Week 2

Box Model- Hand-Built MODFLOW

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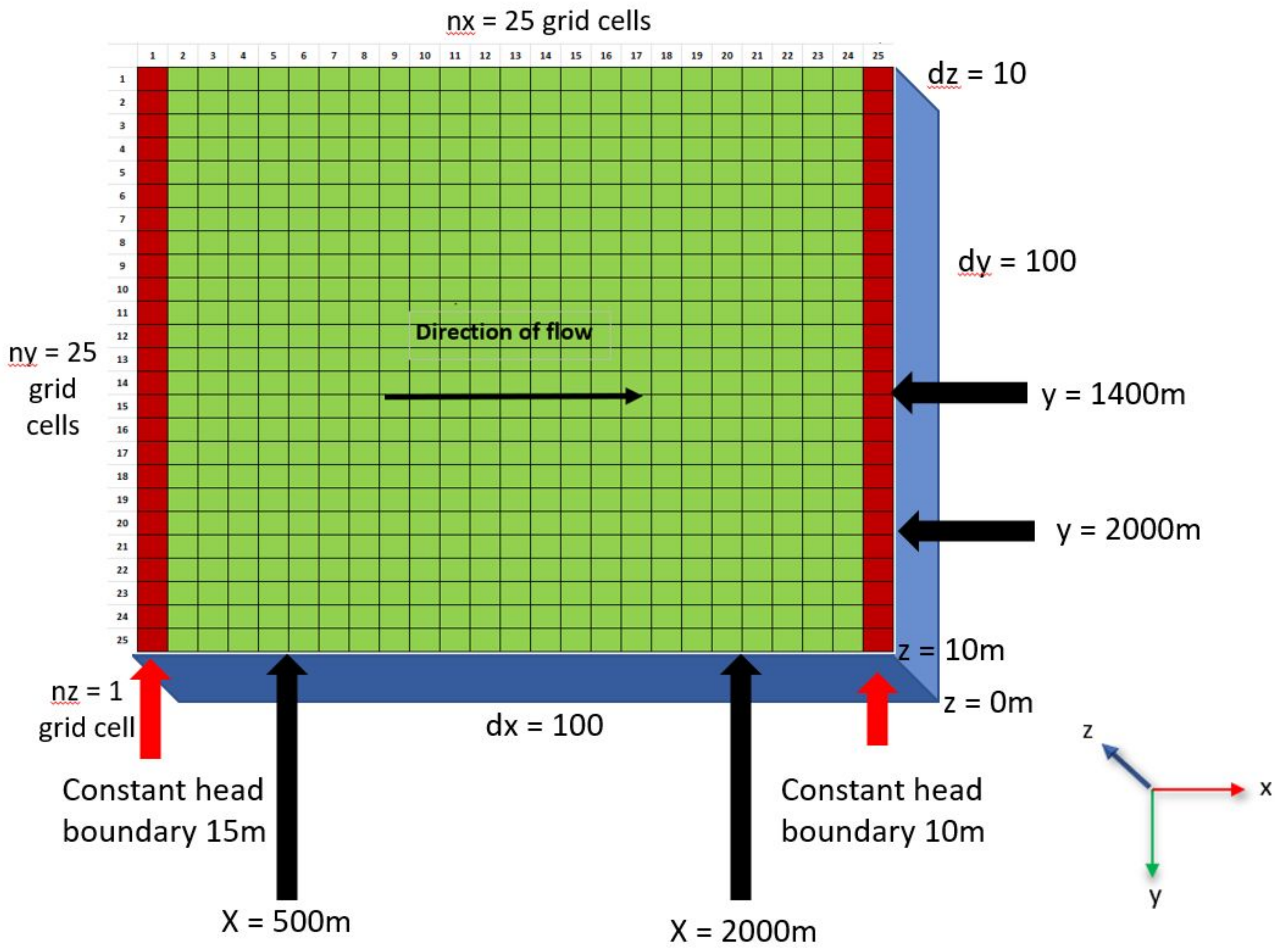
The Challenge #1

Create a conceptual model of the homogenous MODFLOW model: This should be an illustration that shows the locations and values of constant head boundaries, the number of grid cells and their spacing as well as any other model properties. You should also include in here a cross section with your predicted head gradient and direction of flow. You can draw this by hand if you would like.


