Cryptography Lecture 15

Arkady Yerukhimovich

October 21, 2024

Outline

- Exam 1 and the rest of the semester
- 2 Lecture 13 Review
- Constructing Practical Block Ciphers Overview (Chapters 6.Intro, 6.2.Intro)
- 4 Substitution-Permutation Networks (SPN) and AES (Chapters 6.2.1, 6.2.5)

Exam 1 Grades

Exam 1 Stats:

- Max 89
- Mean 61.7
- Median 65.5

Research Project

Important Dates:

- Oct. 25 Project Proposals Due
- Nov. 22 Presentations Due
- Dec. 4 Workshop Day

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Next Steps:

- Form your groups (3 people max) Do this ASAP
- Choose your topic I am happy to help, but you need to start the conversation
- Do the research
- Record 20-minute presentation by due date
- Use Slack to start discussions about projects
- Live discussion of all projects on Dec. 4

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Cryptography

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Lecture 13 Review

- Collision-resistant Hash Functions
- Merkle-Damgard Domain Extension for CRHF
- Applications of Hash Functions
 - Hash-and-MAC
 - Password-based authentication
 - and more

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And Now For Something Completely Different

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- Focus on constructing *practically efficient* pseudorandom objects
- Constructions will be largely combinatorial, and proofs informal

Block Cipher

$$F: \{0,1\}^n \times \{0,1\}^\ell \to \{0,1\}^\ell$$

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 - A cipher with n=256 that can be broken in time 2^{128} is insecure

Block cipher security goals:

• Indistinguishability from a random permutation - PRP

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 - For many block ciphers these properties are not well defined, not needed by all constructions

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Important

Remember that a block cipher is not an encryption scheme

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• Pro: $|F| = 16 \cdot \log(2^8!) \approx 16 \cdot 600 \text{ bits} \approx 3KB << 128 \cdot 2^{128}$

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- Con: If x only changes in one bit, only one block is affected

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- Even a 1-bit change to input must have statistical impact on all bits.

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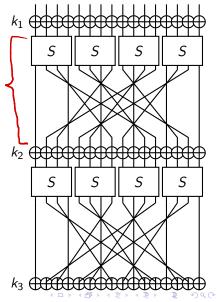
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- Each diffusion step distributes those changes across blocks
- After many iterations, all bits are affected avalanche effect

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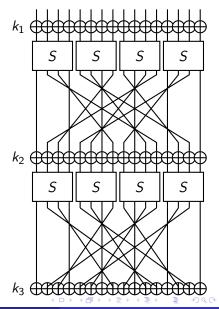
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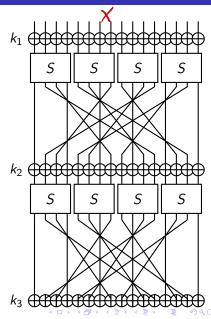


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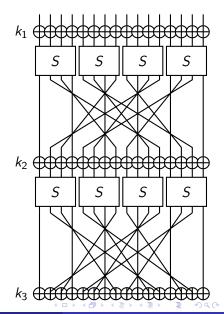
1 Key mixing: Set $x = x \oplus k_i$



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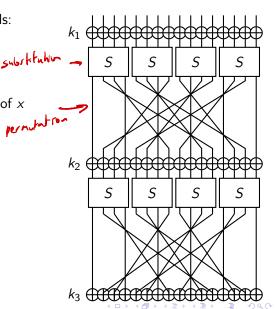
- Substitution:

$$x = S_1(x_1)||\cdots||S_8(x_8)$$



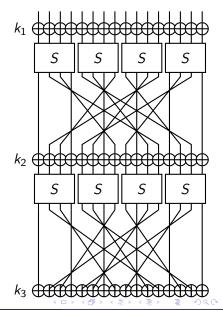
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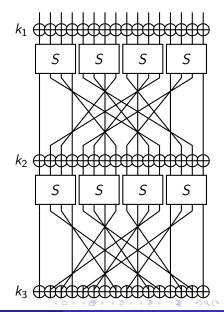


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Observations:

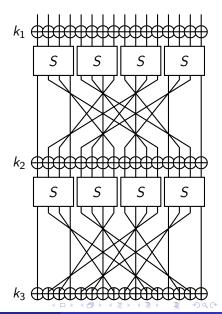
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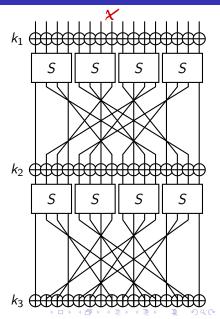
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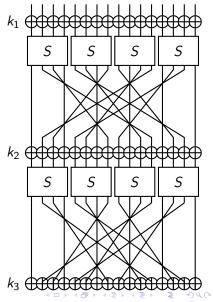
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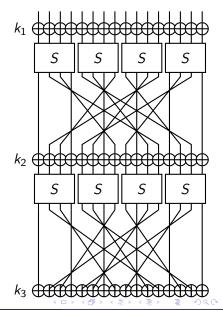
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- Need one more key-mixing step after last round to prevent A from inverting round



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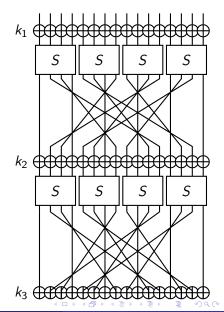
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- Direct application of confusion-diffusion paradigm



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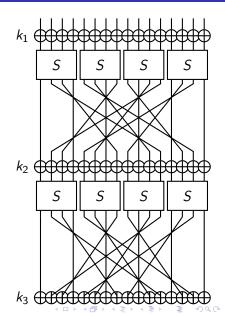


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Key Schedule:

