# Cryptography

## Lecture 25

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December 2, 2024

# Outline


1. Lecture 24 Review
2. Public-Key Crypto Protocol Review
3. Secure Multi-Party Computation (MPC)
4. A Simple MPC Protocol
5. MPC Based on Secret Sharing
6. Defining MPC Security

# Lecture 24 Review

- Signatures from private-key primitives
- One-time signatures
- Going from one-time to standard signatures

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## El Gamal Encryption: $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ with $\mathcal{M} = G$

- $\mathsf{Gen}(1^n)$: $(G, q, g) \leftarrow \mathsf{Gen}(1^n)$, $x \leftarrow \mathbb{Z}_q$, $h = g^x$, $pk = (G, q, g, h)$ and $sk = x$
- $\mathsf{Enc}_{pk}(m)$: $y \leftarrow \mathbb{Z}_q$, compute $c = (g^y, h^y \cdot m)$
- $\mathsf{Dec}_{sk}(c)$: Compute $\hat{m} = c_2 / c_1^x$

# Public-Key Protocols

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## Plain RSA Signature

- Gen: $(N, e, d) \leftarrow \mathrm{GenRSA}(1^n)$, $pk = N, e$, $sk = d$
- $\mathrm{Sign}_{sk}(m \in \mathbb{Z}_N^*)$: $\sigma = [m^d \bmod N]$
- $\mathrm{Verify}_{pk}(m \in \mathbb{Z}_N^*, \sigma \in \mathbb{Z}_N^*)$: Output 1 if and only if $m = [\sigma^e \bmod N]$

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# The Need for Collaboration

**Yao's Millionaire Problem**

**$1.2M**

**$1.6M**

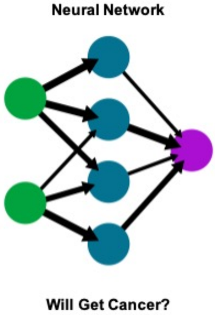
**Who is the richest?**

# The Need for Collaboration

| Name | Age | Smokes | Cancer |
|---|---|---|---|
| Jane Doe | 25 | Y | Y |
| John Doe | 45 | N | Y |

| Name | Age | Smokes | Cancer |
|---|---|---|---|
| Jane Smith | 35 | Y | N |
| John Smith | 85 | Y | Y |

| Name | Age | Smokes | Cancer |
|---|---|---|---|
| Joe Smith | 50 | N | N |
| Jim Doe | 60 | Y | Y |



# The Security Challenge

## Question

How do parties who *do not trust each other* work together to compute joint functions on their private data?

- Without revealing their data or intermediate results to each other
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Secure Multi-Party Computation (MPC or SMC) gives a solution:

- Originally developed in the 1980's
- Originally believed to be purely of theoretical interest
- But, since 2004 there have been many implementations showing real-world value
- Protocol and engineering improvements have yielded 6+ orders of magnitude speed up

# MPC in the Real World

BOSTON UNIVERSITY

UNBOUND (MATH OVER MATTER)

Google

**Gender Pay Equity**

**Cryptographic Key Control**

**Federated Keyboard Prediction**

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# Additive One-Time Pad

Given an Integer $x \in \mathbb{Z}_N$, consider the following scheme:

- Choose $r \leftarrow \mathbb{Z}_N$ at random
- Compute $y = x + r \bmod N$
- Can recover $x$ from $y$ if know $r$

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## Hiding

Given $y$, the value of $x$ is perfectly hidden:

- Every value of $x \in \mathbb{Z}_N$ is equally likely

# An MPC Protocol

## Secure Summation

5 parties $\{P_1, \ldots, P_5\}$ each with private input $x_i \in \mathbb{Z}_{100}$ want to compute

$$y = \sum_{i=1}^{5} x_i$$

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# Additive Secret Sharing

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We wish to distribute a secret value $x \in \mathbb{Z}_N$ among $n$ parties s.t.:

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## Security

Easy to see that any set of $n-1$ parties has no info about $x$

# Computing on Shared Values

Suppose parties hold two secret shared values:

- $[x] = ([x]_1, [x]_2, \ldots, [x]_n)$
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- Problem: We need to compute the cross-terms (e.g., $[x]_1[y]_2$)