nx = 25 grid cells
dz = 10
dy = 100
Direction of flow
ny = 25 grid cells
y = 1400m
y = 2000m
z = 10m
z = 0m
nz = 1 grid cell
dx = 100
Constant head boundary 15m
Constant head boundary 10m
X = 500m
X = 2000m
z
x
y

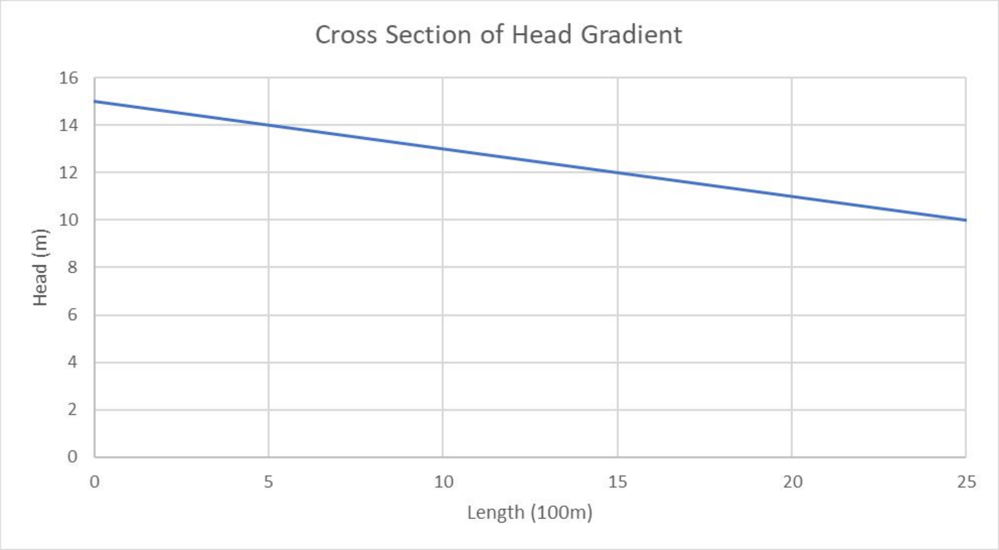
Hydraulic Head Gradient = dH/dl

(15m - 10m)/(0-2500m)

