HOMEWORK #2

The Challenge

1. Your conceptual figure of the baseline model in both plan and cross-section view

Conceptual model of the medium

Head = 15m Column identification Head = 10m

Row identification

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	100	200	300	400	500	600	700	700	800	900	1000	1100	1200	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	00
2	200																							
3	300																							
4	400																							
5	500																							
6	600																							
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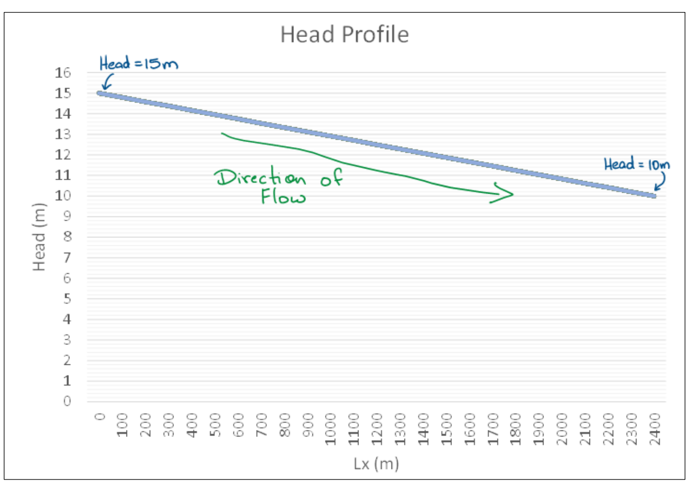
Medium represented by a grid that has 25 rows x 25 columns. Each cell measures 100 m long.

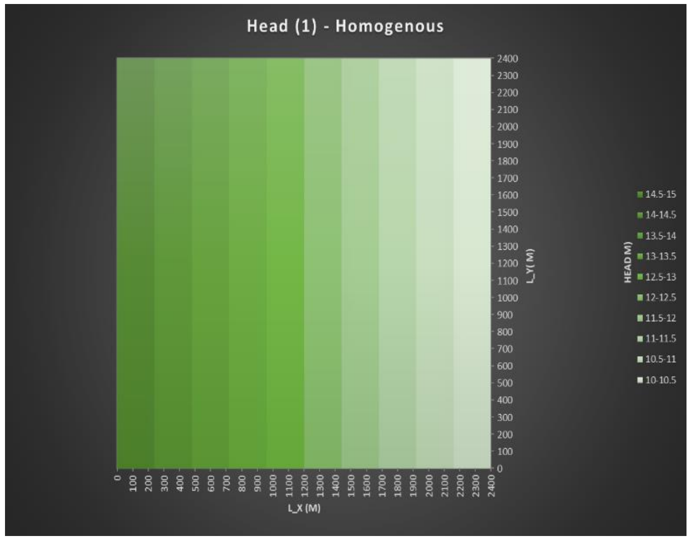
Head Profile View



Head profile of the homogeneous medium, where the flow goes from left to right.

2. Plan view figures of the head distribution for at least 3 cases

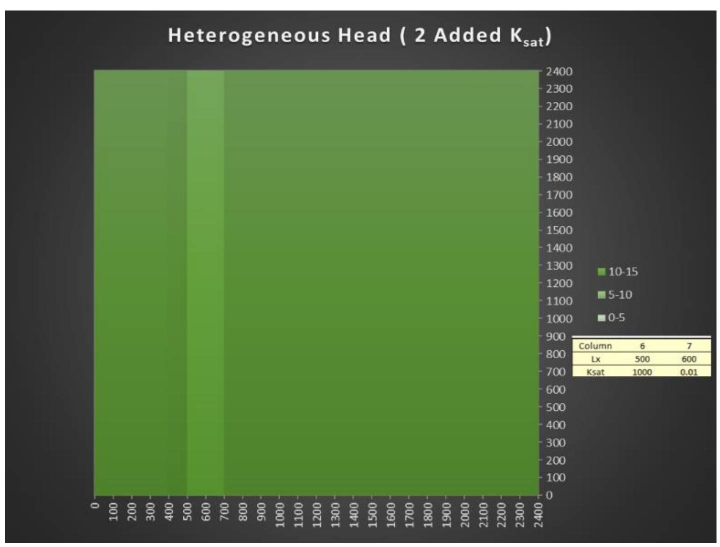
- Homogenous domain



Plain view of the Head of the Homogeneous medium.

