Cryptography

Lecture 15

Arkady Yerukhimovich

October 21, 2024

Outline

1. Exam 1 and the rest of the semester
2. Lecture 13 Review
3. Constructing Practical Block Ciphers – Overview (Chapters 6.Intro, 6.2.Intro)
4. Substitution-Permutation Networks (SPN) and AES (Chapters 6.2.1, 6.2.5)

Exam 1 Grades

Exam 1 Stats:

- Max – 89
- Mean – 61.7
- Median – 65.5

Research Project

Important Dates:

- Oct. 25 – Project Proposals Due
- Nov. 22 – Presentations Due
- Dec. 4 – Workshop Day

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Next Steps:

1. Form your groups (3 people max) – Do this ASAP
2. Choose your topic – I am happy to help, but you need to start the conversation
3. Do the research
4. Record 20-minute presentation by due date
5. Use Slack to start discussions about projects
6. Live discussion of all projects on Dec. 4

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Lecture 13 Review

- Collision-resistant Hash Functions
- Merkle-Damgard Domain Extension for CRHF
- Applications of Hash Functions
  - Hash-and-MAC
  - Password-based authentication
  - and more

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- Focus on constructing *practically efficient* pseudorandom objects
- Constructions will be largely combinatorial, and proofs informal

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Fixed input size strong PRP

$F : \{0,1\}^n \times \{0,1\}^\ell \to \{0,1\}^\ell$

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    - A cipher with $n = 256$ that can be broken in time 2^{128} is insecure

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  - For many block ciphers these properties are not well defined, not needed by all constructions

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Important

Remember that a block cipher is not an encryption scheme

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- Con: If x only changes in one bit, only one block is affected

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- Even a 1-bit change to input must have statistical impact on all bits.

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- After many iterations, all bits are affected – avalanche effect

Outline

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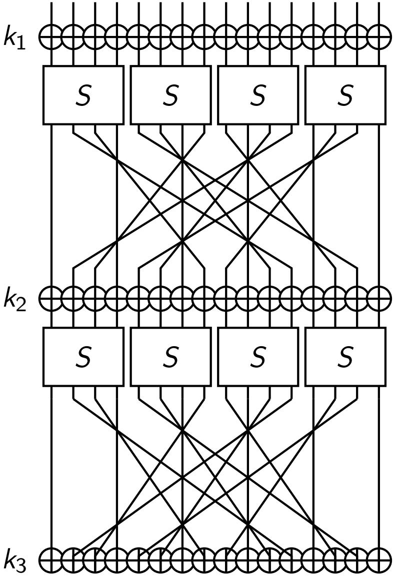

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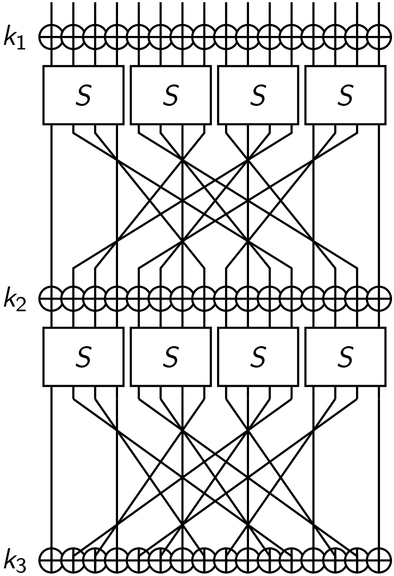

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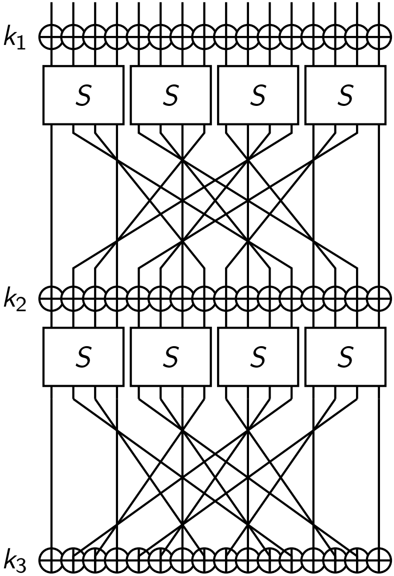

