ground water

Issue Paper/

The Water Budget Myth Revisited: Why Hydrogeologists Model

by John D. Bredehoeft[1]

Abstract/

Within the ground water community, the idea persists that if one can estimate the recharge to a ground water system, one then can determine the size of a sustainable development. Theis addressed this idea in 1940 and showed it to be wrong—yet the myth continues. The size of a sustainable ground water development usually depends on how much of the discharge from the system can be "captured" by the development. Capture is independent of the recharge; it depends on the dynamic response of the aquifer system to the development. Ground water models were created to study the response dynamics of ground water systems; it is one of the principal reasons hydrogeologists model.

Introduction

The idea persists within the ground water community that if one can determine the recharge to an aquifer system then one can determine the maximum magnitude of a sustainable development. One commonly hears the statement, "the pumping must not exceed the recharge (if the development is to be sustainable)."

The idea that the recharge (by which one usually means the virgin recharge before development) is important in determining the magnitude of sustainable development is a myth. A number of hydrogeologists have tried to debunk the myth, starting with Theis (1940) in a paper titled "The Source of Water Derived from Wells: Essential Factors Controlling the Response of an Aquifer to Development." Brown (1963) and Bredehoeft et al. (1982) wrote papers debunking the myth. Unfortunately, the message in Brown's paper was apparent only to those deeply schooled in ground water hydrology. The Bredehoeft et al. paper, while more readily understandable, was published in an obscure National Academy of Science publication that is out of print. At the time the Bredehoeft et al. paper was published, Theis congratulated the authors, commenting that he had intended to write another paper on the subject, but now he did not see the need. Needless to say, in spite of these efforts the myth goes on; it is so ingrained in the community's collective thinking that nothing seems to derail it.

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It is presumptuous and perhaps arrogant of me to imply that the entire community of ground water hydrologists does not understand the principles first set forth by Theis in 1940; clearly this is not the situation. There are good discussions in recent papers that indicate other hydrogeologists understand Theis' message. The 1999 USGS Circular 1186, *Sustainability of Ground-Water Resources* (Alley et al. 1999), states the ideas lucidly. Sophocleous and his colleagues at the Kansas Geological Survey have published extensively on the concept of ground water sustainability; Sophocleous (2000) presents a summary of his ideas that contain the essence of Theis' principles.

On the other hand, I do not find Theis' principles on sustainabilty expressed clearly in the texts on ground water. These ideas were taught to me, early in my career, by my mentors at the U.S. Geological Survey. Also I find in discussions with other ground water professionals that these ideas, even though they are 60 years old, are not clearly understood by many individuals. It is my purpose in this paper to address again the myth that recharge is all important in determining the size of a sustainable ground water development, and show that this idea has no basis in fact.

Analytical Methods in Hydrogeology

Before digital computer modeling codes, hydrogeologists used traditional analytical methods to assess the impacts of wells on ground water systems. The traditional method of analysis used is the principle of superposition. In this approach, one assumes that the hydraulic head (or the water table) before development resulted from the inputs and outputs (recharge and discharge) from the system. One

analyzes the impact of pumping independent of the initial (virgin) hydraulic head. The cone of depression is calculated as a function of time. This cone of depression is then superposed upon the existing hydraulic head (or water table). The resulting head after superposition is the solution to the development.

To make such a superposition calculation, one needs: (1) the transmissivity and storativity distribution within the aquifer, (2) the boundary conditions that will be reached by the cone of depression, and (3) the rate of pumping. Those trained in classical hydraulic theory are well aware of reflection boundaries and image wells to account for the boundary conditions.

Missing from the classical analysis is any mention of recharge. The recharge is taken into account by the initial hydraulic head (or the water table). The initial head is a solution to an initial boundary value problem that includes the recharge and discharge.

Prior to the widespread use of digital computer models most analyses in ground water flow were made using the principles of superposition. This was also the methodology used in the analog computer models of the 1950s, '60s, and '70s. With the advent of digital computer models, it became feasible to specify the varying distributions of recharge and discharge with the idea of solving for the virgin water table. The calculated water table can then be compared to the observed water table (or hydraulic head). To do such an analysis requires knowledge of the distribution of both the virgin rate of recharge and the virgin rate of discharge—in addition to the transmissivity distribution and the boundary conditions.

