Cryptography

Lecture 16

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October 23, 2024

Outline

Lecture 15 Review

- Review of exam and research project
- Blockciphers
- Confusion-Diffusion paradigm and the avalanche effect
- Substitution-Permutation Paradigm
- AES

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AES / Rijndael History

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- Now, standardized and very widely used
- Hardware support (e.g., AES-NI) makes this very fast: 10^8 per second
- This is the right block-cipher to use for most applications

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State: 4×4 array of Bytes

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{pmatrix}$$

AES

1. Add Round Key: View $k_i = k_{11}||k_{12}||\cdots||k_{44}$ with $|k_{ij}| = 1$ Byte

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2. SubBytes: Uses a single 8-bit S-box

$$\begin{pmatrix} S(B_{11}) & S(B_{12}) & S(B_{13}) & S(B_{14}) \\ S(B_{21}) & S(B_{22}) & S(B_{23}) & S(B_{24}) \\ S(B_{31}) & S(B_{32}) & S(B_{33}) & S(B_{34}) \\ S(B_{41}) & S(B_{42}) & S(B_{43}) & S(B_{44}) \end{pmatrix}$$

3. Shift Rows: Shift each row to the left by varying amounts

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- Even this very structured permutation seems enough

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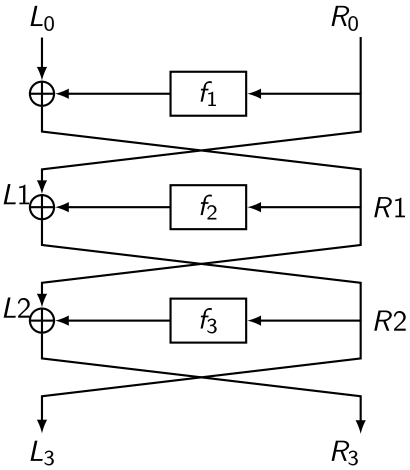
The Challenge

How do we use such f_i to build an invertible function?

Feistel Networks – Making f Invertible



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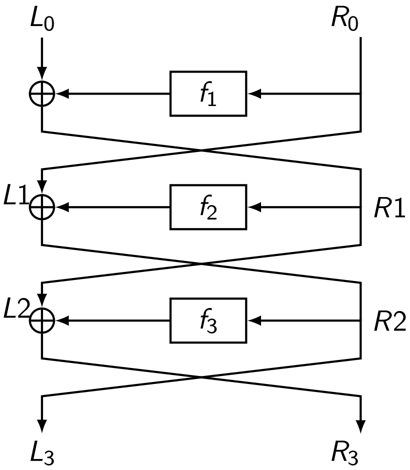

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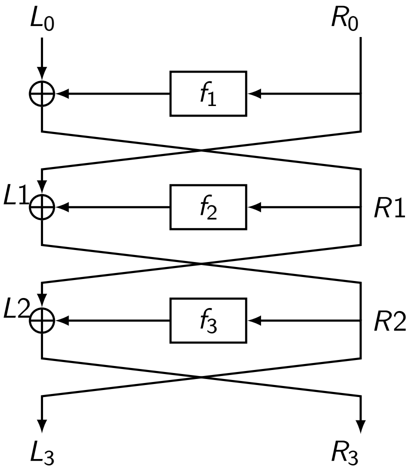

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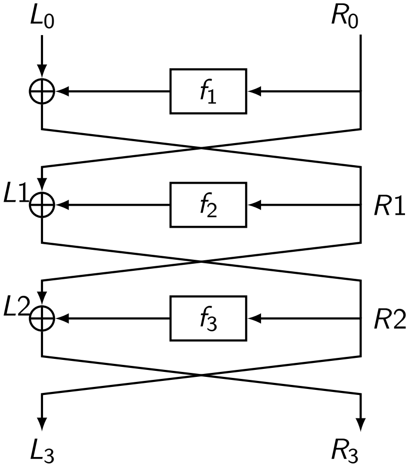

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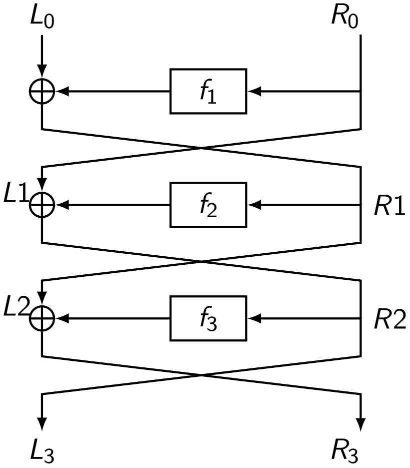

