Foundations of Computing

Lecture 11

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Outline

Lecture 10 Review

- CFG == PDA
  - Construct PDA from CFG
  - Construct CFG from PDA
- CFG Pumping Lemma

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Today

- Midterm review

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The CFG Pumping Lemma

Theorem

If L is a CFL, then there exists a pumping length p s.t. for any $s \in L$, with $|s| \geq p$, s can be divided into 5 pieces $s = uvxyz$ satisfying:

1. For each $i \geq 0$, $uv^i xy^i z \in L$
2. $|vy| > 0$
3. $|vxy| \leq p$

- Last week we used this to prove that

$$L = \{a^n b^n c^n \mid n \geq 0\}$$

is not context-free

Example 2

Consider $L = \{ww \mid w \in \{0,1\}^*\}$, prove L is not CFL

$L = \{ ww^R \}$ is CFL

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Proof:

1. Assume L is CFL, and let p be the pumping length
2. Try 1: Choose $s = 0^p 0^p \in L$

$uvxyz = s$

$v = 0^?$ $\quad y = 0^?$

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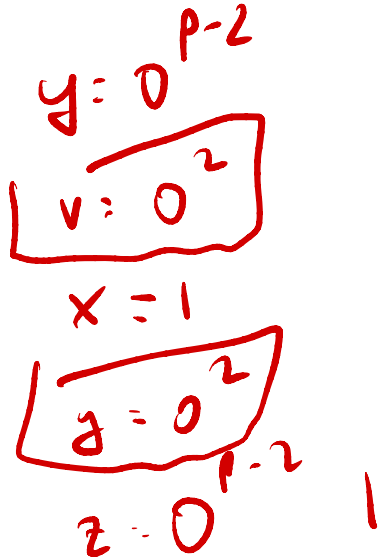

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6. Contradiction – Hence L is not CFL

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   - Know what it means for a DFA to accept a string
   - Know what it means for DFA to accept/recognize a language

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   - NFA to DFA using the finger method

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   - Understand why it is true (state of NFA must repeat)

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   - Understand how to use it.

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   - Be able to build an RE for a language
   - RE to NFA
   - NFA to RE
6. Regular Language Pumping Lemma
   - Remember statement as sequence of quantifiers
   - Understand why it is true (state of NFA must repeat)
   - Understand how to use it.
   - Also know how to prove languages not regular using closure properties
     - IMPORTANT: Make sure you understand how to make this into a proof by contradiction!

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   - Be able to build a PDA given a language description
8. Context-free Grammars (CFG)
   - Remember what this means

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   - Be able to build a PDA given a language description
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   - Be able to construct one from language description

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   - Be able to construct one from language description
   - Remember what a derivation is and what a parse tree is
   - PDA == CFG (at a high level)
9. CFL pumping lemma
   - There will not be any questions on the CFL pumping lemma on the exam
   - But, there will be on the next homework

Exam Format

- 7 questions – most have multiple parts
- Covers most of the material outlined above
- 2 questions requiring proofs, the rest are more constructive
- Some yes/no questions

Don't Forget

- Exam is in class on Thursday 11:10-12:25, don't be late!
- You can bring two 8.5×11 piece of paper
- 5 point bonus for participating in ACM hackathon this weekend

Any Questions?

RE → NFA

$(01)^* \cup (1010)$

$\{0,1,6\}, 0 \to \{7\}$

$\{0,1,6\}, 1 \to \{2\}$