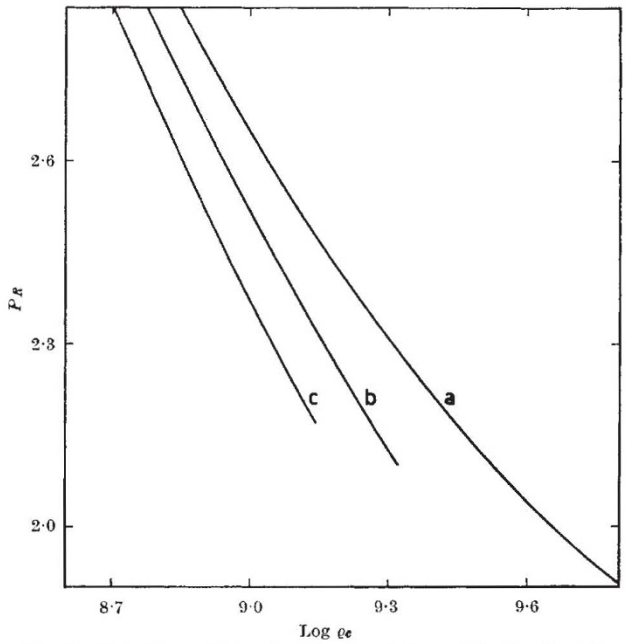

Fig. 1. Pulsation periods of uniformly rotating white dwarfs. The curves labelled *a*, *b* and *c* correspond, respectively, to $J = 3{\cdot}85$ (48), 2·31 (49) and 3·85 (49). The zonal period P_Z is not shown because the effect of rotation is too small to be apparent on this scale.

James[22], all uniformly rotating white dwarfs are secularly stable. The situation is very different for non-uniformly rotating white dwarfs above the Chandrasekhar critical mass. Differential rotation allows much larger values of $T/|W|$ and has, consequently, a much larger effect on the periods. For $\rho_c < 10^{10}$ g cm^{-3}, the zonal period may be as low as 0·25 s. It is worth noting that, for a given mass (not density), rotation increases the periods. This can easily be understood from equations (1) and (2), for large rotation considerably reduces the potential energy and increases the moment of inertia. Finally, let us note that all models given in Table 2 are secularly and ordinarily stable (in the Poincaré sense; our unpublished work).

To sum up, a rotational motion couples the two lowest axisymmetric modes of pulsation of a white dwarf obeying the Chandrasekhar equation of state. Neither of these modes is purely radial, and one of them may have a period as low as 0·25 s for a central density less than 10^{10} g cm^{-3}. Very recently, Durney *et al.*[26] presented calculations indicating that the P_R modes are differentially rotating stars; they also oscillate with a period as short as 0·4 s for a central density of the order of 10^{11}.

This work was supported, in part, by grants from the US Air Force Office of Scientific Research and the US National Science Foundation.

Received July 19, 1968.

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Future of the Population of North-east Scotland: a Statistical Study

by
DAVID J. HALL
Department of Mental Health,
University of Aberdeen

Methods and results are presented of a study of the past and future of the population of the north-east of Scotland.

ESTIMATIONS of future population trends have many uses. Apart from a purely scientific or academic interest, techniques are necessary for the investigation of the outcome of proposed policies concerning population. The principal applications are in the fields of demography and political and economic planning, but there is a growing interest in the application of such estimations to health service planning especially where regional or national plans are envisaged such as the British health service.

In most areas of the world populations are increasing rapidly and contain a high proportion of young people. In these areas official campaigns aimed at population control—chiefly through encouragement of the use of efficient contraception—seek to limit the total size and to maintain a balance between active and dependent members of the community. The ability and effectiveness of such methods in achieving these aims have recently been questioned[1,2].

This study was undertaken to provide information for the planning committee of the North-East Scottish Regional Hospital Board in its consideration of future requirements for psychiatric services in the region. The object was to determine a family of projections for the existing population of the area covered by the regional board in order that the magnitude of the effects of levels of fertility and migration at present, and any changes in these levels may be more fully evaluated.

