Cryptography

Lecture 10

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September 30, 2024

Outline

Lecture 8 Review

- Proof of CPA-security for PRF+OTP
- Modes of operations

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Chosen-ciphertext Attack

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- Want undecrypted messages to remain secure

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PRF+OTP Encryption

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Takeaway

PRF+OTP is not CCA-Secure

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Definition: An encryption scheme $\Pi = (\text{Gen}, \text{Enc}, \text{Dec})$ with message space $\mathcal{M}$ is CCA-secure if for all PPT $\mathcal{A}$ it holds that

$$\Pr[\text{PrivK}^{cca}_{\mathcal{A},\Pi}(n) = 1] \leq 1/2 + \text{negl}(n)$$

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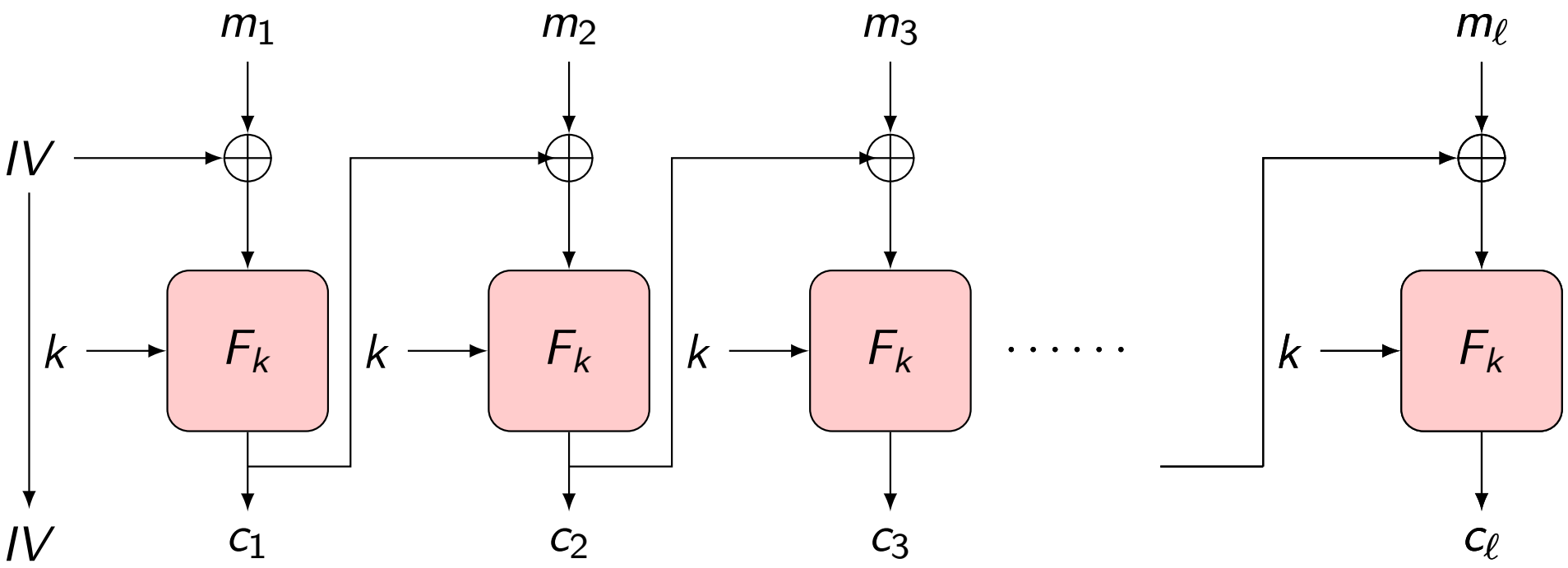
Padding Oracle Attack on CBC Mode



