Cryptography

Lecture 10

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September 30, 2024

Outline

1. Lecture ~~8~~ 9 Review
2. Chosen-ciphertext Attack (CCA) Security (Chapter 3.7)
3. Importance of CCA Security (Chapter 3.7)

Lecture 8 Review

- Proof of CPA-security for PRF+OTP
- Modes of operations

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- Want undecrypted messages to remain secure

Is PRF+OTP CCA Secure?

PRF+OTP Encryption

- $\mathsf{Gen}(1^n)$: $k \leftarrow \{0,1\}^n$
- $\mathsf{Enc}(k,m)$: Choose $r \leftarrow \{0,1\}^n$, output $c = (r, F_k(r) \oplus m)$
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Takeaway

PRF+OTP is not CCA-Secure

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Definition: An encryption scheme $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ with message space $\mathcal{M}$ is CCA-secure if for all PPT $\mathcal{A}$ it holds that

$$\Pr[\mathsf{PrivK}^{cca}_{\mathcal{A},\Pi}(n) = 1] \leq 1/2 + \mathsf{negl}(n)$$

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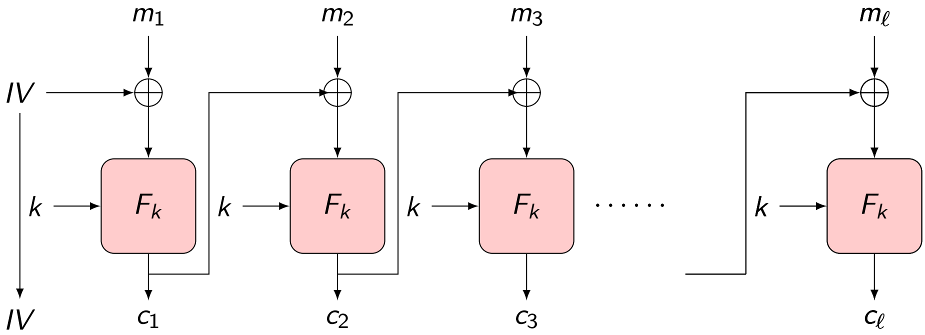

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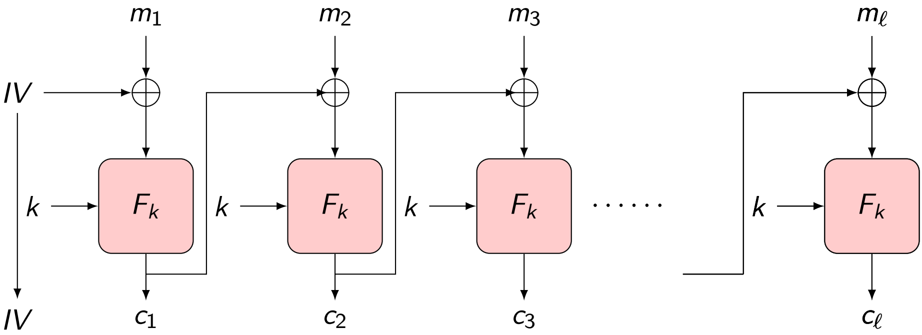

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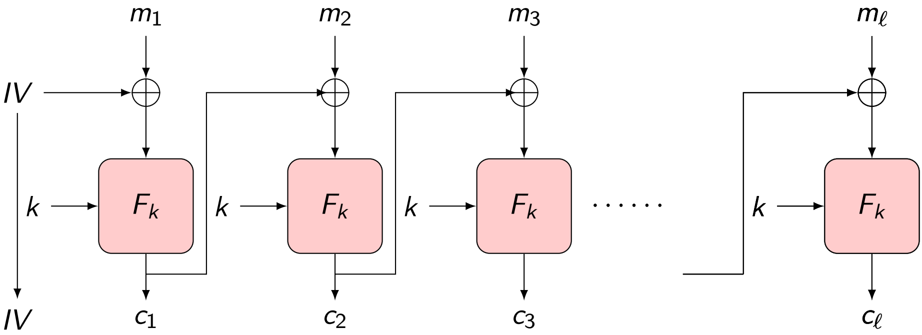

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m_1 m_2

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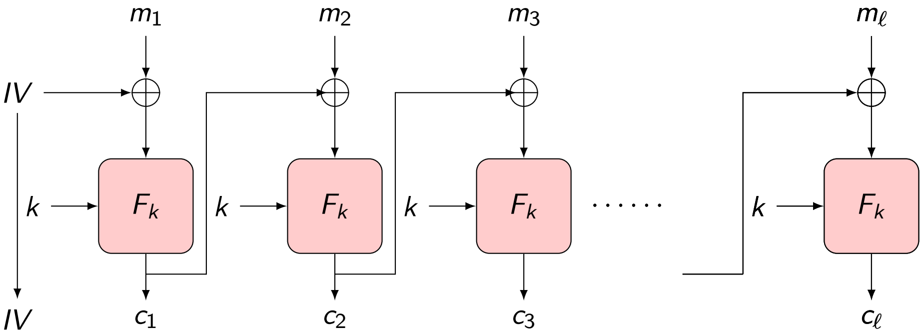

