Jen Steyaert
GW Modeling HW #1

The Challenge

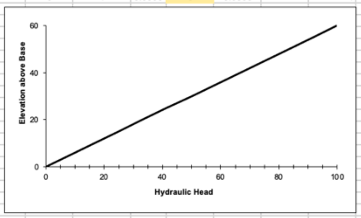
1. Show, based on the flux with depth, that the model is steady state. Repeat this for a homogeneous and for a heterogeneous column.

For the model to be steady state, we would want flux in to be equal to flux out at every discretization as well as across the full length of the model. For a k = 0.0003, we can see below that the q at every layer is equal to 0.0005 and that the overall q is also equal to 0.0005 for a homogeneous base:

Direct solution for flux		
	K	num cells
zone 1	0.0003	12
zone 2	0.01	0
zone 3	0.0001	0
Keq	0.0003	
q	0.0005	

z	K zone cell	K cell	H	q		zone 1	zone 2	zone 3
60	1	0.0003	100			1	0	0
55	1	0.0003	91.66942	0.0005		1	0	0
50	1	0.0003	83.33846	0.0005		1	0	0
45	1	0.0003	75.00701	0.0005		1	0	0
40	1	0.0003	66.67496	0.0005		1	0	0
35	1	0.0003	58.34226	0.0005		1	0	0
30	1	0.0003	50.00893	0.0005		1	0	0
25	1	0.0003	41.675	0.0005		1	0	0
20	1	0.0003	33.34055	0.0005		1	0	0
15	1	0.0003	25.00569	0.0005		1	0	0
10	1	0.0003	16.67055	0.0005		1	0	0
5	1	0.0003	8.335277	0.0005		1	0	0
0	1	0.0003	0	0.0005		1	0	0

Map of node and cell numbers			
	node	cell	
1	-		1
2	-		2
3	-		3
4	-		4
5	-		5
6	-		6
7	-		7
8	-		8
9	-		9
10	-		10
11	-		11
12	-		12
13	-		13



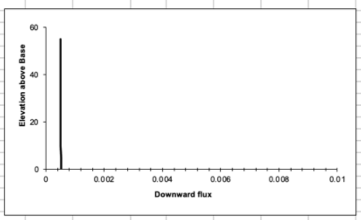
The Challenge:

Create a 1D, vertical steady state model with constant head top and bottom boundaries.

Show, based on the flux with depth, that the model is steady state.
Repeat this for a homogeneous and for a heterogeneous column.

Show that the steady state flux agrees with the direct calculation based on the harmonic mean average K.

Show the steady state head profile for a column with approximately equal-thickness layers with different K values.
Use this profile to explain why the equivalent hydraulic conductivity, Keq, is closer to the lower of the K values.

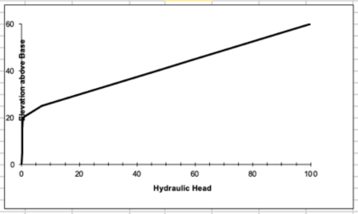


For the heterogeneous mixture, we can see that the flux with depth is also consistent:

Direct solution for flux		
	K	num cells
zone 1	0.0003	7.5
zone 2	0.01	0
zone 3	0.05	4.5
Keq	0.000478	
q	0.000797	

z	K zone cell	K cell	H	q	zone 1	zone 2	zone 3
60	1	0.0003	100		1	0	0
55	1	0.0003	86.71642	0.000797	1	0	0
50	1	0.0003	73.43223	0.000797	1	0	0
45	1	0.0003	60.14734	0.000797	1	0	0
40	1	0.0003	46.86175	0.000797	1	0	0
35	1	0.0003	33.57559	0.000797	1	0	0
30	1	0.0003	20.28906	0.000797	1	0	0
25	1	0.0003	7.002364	0.000797	1	0	0
20	3	0.05	0.319117	0.000797	0	0	1
15	3	0.05	0.239362	0.000798	0	0	1
10	3	0.05	0.159582	0.000798	0	0	1
5	3	0.05	0.079791	0.000798	0	0	1
0	3	0.05	0	0.000798	0	0	1

Map of node and cell numbers			
	node	cell	
1	-		1
2	-		2
3	-		3
4	-		4
5	-		5
6	-		6
7	-		7
8	-		8
9	-		9
10	-		10
11	-		11
12	-		12
13	-		13



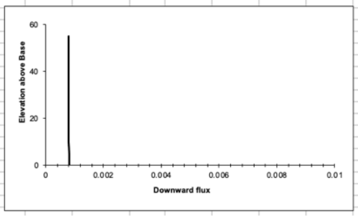
The Challenge:

Create a 1D, vertical steady state model with constant head top and bottom boundaries.

