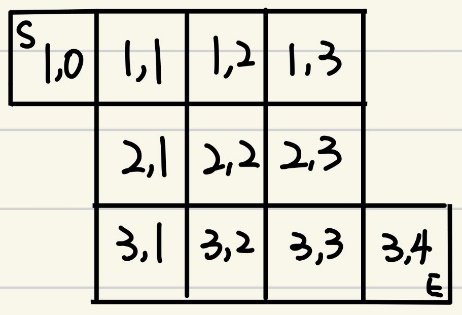
# Group 6: Miuki, Ming, and Joshua

# Project Summary

*This game is about connecting pipes in a 3x3 grid, where the start is in the top left corner and the end is in the bottom right corner. This game aims to connect the starting pipe, the ending pipe, and every pipe in between them (there are 5 types of pipes) by rotating the pipes on the grid. The water can only flow forward and downward. We made a simple diagram to get a better idea of ​​what the entire grid looks like. We will simulate how pipes are placed for the grid to have a solution.*

# Propositions

* Location (p, loc): There is a pipe with a certain pipe orientation at location loc. There are 11 locations available for a 3x3 grid and 3 pipes with a total of 12 pipe orientations.
* NeighborLR (l1, l2): location l1 and l2 is neighbor and they are beside each other
* NeighborUD (l1, l2): location l1 and l2 is neighbour and one is on top of the other
* Connected (l1, l2): location l1 and l2 is connected
* Have\_to\_east (loc): the pipe on loc has an opening facing east
* Have\_to\_south (loc): the pipe on loc has an opening facing south
* Have\_from\_west (loc): the pipe on loc has an opening facing west
* Have\_from\_north (loc): the pipe on loc has an opening facing north

# Constraints

One location:

* There is only one pipe orientation of one pipe at location (1,0) and location (3,4)(the start and end piece)

the opening of the start piece at location (1,0) can only face east

the opening of the end piece at location (3,4) can only face west

* Any location that is not (1,0) or (3,4) will not have a pipe that is only facing east or only facing west
* If there is a pipe at a location, then that location can have a different pip orientation with the same pipe type(eg. A straight pipe at (2,2) can change orientation from NS to EW)
* Location (straight pipe,11)>>add\_exactly\_one (Location(EW,11),Location(NS,11))

(If there is a straight pipe (orientated EW or NS) at location (1,1), then location (1,1) can have exactly one pipe orientation that is either EW or NS, but the other two types of pipes (angled and three way) or any of their orientation can not be at location (1,1))

* For all locations except for (2,2), (1,0), and (3,4), each pipe type at a specific location can only have one orientation( SAT solver choses one out of many) and all other orientations are not allowed.

Ex. Location (3-opening-pipe orientation,11)>> Location(['S','E','W'], 11) or

Location (3-opening-pipe orientation,11)>> (~Location (['N', 'S', 'E'], 11) & ~Location (['N', 'S', 'W'], 11) & ~Location (['N', 'E', 'W'],11))

* A pipe with a specific orientation at a location will not have openings that are not present in the pipe orientation.

(ex. Location (NS,11)>> (~Have\_to\_east (11) **&**~Have\_from\_west (11)))

* If location loc has no opening toward east, then location loc can’t be connected to the grid cell on its right

~ Have\_to\_east (loc)>>~Connected (loc, loc+1)

* If location loc has no opening toward south, then location loc can’t be connected to the grid cell below it

~ Have\_to\_south(loc)>> ~Connected (loc, loc+10)

* Same constraint for opening west and north ^

Two locations:

* If locations differ by exactly 1 or 10, they will be neighbors

(ex. 10 and 11 will be NeighborLR, and 11 and 12 will be NeighborUD)

* Every location that does not differ in 1 or 10 will not be neighbors

(ex. 10 and 34 is not NeighborLR or NeighborUD)

* If two locations are not neighbors, they are not connected (ex. 10 and 34 is not connected)
* No connection upward or left.