The area under consideration has a population of just under half a million and comprises the city of Aberdeen, the counties of Aberdeen, Banff, Kincardine, Moray and the island counties of Orkney and Zetland. The region is unusual in having a population that is declining and

ageing. Furthermore, despite high net rate of emigration, its medical and welfare services have been forthright in advocating and adopting population control practices. Indeed, tubal ligation was first offered to women in certain categories in the late 1930s (ref. 3). By determining the values of the parameters which govern population size—fertility, death and migration—and applying them within a mathematical model of population development, a projection of its future size and structure is obtained. By altering the values of the parameters to simulate changes in the determinants of size—such as the migration rate—the future development of the same starting population in these different conditions can be studied.

Present Structure of the Population

At the 1961 census the region had a population of 479,530 or 9·3 per cent of the total population of Scotland. The city of Aberdeen accounted for 38·7 per cent of the regional population and about 9·8 per cent of the population of the four large cities in Scotland. The regionally indigenous proportion was 85 per cent, and a further 9 per cent was born elsewhere in Scotland. In addition to the city of Aberdeen, there were thirty burghs with a population over one thousand, of which three, Fraserburgh and Peterhead in Aberdeenshire and Elgin in Morayshire, were over ten thousand. If all thirty burghs and the city are regarded as urban, the total urban proportion of the population was 62 per cent; a hundred years before, the whole regional urban population was only 36 per cent.

The density of the population also varied. For the region as a whole there were 109 persons/square mile; for the mainland part, excluding the city, there were 74, and for the Orkney and Shetland Islands there were 39 persons/square mile.

A study of the sex and age distribution of the regional population showed that in common with most population groups, overall, there were more females than males, and also in all age groups except those aged 0–14 years. This youngest age group contained a quarter of the total population, there were 12 per cent to 13 per cent in each 10 year age group up to age 54; 23 per cent were aged 55 and over. In comparison with the population of Scotland, the regional population was slightly older in both sexes.

Changes in the Population, 1801–1961

In the 160 years from 1801 the population of Scotland increased by 222 per cent while that of the region increased by only 185 per cent. In the middle 50 years of this period the Scottish population increased by 55 per cent and that of the region by 21 per cent, but in the past 50 years Scotland's population increased by 9 per cent whereas that of the region fell by nearly 7 per cent. Thus the proportion of the Scottish population formed by the region fell from 16 per cent in 1801 to 9·3 per cent in 1961.

In common with the population of the United Kingdom, the regional population has been ageing. In 1961, 15·5 per cent were over 60 compared with 14 per cent in 1951 and 11 per cent in 1931. The ratio of economically active persons to dependants has consequently been decreasing.

Although the regional population was at a peak in 1911, that of Aberdeen City has continued to increase, although at a much lower rate in the past 50 years than in the previous hundred. In 1801 the city formed only 10 per cent of the regional population, and by 1911, 35 per cent. In contrast, the population of the rest of the mainland declined by 12 per cent in the 50 years after 1911, and the population of the Orkney and Shetland Islands fell by 43 per cent in the hundred years up to 1961. The rate of decline in the islands is also increasing: it was 6·6 per cent in the 20 years before 1951 and 10 per cent in the 10 years after that date.

These demographic changes in the region illustrate well the phenomenon of depopulation, which is one of its major characteristics[4]. Depopulation began at the periphery of the island groups and only twenty-six of the ninety Orkney Islands and nineteen of the ninety Shetland Islands are now inhabited, many only by lighthouse keepers and nature wardens. In spite of the overall decrease in population, the population of Shetland's single urban area increased by 29 per cent in the first half of the twentieth century. Similar but less marked changes have occurred in the mainland part of the region.

Rural depopulation consists of a migration from extreme isolation in the upper glens to relative isolation in the lower glens, from the lower glens to villages, from villages to towns, from the towns to the city, and from the islands to the mainland. Regional depopulation has also been increasing as workers and their families move towards the Glasgow–Edinburgh industrial belt and farther south to England, and has particularly affected skilled workers. Depopulation affects chiefly the younger age groups and may largely account for the weighting towards the higher end of the age range in the region as compared with Scotland. Despite these changes the regional population is undoubtedly relatively static in size and composition in comparison with most other parts of the United Kingdom. This feature, coupled with the remarkably high indigenous proportion, the relative isolation from the rest of Scotland, and perhaps the unique origins of the population in history and pre-history has contributed to the maintenance of the close kinship ties in communities throughout the region. For example, Illsley *et al.*[5] showed that, at the time of their first pregnancy, 89 per cent of Aberdeen-born women had mothers living in the city—indeed, 34 per cent were living in their mother's homes. Five years after the first pregnancy, 22 per cent visited, or were visited by, their mother at least five times a week and a further 40 per cent at least once a week.