With an estimate of the rainfall, there is still no idea of how large the recharge is, except that it cannot exceed some unknown fraction of rainfall. The researcher may know the transmissivity of the aquifer at a few places and the aquifer discharge that makes up the baseflow of streams associated with the aquifer. Based on this set of limited information, a steady-state model analysis is made in an attempt to estimate the transmissivity of the aquifer. This is a common model analysis. In this context, knowledge of the virgin recharge is useful in estimating the transmissivity.

The recharge and the discharge are the inputs and outputs from a ground water system. Both quantities are important in understanding how a particular ground water system functions. However, it is not my purpose in this paper to discuss recharge or discharge. My focus is on how recharge and discharge enter into the determination of the sustainable yield of a ground water system.

In the classical analytical method, the important variables for determining the impacts of pumping are those that describe the dynamic response of the system—the distribution of aquifer diffusivity and the boundary conditions. This argument was the thrust of Brown's 1963 paper. The argument makes sense to one trained in classical analytical methods; it is more obscure to others. Brown's paper made almost no impact. I will attempt to further simplify the mathematical argument.
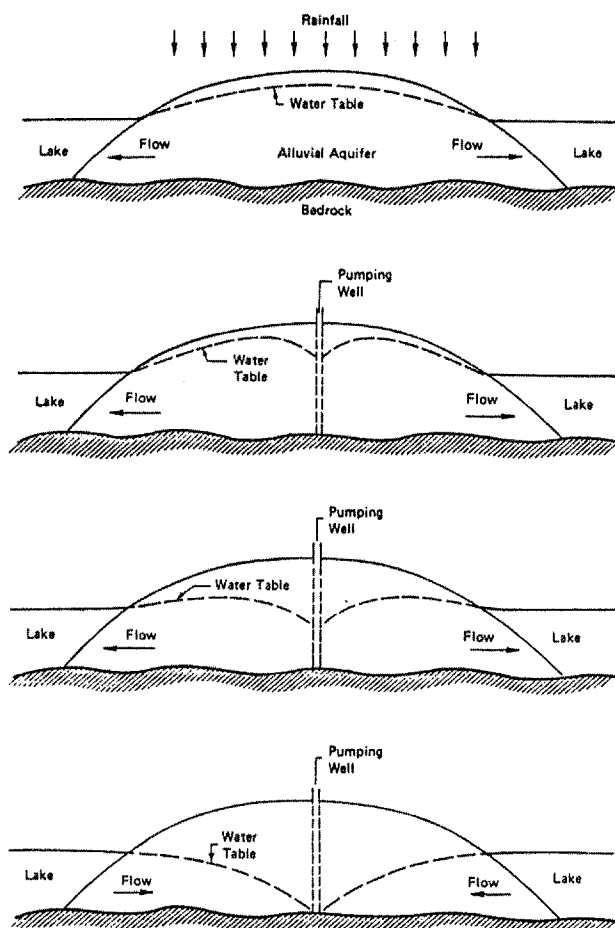

Figure 1. Schematic cross section of an aquifer situated on a circular island in a fresh water lake that is being developed by pumping. (Reprinted with permission from *Scientific Basis of Water-Resource Management.* Copyright 1982 by the National Academy of Sciences. Courtesy of the National Academy Press, Washington, D.C.)

The Water Budget

To illustrate the basic premise, I want to consider a simple aquifer system. A permeable alluvial aquifer underlies a circular island in a fresh water lake. Our intent is to develop a well on the island. The island aquifer is shown schematically in various stages of development in Figure 1.