dH/dl = -0.002

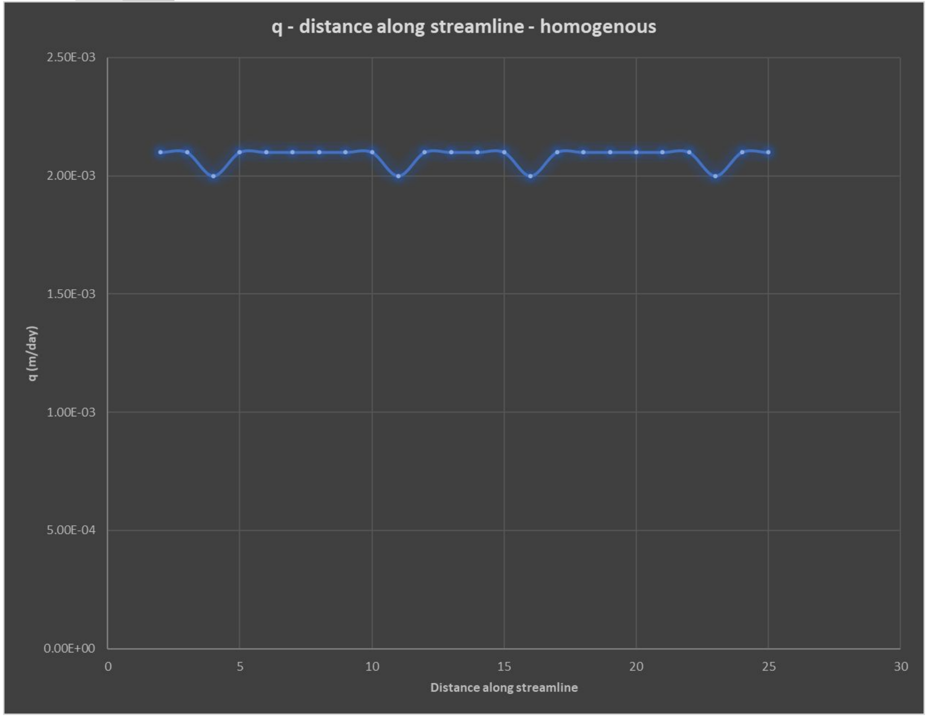
q = -K (dH/dl) ; K = 1m/d

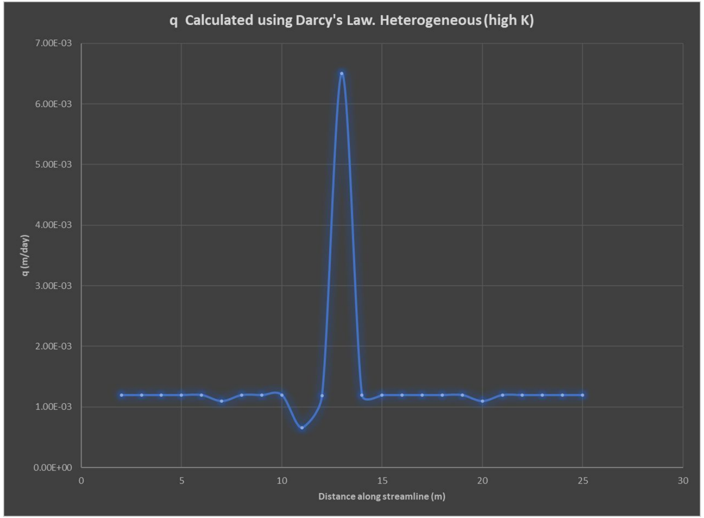
q = 0.002 m/d

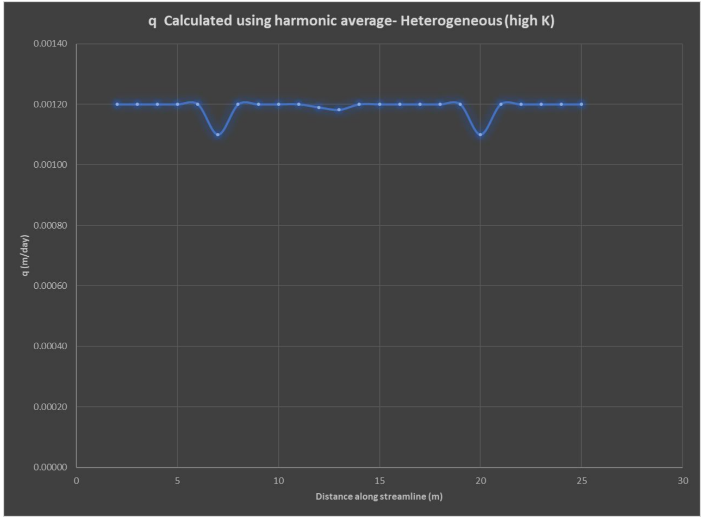


The Challenge #2

Show, based on the flux with horizontal distance from a constant head boundary, that the model is steady state. Repeat this for a homogenous and a heterogenous cases where you place different K values in series in the direction of flow (Note: to modify the K values you should change the .bcf file, just be careful because spacing matters! Note 2: see the excel sheet for an example calculating flux. Keep in mind that that heads are calculated at the center of a cell and the K values are defined across the entirety of a cell)

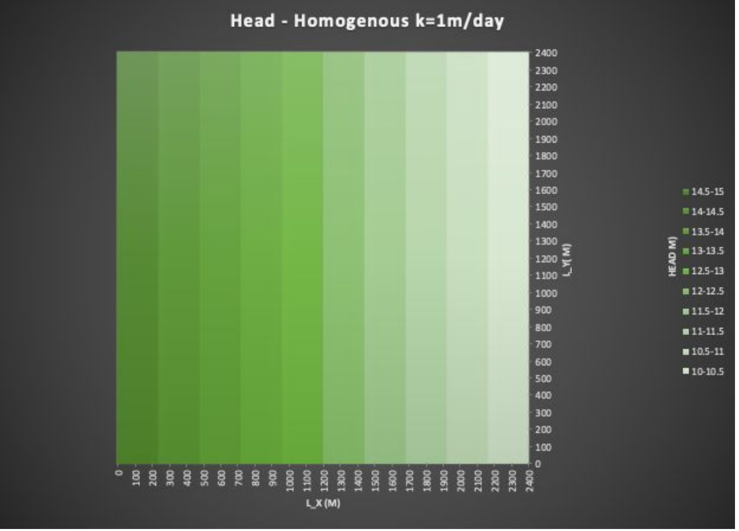

q - distance along streamline - homogenous
2.50E-03
2.00E-03
1.50E-03
1.00E-03
5.00E-04
0.00E+00
q (m/day)
0
5
10
15
20
25
30
Distance along streamline