Show, based on the flux with depth, that the model is steady state.
Repeat this for a homogeneous and for a heterogeneous column.

Show that the steady state flux agrees with the direct calculation based on the harmonic mean average K.

Show the steady state head profile for a column with approximately equal-thickness layers with different K values.
Use this profile to explain why the equivalent hydraulic conductivity, Keq, is closer to the lower of the K values.



2. Show that the steady state flux agrees with the direct calculation based on the harmonic mean average K. Write the equation defining the direct calculation of the flux.
   Please see above figure to show that the steady state flux agrees with the direct calculation. The equation for the harmonic mean is:

$$K_{eff} = \left(sum(number_{allcells})\right) / \left(\left(\frac{n_1}{k1}\right) + \left(\frac{n_2}{k_2}\right) + \frac{n_3}{k_3}\right))$$

$$Therefore\ the\ equation\ for\ the\ total\ flus\ is:$$

$$q_{total} = -K_{eff} * \frac{dH}{dL}$$

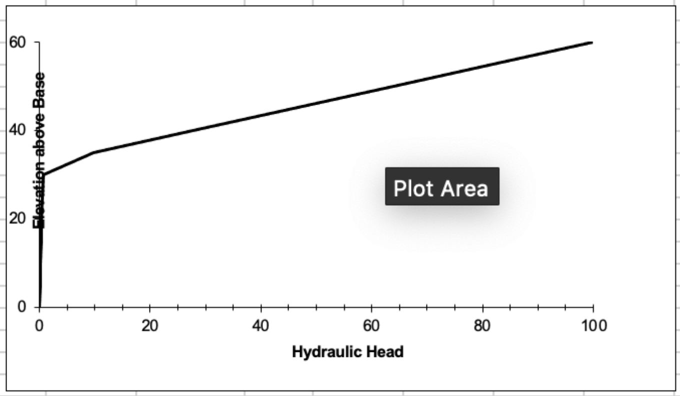
   Where dH is the total head (top- bottom) and dL is the total length (top-bottom).

3. Show the steady state head profile for a column with approximately equal-thickness layers that have different K values.

Direct solution for flux

	K	num cells
zone 1	0.0003	5.5
zone 2	0.01	0
zone 3	0.05	6.5
Keq	0.00065	
q	0.001083	

z	K zone cell	K cell	H	q	zone 1	zone 2	zone 3
60	1	3E-04	100		1	0	0
55	1	3E-04	81.9478	0.001083	1	0	0
50	1	3E-04	63.89472	0.001083	1	0	0
45	1	3E-04	45.84082	0.001083	1	0	0
40	1	3E-04	27.78637	0.001083	1	0	0
35	1	3E-04	9.731732	0.001083	1	0	0
30	3	0.05	0.650226	0.001083	0	0	1
25	3	0.05	0.541879	0.001083	0	0	1
20	3	0.05	0.433518	0.001084	0	0	1
15	3	0.05	0.325146	0.001084	0	0	1
10	3	0.05	0.216766	0.001084	0	0	1
5	3	0.05	0.108383	0.001084	0	0	1
0	3	0.05	0	0.001084	0	0	1



4. Use the head profile to explain WHY the equivalent hydraulic conductivity, Keq, is closer to the lower of the two K values.

Since K is defined as the ability of water to flow through porous medium, the flow and flux will be dependent on the K values. For our column, we have a low K value on top of a high K value. In looking at the hydraulic head across the column, we see that more head is lost in the low K area compared to the high K area. This is because our system is at steady state and the flux entering and leaving each cell needs to be the same. Since the lower K value has a stronger impact on the hydraulic head of the system, the Keq should skew towards the lower K value.