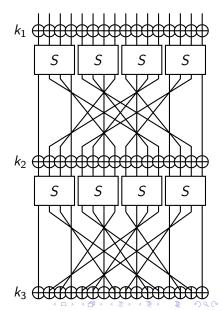
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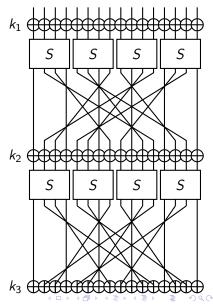
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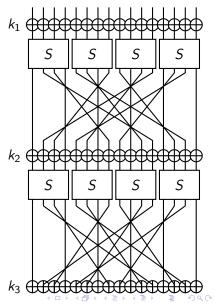
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- r-round SPN needs r + 1 keys



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- Hence, run for >> 6 rounds

AES / Rijndael History

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- This is the right block-cipher to use for most applications

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State: 4×4 array of Bytes

$$\begin{pmatrix}
B_{11} & B_{12} & B_{13} & B_{14} \\
B_{21} & B_{22} & B_{23} & B_{24} \\
B_{31} & B_{32} & B_{33} & B_{34} \\
B_{41} & B_{42} & B_{43} & B_{44}
\end{pmatrix}$$

1 Add Round Key: View $k_i = k_{11} ||k_{12}|| \cdots ||k_{44}|$ with $|k_{ij}| = 1$ Byte

$$\begin{pmatrix} B_{11} \oplus k_{11} & B_{12} \oplus k_{12} & B_{13} \oplus k_{13} & B_{14} \oplus k_{14} \\ B_{21} \oplus k_{21} & B_{22} \oplus k_{22} & B_{23} \oplus k_{23} & B_{24} \oplus k_{24} \\ B_{31} \oplus k_{31} & B_{32} \oplus k_{32} & B_{33} \oplus k_{33} & B_{34} \oplus k_{34} \\ B_{41} \oplus k_{41} & B_{42} \oplus k_{42} & B_{43} \oplus k_{43} & B_{44} \oplus k_{44} \end{pmatrix}$$

4 Add Round Key: View $k_i = k_{11} ||k_{12}|| \cdots ||k_{44}|$ with $|k_{ij}| = 1$ Byte

$$\begin{pmatrix} B_{11} \oplus k_{11} & B_{12} \oplus k_{12} & B_{13} \oplus k_{13} & B_{14} \oplus k_{14} \\ B_{21} \oplus k_{21} & B_{22} \oplus k_{22} & B_{23} \oplus k_{23} & B_{24} \oplus k_{24} \\ B_{31} \oplus k_{31} & B_{32} \oplus k_{32} & B_{33} \oplus k_{33} & B_{34} \oplus k_{34} \\ B_{41} \oplus k_{41} & B_{42} \oplus k_{42} & B_{43} \oplus k_{43} & B_{44} \oplus k_{44} \end{pmatrix}$$

SubBytes: Uses a single 8-bit S-box

$$\begin{pmatrix} S(B_{11}) & S(B_{12}) & S(B_{13}) & S(B_{14}) \\ S(B_{21}) & S(B_{22}) & S(B_{23}) & S(B_{24}) \\ S(B_{31}) & S(B_{32}) & S(B_{33}) & S(B_{34}) \\ S(B_{41}) & S(B_{42}) & S(B_{43}) & S(B_{44}) \end{pmatrix}$$

Shift Rows: Shift each row to the left by varying amounts

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} & (0 \text{ shift}) \\ B_{22} & B_{23} & B_{24} & B_{21} & (1 \text{ shift}) \\ B_{33} & B_{34} & B_{31} & B_{32} & (2 \text{ shift}) \\ B_{44} & B_{41} & B_{42} & B_{43} & (3 \text{ shift}) \end{pmatrix}$$

Shift Rows: Shift each row to the left by varying amounts

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} & (0 \text{ shift}) \\ B_{22} & B_{23} & B_{24} & B_{21} & (1 \text{ shift}) \\ B_{33} & B_{34} & B_{31} & B_{32} & (2 \text{ shift}) \\ B_{44} & B_{41} & B_{42} & B_{43} & (3 \text{ shift}) \end{pmatrix}$$

MixColumns: Apply invertible transformation to Bytes in each column

Shift Rows: Shift each row to the left by varying amounts

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} & (0 \text{ shift}) \\ B_{22} & B_{23} & B_{24} & B_{21} & (1 \text{ shift}) \\ B_{33} & B_{34} & B_{31} & B_{32} & (2 \text{ shift}) \\ B_{44} & B_{41} & B_{42} & B_{43} & (3 \text{ shift}) \end{pmatrix}$$

MixColumns: Apply invertible transformation to Bytes in each column

Observations:

Shift Rows: Shift each row to the left by varying amounts

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} & (0 \text{ shift}) \\ B_{22} & B_{23} & B_{24} & B_{21} & (1 \text{ shift}) \\ B_{33} & B_{34} & B_{31} & B_{32} & (2 \text{ shift}) \\ B_{44} & B_{41} & B_{42} & B_{43} & (3 \text{ shift}) \end{pmatrix}$$

MixColumns: Apply invertible transformation to Bytes in each column

Observations:

Steps 3 and 4 correspond to permutation step

Shift Rows: Shift each row to the left by varying amounts

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} & (0 \text{ shift}) \\ B_{22} & B_{23} & B_{24} & B_{21} & (1 \text{ shift}) \\ B_{33} & B_{34} & B_{31} & B_{32} & (2 \text{ shift}) \\ B_{44} & B_{41} & B_{42} & B_{43} & (3 \text{ shift}) \end{pmatrix}$$

MixColumns: Apply invertible transformation to Bytes in each column

Observations:

- Steps 3 and 4 correspond to permutation step
- Even this very structured permutation seems enough