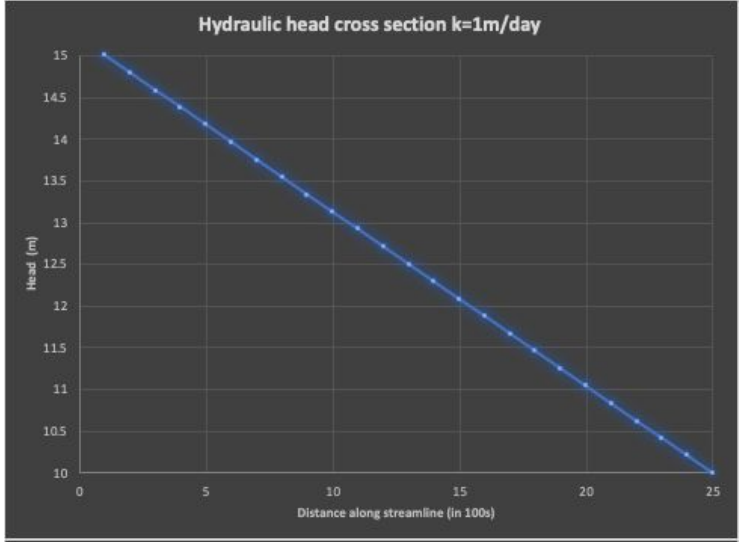

q Calculated using Darcy's Law. Heterogeneous (high K)
7.00E-03
6.00E-03
5.00E-03
4.00E-03
3.00E-03
2.00E-03
1.00E-03
0.00E+00
q (m/day)
0
5
10
15
20
25
30
Distance along streamline (m)

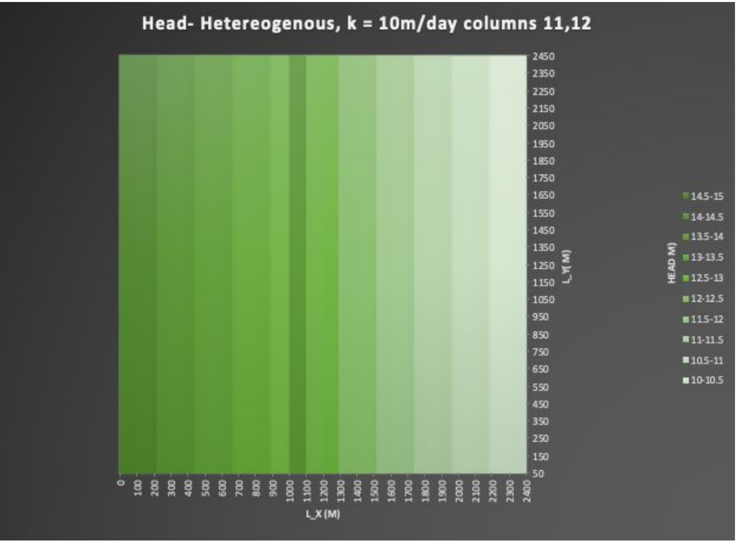

q Calculated using harmonic average- Heterogeneous (high K)
0.00140
0.00120
0.00100
0.00080
0.00060
0.00040
0.00020
0.00000
q (m/day)
0
5
10
15
20
25
30
Distance along streamline (m)

