Cryptography
Lecture 5

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Outline

1. Lecture 4 Review

2. Security of PRG+OTP (Chapter 3.3.3)

- PRGs
- Proofs by reduction

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2. Security of PRG+OTP (Chapter 3.3.3)

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- $\mathcal{A}_r$ runs $\mathcal{A}_c$ generating challenge c using r, observes if $\mathcal{A}_c$ wins, and if so outputs "PRG".

Security of PRG+OTP: Building $\mathcal{A}_r$

PRG+OTP Encryption

- $\mathsf{Gen}(1^n)$: $k \leftarrow \{0,1\}^n$
- $\mathsf{Enc}(k,m)$: $c = G(k) \oplus m$
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$PRG_{\mathcal{D},G}(n)$

- The challenger chooses $b \leftarrow \{0,1\}$.
  If $b = 0$, he chooses $r \leftarrow \{0,1\}^{l(n)}$;
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  He gives r to $\mathcal{D}$.
- On input r, the distinguisher $\mathcal{D}$ outputs a guess b'
- $PRG_{\mathcal{D},G}(n) = 1$ (i.e., $\mathcal{D}$ wins) if $b' = b$

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- $\mathcal{A}$ outputs two messages $m_0, m_1 \in \mathcal{M}$
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  - $\mathcal{A}_r$ gives c to $\mathcal{A}_c$ and gets bit b'
  - $\mathcal{A}_r$ outputs 1 ("PRG") if $b = b'$ and 0 otherwise

Security of PRG+OTP: Analysis

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  - $\Pr[\mathcal{A}_r(r) = 1] = \Pr[\mathcal{A}_c \text{ outputs } b' = b] = 1/2$
- Case 2: $r = G(s)$
  - $\mathcal{A}_c$ receives $c = r \oplus m_b$ with $r = G(s)$, this is OTP+PRG

Security of PRG+OTP: Analysis

PRG+OTP Encryption

- Gen(1^n): $k \leftarrow \{0,1\}^n$
- Enc(k, m): $c = G(k) \oplus m$
- Dec(k, c): $m = G(k) \oplus c$

$PRG_{\mathcal{D},G}(n)$

- The challenger chooses $b \leftarrow \{0,1\}$.
  If $b = 0$, he chooses $r \leftarrow \{0,1\}^{l(n)}$;
  if $b = 1$, he chooses $s \leftarrow \{0,1\}^n$, and computes $r = G(s)$.
  He gives r to $\mathcal{D}$.
- On input r, the distinguisher $\mathcal{D}$ outputs a guess b'
- $PRG_{\mathcal{D},G}(n) = 1$ (i.e., $\mathcal{D}$ wins) if $b' = b$

$\mathsf{PrivK}^{eav}_{\mathcal{A},\Pi}$

- $\mathcal{A}$ outputs two messages $m_0, m_1 \in \mathcal{M}$
- The challenger chooses $k \leftarrow$ Gen, $b \leftarrow \{0,1\}$, computes $c \leftarrow \mathsf{Enc}_k(m_b)$ and gives c to $\mathcal{A}$
- $\mathcal{A}$ outputs a guess bit b'
- We say that $\mathsf{PrivK}^{eav}_{\mathcal{A},\Pi} = 1$ (i.e., $\mathcal{A}$ wins) if $b' = b$.

Need to analyze $\Pr[\mathcal{A}_r$ WINS$]$ ($\Pr[PRG_{\mathcal{A}_r,G}(n) = 1]$)

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  - $\Pr[\mathcal{A}_r(r) = 1] = \Pr[\mathcal{A}_c$ outputs $b' = b] = 1/2$
- Case 2: $r = G(s)$
  - $\mathcal{A}_c$ receives $c = r \oplus m_b$ with $r = G(s)$, this is OTP+PRG
  - $\Pr[\mathcal{A}_r(r) = 1] = \Pr[\mathcal{A}_c$ outputs $b' = b] =$
    $= \Pr[PrivK^{eav}_{\mathcal{A}_c,\Pi}(1^n) = 1] \geq 1/2 + 1/\mathrm{poly}(n)$

Security of PRG+OTP: Analysis

PRG+OTP Encryption

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Need to analyze $\Pr[\mathcal{A}_r \text{ WINS}]$ ($\Pr[PRG_{\mathcal{A}_r,G}(n) = 1]$)

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  - $\Pr[\mathcal{A}_r(r) = 1] = \Pr[\mathcal{A}_c \text{ outputs } b' = b] = 1/2$
- Case 2: $r = G(s)$
  - $\mathcal{A}_c$ receives $c = r \oplus m_b$ with $r = G(s)$, this is OTP+PRG
  - $\Pr[\mathcal{A}_r(r) = 1] = \Pr[\mathcal{A}_c \text{ outputs } b' = b] =$
    $= \Pr[PrivK^{eav}_{\mathcal{A}_c,\Pi}(1^n) = 1] \geq 1/2 + 1/\text{poly}(n)$
- Summing these together, we get

$$\begin{aligned} \Pr[PRG_{\mathcal{A}_r,G}(1^n) = 1] &\geq 1/2 \cdot 1/2 + 1/2 \cdot (1/2 + 1/\text{poly}(n)) \\ &= 1/2 + 1/(2\text{poly}(n)) \end{aligned}$$

Contradiction!

Where Are We Now

- Features of PRG+OTP encryption
    - Can encrypt messages of arbitrary length, just need PRG with enough stretch.
    - Achieve security against an eavesdropper

Where Are We Now

- Features of PRG+OTP encryption
  - Can encrypt messages of arbitrary length, just need PRG with enough stretch.
  - Achieve security against an eavesdropper
- Limitations of PRG+OTP encryption
  - Can only see one encryption
  - If see two, can tell whether they are equal