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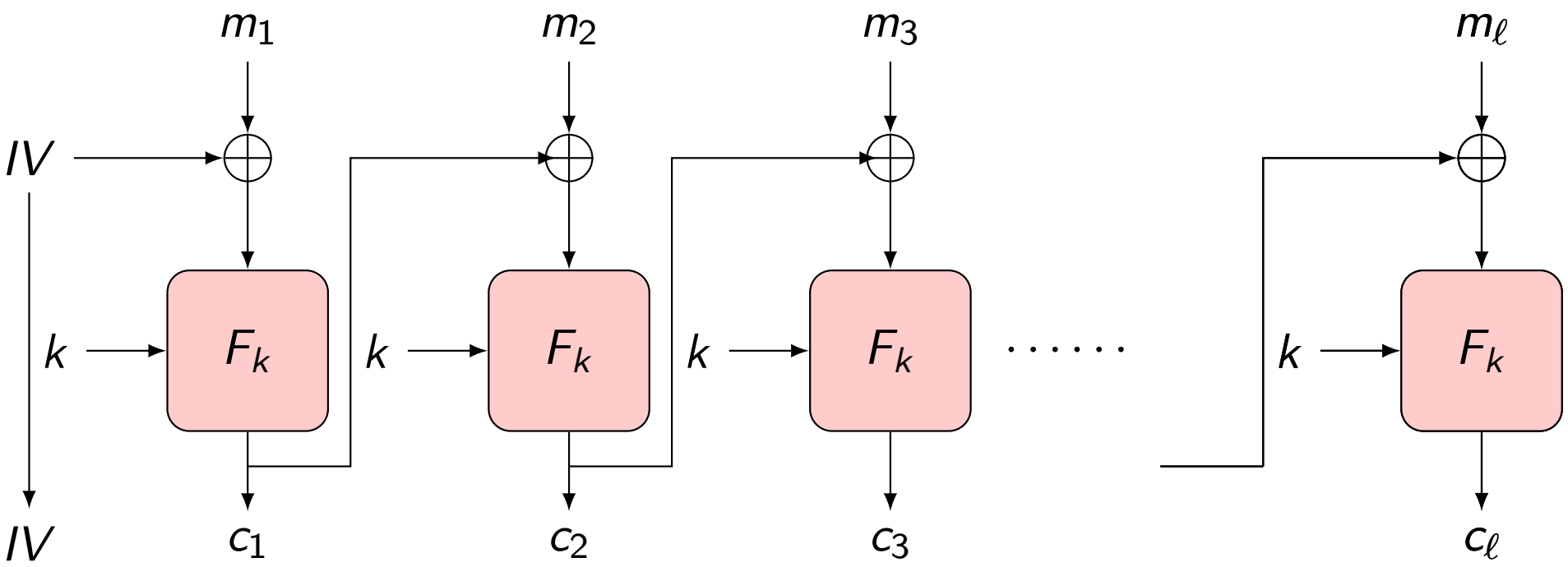