Before development, recharge from rainfall creates a water table. The recharge over the island is balanced by discharge from the permeable aquifer directly to the lake (Figure 1—top cross section). We can write the following water balance for virgin conditions on our island:

$$R_0 = D_0 \qquad \text{or} \qquad R_0 - D_0 = 0$$

where R_0 is the virgin recharge (this is the recharge generally referred to in the myth), and D_0 is the virgin discharge. A water table develops on the island in response to the distribution of recharge and discharge and the transmissivity of the alluvial aquifer (Figure 1—top cross section).

The discharge to the lake can be obtained at any point along the shore by applying Darcy's law:

$$d = T \, (dh/dl)$$

where d is the discharge through the aquifer at any point along the shore; T is the transmissivity at the same point; and dh/dl is the gradient in the water table at that point. If

we integrate the point discharge along the entire shoreline of the island we obtain the total discharge from the island:

$$\int T \, (dh/dl) \, ds = D_0$$

We now go into the middle of the island, install a well and initiate pumping (Figure 1—second cross section). At any new time, we can write a new water balance for the island:

$$(R_0 + \Delta R_0) - (D_0 + \Delta D_0) - P + dV/dt = 0$$

where ΔR_0 is the change in the virgin rate of recharge caused by our pumping; ΔD_0 is the change in the virgin rate of discharge caused by the pumping; P is the rate of pumping; and dV/dt is the rate at which we are removing water from ground water storage on the island.

We know that the virgin rate of recharge, R_0, is equal to the virgin rate of discharge, D_0, so our water budget equation following the initiation of pumping reduces to

$$\Delta R_0 - \Delta D_0 - P + dV/dt = 0$$
or
$$\Delta R_0 - \Delta D_0 - P = dV/dt$$

For a sustainable development, we want the rate of water taken from storage to be zero; in other words, we define sustainability as

$$dV/dt = 0$$

Now our water budget for sustainable development is

$$\Delta R_0 - \Delta D_0 = P$$

We are now stating that, to reach a sustainable development, the pumping must be balanced by a change in the virgin rate of recharge, ΔR_0, and/or a change in the virgin rate of discharge, ΔD_0, caused by the pumping. Traditionally, the sum of the change in recharge and the change in discharge caused by the pumping, the quantity $(\Delta R_0 - \Delta D_0)$, is defined as the "capture" attributable to the pumping. To be a sustainable development, the rate of pumping must equal the rate of capture.

Notice that to determine sustainability we do not need to know the recharge. The recharge may be of interest, as are all the facets of the hydrologic budget, but it is not a determining factor in our analysis.

Recharge is often a function of external conditions—such as rainfall, vegetation, and soil permeability. In many, if not most, ground water situations, the rate of recharge cannot be impacted by the pumping; in other words, in terms of our water budget,

$$\Delta R_0 = 0$$

In most situations, sustainability of a ground water development occurs when the pumping captures an equal amount of virgin discharge:

$$P = \Delta D_0$$

Let's return to the island aquifer and see how the capture occurs conceptually. When we start to pump, a cone of depression is created. Figure 1 (second cross section) shows the cone of depression at an early stage in the development of our island aquifer. The natural discharge from the island does not start to change until the cone of depression changes the slope in the water table at the shore of the island; remember: Darcy's law controls the discharge at the shoreline. Until the slope of the water table at the shoreline is changed by the pumping, the natural discharge continues at its virgin rate. Until the point in time that the cone reaches the shore and changes the water table gradient significantly, all water pumped from the well is supplied totally from storage in the aquifer. In other words, the cone of depression must reach the shoreline before the natural discharge is impacted (Figure 1—third cross section). The rate at which the cone of depression develops, reaches the shoreline, and then changes the slope of the water table there depends on the dynamics of the aquifer system—transmissivity, storativity (or specific yield), and boundary conditions. The rate of capture in a ground water system is a problem in the dynamics of the system. Capture has nothing to do with the virgin rate of recharge; the recharge is irrelevant in determining the rate of capture.

Figure 1 (third cross section) shows the water table in our island aquifer at a point in time when the natural discharge is almost eliminated; the slope of the water table is almost flat at the shoreline. I deliberately created an aquifer system in which one can induce water to flow from the lake into the aquifer (Figure 1—fourth cross section). In this instance, the sustainable development can exceed the virgin recharge (or the virgin discharge). This again suggests that the recharge is not a relevant input in determining the magnitude of a sustainable development.