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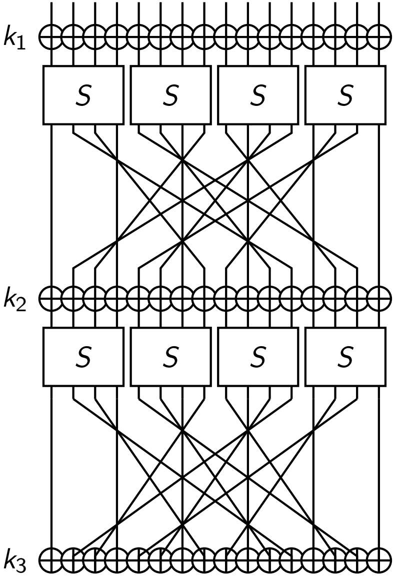

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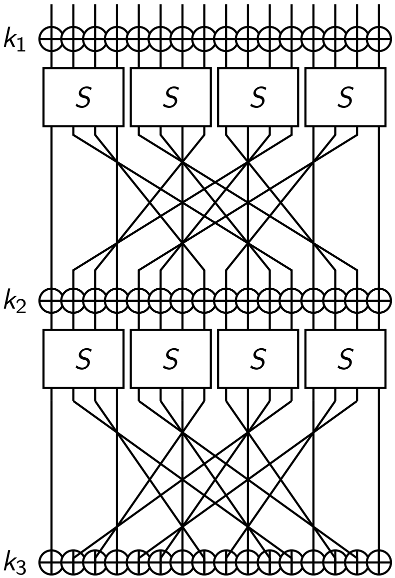

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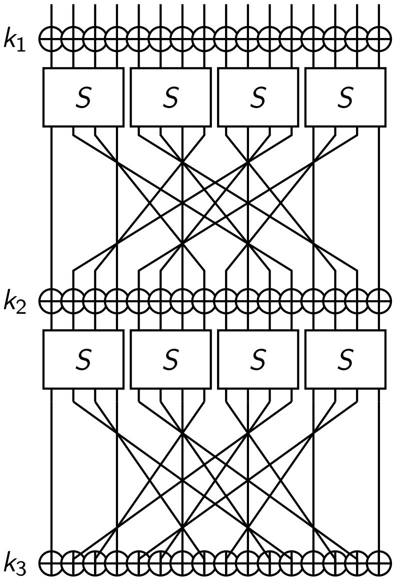

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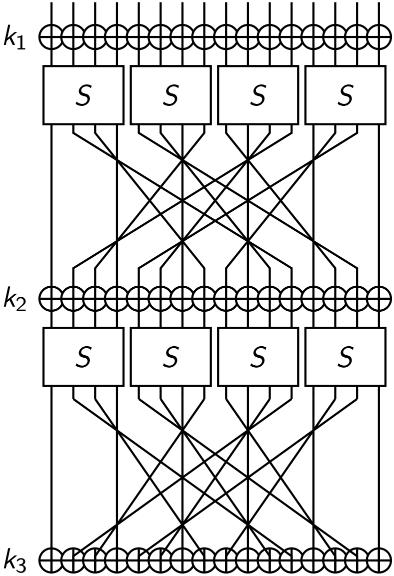

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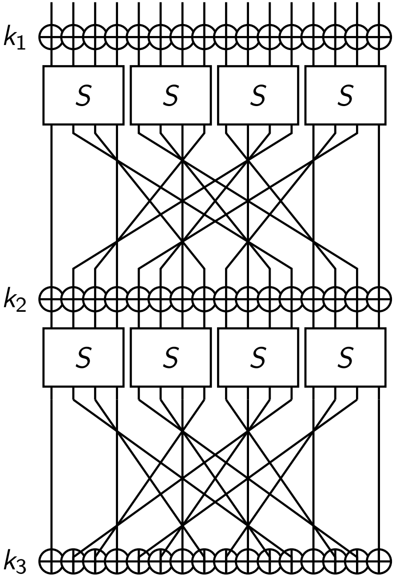