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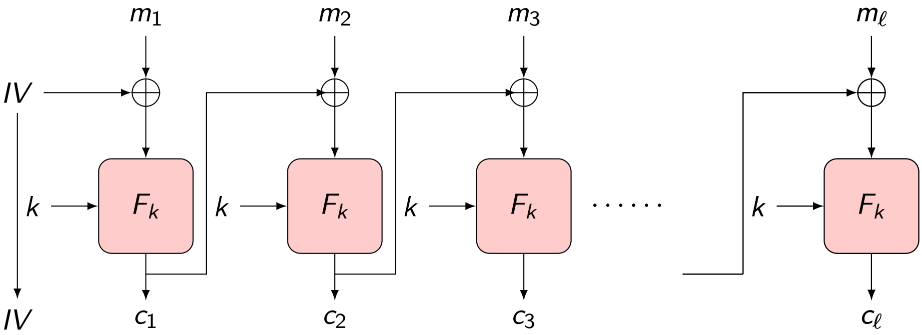

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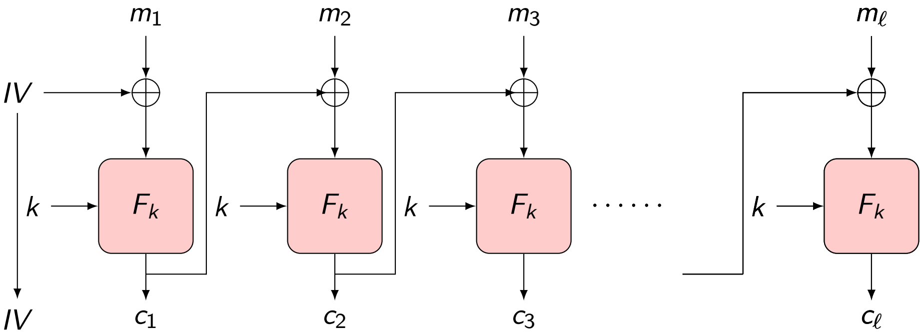

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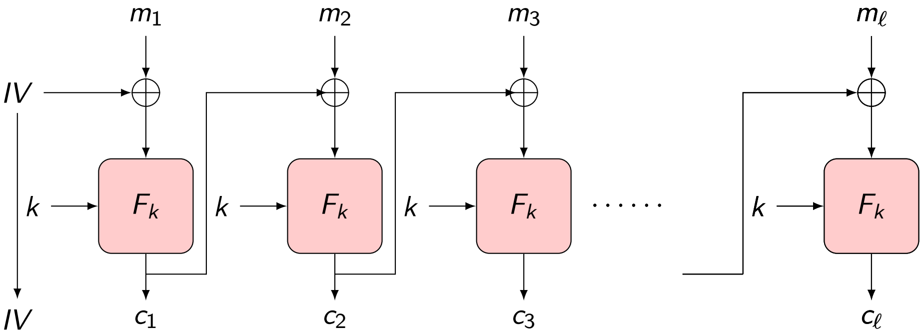

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- Decryption can then remove padding and return m
  - If padding incorrect, return "bad padding" error

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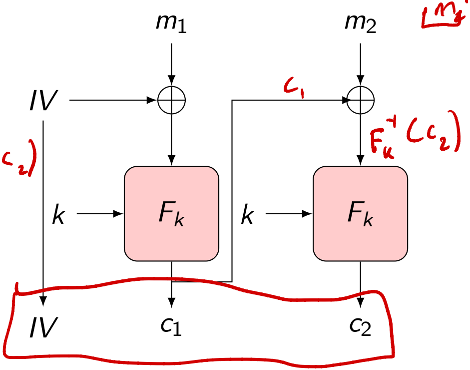
$m_2 \oplus \delta = (c_1 \oplus \delta) \oplus F_k^{-1}(c_2)$



- Consider encryption of a 2-block message m

Quiz

You will now develop an attack on this mode of operations.

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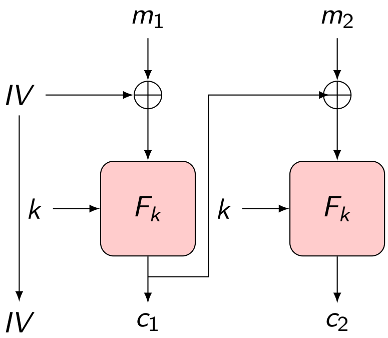

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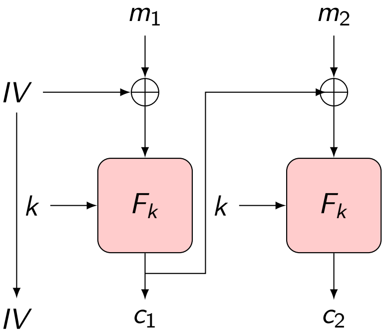

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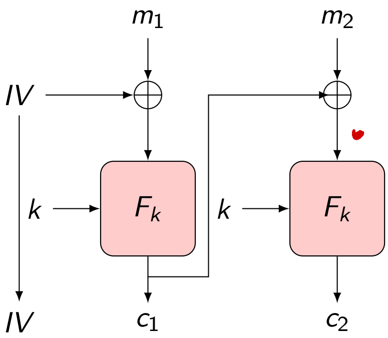

- Consider encryption of a 2-block message m
- Note that $m_2 = F_k^{-1}(c_2) \oplus c_1$
  - If we change c_1 to $c_1' = c_1 \oplus \delta$ without changing c_2 then, we change m_2 to $m_2' = m_2 \oplus \delta$

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Observation: We know that m_2 ends in $(0xb)$ repeated b times

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- Change Bytes $2, \ldots, L$ until first time we get error, this is first Byte of padding

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Step 2: Using knowledge of $b = |pad|$, decrypt m

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- Change c_1 (and thus also m_2) by δ_i defined as

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- Can mount similar attack to decrypt m_1

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Warning

Be very careful with error messages in crypto constructions