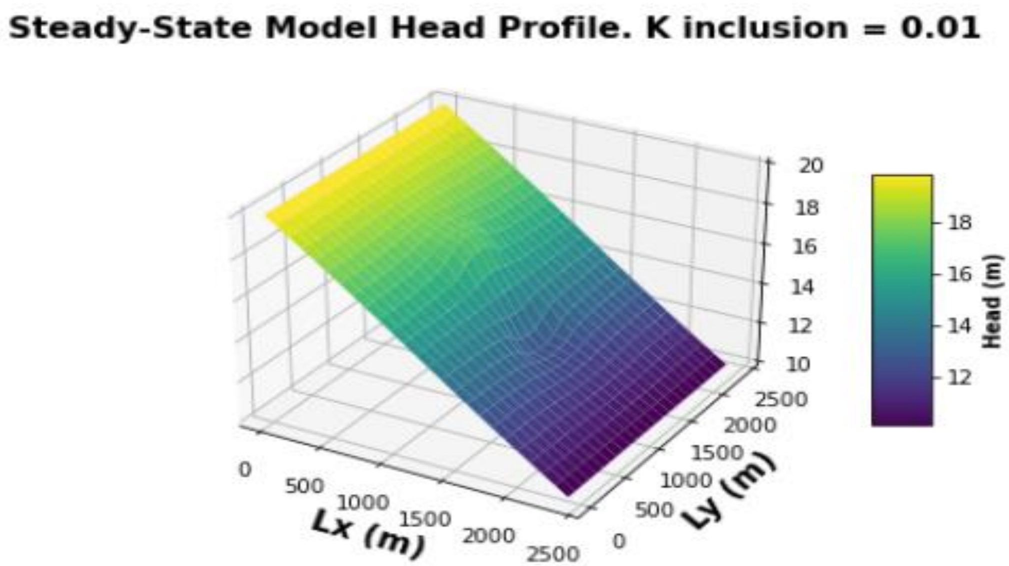
- Heterogenous domain where K varies in the direction of flow

Excel plain view of the Head of the medium with two columns changed their K_{sat} values:
Column 6: K_{sat} = 1000 m/d
Column 7: K_{sat} = 0.01 m/d
Rest of the domain: K_{sat} = 1 m/d



Heterogeneous medium plain videw, with 2 colums incrusted with different Ksat values from the rest.

