HOMEWORK #1

The Challenge

1. Show, based on the flux with depth, that the model is steady state. Repeat this for a homogeneous and for a heterogeneous column.

Direct solution for flux

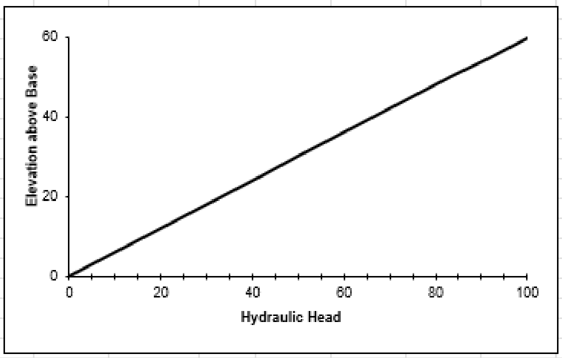
	K	num cells
zone 1	0.02	0
zone 2	0.01	0
zone 3	0.0001	12
Keq	0.0001	
q	0.00017	

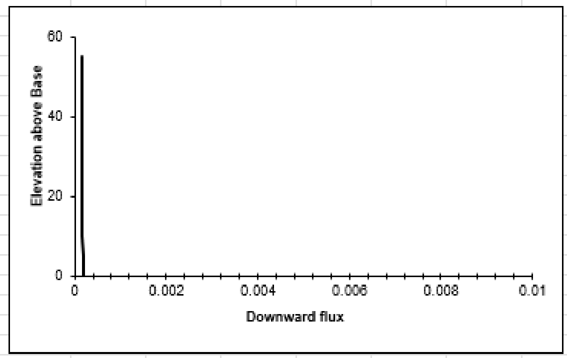
HOMOGENEOUS COLUMN

z	K zone cell	K cell	H	q	zone 1	zone 2	zone 3
60	3	1E-04	100		0	0	1
55	3	1E-04	91.6702	0.00017	0	0	1
50	3	1E-04	83.34	0.00017	0	0	1
45	3	1E-04	75.0091	0.00017	0	0	1
40	3	1E-04	66.6774	0.00017	0	0	1
35	3	1E-04	58.3449	0.00017	0	0	1
30	3	1E-04	50.0115	0.00017	0	0	1
25	3	1E-04	41.6774	0.00017	0	0	1
20	3	1E-04	33.3427	0.00017	0	0	1
15	3	1E-04	25.0074	0.00017	0	0	1
10	3	1E-04	16.6717	0.00017	0	0	1
5	3	1E-04	8.33585	0.00017	0	0	1
0	3	1E-04	0	0.00017	0	0	1

Map of node and cell numbers

	node	cell	
1	-		1
2	-		2
3	-		3
4	-		4
5	-		5
6	-		6
7	-		7
8	-		8
9	-		9
10	-		10
11	-		11
12	-		12
13	-		13





2. Show that the steady state flux agrees with the direct calculation based on the harmonic mean average K. Write the equation defining the direct calculation of the flux.

Darcy's Equation for Groundwater Flux

q = -k * (dH/dl)

Harmonic Hydraulic Conductivity

$$K\ \text{harmonic} = \frac{\sum \text{cells total}}{\sum \frac{\#\text{cells}}{K}}$$

$$K\text{harmonic} = \frac{12}{\left[\frac{4.5}{0.02} + \frac{3.5}{0.01} + \frac{4}{0.0001}\right]}$$

The lower the "K", the bigger the denominator of the entire fraction gets, hence the K harmonic is skewed to the lowest value.

3. Show the steady-state head profile for a column with approximately equal-thickness layers that have different K values.

Direct solution for flux

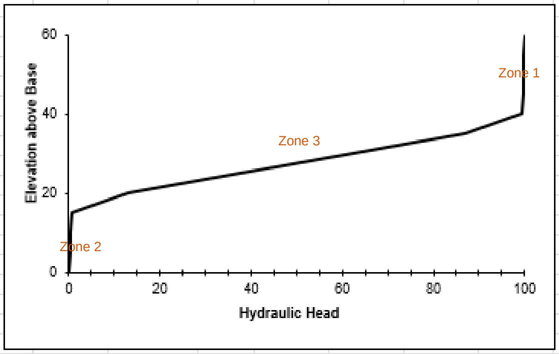
	K	num cells
zone 1	0.02	4.5
zone 2	0.01	3.5
zone 3	0.0001	4
Keq	0.0003	
q	0.00049	

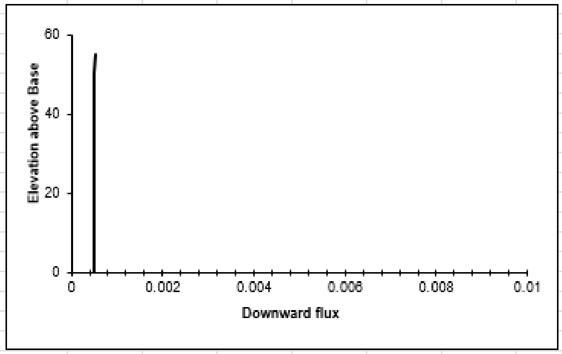
HETEROGENEOUS COLUMN

z	K zone cell	K cell	H	q	zone 1	zone 2	zone 3
60	1	0.02	100		1	0	0
55	1	0.02	99.8751	0.0005	1	0	0
50	1	0.02	99.7506	0.0005	1	0	0
45	1	0.02	99.6268	0.0005	1	0	0
40	1	0.02	99.5036	0.00049	1	0	0
35	3	1E-04	87.1192	0.00049	0	0	1
30	3	1E-04	62.4742	0.00049	0	0	1
25	3	1E-04	37.8295	0.00049	0	0	1
20	3	1E-04	13.1848	0.00049	0	0	1
15	2	0.01	0.73931	0.00049	0	1	0
10	2	0.01	0.49287	0.00049	0	1	0
5	2	0.01	0.24643	0.00049	0	1	0
0	2	0.01	0	0.00049	0	1	0

