Foundations of Computing

Lecture 1

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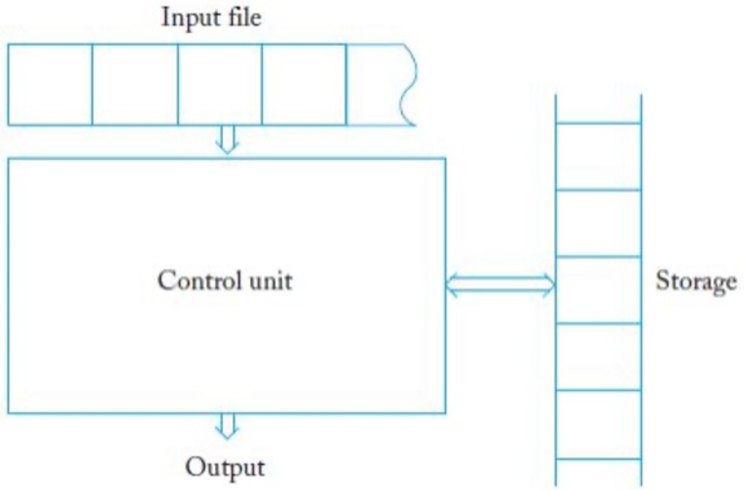
Modeling Computation



Outline

1. Strings, Languages, and Automata

2. Deterministic Finite Automata (DFA)

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- Operations on Strings
  - Concatenation: $vw = abaabaaa$
  - Reverse: $w^R = aaaba$
  - Repeat: $v^2 = abaaba$ and $v^0 = \lambda$

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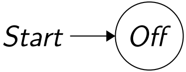
We will often be interested in languages recognized by a particular "computer".

A Simple Example: A Light Switch



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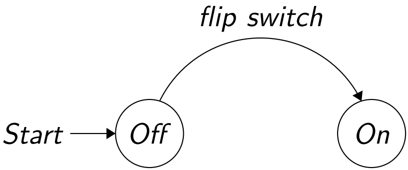

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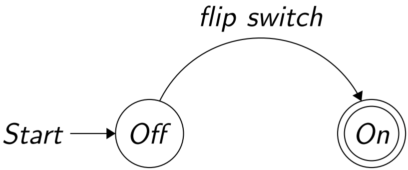

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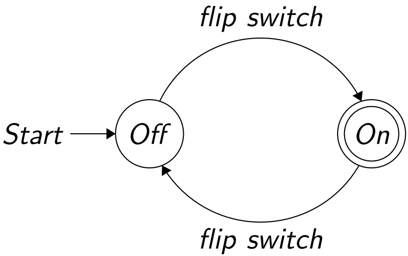

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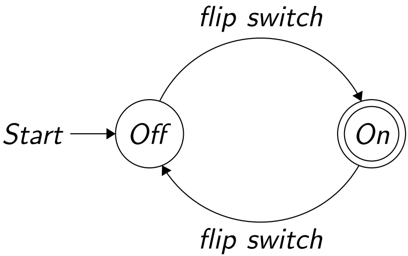

Viewing this as a language

L_{light} = {set of all flip sequences resulting in the light being on}
L_{light} = {1 flip, 3 flips, 5 flips, ...}

Finite Automata

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A note of input size

An automaton must be able to accept input of arbitrary length. The length of the input may be much larger than the number of states.

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- Pushdown automata
  - Add the simplest form of memory to a Finite State Machine
- Turing Machines
  - Add unrestricted memory to a Finite State Machine
  - Believed to be as powerful as any other model of computation
  - This will be the main model of computation used in computability and complexity theory

Outline

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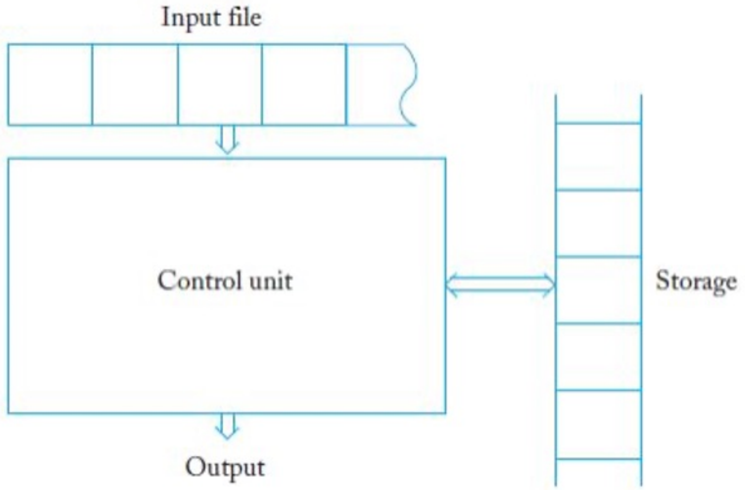