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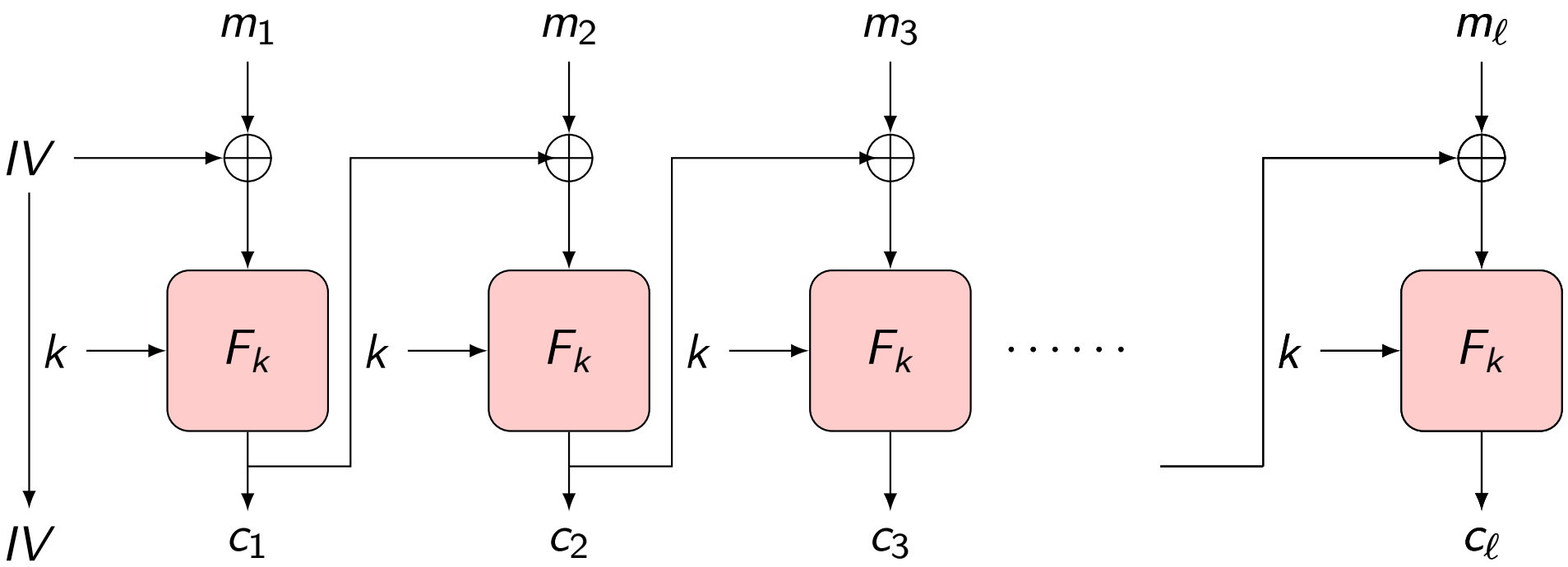

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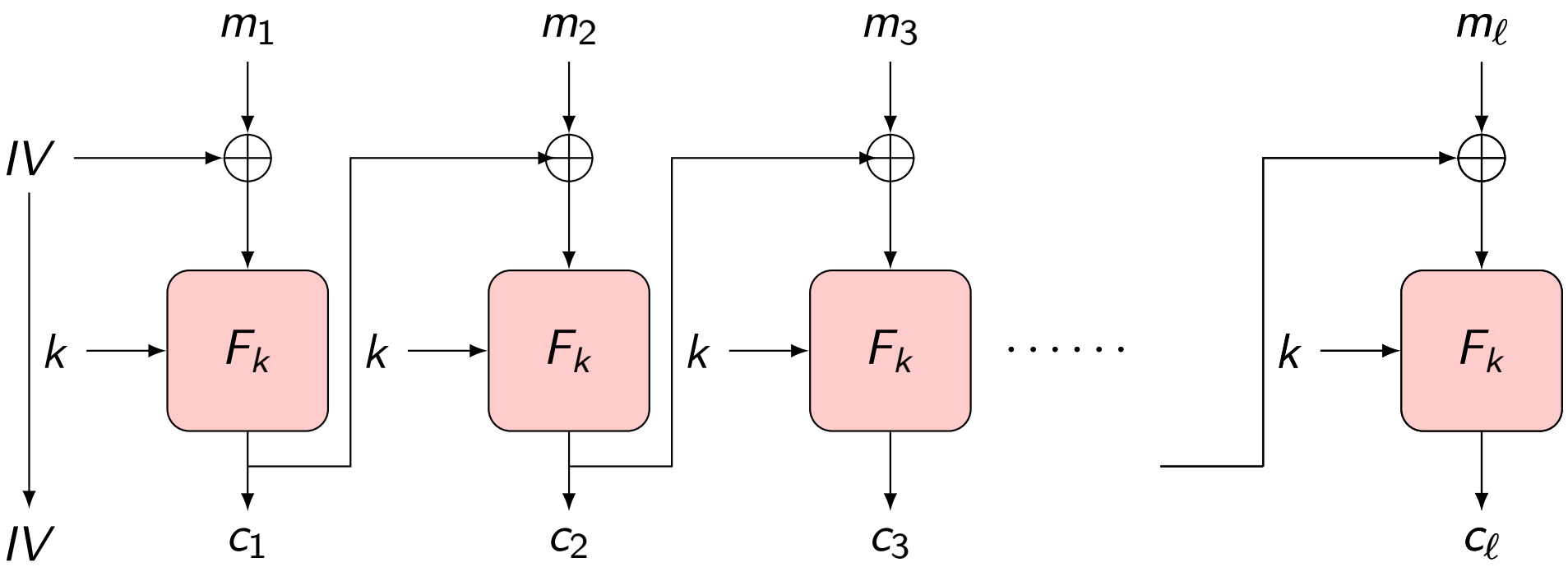