(Connected(l1,l2) **~** Connected(l2,l1) where l2>l1)

Whole grid

-there is a solution if there are a connection from start to end

(Connected (10, 11) & Connected (11, 12) & … & Connected (33, 34)) | … | (Connected (10, 11) & Connected (11, 21) & … & Connected (33, 34))

# Model Exploration

* For the model exploration, we first tried to set first row all to straight pipe with the function no\_sol\_with\_row\_strai (). According to our model, this should have no solution

A screenshot of a computer

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A picture of the no\_sol\_with\_row\_strai in code

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Output when execute no\_sol\_with\_row\_strai ()

This setup does not have a solution because even though element in each row have connection between them, but there are no connection between rows which is needed for the constraint for the whole grid.

#may need detail

* Remove a pipe in solution route when it only has one solution

A screen shot of a computer

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On the left is a test case we used for our model and on the right, it shows that this test case has solutions and display all the connections this test case has. And there are one solution routes for this one is: location 10 to 11 to 21 to 31 to 32 to 33 to 34.

For the model exploration, we want to remove a pipe in solution route when it only has one solution. Then, according to our model, this should not have a solution

A screen shot of a computer program

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Code for the function and output of executing the function

But instead of saying no solution for this grid, solver did give a solution that is not right (it has connection for grid cell 21). Since the condition for two grid cells to be connected is if they are neighbours and the pipes on them have opening towards each other, and 21 has no pipe on it means that there should be no connection to it and from it. We soon realized that we are missing a constraint and a proposition that checks if there are empty grid cells. Before this function, we assumed that all gird cells have a pipe on it.

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Add constraint to check if any grid cell is empty

A screen shot of a computer code

Description automatically generated

Add constraint: check all location that is neighbour, if any location in a neighbour pair empty, there should be no connection to that cell

After adding the codes in the pictures, now executing the function empty\_grid\_cell () give us an output saying there are no solutions.

#disconnect\_at\_beginning()

We wanted to make sure that if the beginning pipe was not connected that there would be no solution because it violates the constraint we have put in for this game. This is a relatively simple model exploration and only needed to make sure that if the starting pipe and the pipe at position 1,1 does not connect, then there is no solution to be found.

A constraint is added to check if this is the case.

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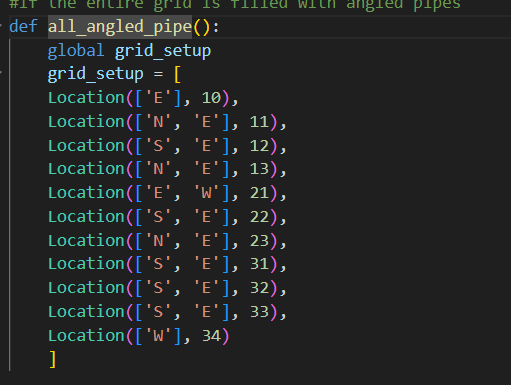
The final result is that there is no solution after testing it with a test case.

The reason why there is no solution when the 10 and 11 does not connect is because in order for a solution to appear, there must be a path connecting from 10 all the way to 34 (Connected(10,11)/\.../\Connected(33,34). Since there is no connection between 10 and 11, there is no solution.

#all\_angled\_pipe()

This is a edge case where we check whether there is a solution if the entire grid is filled with angled pipes other than the starting and ending pipe at 10 and 34. There should be no solution because due to our constraint that pipes can only be connected from left to right and from up to down, there is no path

in which all angled pipe can connect from start to finish.



This is the grid in which the entire grid is filled with only angled pipes. The result should be no solution.



The reason why there is no solution is because our grid is limited to only going down and going right, meaning that it is not possible to connect from 10 to 34. Some kind of movement to the left or upwards is required in order for there to be a solution from 10 to 34 (pipes are not allowed to be connected right to left or from down to up, only left to right connections and up to down connections are allowed.)

# Jape Proof Ideas

1. A screenshot of a computer

   Description automatically generatedIf there is a straight pipe on grid ‘13’, then the route containing this grid is not feasible.