- Heterogenous domain with a box of low K in the middle of the domain

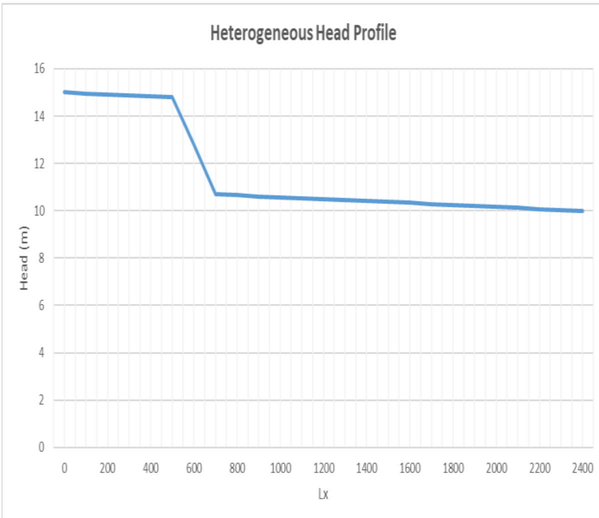


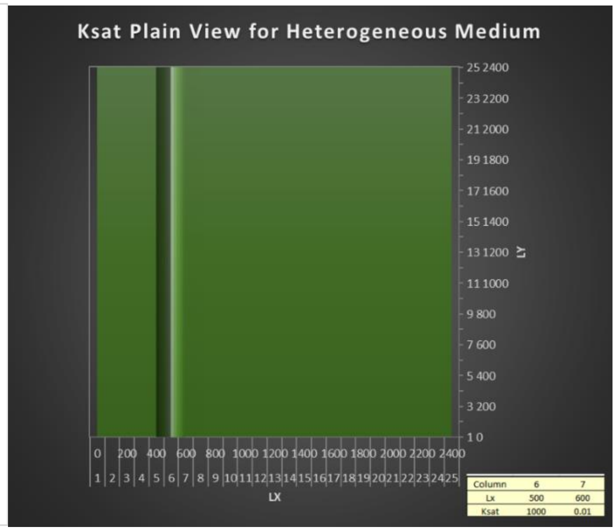
3D Head view using FloPy, about an inclusion of Ksat=0.01m/d in the middle of the domain.

Optional figure you might want to add:

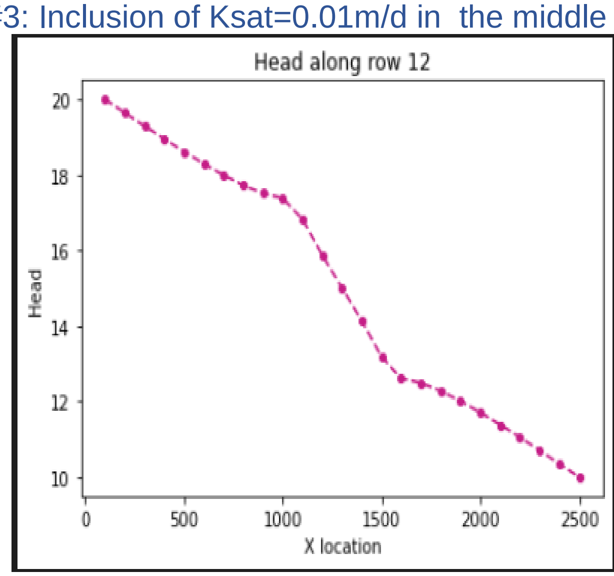
1. Cross sections of hydraulic Head
2. Plan views of the hydraulic conductivity that goes with each output

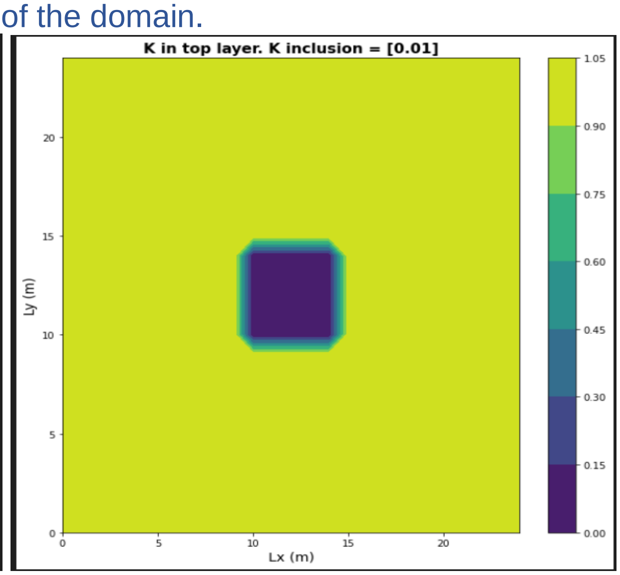
Case #2: Two columns changed their K_{sat} values.
Column 6: K_{sat} = 1000 m/d
Column 7: K_{sat} = 0.01 m/d





Case #3: Inclusion of Ksat=0.01m/d in the middle of the domain.