Often the geometry of the aquifer restricts the capture. For example, were the aquifer on the island to be thin, we might run out of water at the pump long before we could capture any fraction of the discharge. In this case all water pumped would come from storage. It would be "mined." In the island example, with a thin aquifer, the well could run dry before it could impact the discharge at the shoreline. Notice in Figure 1 (fourth cross section) that I have drawn the situation where the drawdown reached the bottom of the aquifer; the aquifer geometry and diffusivity limit the potential drawdown at the well. This again points out that the dynamic response of the aquifer system is all-important to determining the impacts of development. It is for these reasons that hydrogeologists are concerned with the dynamics of aquifer system response. Hydrogeologists model aquifers in an attempt to understand their dynamics.

Clearly, the circular island aquifer is a simple system. Even so, the principles explained in terms of this simple aquifer apply to all ground water systems. It is the dynamics of how capture takes place in an aquifer that ultimately determines how large a sustainable ground water development can be.

Water Law in the West

Nevada recognized in the early 1900s that the water supply for many of the valleys within the state would have

to come totally from local ground water. Enlightened individuals in Nevada decided to attempt to make the ground water supply within these valleys sustainable. The total discharge in many of the closed valleys in Nevada is by evaporation from the playas and from the transpiration (evapotranspiration [ET]) of phreatophytic plants that tap the water table. Nevada was willing to let the ground water pumping capture both the evaporation of ground water and the ground water that went to support the phreatophytic plants. This thinking led to the Nevada Doctrine that ground water pumping must not exceed the recharge. Perhaps the Nevada Doctrine perpetuates the myth. In reality the Nevada Doctrine is a roundabout statement that the development must not exceed the potential capture of ET (because as shown previously, the virgin ET is equal to the virgin rate of recharge).

As an aside, it has been difficult for the state engineer in Nevada to administer this doctrine in places of heavy urbanization such as Las Vegas, even though Nevada law codified the doctrine. The law also has been difficult to administer where discharge from a valley occurs as perennial streamflow (surface water) that is already appropriated.

The case of the perennial stream with an associated aquifer raises the problem of stream depletion, where pumping impacts streamflow that is appropriated by downstream users. Again, stream depletion is a dynamic ground water problem in capture—all the principles of the simple island example apply. Western water law recognizes the process of stream depletion with varying degrees of success—from zero to full recognition, depending upon the particular state.

Aquifer Dynamics and Models

Since the development of the Theis equation in 1935, hydrogeologists have been concerned with the dynamics of aquifer response to stress: pumping or recharge. Once Theis (1935) and later Jacob (1940) showed the analogy of ground water flow to heat flow, the ground water community has been busy solving the appropriate boundary value problems that describe various schemes of development. This endeavor has gone through several stages.

The 1940s and 1950s were a time during which the ground water profession was concerned with solving the problems of flow to a single well. Numerous solutions to the single well problem were produced. These solutions were used both to predict the response of the aquifer system and to estimate aquifer properties—transmissivity (or permeability) and storativity.

Hydrogeologists of that day saw the limitations in analyzing wells and sought a more robust methodology by which to analyze an entire aquifer, including complex boundary conditions and aquifer heterogeneity. The search led a group at the U.S. Geological Survey (USGS) to invent the analog model in the 1950s; the genius behind this development was Herb Skibitski, one of those individuals who rarely published. The new tool was the electric analog computer model of the aquifer. The model consisted of a finite-difference network of resistors and capacitors. In the
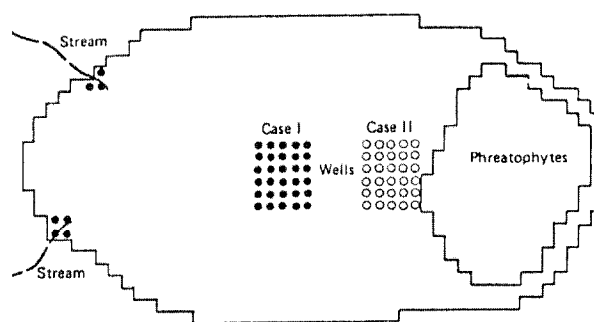