The following assumptions are placed into Jape to set things up.

1. P→(P1∨P2): Straight pipes imply vertical and horizontal pipes.
2. P1∧A3: vertical pipe in grid ‘13’, same with P2.
3. ¬C: the grid ‘12’, ‘13’, and ‘23’ are not connected.
4. R: The result does not contain a route for ‘13’.
5. There is a "chain of connections" from i1 to i3 implies that any two grid spaces along the way are connected.

The following assumptions are placed into Jape to set things up.

1. ∀x.∀y.(PF(x,y)→PC(x,y)) : If the water can flow from grid x to grid y, it means they are connected.
2. ∀x.∃y.∀z.(PF(x,z)→(PF(x,y)∧PF(y,z))) : If water can flow from grid x to grid z, then there exists a grid y connecting them.
3. Set up the initial configuration: actual i1, actual i2, actual i3

A screenshot of a computer

Description automatically generatedWe want to prove: ∃x.∃y.PC(x,y) There exists two neighboring grids (neighbor up-down or neighbor left-right) on the route that are connected.

1. If the first grid is connected to the second grid. However, the second grid is not connected to the third grid. This route is not connected.

The following assumptions are placed into Jape to set things up.

1. ∀x.∀y. (PC(x,y)→PF(x,y)) : If grids x and y (neighbor up-down or neighbor left-right) are connected, water can flow from grid x to grid y.
2. ∀x.∀y.∀z.((PF(x,y)∧PF(y,z))→PF(x,z): If water can go through from grid x grid to y and from grid y to grid z, then water can flow from grid x to grid z.
3. ∀x.∀y.(PC(x,y)→PF(x,y)): if grids x and y are not connected, it means water can not flow from x to y.
4. Set up the initial configuration: actual i1, actual i2, actual i3, PC (i1, i2), PC (i2, i3)
5. ∀x.∀y.∀z.((PF(x,y)∧PF(y,z))→PF(x,z): If water cannot flow from y to z, even if it can flow from x to y, it cannot eventually flow from x to z.

A screenshot of a computer

Description automatically generatedwe want to prove: PF (i1, i3) Water can not go through in this route.

# First-Order Extension

* **Placed (p, l)**: pipe p is placed at location l.
* **Location (l)**: l is on the location.
* **East (l):** the pipe on loc has an opening facing east.
* **West (l):** the pipe on loc has an opening facing west.
* **Connect (l1, l2):** The pipes are connected in the neighbor location.
* **Route(r):** r is a route.
* **Start (r, s):** s is the starting location of route r
* **End (r, e):** e is the ending location of route r
* **Orientation(p, o):** Pipe p has orientation o.

- Every Pipe is Placed in Exactly One location:

One location can only have one pipe.

1. No two different pipes can be placed at the same location.

∀p1​. ∀p2​. ∀l.((Placed(p1​,l)Placed(p2​,l))→(p1​=p2)​)

1. If a pipe is placed at a location, the location must exist on the board.

∀p.∀l.(Placed(p, l)→Loc(l))

The final constraint will be (1)(2)

* Pipe rules after the setup

Any pipe at the location can have different pipe orientations.

∀p.∀l. o1. o2.(placed(p,l)(Orientation(p,o1)Orientation(p,o2))

* Start and End Pipe Placement:

The starting pipe must be at the top-left corner facing East, and the ending pipe at the bottom-right corner facing West.

l. Placed(Start(r, s), East(‘01’))Placed(End(r, e), West(‘34’))

* Route connected

For both grid and pipe, it cannot connect to the left and up grid, it can only go right or down. (It can avoid self-loop and will be easier to find the solution.)

Connected (l1, l2) **~** Connected (l2, l1)

* Condition of solution

For one of the routes, if there is a connection from start to end, there will be a solution.

∀r. (Route(r)∃s.∃e. (Start(r, s)End(r, e)Connected(s, e))→Solution(r)).

# Useful Notation

*Feel free to copy/paste the symbols here and remove this section before submitting.*