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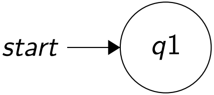

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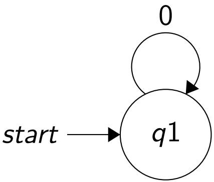

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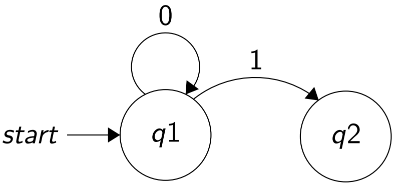

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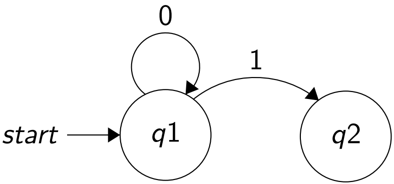

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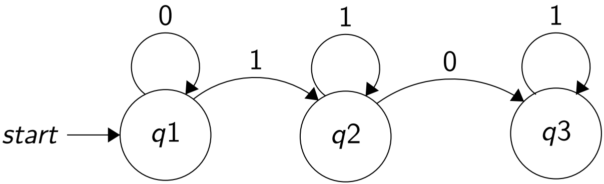

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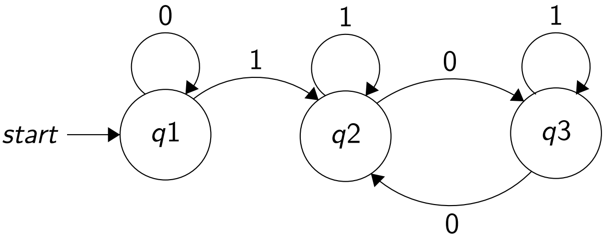

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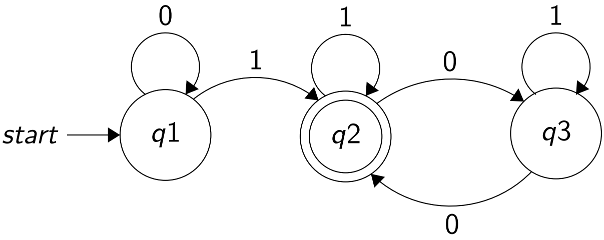

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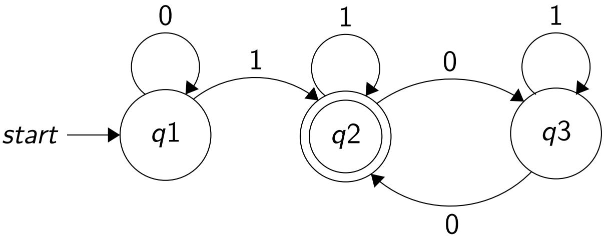


Computation on string $x = 1101$

1. Start in state $q1$
2. read 1, follow transition to $q2$
3. read 1, follow transition to $q2$
4. read 0, follow transition to $q3$
5. read 1, follow transition to $q2$
6. "accept" (output 1) because $q2$ is an accept state

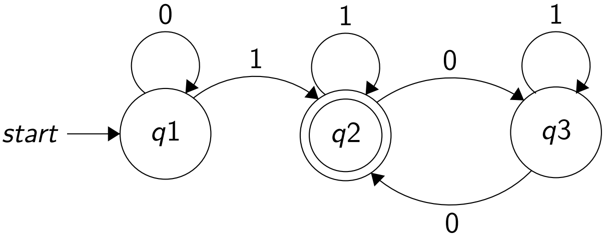
Finite Automaton – Formal Definition

Finite Automaton

A finite automaton is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where:

- Q is a finite set of states
- Σ is a finite input alphabet
- $\delta : Q \times \Sigma \to Q$ is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states

Example Automaton



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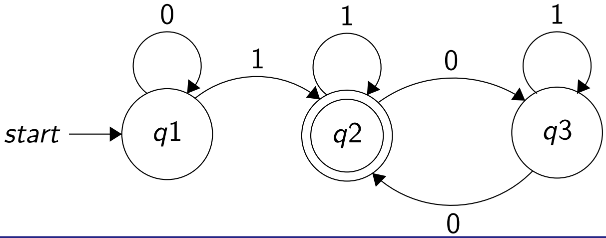