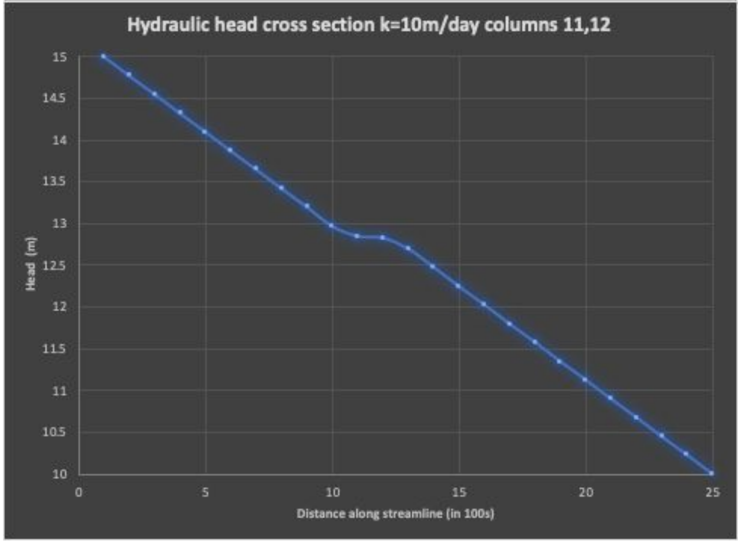
The Challenge #3

Show the steady state head contour in plan view for the homogeneous and heterogeneous (zones in series) condition. Use this plot to defend a contention that flow is 1D. Then, drawing on your first assignment, use the results to explain WHY the equivalent hydraulic conductivity, Keq, is closer to the lower of the two K values.


Head - Homogenous k=1m/day
L_X (M)
L_Y(M)
HEAD M)
14.5-15
14-14.5
13.5-14
13-13.5
12.5-13
12-12.5
11.5-12
11-11.5
10.5-11
10-10.5


Hydraulic head cross section k=1m/day
Head (m)
Distance along streamline (in 100s)


Head- Hetereogenous, k = 10m/day columns 11,12
L_X (M)
L_Y(M)
HEAD M)
14.5-15
14-14.5
13.5-14
13-13.5
12.5-13
12-12.5
11.5-12
11-11.5
10.5-11
10-10.5


Hydraulic head cross section k=10m/day columns 11,12
Head (m)
Distance along streamline (in 100s)

How is flow 1D?

- Assume no flow y or z direction
- Saturated conditions
- Dimensional analysis

Modflow output = no change in head values in the y direction, only along the (x) horizontal direction. The z layer is only 1 unit deep so no flow is possible

		1	2	3	4
		0	100	200	300
1	0	15	14.79	14.58	14.38
2	100	15	14.79	14.58	14.38
3	200	15	14.79	14.58	14.38
4	300	15	14.79	14.58	14.38
5	400	15	14.79	14.58	14.38
6	500	15	14.79	14.58	14.38
7	600	15	14.79	14.58	14.38
8	700	15	14.79	14.58	14.38
9	800	15	14.79	14.58	14.38
10	900	15	14.79	14.58	14.38
11	1000	15	14.79	14.58	14.38
12	1100	15	14.79	14.58	14.38
13	1200	15	14.79	14.58	14.38
14	1300	15	14.79	14.58	14.38
15	1400	15	14.79	14.58	14.38
16	1500	15	14.79	14.58	14.38
17	1600	15	14.79	14.58	14.38
18	1700	15	14.79	14.58	14.38
19	1800	15	14.79	14.58	14.38
20	1900	15	14.79	14.58	14.38
21	2000	15	14.79	14.58	14.38
22	2100	15	14.79	14.58	14.38
23	2200	15	14.79	14.58	14.38
24	2300	15	14.79	14.58	14.38
25	2400	15	14.79	14.58	14.38

SPECIFIC DISCHARGE, flux

$q = \frac{Q}{A}$ $\quad Q = L^3/T \quad A = L^2$

$$q = \frac{L^3/T}{L^2} = \frac{L}{T}$$

$q = -K\frac{dh}{dL} \quad -K = \frac{L}{T} \quad dL = L \quad q = \frac{L}{T}$

$$dh = \frac{q \cdot dL}{-K} = \frac{\cancel{\frac{L}{T}} \cdot L}{\cancel{\frac{L}{T}}}$$

$$\boxed{dh = L = 1D}$$

Use the results to explain WHY the equivalent hydraulic conductivity, Keq, is closer to the lower of the two K values.

Using a simple scenario we can illustrate how smaller k values influence the overall value of Keq to be skewed towards the smaller k value.