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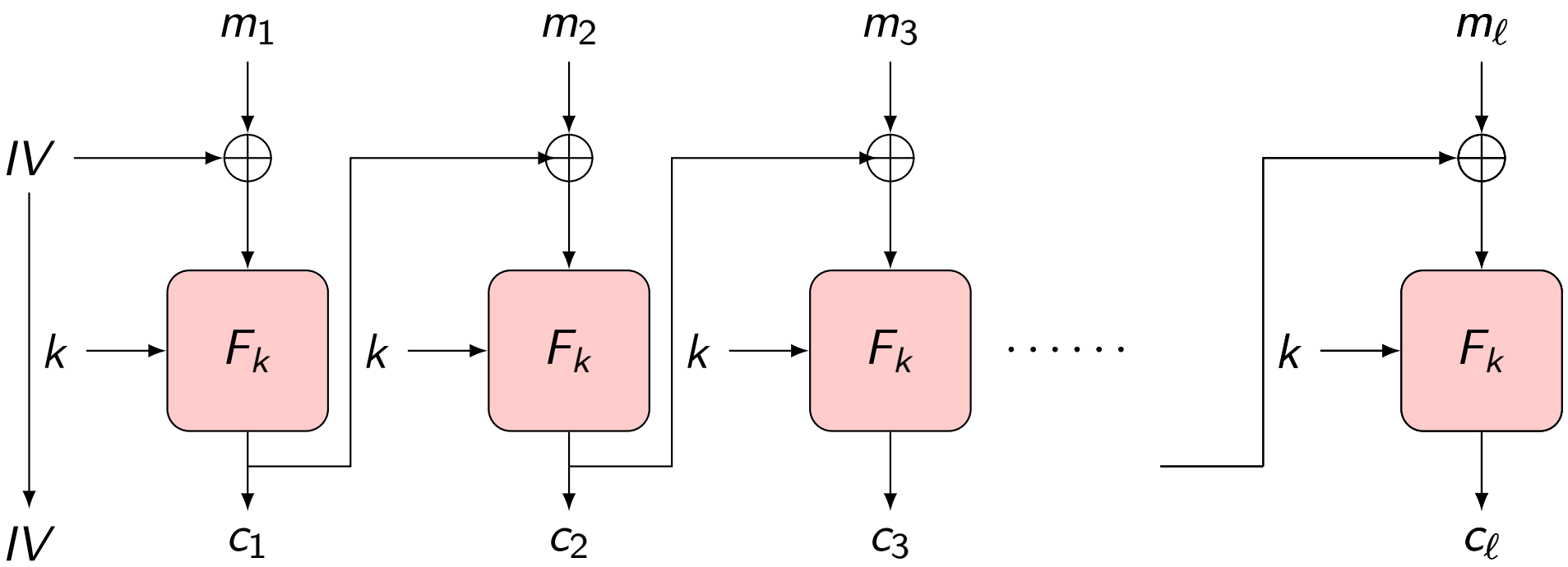