Figure 2. Plan view of a hypothetical closed basin aquifer that is being developed. (Reprinted with permission from *Scientific Basis of Water-Resource Management.* Copyright 1982 by the National Academy of Sciences. Courtesy of the National Academy Press, Washington, D.C.)

analog computer, aquifer transmissivity is represented by the network of resistors; the storativity is represented by the network of capacitors. The resulting resistor-capacitor network is excited by electrical function generators that simulate pumping or other stresses. Voltage is equivalent to hydraulic head in the analog computer; electrical current is equivalent to the flow of water.

In reality, these were elegant finite-difference computer models of aquifer systems. By 1960, the USGS had a facility in Phoenix, Arizona, where analog models of aquifers were routinely built on a production basis. Some of these analog models had multiple aquifers; some had as many as 250,000 nodes. At the time, it was infeasible to solve the same problems with digital computers; the digital computers of the day were too small and too slow. However, by 1970 the power of digital computers increased to the point that digital aquifer models could begin to compete with the analog models. By 1980 digital computer models had replaced the analog models, even at the USGS. The models of the 1980s have now grown to include solute transport, pre- and postprocessors, and automatic parameter estimation. By far the vast majority of ground water flow problems are simulated using the USGS code MODFOW; there is a new version MODFLOW 2000.

The ground water model is a tool with which to investigate the dynamics of realistic aquifer systems. As suggested previously, it is only through the study and understanding of aquifer dynamics that one can determine the impact of an imposed stress on an aquifer system.

Dynamics of a Basin and Range Aquifer

To illustrate the dynamic response of aquifers, I will use closed basin aquifers such as those in the Basin and Range of Nevada as the prototypes. The aquifer geometry is illustrated in plan view in Figure 2. The basin is approximately 50 miles in length by 25 miles in width. At the upper end of the valley, two streams emerge from the nearby mountains and recharge the aquifer at an average combined rate of 100 cfs; approximately 70,000 acre-feet annually. At the lower end of the valley, an area of phreatophyte vegetation discharges ground water as ET at an average rate of 100 cfs. The system before development is in balance; 100 cfs is being recharged, and 100 cfs is being discharged by ET.

Table 1
Aquifer Properties for Our Hypothetical
Basin and Range Aquifers

Basin size	50×25 miles (Figure 2)
Cell dimensions	1×1 mile
Hydraulic conductivity	0.0005 and 0.00025 ft/sec
Saturated thickness	2000 ft
transmissivity	1.0 and 0.5 ft²/sec (approx-imately 90,000 and 40,000 ft²/day—both highly trans-missive)
Storage coefficient	0.1%–10% specific yield
Phreatophyte area	170 mi²
Average consumption	100 cfs
Wellfield area	30 mi²
Average pumping	100 cfs
Recharge	100 cfs
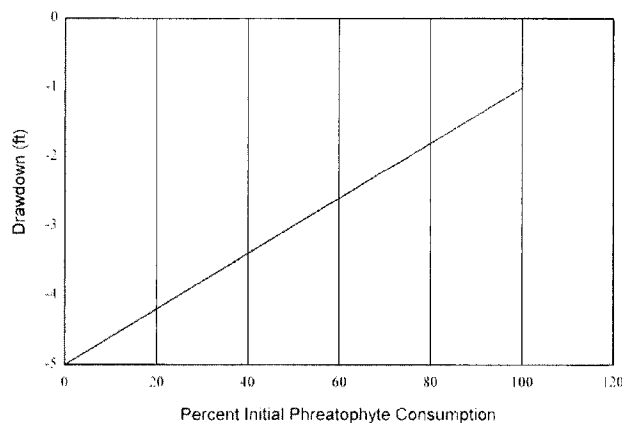
Figure 3. Linear function relating phreatophyte use to draw-down in the aquifer.