Consider a model with 20 cells, there are 10 cells with k value= 10 and 10 cells where k = .1 . To calculate Keq we use this harmonic average

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}} = \left(\frac{\sum_{i=1}^{n} x_i^{-1}}{n} \right)^{-1}$$

20 / ((10/10) +(10/.1)) = 20 / ((1+ 100)) = Keq = 0.198

A more extreme example where only one cell out of 20 = .1 :

20 / ((19/10) +(1/.1)) = 20 / ((1.9 + 10)) = Keq = 1.68

The Challenge #4

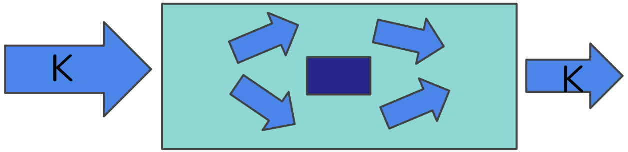
Build a model based on a homogeneous domain with a square region of lower K in the middle of the domain. What can you learn based on your explanation of what controls the effective K for a 1D flow system now that you are applying it to a 2D system? What do you think the Keq of this entire system would be compared to the high and low K values? Explain why it is much more difficult to develop a direct solution for this 2D system than it was for a 1D system (including the zones placed in series).

What can you learn based on your explanation of what controls the effective K for a 1D flow system now that you are applying it to a 2D system? What do you think the Keq of this entire system would be compared to the high and low K values?

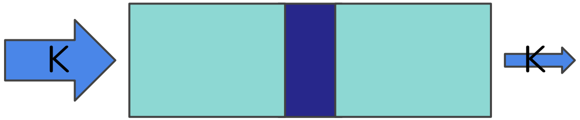
For Discussion

It's possible Keq would skew towards the high K value when we create a square blockage.

2D Flow



1D Flow



Another perspective:

Keq in 2d will **still** be skewed towards the smaller k value based on the harmonic average equation:

										Keq
1	1	1	1	1	1	1	1	1	1	0.40983607
1	1	1	1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	
1	1	1	0.1	0.1	0.1	0.1	1	1	1	
1	1	1	0.1	0.1	0.1	0.1	1	1	1	
1	1	1	0.1	0.1	0.1	0.1	1	1	1	
1	1	1	0.1	0.1	0.1	0.1	1	1	1	
1	1	1	1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	

										Keq
1	1	1	1	1	1	1	1	1	1	0.73529
1	1	1	1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	
1	1	1	0.1	0.1	1	1	1	1	1	
1	1	1	0.1	0.1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	
1	1	1	1	1	1	1	1	1	1	

Freeze and Cherry, 1979

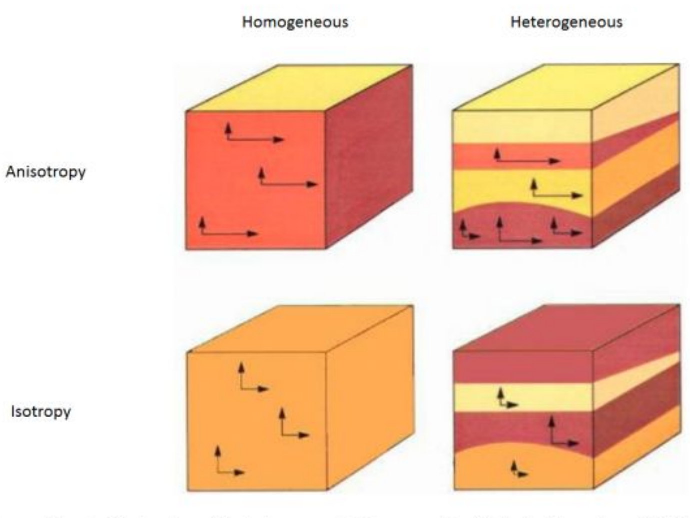


Figure 1.1: An illustration of anisotropy and heterogeneity (Ikelle & Amundsen, 2005)

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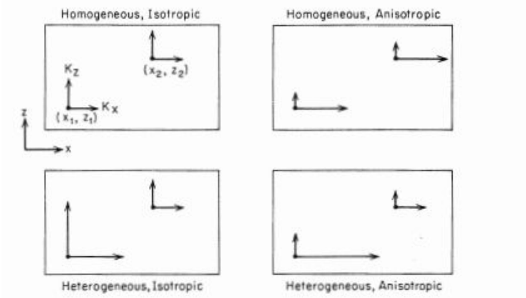

Figure 2.8 Four possible combinations of heterogeneity and anisotropy.