Map of node and cell numbers

	node	cell	
1	-		1
2	-		2
3	-		3
4	-		4
5	-		5
6	-		6
7	-		7
8	-		8
9	-		9
10	-		10
11	-		11
12	-		12
13	-		13





4. Use the head profile to explain WHY the equivalent hydraulic conductivity, Keq, is closer to the lower of the two K values.

The hydraulic conductivity is the slope of the line. We can see that the line has the same slope all over when there is a homogeneous system. However, when we have heterogeneous conditions, the line changes its slope three times, explaining the three mediums involved with different k values.

Discussion questions:

Q1./ What are boundary conditions? Answer this both conceptually and mathematically.

A/. Conceptual explanation: Boundary conditions are circumstances that let us define the borders of our system to facilitate our calculations; they can be assumptions or facts like a constant head due to the presence of a long lake beside the aquifer or a zero flux due to the aquifer is confined.
Mathematical explanation:
$dh/dz = 0$
$q = 0$

Q2./ What are model parameters? How do they (and don't they) represent the actual subsurface?

A/. Model parameters are values that we use to build a representation of a situation, a model, and they are based on the data we already have. An example of how a parameter represents the subsurface is when we use the hydraulic conductivity to measure how fast the water moves through the medium in Darcy's Law, this parameter is the slope of the curve made by putting the flux in the vertical axis and the change in the head in the horizontal axis.

Q3./ What are steady-state conditions and how can they be identified from the Excel model results?

A/. Steady-state conditions happen when the flow that enters the system is equal to the flow out of the system. In the Excel model results, we can identify them on the graph that shows the volumetric flux as a vertical line; this tells us that no matter the elevation, the flow will stay constant.

Q4./ Can you imagine how the model inputs could be stored in separate files rather than other spreadsheet cells? Describe the flow of information from a file that describes the other files that contain model-specific information about the system.

A/. I think what the model does is that it creates functions that call the other files and process the information within them through operations given to each variable stored.
The flow of information will be like this:

- The program creates a domain to represent the current situation.
- The database with all the properties of the soil, and hydrology is read to use in next operations.
- An equation is reproduced using the values extracted from the database.
- A file is created to save the final result of the operation and store all the variables generated in another file.

Q5./ What is an iterative solution? Can you explain it to a hydrologist who is not a modeler? Can you describe (or imagine) how Excel finds the solution?

A/. An iterative solution is a mathematical operation that is repeated a determined number of times or until it complies with the condition required; the result of this operation is entered again and again into the loop to make the same calculation until the limit of repetitions is reached, or the condition mentioned above is completed.

Q6./ What is a direct solution? What are its (dis)advantages compared to an iterative (numerical) solution?

A/. A direct solution tries to obtain the value of the unknown variable by solving each equation of the system until it gets the result. However, an iterative solution gives a random value to the unknown variable to try to solve the equations that govern the system until the residual complies with the specified tolerance of the model.
The disadvantage of a direct solution is that it can take a long time to actually get a result because it solves the equation for every cell, so it can get computationally expensive.

Glossary questions:

Q1./ What is a model? What is a groundwater model? How are groundwater models used?

A/. A model is a simplified representation of a system or circumstance that tries to be as accurate as possible with reality. A model can be: a conceptual model, a physics-based model, a mathematical model, a computational model.
A groundwater model is a representation of the hydrogeologic system, how the water is stored, or how does it travels through the different layers that compound the soil underneath us.
Models can be used to let us know how the groundwater moves through the geologic layers; it also tells us how fast it moves, how much water is it stored on the medium, how much can we extract from it, or the quality of that water too.

Q2./ What are model parameters? How do they (and don't they) represent the actual subsurface?

A/. Model parameters are values that we use to build a representation of a situation, a model, and they are based on the data we already have. An example of how a parameter represents the subsurface is when we use the hydraulic conductivity to measure how fast the water moves through the medium in Darcy's Law, this parameter is the slope of the curve made by putting the flux in the vertical axis and the change in the head in the horizontal axis.

Q3./ What does it mean for a model to be in steady-state? Discuss your answer with respect to both heads and fluxes. What is the utility of steady-state solutions in the practice of groundwater modeling (i.e., when and why do we use steady-state solutions)?

A/. A steady-state system is the one where the flow in is equal to the flow out, but for that to occur, some conditions need to happen, for example:
If we have a constant flux, the head will vary depending on the position into the domain; for the intention to be the same volumetric flow entering and going out of the system.

Modeling using steady-state conditions can be helpful in modeling because they facilitate the calculations by decreasing the number of unknown variables in the system.

We can use steady-state conditions when we have constant flows entering and going out of the domain, like when we have a vast lake feeding the system with a constant flow and a discharge area on the other side that has the same flow rate going out.