In summary, the north-east region may be regarded as having an isolated, indigenous, stable but slowly declining, slowly urbanizing population of close-knit family and kinship structure, long and idiosyncratic tradition, and unique origins.

Computing Projections

Because of the differences in mortality, emigration and fertility between town and country, two entirely separate projections were computed, one for the city of Aberdeen and one for the remainder of the north-east region, namely, the counties of Aberdeenshire, Banff, Kincardineshire, Moray, Orkney and Zetland. Each projection was carried out by identical methods using data specific to the area under consideration.

There are only two ways of leaving a given population: by death or by emigration; similarly there are only two ways of entering it: by birth or by immigration. During a period of time the individuals who constitute the population will change and those who remain will become older. The population is composed of groups of individuals who, over the course of the life cycle, increase and decrease in numbers and in the ratio between the sexes. It is the estimation of these constituent elements as well as their sum which is necessary to the construction of a projection of future structure which in turn is of practical value in planning.

The rates at which various factors, such as the death rate or the net rate of migration, influence the changes in numbers which occur can be calculated from past experience of the population. On the assumption that these rates are relatively invariant over the projection period, their application to the existing population will yield the subsequent population. The detail with which the age and sex structure of the subsequent population can be specified depends on the detail in which the various rates can be obtained.

Ten age groups for each sex were chosen for which data were available from the Registrar General's annual reports as follows: 0–4, 5–14, 15–24, 25–34, 35–44, 45–54,

55–64, 65–74, 75–84, 85 and over[6]. The projection was calculated for intervals of 10 years so that all but the first and last age groups—after subtraction of deaths and emigrants—became the next oldest group in the subsequent projection year.

All the rates and ratios used in the calculation remained unchanged throughout the projection. Thus no allowance was made for the possibility of these rates changing in the course of the projection. Births during each interval of 10 years were calculated as the product of the fertility ratio (live births/woman aged 15 to 44) and the number of women aged 15 to 44. Deaths in each age group were calculated using age specific death rates. Net migration—emigration minus immigration—was calculated using age specific net migration rates. The mathematics of the method are detailed in the appendix. The base population from which the projection was calculated was that reported from the 1961 census[7].

One year death rates specific to each age–sex group were calculated by taking the mean number of deaths/year for the years 1961 to 1965 and dividing by the census population for 1961. The alternative approach of constructing the mean of annual rates was avoided because existing estimates of the population in the years other than the census year were less reliable than using the figures for the census year as their estimate.

The 5 year fertility ratio was the ratio of the births in the five years 1961 to 1965, to the female 1961 census population aged 15 to 44. Alternative estimates of the fertility ratio were calculated by dividing the births in the years 1960 to 1964 by the Registrar General's mid-year estimate of the female population aged 15 to 44 in 1960 and again using the births in the years 1959 to 1963 and the population in 1959. Compared with the values reported from the 1961 census of the females aged 15 to 44, the Registrar General's estimates for the two previous years were too high. When these were corrected the three values of the fertility ratio accorded closely.

The sex ratio, the number of boys born divided by the number of girls born, determined as 1·06062, was the mean value of the years 1959 to 1965 for the whole of Scotland. The value for the whole country was used in preference to the observed values for the two projection areas because it was thought to be a better estimate of these values by reason of the very much larger number of observations on which it was based. In the estimates for Scotland the Registrar General used the value of 1·06 (ref. 8). Examples of actual mean values for the 5 years 1961 to 1965 were: for Aberdeen City, 1·06084; for Aberdeen County, 1·05893; and for Moray County, 1·0610.

