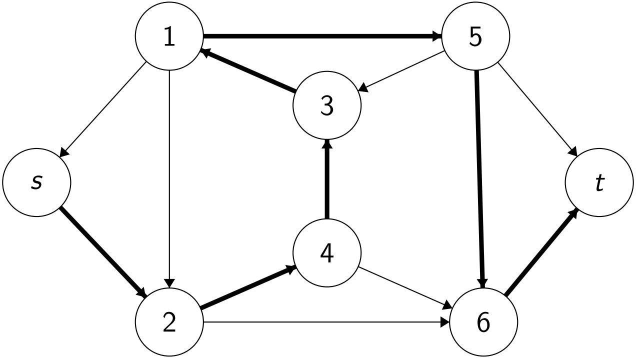

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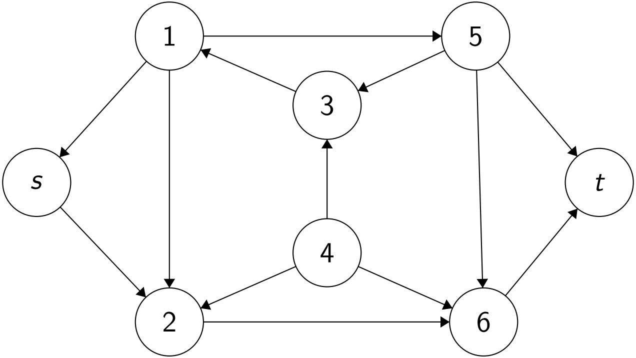
But, not every graph has a Hamiltonian Path.



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Polynomial Verifiability

However, given a path from s to t, can easily verify whether it is Hamiltonian in polynomial time.

Satisfiability

Boolean Formula

A Boolean formula is an expression inolving Boolean variables and logic operations AND ($\wedge$), OR ($\vee$), and NOT ($\neg$ or $\overline{x}$).

$$\phi = (\overline{x} \wedge y) \vee (x \wedge \overline{z})$$

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Polynomial Verifiability

However, given an assignment (i.e., values for all the variables), can easily verify whether ϕ is satisfied by this assignment in polynomial time.

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- String w is called a witness that $x \in L$

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- $\mathcal{NP}$ is the class of problems where you can verify a solution in poly-time
- Question: $\mathcal{P} \stackrel{?}{=} \mathcal{NP}$

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- ($\Leftarrow$) A verifier simulates the NTM by using the accepting branch as the witness

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We can define the class of languages decided by poly-time NTMs

Definition

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Problems in $\mathcal{NP}$ – Example 1

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$$CLIQUE = \{\langle G, k\rangle \mid G \text{ is an undirected graph with a } k\text{-clique}\}$$

Problems in $\mathcal{NP}$ – Example 2

Subset Sum

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$$SUBSET - SUM = \{\langle S, t\rangle \quad | \quad S = \{x_1, \ldots, x_k\} \text{ and for some } \{y_1, \ldots, y_l\} \subseteq \{x_1, \ldots, x_k\}, \sum y_i = t\}$$

The Million Dollar Question

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- Is it easier to verify a solution than to find that solution?
- This is the biggest open question in complexity theory

Let's Try to Answer It

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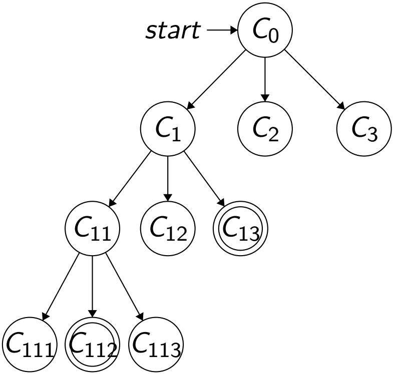
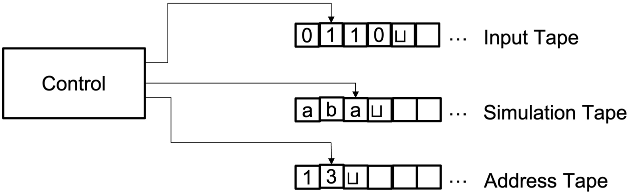


- Recall that an execution of a DTM is a sequence of configurations
- Execution of an NTM is a tree of configurations (branches correspond to non-deterministic choices)
- If any node in the tree is an accept node, the NTM accepts
- To simulate an NTM by a DTM, need to try all configurations in the tree to see if we find an accepting one

Simulating NTM on a 3-tape DTM



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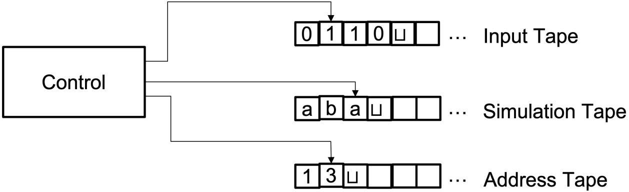

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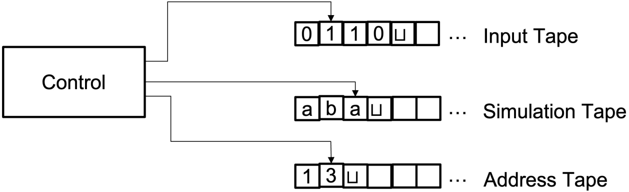

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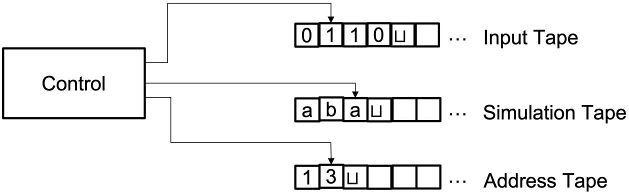

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3. Address tape – use to store which nondeterministic branch you are on

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- Resulting DTM runs in exponential time

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- We will show this using reductions – Yay!