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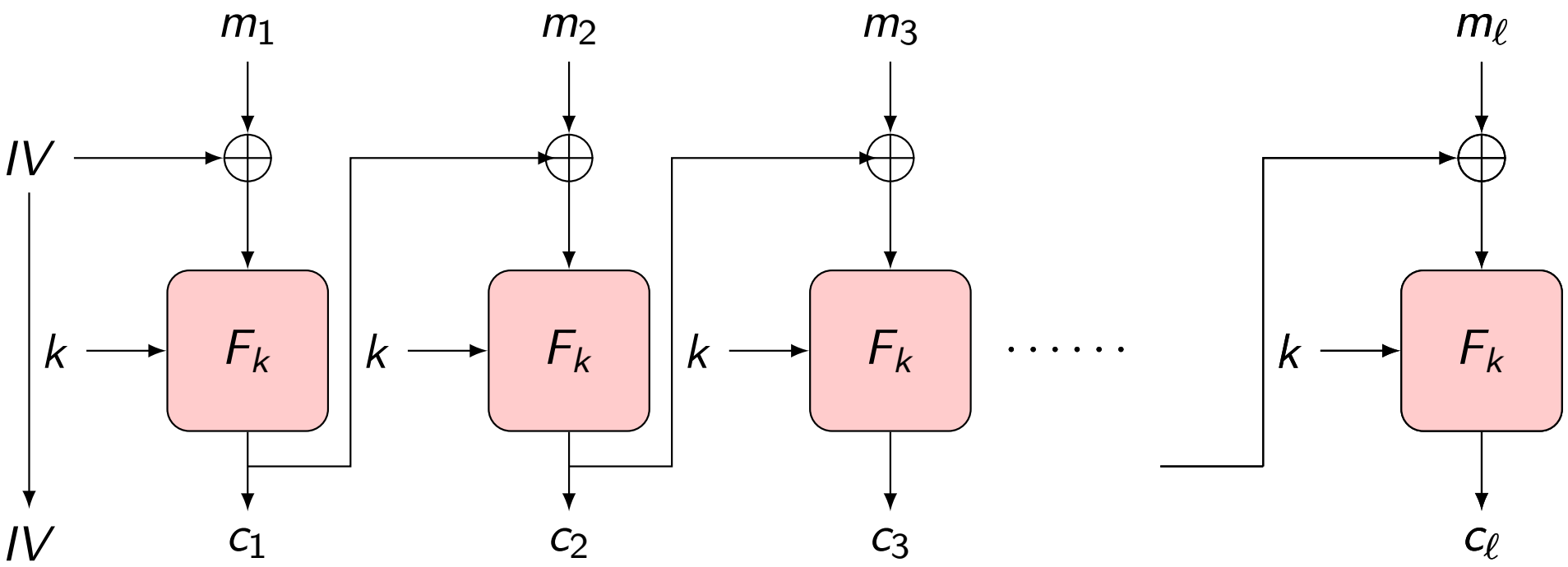

