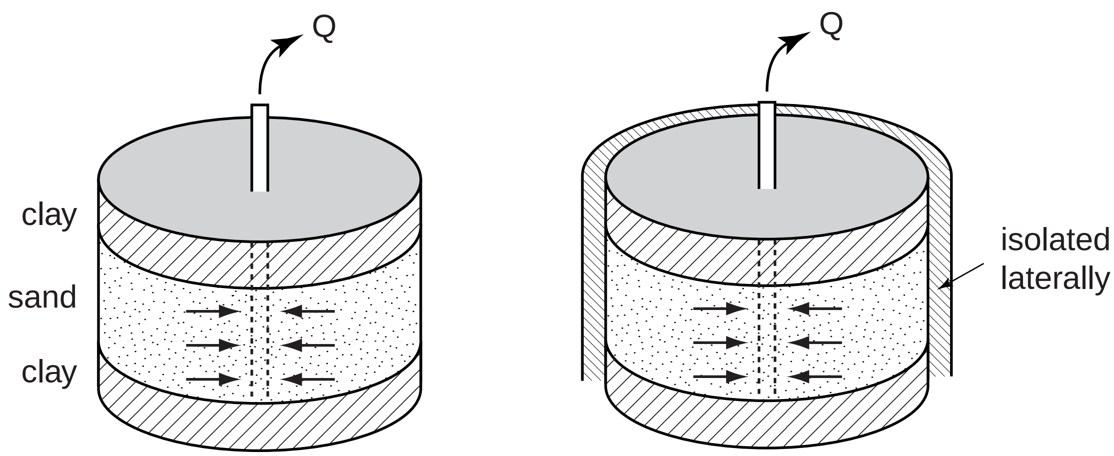
Read Freeze & Cherry, Ch. 2.8, 2.9, 2.10

PART 6 Storage of water

We will use the usual mass balance in a reservoir: Qin - Qout = Δ storage

Look at a pumping well in a confined aquifer (Figure below). If the aquifer is unbounded on the sides (that is, if it is confined on top and bottom, but not on the sides), water comes from the sides (we call this flux of water lateral flow). Now look at a totally isolated system on all sides. Pumped water comes from storage (Δ storage < 0).



isolated only on top and bottom

isolated on top and bottom, and on the sides

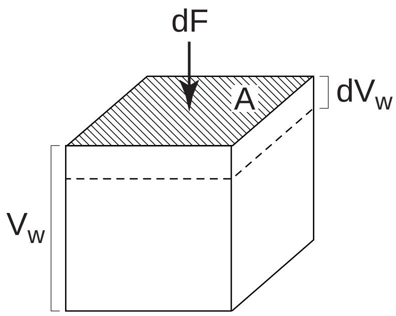
Where does the water come from?

We get water due to compressibility of water and rearrangement of soil particles (recall different packing of clasts in a porous medium).

Specifically, the sources of water are:

(1) water expands;
(2) soil particles expand (negligible source);
(3) matrix consolidates (e.g., grains rearrange).

Expansion of water



Consider initial volume of water V_w. Apply force dF (e.g., weight of rocks above). The result is pressure dP on area A: dP = dF/A

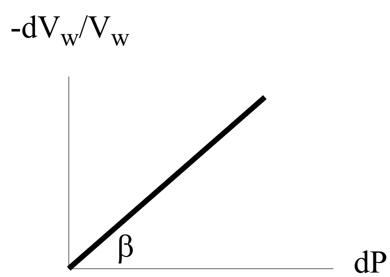
Water is compressed from initial volume V_w by the amount dV_w

$dV_w = -V_w \cdot dP \cdot \beta$

where β is the compressibility coefficient for water (see "Properties and types of water" on page 11; units are those of inverse of pressure, i.e., m^2/N).

Plotting this, we can get β:

$$\beta = -\frac{1}{V_w} \cdot \frac{dV_w}{dP}$$



valid for solid, non-porous materials.

For porous media, we can write:

$dV_w = dV_T$

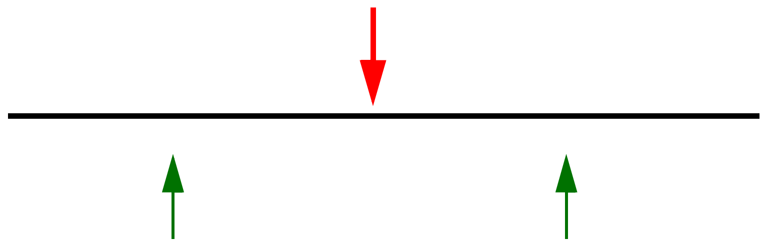
where total volume $V_T = V_{total} = V_{void} + V_{solids}$; therefore, $V_w = V_T \cdot n$