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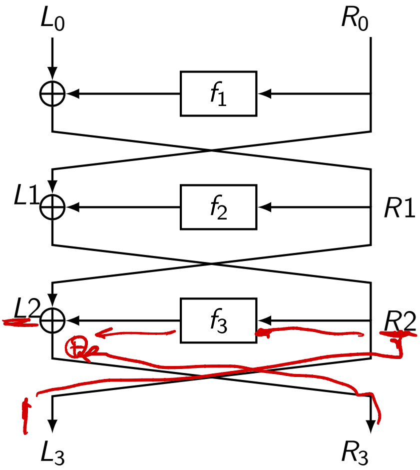
$R_2 = L_3$



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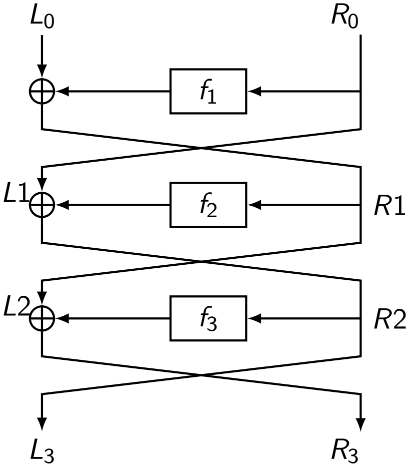

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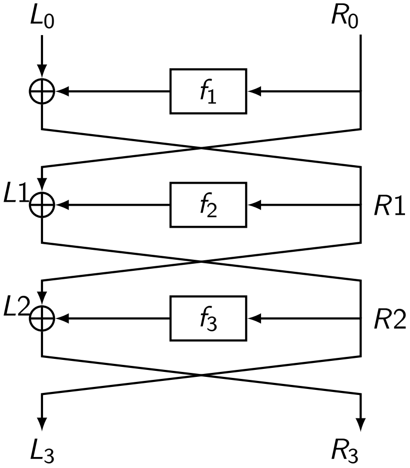

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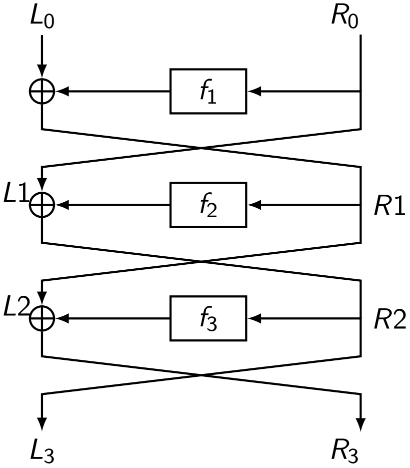

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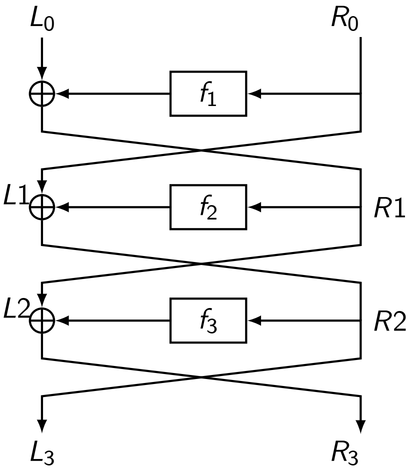

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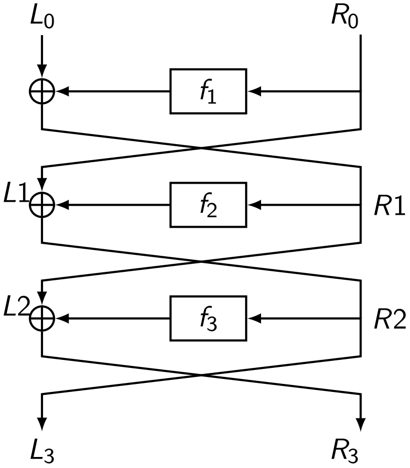

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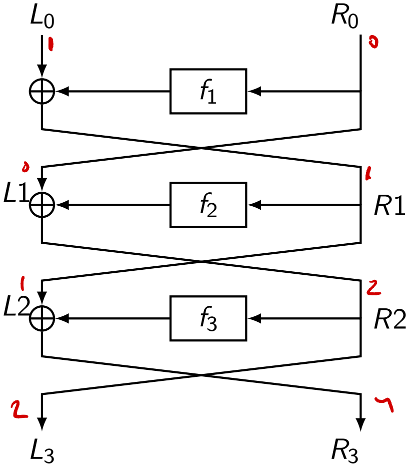
- Usually f_i derived from global public f and round key k_i



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- We will talk about how to extend the key later

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- Which bits chosen in each round is public

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Mixing Permutation:

- Output from any S-Box, affects input to 6 S-Boxes in next round

DES Avalanche Effect

$$
\begin{array}{lccccccccc}
 & (L_0, R_0) & & (L_1, R_1) & & (L_2, R_2) & & (L_3, R_3) & \\
 & (L'_0, R'_0) & \rightarrow & (L'_1, R'_1) & \rightarrow & (L'_2, R'_2) & \rightarrow & (L'_3, R'_3) & \cdots \\
\#\ \text{Bits different} & 1, 0 & & 0, 1 & & 1, 2 & & 2, 4 &
\end{array}
$$

- After 7 rounds all 32 bits in R affected
- After 8 rounds all 32 bits in L affected

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Question

How can we increase the key length of a block cipher?