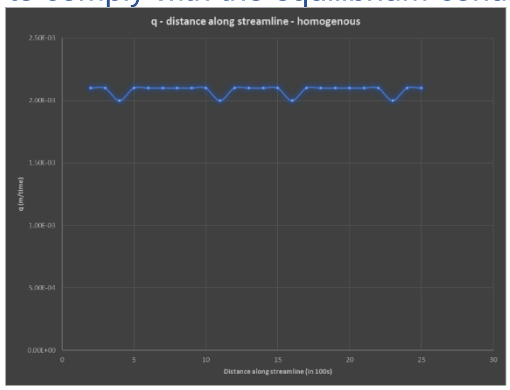




Discussion

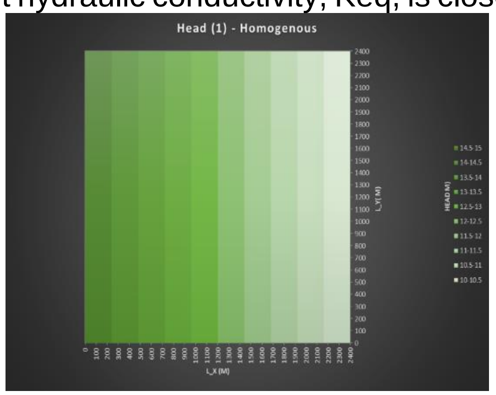
2. Show, based on the flux with horizontal distance from a constant head boundary, that the model is steady state.

In the graph we can see that the model is steady-states because it fluctuates its flux to maintain the constant head in the boundaries, to comply with the equilibrium condition of steady-state.



Flow profile in the boundaries.

3. Use the steady state head contour in plan view for the homogenenous and heterogeneous to defend a contention that flow is 1D. Then, drawing on your first assignment, use the results to explain WHY the equivalent hydraulic conductivity, Keq, is closer to the lower of the two K values.



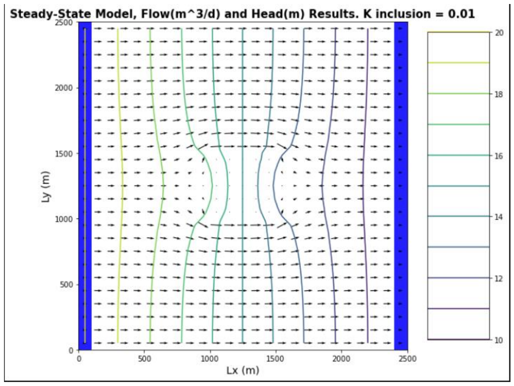
We can see that all the Y axis has the same magnitude of head from top to bottom, however we see that the head do varies in the x axis from left to right boundary, that is telling us that the flow is just 1D and it goes from the left boundary to the right boundary.

4. What can you learn based on your explanation of what controls the effective K for a 1D flow system now that you are applying it to a 2D system? What do you think the Keq of this entire system would be compared to the high and low K values? Explain why it is much more difficult to develop a direct solution for this 2D system than it was for a 1D system (including the zones placed in series).

The effective K in Modflow is biased to the lowest value of Ksat in the system due to it uses the harmonic average of the Ksat, and as showed in the equation below, no matter that 600 cells have the 1.0 m/day Ksat value, the only 25 cells that have the 0.01 m/day Ksat values are highly affecting the K effective of the entire domain, reducing it until 0.20 m/day value.

$$K\ \text{harmonic} = \frac{\sum_i \text{cells total}}{\sum_i \frac{\#\text{cells}}{K}} = \frac{625}{\left[\frac{600}{1} + \frac{25}{0.01}\right]} = 0.20\ \text{m/day}$$

The inclusion of K_{sat}=0.01m/day in the middle of the domain makes the system to change from being 1D to 2D, because as the hydraulic conductivity I there is lower, the flow just avoid that point and seeks the easier and faster way to flow through, so it goes around that inclusion, moving now in two directions (x and y), instead of moving just to in one direction (x axis). We can see this phenomenon better in the picture below made with FloPy.



Flow and Head contour lines of a domain with an inclusion of 5x5 in the middle with K_{sat}=0.01m/d.

5. For steady state conditions, there are equivalent Type I and Type II boundary conditions. What would the Type II boundary condition be that would result in the same equipotentials for the first model? What is the value of the constant flux? What about the second model? What are the values of the constant flux on the left and right boundaries? What is fundamentally different about the equivalent Type II boundary for the third model compared to the first two?

