Refs = Disjoint set slide P.18

5. First for loop runs $|V|$ times, so it's in total

$O(|V|) + |V|$ make-set() time

Second for loop runs $|E|$ times. In worst case, union() runs $|V|-1$ times and find-set() always run $2 \cdot |E|$ times.

So total time complexity is $O(1) + O(|V|) + O(|E|)$ $+ |V|$ make-set() $+ (|V|-1)$ union() $+ 2 \cdot |E|$ find-set().

Using union-by-rank + path compression makes those disjoint set operations take $O((|V|+|E|) \alpha(|V|+|E|))$

Therefore total time complexity $= O(|V|+|E| \, \alpha(|V|+|E|))$

$\alpha(x)$ is inverse function of ackermann's function.