Figure 4. Plots of phreatophyte use vs. time.
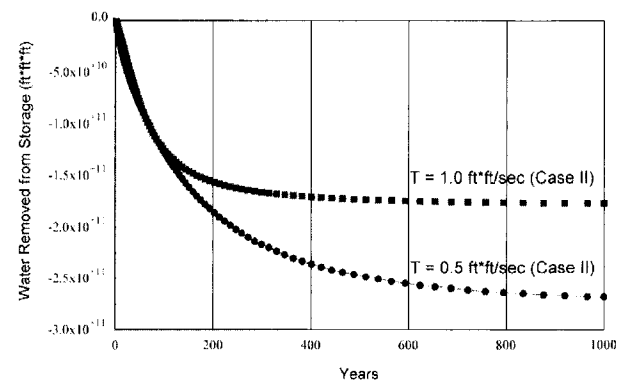
Figure 5. Plots of the change in storage vs. time.

To simulate a well development in this aquifer, I will make the size of the development equal to the recharge (and the discharge) 100 cfs. We consider two locations for our wellfield, shown as Case I and Case II in Figure 2. The Case II wellfield is closer to the area of phreatophyte veg-etation. To simulate the system, we need aquifer properties; the aquifer properties are specified in Table 1.

In our hypothetical system, we will eliminate phreato-phyte ground water consumption as the pumping lowers the water table in the area containing phreatopyhtes. I deliberately created a ground water system in which cap-ture of ET can occur. A linear function is used to cut off the phreatophyte consumption. As the water table drops from 1 to 5 feet, we linearly reduce the phreatophyte use of ground water—the function is shown in Figure 3. The reduction in phreatophyte use does not start until the ground water declines 1 foot; by the time the water table drops 5 feet, the phreatophyte use is eliminated in that cell. The phreatopy-
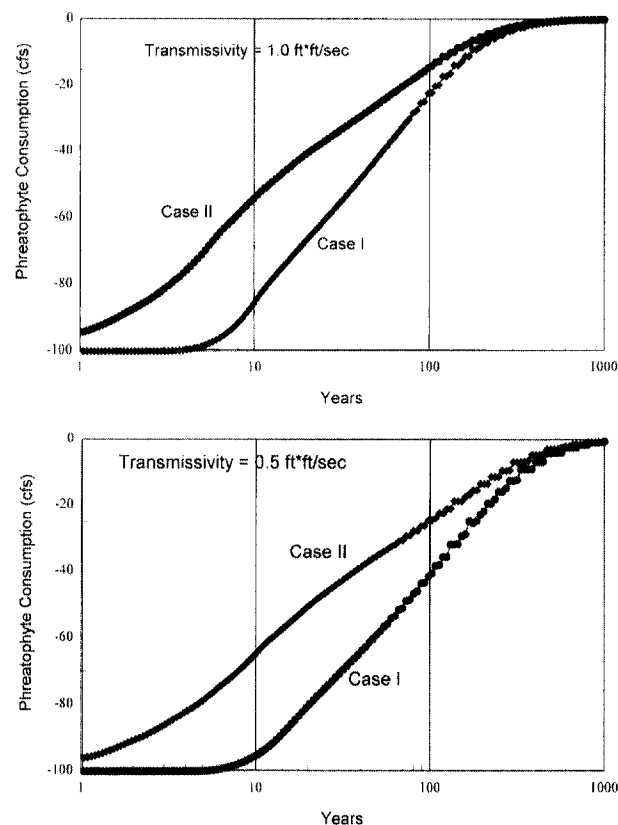
hte reduction function is applied cell by cell in the model.

For this system to reach a new state of sustainable yield, the phreatophyte consumption must be eliminated entirely. Using the model, we can examine the phreato-phyte use as a function of time. Figure 4 is a plot of the phreatophyte use in our system versus time since pumping was initiated. I have considered two transmissivities for the hypothetical system (1.0 and 0.5 ft²/sec); both are high transmissivities. In the higher transmissivity aquifer, the phreatophyte consumption is very small after 400 years; in other words, the system has reached a new steady state in approximately 400 years. The new steady state is a sustain-able development. In the lower transmissivity case, it takes approximately 900 to 1000 years for the phreatophyte con-sumption to be become very small.

