Foundations of Computing

Lecture 1

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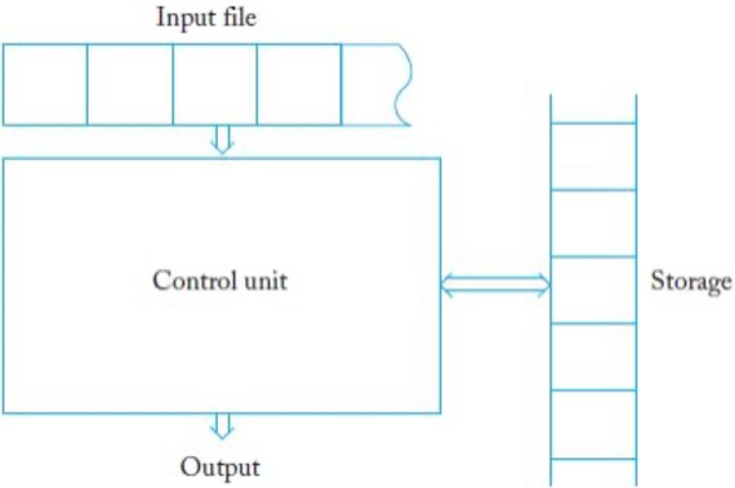
Modeling Computation



Outline

1. Strings, Languages, and Automata
2. Deterministic Finite Automata (DFA)

Strings

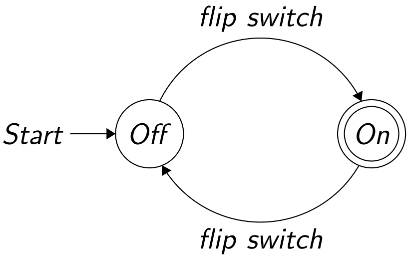
- Alphabet Σ: Set of symbols
  - Ex: $\Sigma = \{a, b\}$, $\Sigma = \{0, 1\}$
- String: finite sequence of symbols from Σ
  - ex: $v = aba$, $w = abaaa$
  - ex: $v = 001$, $w = 11001$
  - λ – empty string
  - Length of a string: $|v| = 3$ and $|\lambda| = 0$
- Operations on Strings
  - Concatenation: $vw = abaabaaa$
  - Reverse: $w^R = aaaba$
  - Repeat: $v^2 = abaaba$ and $v^0 = \lambda$

Languages

- Language L: Set of strings
  - Usually meant to capture strings that satisfy some property
- More formally:
  Σ^* = set of all strings formed by concatenating zero or more symbols from Σ
  - Ex: If $\Sigma = \{0, 1\}$ then $\Sigma* = \{$ all binary strings, including empty string$\}$
- Examples: $L_1 = \{ab, aa\}$ and $L2 = \{a^n b^n : n \geq 0\}$

We will define computation as deciding membership in a language.

A Simple Example: A Light Switch



Viewing this as a language

$L_{light} = \{$ set of all flip sequences resulting in the light being on$\}$
$L_{light} = \{$1 flip, 3 flips, 5 flips, ...$\}$

Automata

- An automaton is an abstract model of a computing device
- An automaton consists of:
  - An input mechanism
  - A control unit
  - Possibly, a storage mechanism
  - Possibly, an output mechanism
- Control unit can be in any number of internal states, as determined by a next-state or transition function
- There are a finite number of states

Automata we will study

- Finite Automata (Deterministic and Non-deterministic)
  - These model Finite State Machines with no memory
- Pushdown automata
  - Add the simplest form of memory to a Finite state machine
- Turing Machines
  - These model today's computers in terms of computational ability
  - This will be the main model of computation used in computability and complexity theory

