

**CS 3313**

**Foundations of Computing:**

**Lab 7**

# Turing Machine

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  - A tape input in  $\Gamma$

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  - $q'$  – a new state
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- Convention
  - Just as with NFAs and PDAs, if transition is undefined TM goes to reject state
  - If TM enters reject state it halts and rejects
  - If TM enters accept state, it halts and accepts

# Church-Turing Thesis

Anything that can be computed by an algorithm  
can be computed by a Turing Machine

# Three Steps to Build a TM

## 1. Write an algorithm

- Describe at a high-level the logic for recognizing L
- By Church-Turing thesis, this describes a TM

## 2. Write a Turing-Machine algorithm

- Specify what happens to the tape (i.e., scan the tape until the first 1 and write a 0)
- Don't need to specify the control states
- This is usually enough – unless asked for the formal specification

## 3. Write the full specification

- Includes full specification of transition function and states of control machine (remember that this is a DFA/NFA)

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  2. Check that the string before and after the midpoint are the same
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  - How do we find the midpoint?
  - How do we check the two strings are equal (we already saw how to do this in class)?



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- Composing TMs
  - Just like we compose algorithms
  - Design TM for step 1, and then step 2 and call one after the other

## Exercise 1: $L = \{ww \mid w \in \{a, b\}^*\}$

- Step 2: Write a Turing-Machine Algorithm
  - Describe what how to manipulate the tape

## Exercise 1: $L = \{ww \mid w \in \{a, b\}^*\}$

- Step 3: Write a full description
  - Give the transition diagram for the TM

## Exercise 2: $L = \{ a^i b^j c^i d^j \mid i, j > 0 \}$

- Built TM to decide L