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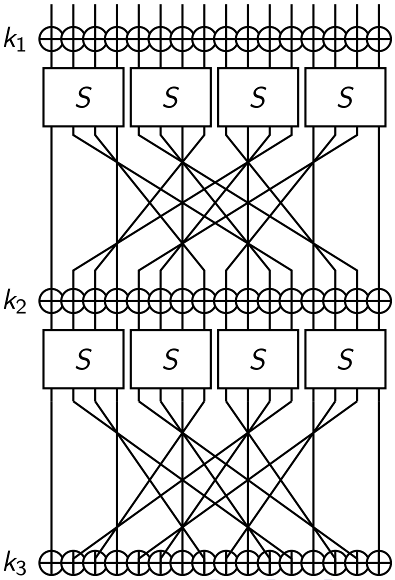

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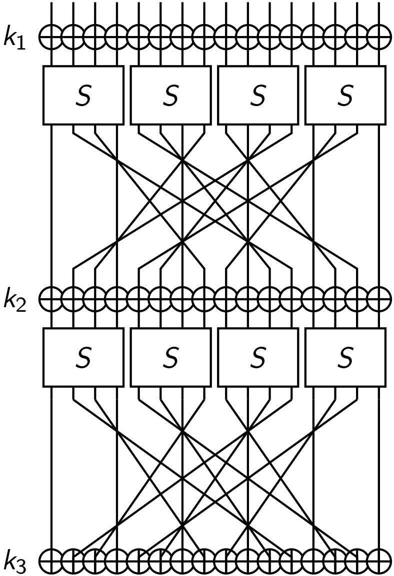

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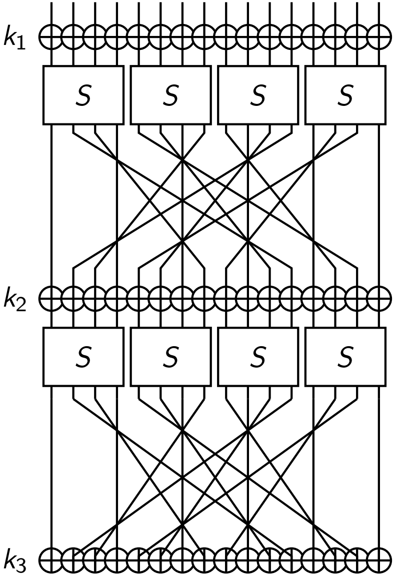
- Direct application of confusion-diffusion paradigm



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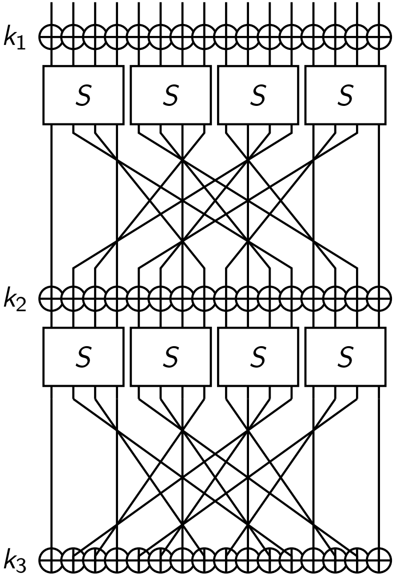

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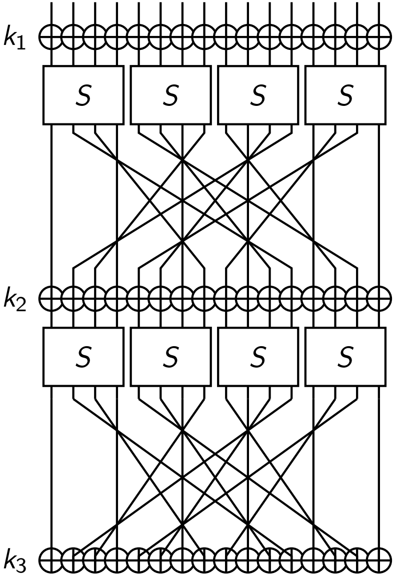

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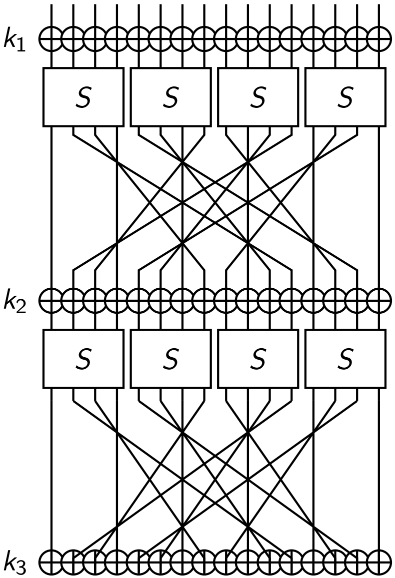