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- Security: Since $a$ and $b$ are random $(x + a)$, $(y + b)$ reveal nothing about $x$, $y$

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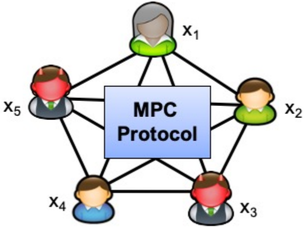


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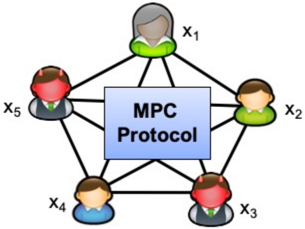


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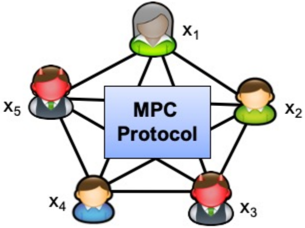


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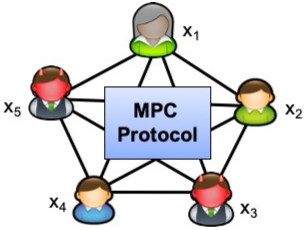


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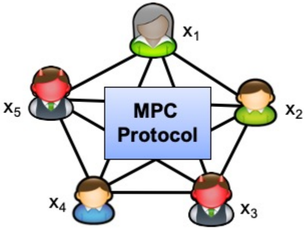


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- Adversary ($\mathcal{A}$) controls some of the parties

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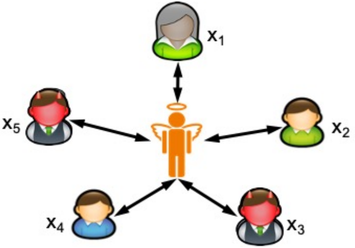
### Defining Security

We could give a security definition for each of these, but instead take a different approach.

# Real-Ideal Paradigm

Ideal World

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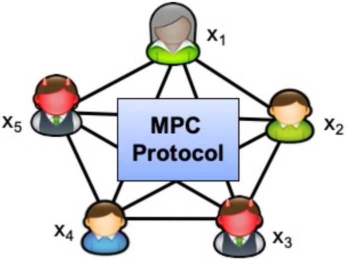


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MPC protocol emulates ideal-world execution

- Whatever security holds in ideal world holds in the real world
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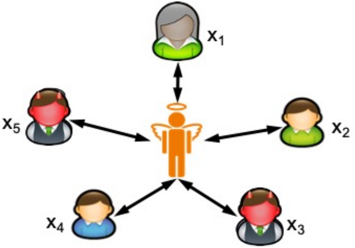


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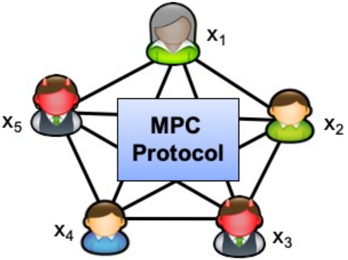


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# Simulating the Sharing-based MPC protocol

- Assume that $\mathcal{A}$ corrupts $t < n$ parties

# Simulating the Sharing-based MPC protocol

- Assume that $\mathcal{A}$ corrupts $t < n$ parties
- Then, all $\mathcal{A}$ even sees are random shares sent to parties he corrupts
- Since he never sees all $n$, these are completely random
- Simulator, can just send random values to $\mathcal{A}$ to simulate his view (distributed the same as real protocol)

# Limitations of Security Definition

## Correctness of inputs

- MPC allows adversary to choose his own inputs
- No notion of "valid" input
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- But, MPC does not consider what the output reveals
- In particular, output may reveal parties' inputs: e.g. C(x,y) = x+y
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Both of these limitations also happen in the ideal world

# Adversary Types

Adversary type:

- Semi-honest (honest but curious) – follows protocol, but tries to learn more
- Malicious – may deviate from protocol arbitrarily
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Adversary threshold:

- $t < n/2$ – honest majority
- $t < n$ – dishonest majority (particularly important for 2PC)
- $t < n/3$ – allows highly optimized protocols