Net rates of migration were unobtainable from any official source so that it was necessary to use the projection itself, projecting from one census to the next with zero net migration rates and subtracting the later census figure from the projected value to obtain the number of emigrants. This exercise was first carried out using the 1961 census as starting date and projecting to the 1966 census[9]. The rates thus derived were unreasonable, however, varying wildly from one age group to the next and between the sexes within an age group. The cause of the difficulty was the sample nature of the 1966 census. A value in the census of 20,000 which was the order of magnitude of a 10 year age group was ten times the enumerated value of 2,000, which had a standard error of $\sqrt{2000} = 45$. The 95 per cent confidence interval to contain the correct population value was $(2{,}000 \pm 2 \times 45) \times 10$ or 20,900 to 19,100. The relative sampling error $\left(\frac{2 \times 45}{2{,}000} = 4{\cdot}5 \text{ per cent}\right)$ was of a greater order of magnitude than the net migration rates that were being estimated. The 1966 sample census therefore could not be used for this purpose. The exercise was repeated using the 1951 full census as base and projecting to 1961 (ref. 7). The results were wholly reasonable and were used in the subsequent projections proper. The 1951 to 1961 projection was carried out with data valid for the period 1961 to 1965 as described. Following a general trend in national and local death rates and fertility ratios, it can be assumed that these rates were both very slightly higher in the period 1951 to 1961 than between 1961 and 1965. Consequently the estimates of net migration rates obtained were very slightly too high. The size of the error was too small to have had any important effect on the projections.

Parameters for the Projections

The 1961 population is shown by the sex and age groups for each of the two areas under consideration in Table 1. This table also shows the 10 year death rates, that is, the proportion of persons in each age group who could be expected to die within 10 years, and the net rate of migration during the 10 years. The city population of 185,390 persons had an overall ratio of 100 males to 117 females. The ratio decreased steadily from 100 males to 96 females in the age group 0 to 4, to 100 males to 240 females in the age group 85 and over. The only deviation from this steady decline was in the age group 15 to 24 where the ratio was 100 males to 120 females. Linear interpolation between the two adjacent age groups gave a value of around 100 males to 103 females. This deviation is explained by net emigration of males and net immigration of females in the preceding age groups, for the rates for each ten year age group apply over the 10 years until they become the next age group.

Table 1. POPULATION OF NORTH-EAST REGION IN 1961. TEN YEAR DEATH AND NET MIGRATION RATES

Males

Age	Population				Ten year death rate		Ten year migration rate*	
	City		Country					
	No.	Per cent	No.	Per cent	City	Country	City	Country
0–4	7,837	9·2	12,745	8·9	0·0191	0·0204	0·0285	0·0767
5–14	15,283	17·9	26,032	18·2	0·0052	0·0093	0·1190	0·2246
15–24	11,412	13·4	19,879	13·9	0·0093	0·0134	0·0452	0·2159
25–34	11,261	13·2	18,206	12·7	0·0190	0·0192	0·0705	0·1070
35–44	10,999	12·9	17·571	12·3	0·0493	0·0437	0·0218	0·0680
45–54	11,567	13·6	18,295	12·8	0·1454	0·1168	−0·0032	0·0244
55–64	9,599	11·2	15,212	10·6	0·3403	0·2767	−0·0132	−0·0288
65–74	5,060	5·9	9,598	6·7	0·6085	0·5532	−0·0318	−0·0089
75–84	2,034	2·4	4,564	3·2	0·8927	0·8739	−0·0145	−0·0321
85 and over	303	0·4	764	0·5	0·9667	0·9591	0	0
Total	85,355	100·1	142,866	99·8	—	—	—	—

Females

Age	Population				Ten year death rate		Ten year migration rate*	
	City		Country					
	No.	Per cent	No.	Per cent	City	Country	City	Country
0–4	7,488	7·5	12,067	8·0	0·0127	0·0148	0·0296	0·0895
5–14	14,668	14·7	24,718	16·3	0·0029	0·0040	−0·0708	0·2219
15–24	13,693	13·7	19,194	12·7	0·0054	0·0063	0·1667	0·1652
25–34	12,427	12·4	18,462	12·2	0·0131	0·0123	0·0736	0·0903
35–44	12,510	12·5	18,461	12·2	0·0308	0·0308	0·0199	0·0642
45–54	13,619	13·6	19,475	12·9	0·0789	0·0750	−0·0018	0·0447
55–64	12,541	12·5	17,559	11·6	0·2007	0·1854	−0·0130	0·1078
65–74	8,315	8·3	12,846	8·5	0·4560	0·3975	−0·0050	0·0594
75–84	4,046	4·0	7,013	4·6	0·8205	0·7852	−0·0067	−0·0126
85 and over	728	0·7	1,479	1·0	0·9344	0·9232	0	0
Total	100,035	99·9	151,274	100·0	—	—	—	—

* Emigration is positive and immigration negative.

