Problem 1

> Refs:
>
> [1] https://www.cs.duke.edu/courses/cps102/spring09/Lectures/L-18.pdf

1.

Probability of no collision q = (# of arrangements with no collision) / (# of all possible arrangements)

$$p' = \frac{P_n^{n^2}}{(n^2)^n} = \frac{(n^2)!}{n^{2n} \cdot (n^2 - n)!}$$

Then , the probability of any collision p :

$$p = 1 - p' = 1 - \frac{(n^2)!}{n^{2n} \cdot (n^2 - n)!}$$

2.

> Using P instead of $|P|$ for convenience

Let ϵ = expected # of queries needed and $F(n)$ = # of collisions after n queries.
Then ϵ should satisfy:

$$\epsilon - E(F(\epsilon)) = \frac{P}{4}$$

To calculate $E(F)$, we need $E(G)$, where $G(n)$ = # of empty slots after n queries. By empty slot I mean a hashed value that hasn't occurred yet.
The probability of a slot being empty after n queries is $(1 - \frac{1}{P})^n$, and since we are using a uniform hash, we have:

$$E(G) = P \cdot (1 - \frac{1}{P})^n$$

And # of queries without collision $=$ # of not-empty slot $= P - E(G)$.
Then # of collisions $= n - (P - E(G))$

$$
\begin{aligned}
E(F) &= n - P + E(G) \\
&= n - P + P(1 - \frac{1}{P})^n \\
&= n - P(1 - (1 - \frac{1}{P})^n)
\end{aligned}
$$

Plugin this back to the equation:

$$
\begin{aligned}
\epsilon - (\epsilon - P(1 - (1 - \frac{1}{P})^\epsilon)) &= \frac{P}{4} \\
P(1 - (1 - \frac{1}{P})^\epsilon) &= \frac{P}{4} \\
(1 - \frac{1}{P})^\epsilon &= \frac{3}{4} \\
\epsilon &= \frac{\ln \frac{3}{4}}{\ln (1 - \frac{1}{P})}
\end{aligned}
$$

3.

- Open addressing with linear probing

keys to be inserted \ index	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
18								18			
34		34						18			
9		34						18		9	
37		34			37			18		9	
40		34			37			18	40	9	
32		34			37			18	40	9	32
89		34	89		37			18	40	9	32

- Open addressing with double hashing

keys to be inserted \ index	0	1	2	3	4	5	6	7	8	9	10
18								18			
34		34						18			
9		34						18		9	
37		34			37			18		9	
40		34			37			18	40	9	
32		34			37			18	40	9	32
89	89	34			37			18	40	9	32

4.

- Table T_1, using $h_1(k)$

keys to be inserted \ index	0	1	2	3	4	5	6
6							6
31				31			6
2			2	31			6
41			2	31			41
30			30	31			6
45			30	45			6
44			44	31			6

- Table T_2, using $h_2(k)$

keys to be inserted \ index	0	1	2	3	4	5	6
6							
31							
2							
41	6						
30	2					41	
45	2				31	41	
44	2				30	41	45