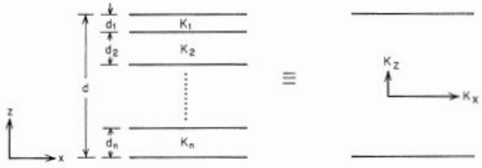

Figure 2.9 Relation between layered heterogeneity and anisotropy.

Explain why it is much more difficult to develop a direct solution for this 2D system than it was for a 1D system (including the zones placed in series).

Consider an arbitrary flowline in the xz plane in a homogeneous, anisotropic medium with principal hydraulic conductivities K_x and K_z [Figure 2.10(a)]. Along the flowline

$$v_s = -K_s \frac{\partial h}{\partial s} \tag{2.35}$$

where K_s is unknown, although it presumably lies in the range $K_x - K_z$. We can separate v_s into its components v_x and v_z, where

$$\begin{aligned} v_x &= -K_x \frac{\partial h}{\partial x} = v_s \cos\theta \\ v_z &= -K_z \frac{\partial h}{\partial z} = v_s \sin\theta \end{aligned} \tag{2.36}$$

Now, since $h = h(x, z)$,

$$\frac{\partial h}{\partial s} = \frac{\partial h}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial h}{\partial z} \cdot \frac{\partial z}{\partial s} \tag{2.37}$$

Geometrically, $\partial x/\partial s = \cos\theta$ and $\partial z/\partial s = \sin\theta$. Substituting these relationships together with Eqs. (2.35) and (2.36) in Eq. (2.37) and simplifying yields

$$\frac{1}{K_s} = \frac{\cos^2\theta}{K_x} + \frac{\sin^2\theta}{K_z} \tag{2.38}$$

This equation relates the principal conductivity components K_x and K_z to the resultant K_s in any angular direction θ. If we put Eq. (2.38) into rectangular coordinates by setting $x = r\cos\theta$ and $z = r\sin\theta$, we get

$$\frac{r^2}{K_s} = \frac{x^2}{K_x} + \frac{z^2}{K_z} \tag{2.39}$$

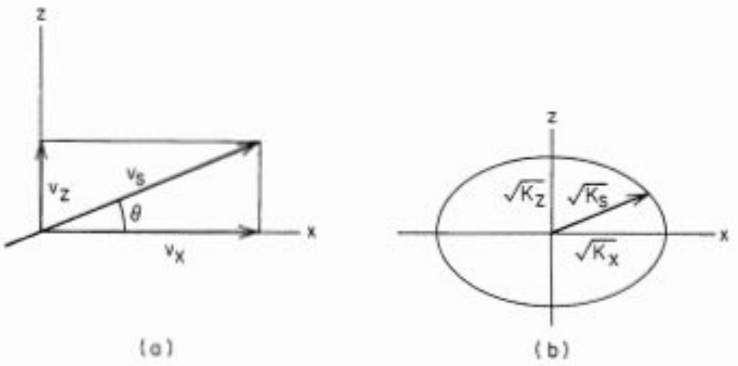

Figure 2.10 (a) Specific discharge v_s in an arbitrary direction of flow. (b) Hydraulic conductivity ellipse.

There is more to account for in 2D, leading to more difficulty for calculating direct solutions

The Challenge #5

For steady state conditions, there are equivalent Type I and Type II boundary conditions. What would the Type II boundary condition be that would result in the same equipotentials for the first model? What is the value of the constant flux? What about the second model? What are the values of the constant flux on the left and right boundaries? What is fundamentally different about the equivalent Type II boundary for the third model compared to the first two?

Type 2 = constant flux

Type 1 = constant head

1st Model

-Constant flux boundaries on both sides would produce the same equipotential profile as the constant head boundary condition run of our model

The left and right q values would both be 2.1 E-3 m/day

2nd Model

-Constant flux would also produce the same equipotential profile in this case, however the flux needed will differ from the 1st model, but remains identical on both the left and right.

The left and right q value would both be 1.2 E-3 m/day for higher K layer scenario

3rd Model

-Constant flux would work in this case as well, but depending on where the profile is taken in the model space, the left and right boundary may not have the same constant flux

If taken through the center of the square, q_{left}= 5 E-4 m/day, q_{right}= 1.5 E-3 m/day

If taken through the edge of our model area, q becomes 2.1 E-3 on both sides, just as in the homogenous case