$DES_K(m)$

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1. For each $k_1 \in \{0,1\}^n$, compute $z = F_{k_1}(x)$, store (z, k_1) in table L

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k_1	$F_{k_1}(x)$
$\vdots$	$\vdots$

$F_{k_1^*}(x)$

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- This attack takes $O(2^n)$ time, so we are no better off than we were with just single encryption.

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Outline

Davies-Meyer

Collision-resistant hash function from an (ideal) block-cipher:

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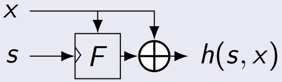

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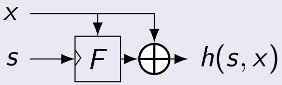

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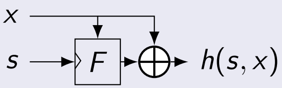

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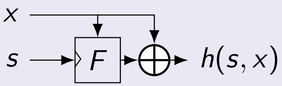

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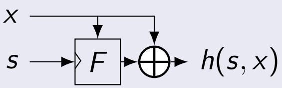

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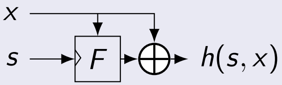

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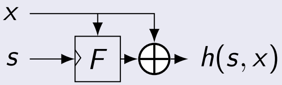

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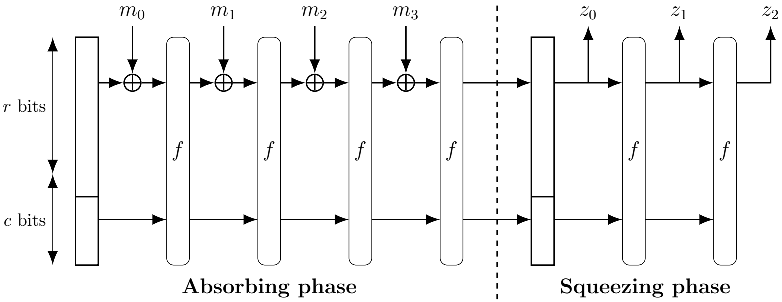
- But, there are block-ciphers for which this is believed to be secure

Sponge Function (Keccak/SHA-3)



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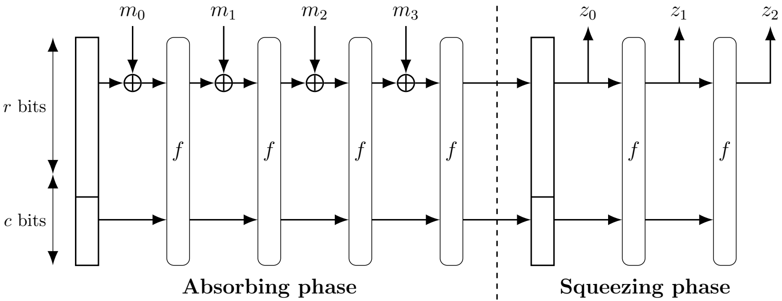

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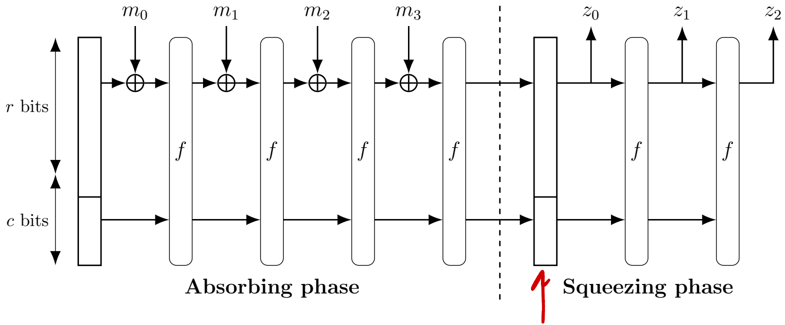

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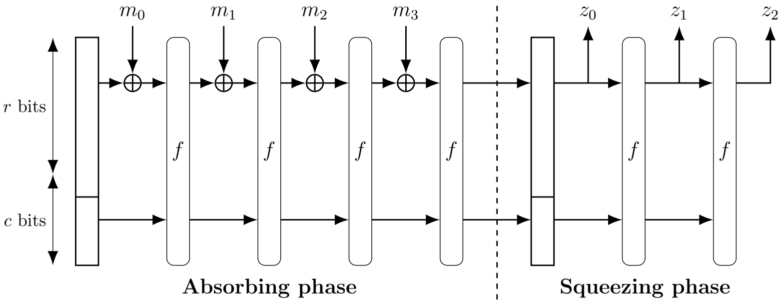

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- Looks like a random function if f is random permutation