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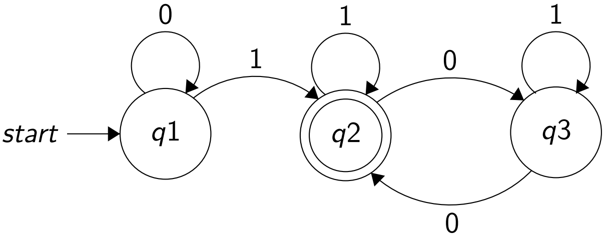

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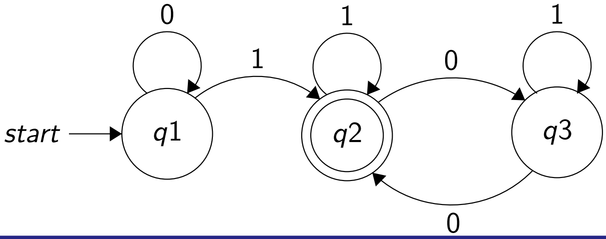

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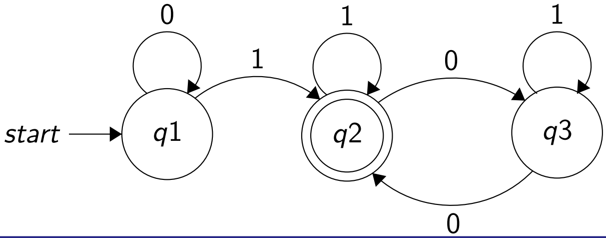

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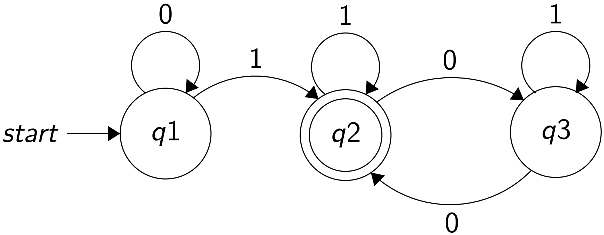

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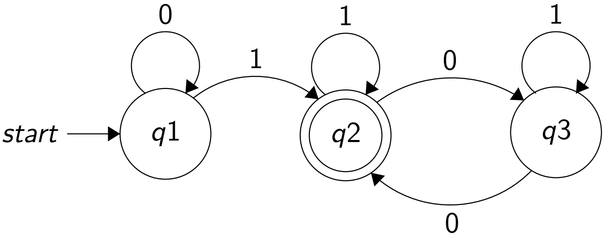

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- $F = \{q2\}$

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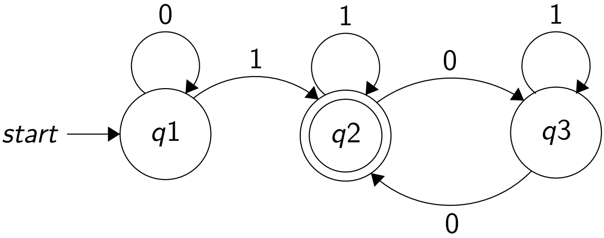

Accepting a string

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- We already saw that this M accepts 1101
- What other strings does M accept?

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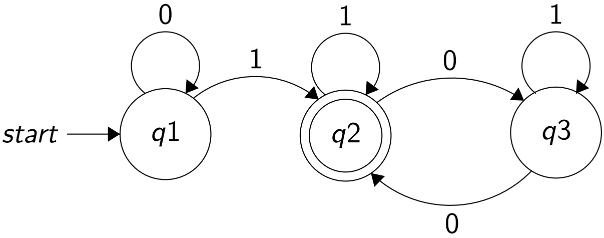

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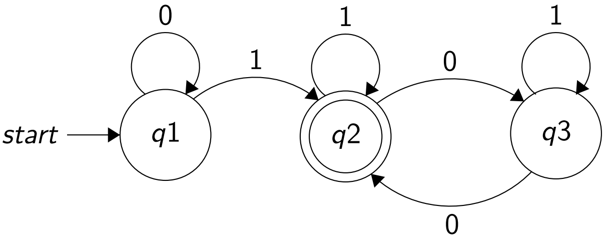
Accepting a language

- M accepts a language L if it accepts ALL strings in L and NO strings not in L
- Every M accepts exactly one language $L(M)$

What language does M accept?



$L(M)$:

- String must contain at least one 1
- After the first string of 1's, there must be an even number of 0's or no 0's

Why study this?

- Finite Automata are one of the most basic models of computation
- Turns out they capture some very useful functionalities
  - We will see next week, that finite automata correspond to regular expressions

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  - Review languages/strings/graphs
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- Your to do list:
  - Sign up for Gradescope
  - Sign up for Piazza
  - (optional) Download and install JFLAP (check tutorial on course webpage)