The country population of 294,140 persons had an overall ratio of 100 males to 106 females—a considerably lower ratio than that of the city. There was a monotonic change from 100 males to 95 females in the age group 0 to 4, to 100 males to 194 females in the age group 85 and over. For each age group the ratio was lower in the city than in the country.

Differences in the age distributions between the city and country populations were slight although there was a small excess of elderly persons in the country at the expense of the middle-aged. Comparison of the age specific death rates in the city and country revealed a similar pattern for both males and females although, except for the very young, the differences in the rates were much smaller among females than among males. Over the three youngest age groups the death rate for the city was about three-quarters of that for the country. Death rates for both sexes aged 25 to 34 and for females aged 35 to 44 were similar. For older age groups the city death rate

was always higher than the country rate, reaching a maximum relative difference in the 45 to 54 age group for males and in the 65 to 74 age group for females.

The general feature of the net migration rates was that they were comparatively high in the 0 to 4 age group, reached their peak in one of the two subsequent age groups, and then declined steadily, being reversed to become a low level of net immigration in the elderly. For both males and females the country rate was higher than the city rate. There were three exceptions to these generalizations. (1) Among city males aged 15 to 24 emigration was about half the expected value. This could be explained by a high rate of immigration of young men from the country to the city at this age. (2) For a similar reason there was a small net immigration of city females aged 5 to 14. (3) From the overall trend the net rate of emigration for country females aged 55 to 64 was much higher than expected. A possible explanation is that these women, many of whom will have been widowed, tended to move in order to have better facilities in the city, or to be nearer their grown-up children living in the city.

The annual fertility ratios for Aberdeen City, the north-east counties and Scotland were all evaluated for the years 1959 to 1965. The pattern was the same for each year with the Aberdeen City value lowest, followed by the county value, followed closely by the Scottish value. The means of the ratios for the seven years were 0·0823, 0·0950 and 0·0967 respectively.

Findings of the Projection

For each of the two areas, five different projections were carried out, all for the century following 1961: (i) a current estimate, that is to say, the projection was performed with the best available data at present; (ii) a current estimate, but with all net migration rates put to zero; (iii) a current estimate but with all net migration rates doubled; (iv) a current estimate but with the fertility ratio put to the lowest city level of 1961–1965 for the city projection and at the mean city level for the country projection (low fertility); (v) a current estimate but with the fertility ratio for the city put to the mean country level, a value considerably higher than that of the city, and that for the country at its highest level of the years 1961–1965 (high fertility).

Table 2. FIVE PROJECTIONS OF THE NORTH-EAST REGION POPULATION, 1961–2061: NUMBERS OF PERSONS

			City		
Year	Current estimate	Zero net migration	Double net migration	Low fertility	High fertility
1961	185,390	185,390	185,390	185,390	185,390
1981	179,682	201,550	159,460	177,157	187,308
2021	164,214	246,464	104,826	154,967	194,072
2061	150,949	297,681	70,277	134,649	208,531
			Country		
Year	Current estimate	Zero net migration	Double net migration	Low fertility	High fertility
1961	294,140	294,140	294,140	294,140	294,140
1981	249,219	330,051	182,576	239,541	252,445
2021	169,207	453,754	53,367	145,049	177,764
2061	115,883	634,484	15,333	85,240	127,608