Type II boundary conditions is about having a constant flux, an for the head to be equipotential at all domain, and an example where this can be comply is:

- An aquifer surrounded by huge water bodies, that are recharging the aquifer at the same rate that the aquifer is being pumped out, maintaining the same head everywhere. So the system will have constant fluxes and constant head.

Case 1 Homogeneous domain:

$$q = -K_{sat} \cdot \left(\frac{dh}{dl}\right)$$

$$q = \left(1 \text{ m/d}\right)\left(\frac{15m - 10m}{2400m}\right)$$

$$q = 2.08 \times 10^{-3} \text{ m/d}$$

Case 2 Heterogeneous domain with parallel K_{sat}:

I got lost over here with the excel, need to think more about it. (trying to catch up, and as I'm running out of time, I will ask this in class better 🙂)

Case 3 Heterogeneous domain with an inclusion in the middle of the domain:

Now it is harder to calculate the flux because we need to account in 2 dimensions.

Same for this one! I will ask in class.

Glossary Questions:

1. What is MODFLOW? What is a MODFLOW package (provide at least 2 examples)? What are the inputs to a MODFLOW model?

MODFLOW is a program that models groundwater systems in 3D, and it uses finite-difference approximation to solve the groundwater flow equation. It can represent different conditions like steady flow and non-steady flow. One crucial fact is that MODFLOW always represents Saturated conditions, using Darcy's Law; it does NOT resolve Richards' Equation for Unsaturated flow.
The packages of MODFLOW are tools that provide functions to process the data and execute calculations; some examples are:

- RCH1 -- Recharge Package
- EVT1 -- Evapotranspiration Package

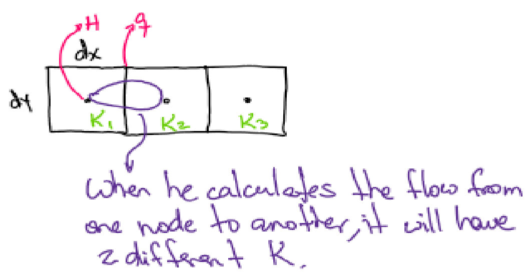
The inputs of MODFLOW include the hydraulic properties of the system, like hydraulic conductivity, boundary conditions like constant heads, layers, the distance of each cell.

2. What is the relationship between head gradients and hydraulic conductivity in steady state systems?

In steady-state conditions, the relation between head gradient and hydraulic conductivity is an inversely proportional one. Darcy's Law expresses that: $\boxed{q = -K_{sat}*dH}$, so in order to maintain a steady-state condition (constant flux), the Head will be low when the Ksat is high; on the other hand, the Head will increase when the K_{sat} decreases.

3. What is a model node? A model cell? Use a simple diagram to show the relationship between heads defined at nodes and properties defined in cells.

A model node is a reference point in the middle of a cell where the calculations are done. A model cell is an area that is part of a vast domain, and depending on the model's resolution, the size of each cell will change.



4. What is the difference between Type I and Type II boundary conditions, and under what conditions might you use each? Provide at least 2 examples for locations where we might use Type I or Type II boundaries to represent a feature in the real world.

- Type I boundary condition is when we have a specified head; on the other hand, Type II Boundary Condition is when we have a specified flux instead.
- To use this type of boundary, we will use Type 1 for the nodes of the cells, as there is the place where they are calculated, but for the Type II boundary conditions, it will be used in the fronts of the cell because there is where the fluxes are calculated.
- Examples of types of the boundary are as follows:

- Type 1 (Constant Head): when we have a confined aquifer that is giving us a head pressure in that boundary, or a lake of the sea located in that boundary too.
- Type II (Constant flux): it can be when there is a constant recharge flow rate in the edge produced by a recharge well or produced by the discharge of a pipe at a constant flux that recharges the aquifer; another example of Type II boundary condition is a pumping well that extracts water at the same rate always.