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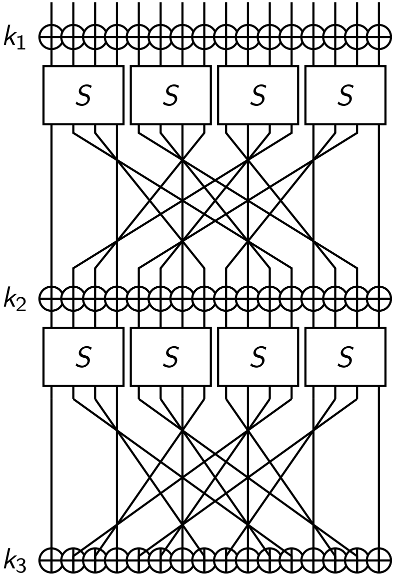

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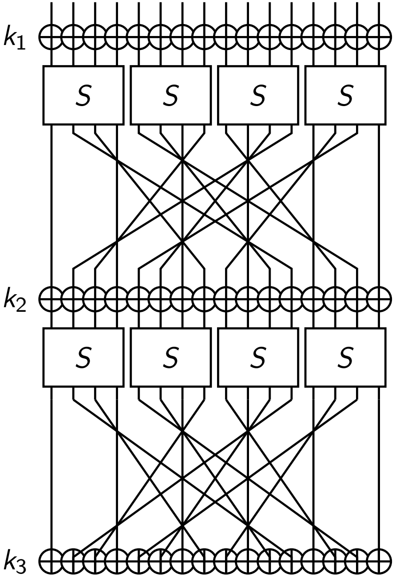
- r-round SPN needs $r + 1$ keys



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- Hence, run for $>> 6$ rounds

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- Winner of NIST blockcipher competition in 2000
- Now, standardized and very widely used
- Hardware support (e.g., AES-NI) makes this very fast: 10^8 per second
- This is the right block-cipher to use for most applications

AES

Available Versions: $\ell = 128$(16 Bytes), $n = 128, 192, 256$

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State: 4×4 array of Bytes

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{pmatrix}$$

AES

1. Add Round Key: View $k_i = k_{11}||k_{12}||\cdots||k_{44}$ with $|k_{ij}| = 1$ Byte

$$\begin{pmatrix} B_{11} \oplus k_{11} & B_{12} \oplus k_{12} & B_{13} \oplus k_{13} & B_{14} \oplus k_{14} \\ B_{21} \oplus k_{21} & B_{22} \oplus k_{22} & B_{23} \oplus k_{23} & B_{24} \oplus k_{24} \\ B_{31} \oplus k_{31} & B_{32} \oplus k_{32} & B_{33} \oplus k_{33} & B_{34} \oplus k_{34} \\ B_{41} \oplus k_{41} & B_{42} \oplus k_{42} & B_{43} \oplus k_{43} & B_{44} \oplus k_{44} \end{pmatrix}$$

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2. SubBytes: Uses a single 8-bit S-box

$$\begin{pmatrix} S(B_{11}) & S(B_{12}) & S(B_{13}) & S(B_{14}) \\ S(B_{21}) & S(B_{22}) & S(B_{23}) & S(B_{24}) \\ S(B_{31}) & S(B_{32}) & S(B_{33}) & S(B_{34}) \\ S(B_{41}) & S(B_{42}) & S(B_{43}) & S(B_{44}) \end{pmatrix}$$

AES

3. Shift Rows: Shift each row to the left by varying amounts

$$\begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} & \text{(0 shift)} \\ B_{22} & B_{23} & B_{24} & B_{21} & \text{(1 shift)} \\ B_{33} & B_{34} & B_{31} & B_{32} & \text{(2 shift)} \\ B_{44} & B_{41} & B_{42} & B_{43} & \text{(3 shift)} \end{pmatrix}$$

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4. MixColumns: Apply invertible transformation to Bytes in each column

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Observations:

- Steps 3 and 4 correspond to permutation step
- Even this very structured permutation seems enough