The total males and females for the base year 1961 and projected numbers for the years 1981, 2021, 2061 are shown in Table 2. It will be seen that (i) under the stated conditions and with the parameters of the current estimate, the city population would decrease to 81 per cent of its 1961 value over one hundred years and the country to 39 per cent. (ii) Should emigration cease, both the populations would increase in size by one and a half times and two times, respectively. (iii) If emigration doubled both city and country would very rapidly lose population. (iv) A decrease in fertility would reduce the "current estimate" projection by a further 10 per cent of the 1961 value in both cases. The decrease in the fertility ratio, 5 year rate, from 0·48 to 0·42 in the country was, however, greater than the drop from 0·42 to 0·40 in the city. (v) The projection involving a rise in fertility was the only one in which city and country populations moved in opposite directions. The city, with a rise in fertility from 0·42 to the country level of 0·48 would increase in population over one hundred years by one-eighth. The country, with a rise from 0·48 to 0·50, would decrease to 43 per cent of the 1961 population.

Age Distribution

The distribution of the city population in 2061 by age group (Fig. 1) shows the increasing proportion of elderly in the population as either net migration increases or fertility decreases. Over a hundred years the "current estimate" of the male population of the city yielded an increase in the proportion aged 65 and over from 8·6 per cent to 11·0 per cent. The 15 to 34 age group would also increase from 26·6 per cent to 28·3 per cent but the proportion aged 35 to 44 would not change. A similar pattern can be seen in the "zero net migration" projection although the emphasis would shift from the elderly group, which would rise only to 9·0 per cent, to the young men, where the increase would extend to include the 35 to 44 age group, with a rise of 4·3 per cent. The "double net migration" projection magnified the effects of the "current estimate"; in this case the over 65 male population would increase to 13·7 per cent and all other age groups up to age 54 would decrease in proportion. In particular, the rise in the 15 to 34 age group noted in the other two projections would not occur. The "high fertility" projection would lead to only a very slight rise in the proportion of aged men (0·7 per cent). The main changes would be the increase of the 15 to 34 age group to 29·3 per cent and the decrease of the 45 to 64 age group from 24·8 per cent to 21·2 per cent. The "low fertility" projection resulted in a rise of 3 per cent in the proportion of aged men with a corresponding decrease in boys under age 15.

The results for females in the city were similar to those for the males. In addition to the features of the results for males in the "current estimate" the females aged 45 to 64 decreased more markedly from 26·1 per cent to 22·7 per cent. In the "zero net migration" projection females up to age 44 increased by 3·7 per cent, those aged 45 to 64 decreased correspondingly and there was no change in the proportion of aged. With "double net migration" the influx of young females from country to city alluded to previously would cause females aged 15 to 24 to increase their share of the total; with this exception there would be a decrease in the proportion for all age groups up to age 64. With "high fertility" all ages up to 34 would increase their share with consequent decrease in

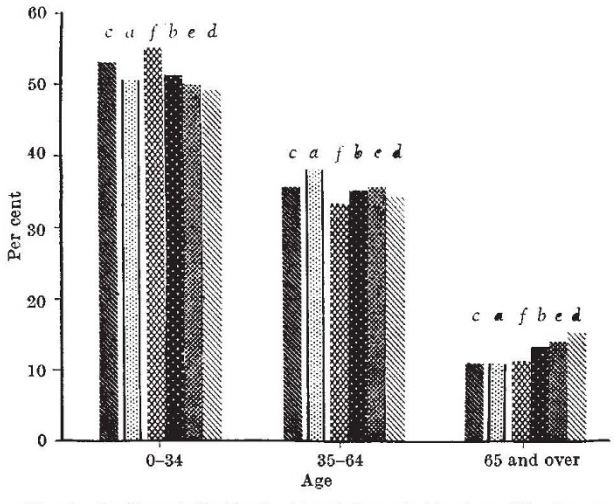

Fig. 1. North-east Scotland. Population of Aberdeen City by age groups. Percentage distribution in 1961 and from five projections in 2061. *a*, 1961 value; *b*, current estimate; *c*, zero net migration; *d*, double net migration; *e*, low fertility; *f*, high fertility.

