CS 3313
Foundations of Computing:

Lab 3: Regular Expessions Review and the Pumping Lemma

Outline

Proving Languages not Regular

- NFA/DFA Pumping Lemma
- Using Closure Properties

How to prove a language is not regular…
The Pumping Lemma for Regular Languages

For every regular language L

There is an integer p, such that (note; you cannot fix p)

For every string w in L of length $\geq p$ *(you can choose w)*

We can write $w = xyz$ such that:

1. $|xy| \leq p$ *(this lets you focus on pumping within first p symbols)*
2. $|y| > 0$ *(y cannot be empty)*
3. For all $i \geq 0$, xy^iz is in L. *(to get contradiction find one value of i where pumped string is not in L)*

Pumping Lemma as an Adversarial Game

1. Player 1 (me) picks language L to be proved nonregular
    - ❖ Prove $L = \{ss^R \mid s \in \{a,b\}^*\}$ is not regular.

2. Player 2 picks p, but doesn't tell me what p is, player 1 must win for all values of p
3. Player 1 picks a string w, which may depend on p, and must be of length at least p

    ➢ Assume L is regular. Let w $= a^p b^1 b^1 a^p \in L$, i.e., s $= a^p b^1$; as well as $|s| \geq p$.

Note: Words in purple are the example wordings we use in this type of proofs.

Pumping Lemma as an Adversarial Game

4. Player 2 divides w into xyz s.t. |y|>0 and |xy|<=p
   - He does not tell player 1 this division, player 1's strategy must work for all choices

➢ Then by the Pumping Lemma, w can be divided into three parts $w = xyz$, such that $x = a^{\alpha}, y = a^{\beta}, z = a^{p-\alpha-\beta}b^1b^1a^p$, where $\beta \geq 1, (\alpha + \beta) \leq p$.

5. Player 1 "wins" by picking an integer k>=0, which may be a function of p,x,y, and z, such that $xy^kz \notin L$

➢ Now, consider $k = 0$. Then the string after the pumping becomes $w' = xy^0z = xz = a^{p-\beta}b^1b^1a^p$. Note that since $\beta \geq 1$, there's no way for w' to be in the form of a string followed by its reverse; hence $w' \notin L$. *Contradiction*. $\Longrightarrow L$ not regular.

Pumping Lemma Remarks

- How do we know what string we need to choose?
  - **Trial and Error** and some eureka
  - $L = \{ss^R \mid s \in \{a, b\}^*\}$, if we'd chosen w $= a^n a^n$, then for $w' = a^{n-\beta} a^n$, then adversary can just choose $\beta \geq 1$ to be of even length, such that $w' = s' s'^R$. So, choosing such a w has no use for us.
  - $L = \{a^n b^m \mid m \neq n,\ n, m \geq 1\}$, if we choose $w = a^p b^{p+1}$ or $w = a^p b^{2p}$, can we find some integer k such for $w' = xy^k z$, number of a's equals to number of b's. [We saw this in class]

Exercise 1: Pumping Lemma

Exercise: Prove that $L = \{a^m b^n \mid m < n\}$ is not regular.

1. What string w should we choose?
2. What does the pumping lemma tell us?
3. How to complete the proof?

Exercise 2: Pumping Lemma

Exercise: Prove that $L = \{0^m 1^n \mid m \neq 2n\}$ is not regular.

1. What string w should we choose?
2. What does the pumping lemma tell us?
3. How to complete the proof?

4. Remember, it’s okay if you don’t pick w correctly on the first try!

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Closure Properties of Regular Languages

We have proven that regular languages are closed under a number of operation:

1. $\overline{L}$ is regular if L is
2. $L_1 \cup L_2$ is regular if L_1 , L_2 are
3. $L_1 \cap L_2$ is regular if L_1 , L_2 are
4. $L_1 \| L_2$ is regular if L_1, L_2 are
5. L^R is regular if L is
6. L* is regular if L is
7. NOPREFIX, NOEXTEND
8. There are many more

Proving Non-Regularity Using Closure

To prove L' is not regular:

1. Assume L' is regular
2. Show that if L' is regular than by closure, we get that some language L is regular
3. If we know that L is not regular, this is a contradiction.

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<u>Example</u>:

Prove that $L = \{0^n 1^n \cup 1^n 0^n\}$ is not regular

1. Assume L is regular
2. Observe that $L' = \{0^* 1^*\}$ is regular
3. $\{0^n 1^n\} = L \cap L'$, so if L is regular we have a contradiction

Exercise 3: Closure Properties

Exercise: Prove that $L = \{0^n 1^{n-3}\}$ is not regular.

1. What do we assume?
2. What closure property should we use here?
3. How do we get to contradiction?