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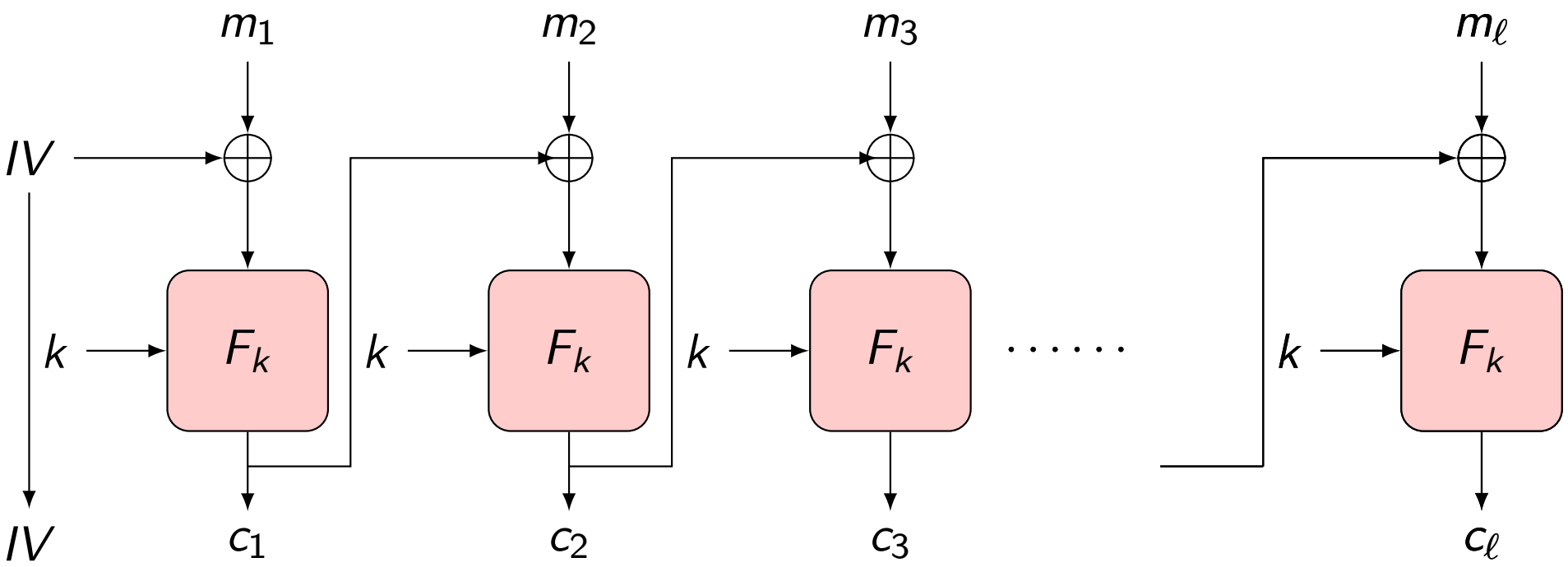

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- Decryption can then remove padding and return m
  - If padding incorrect, return "bad padding" error

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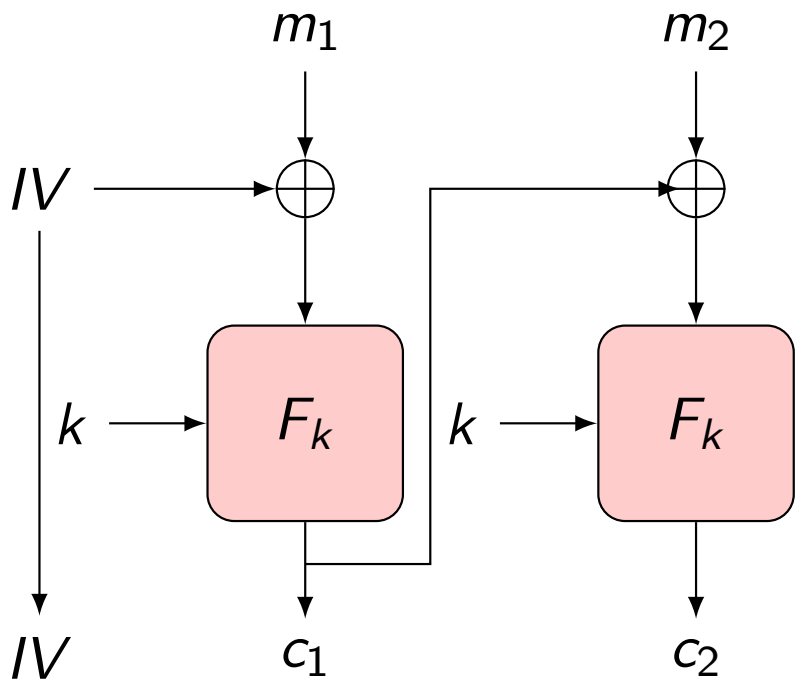

- Consider encryption of a 2-block message m

Quiz

You will now develop an attack on this mode of operations.