In both aquifers, the phreatophytes are impacted faster where the pumping is closer to the phreatopytes (Case II). The point of considering Cases I and II is to show that the location of the pumping makes a difference in the dynamic response of the system. Most individuals, even trained hydrogeologists, are surprised at how slowly a water-table ground water system, like both the two systems simulated, responds to development.

We can look at the output from the model another way by examining the total amount of water removed from storage in our aquifers (Figure 5). In the high transmissivity aquifer, the amount of water removed from storage stabilizes in ~400 to 500 years, indicating we have reached a new steady state. Figure 5 shows that something of the order of 10^{11} cubic feet (approximately 3 million acre-feet) of water has been permanently removed from storage as the system changed to reach this new steady-state condition. This illustrates the important point that water must be removed from storage to reach a new steady state (sustainable) condition. In the lower transmissivity aquifer, water is still being removed from storage at 1000 years, and we have not yet reached a new steady state. In the lower transmissivity aquifer, ~5.7 million acre-feet of water have been removed from storage in 1000 years of pumping. Figure 5 again illustrates how slowly a water table aquifer responds.

It is important to notice that, even though the two developments (Case I and Case II) are equal in size, the aquifer responds differently depending on where the developments are sited. This again emphasizes the importance of studying the dynamics of the aquifer response: the response is different depending on where the development is located.

This example of our rather simple basin and range aquifer illustrates the importance of understanding the dynamics of aquifer systems. Again, while this is a simple example, the principles illustrated apply to aquifers everywhere. It is the rate at which the phreatopyte consumption can be captured that determines how this system reaches sustainability; this is a dynamic process. Capture always entails the dynamics of the aquifer system.

Conclusions

The idea that knowing the recharge (by which one generally means the virgin rate of recharge) is important in determining the size of a sustainable ground water development is a myth. This idea has no basis in fact.

The important entity in determining how a ground water system reaches a new equilibrium is capture. How capture occurs in an aquifer system is a dynamic process. For this reason, hydrologists are occupied in studying aquifer dynamics. The principal tool for these investigations is the ground water model.

These ideas are not new; Theis spelled them out in 1940. Somehow the ground water community seems to lose sight of these fundamental principles.

References

Alley, W.M., T.E. Reilly, and O.L. Frank. 1999. Sustainability of ground-water resources. U.S. Geological Survey Circular 1186.

Bredehoeft, J.D., S.S. Papadopulos, and H.H. Cooper Jr. 1982. The water budget myth. In *Scientific Basis of Water Resource Management*, Studies in Geophysics, 51–57. Washington, D.C.: National Academy Press.

Brown, R.H. 1963. The cone of depression and the area of diversion around a discharging well in an infinite strip aquifer subject to uniform recharge. U.S. Geological Survey Water-Supply Paper 1545C.

Jacob, C.E. 1940. On the flow of water in an elastic artesian aquifer. *Transactions of American Geophysical Union*, part 2: 585–586.

Sophocleous, M. 2000. From safe yield to sustainable development of water resources: The Kansas experience. *Journal of Hydrology* 235, 27-43.

Theis, C.V. 1935. The relation between lowering the piezometric surface and the rate and duration of discharge of a well using ground water storage. *Transactions of American Geophysical Union*, 16th annual meeting, part 2, 519–524.

Theis, C.V. 1940. The source of water derived from wells: Essential factors controlling the response of an aquifer to development. *Civil Engineer* 10, 277-280.

Appendix

Conversion of Relevant Units—English versus Metric

1 foot	=	0.305 m
1 mile	=	1.61 km
1 square foot	=	0.0929 m^2
1 square mile	=	2.59 km^2
1 acre-foot	=	1234 m^3
1 cubic foot per second (cfs)	=	0.0283 m^3/sec