Now we calculate the amount of water from water expansion (hence, double subscript w on the left-hand side of the equation below):

$$dV_{w_w} = -V_T \cdot n \cdot \beta \cdot dP = -V_T \cdot n \cdot \beta \cdot \rho \cdot g \cdot dh$$

Consolidation of matrix

σ = total stress = overburden



P = interstitial pressure
(neutral stress)

σ_e = effective stress
(grain-to-grain stress
or skeleton stress
or matrix transmitted
stress)

Total stress must be supported by pressure and effective stress: $\boldsymbol{\sigma = \sigma_e + P}$

Thus, $\sigma_e = \sigma - P$

And, in terms of differences: $d\sigma_e = d\sigma - dP$

But the total stress does not change (unless new constructions are built or host rock is mined), i.e., $d\sigma = 0$. Thus, we have:

$\boldsymbol{d\sigma_e = - dP}$

Introduce coefficient of compressibility of matrix, α:

$$\alpha = -\frac{1}{V_T} \cdot \frac{dV_T}{d\sigma_e}$$

and

$$dV_T = -\alpha \cdot V_T \cdot d\sigma_e = \alpha \cdot V_T \cdot dP$$

Amount of water produced

$dV_w = -dV_T = \alpha V_T d\sigma_e$

Recall $h = \psi + z$, and assume z = constant (we can always choose an arbitrary datum). Then:

$dh = dp/\rho g$

and thus $dp = \rho g dh$

Substitute dP into dV_w above to get the amount of water from consolidation of matrix

$$dV_{w_m} = -\alpha \cdot V_T \cdot \rho \cdot g \cdot dh$$

Total amount of water produced is the sum of water from consolidation of matrix and from expansion of water, i.e.:

$$dV_w = dV_{w_m} + dV_{w_w}$$

$$dV_w = -\alpha \cdot V_T \cdot \rho \cdot g \cdot dh - \beta \cdot n \cdot V_T \cdot \rho \cdot g \cdot dh$$

$$dV_w = -(\alpha + \beta \cdot n) \cdot V_T \cdot \rho \cdot g \cdot dh$$

→ The above formula is never used. Why?

Define **specific storage** as the volume of water released per unit volume of aquifer per unit decline of hydraulic head:

$$S_S = \frac{dV_w}{V_T \cdot (-dh)}$$

$$S_S = \frac{-(\alpha + n\beta) \cdot V_T \cdot \rho \cdot g \cdot dh}{-V_T \cdot dh}$$

$$\mathbf{S_S = (\alpha + n\beta) \cdot \rho g = (\alpha + n\beta) \cdot \gamma}$$

This is **specific storage coefficient**.

Aquifer storage coefficient or **storativity** (S) is defined as the amount of water released by unit area of aquifer per unit decline of hydraulic head:

$S = Ss \cdot b$ [dimensionless]

where b = thickness of the aquifer

Storativity is simply the amount of water obtained from the vertical section of the aquifer, i.e., it is a **vertically integrated** quantity. Storativity applies only for horizontal flow (1-D or 2-D, but never where there is a vertical component of flow!)

Some values:

water: $\beta = 4.4E\text{-}10\ m^2/N$

clay: $\alpha = 10^{-6}$ to $10^{-8}\ m^2/N$

sand: $\alpha = 10^{-7}$ to $10^{-9}\ m^2/N$

jointed rock: $\alpha = 10^{-8}$ to $10^{-10}\ m^2/N$

solid rock: $\alpha = 10^{-9}$ to $10^{-11}\ m^2/N$

Typical values of Ss: about 3E-6 m^{-1}

Typical values of S: 3E-6 times aquifer thickness

Confined aquifer:

Storativity is from 5E-5 to 1E-3

We see that only very small amount of water is released due to compressibility, but because areas of confined aquifers are large, the total amount can be very large.

Phreatic aquifer:

More water released by unit volume per unit decline of water table. Why?

Pores are drained. In clean sand it may be 30-40% of the total volume that is drainable.

Specific yield = S_y = volume of water drained per unit area of phreatic aquifer per unit decline of water table.

$S_y < n$

$S_y = n$ - specific retention

Specific retention = S_R = amount of water that remains in porous medium after gravity draining, i.e., due to chemistry etc. Specific retention is high in clays, low in sands.

Examples:

	n (%)	S_y (%)	S_R (%)
Clay	40	10	30
Sand	20	16	4
Gravel	25	24	1

Total storage

In phreatic aquifers the total storage term is the sum of elastic storage and specific yield:

$S = S_y + S_S \cdot b$

In confined aquifers the total storage is the elastic storage:

$S = S_S \cdot b$