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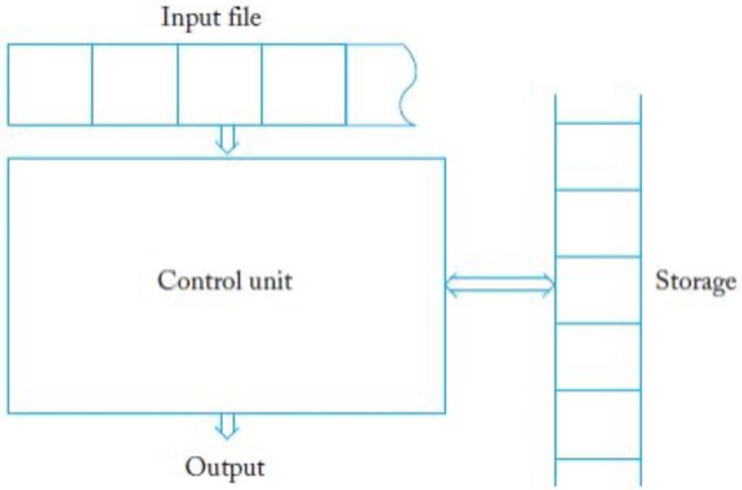
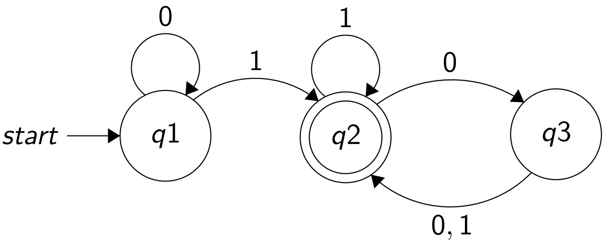
Modeling Computation



Finite Automata by Picture



Computation on string $x = 1101$

1. Start in state $q1$
2. read 1, follow transition to $q2$
3. read 1, follow transition to $q2$
4. read 0, follow transition to $q3$
5. read 1, follow transition to $q2$
6. "accept" (output 1) because $q2$ is an accept state

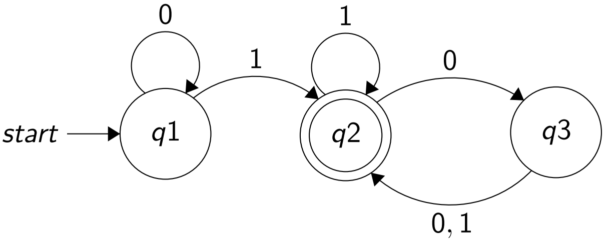
Finite Automaton – Formal Definition

Finite Automaton

A finite automaton is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where:

- Q is a finite set of states
- Σ is a finite input alphabet
- $\delta : Q \times \Sigma \to Q$ is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states

Example Automaton



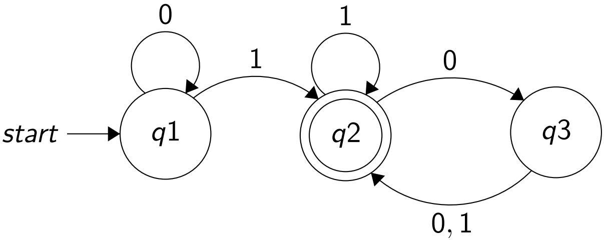
Defining this formally: $M = (Q, \Sigma, \delta, q1, F)$

- $Q = \{q1, q2, q3\}$
- $\Sigma = \{0, 1\}$
- $\delta =$

	0	1
q1	q1	q2
q2	q3	q2
q3	q2	q2

- $q1$ is the start state
- $F = \{q2\}$

Language accepted by M



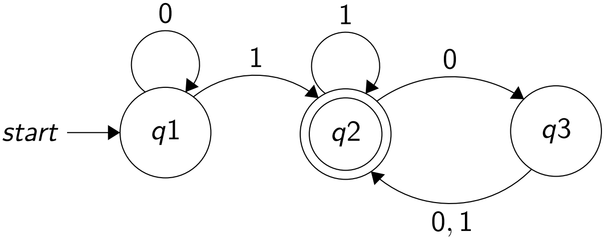
Accepting a string

- M accepts a string x (over Σ) if $M(x)$ stops in an accept state
- We already saw that this M accepts 1101
- What other strings does M accept?

Accepting a language

- M accepts a language L if it accepts ALL strings in L and NO strings not in L
- Every M accepts exactly one language $L(M)$

What language does M accept?



$L(M)$:

- String must contain at least one 1
- After the last 1, there must be an even number of 0's

Why study this?

- Finite Automata are one of the most basic models of computation
- Turns out they capture some very useful functionalities
  - We will see next week, that finite automata correspond to regular expressions

Next...

- Labs this week:
  - Review of proof techniques
  - Review languages/strings/graphs
  - In-class exercises
- Thursday lecture:
  - More about finite automata and their properties
- Your to do list:
  - Sign up for Piazza
  - (optional) Download and install JFLAP (check tutorial on course webpage)