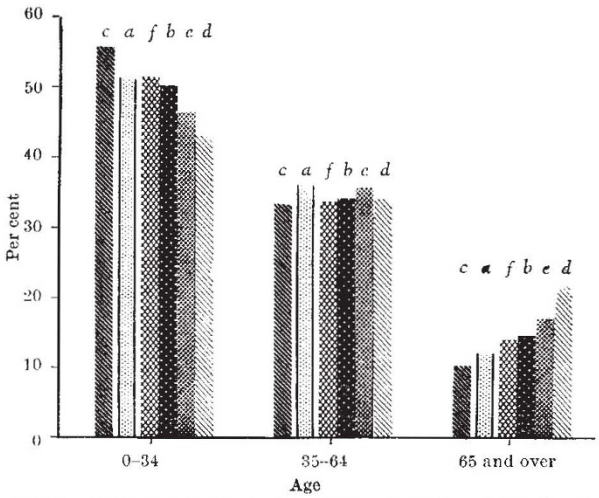

Fig. 2. North-east Scotland. Population of north-east counties by age groups. Percentage distribution in 1961 and from five projections in 2061. *a–f*, Same as Fig. 1.

succeeding ages up to 74. For "low fertility" the increase in the proportion of aged females was the same as that of the males, and the decrease in girls under age 15 was not as great as in the males.

In the country projections (Fig. 2) the markedly increased proportion of elderly people in each of the city "current estimate", "double net migration" and "low fertility" calculations occurred also. In addition the "double net migration" and "low fertility" projections for the country showed large changes in their age distributions. Males aged 55 and over would increase from 21·1 per cent to 35·2 per cent in the former and to 28·1 per cent in the latter with consequent decreases in the under 55 age range.

The "zero net migration" country projection was the only one where the proportion of elderly decreased, the proportion of males aged 45 and over dropping from 33·9 per cent to 29·8 per cent. Correspondingly the proportions for age groups under 45 increased, with the exception of youths aged 5–14. The projection of females was qualitatively the same as that of the males but the changes were smaller. The "high fertility" projection was similar to the "current estimate" but with less marked changes.

In summary, taking the "current estimate" as reference, the "low fertility" and "double net migration" projections would result in an increasing proportion of aged persons with a correspondingly lower proportion of younger people, whereas the "high fertility" and "zero net migration" projections would yield a decreasing proportion of elderly.

The existing (1961) distribution lay somewhere between the "high fertility" and "zero net migration" distributions. This broad overall picture held true for each of the four location–sex projections. The spread or divergence of the proportion from a median proportion was greater for the north-east counties than for the city of Aberdeen and was slightly greater for males than for females. The age at which the proportion changed from higher to lower varied slightly between the location–sex distributions; again taking the "current estimate" as reference for both city and country males, the bounding projections of zero and double net migration converged at approximately age 52 and the inner projections of high and low fertility crossed at approximately 37. The corresponding pairs of ages for city females were 54 and 34, and for country females 46 and 38.

Sex Distribution

Only three different estimates of the outcome of the sex-ratio over the age groups could be derived from these projections. Because of the nature of the method, changes in fertility had no effect on the sex ratio. The three estimates obtained were the "current estimate" which, in terms of the sex ratio was equivalent to both the high and low fertility projections, the "zero net migration" and the "double net migration" projections.

The general result was that at low ages the sex ratio was just over 50 per cent males and this percentage decreased with increasing age. The special characteristics of the region would impose themselves on this pattern. In the city the high net rate of emigration of young men aged 15 to 24 would cause a sharp drop in the percentage of this age group which stood at 45·5 per cent in 1961. After a hundred years the "current estimate" gave 46·1 per cent for this value, "zero net migration" gave the value for this age group of 51 per cent. "Double net migration" would have, of course, a severe effect and the value would fall to 40·9 per cent. Comparison with the 1961 values shows that for all parameters for all age groups, bar the 15 to 24 year olds and the oldest age groups, the percentage of males would rise; the rank order for the different projections, highest percentage of males to lowest, was "zero net migration", "current estimate", "double net migration" and 1961 values.

For the country the "zero net migration" projection again yielded the expected standard decline. Up to age 55 the "current estimate" and 1961 values ran closely together at up to 5 per cent below the "zero net migration" level. At age 55 and above, the higher net immigration of males led to a rise in the percentage of males in the "current estimate". The "double net migration" projection, of course, emphasized this effect and the proportion of males at ages 70 and 80 rose to over 50 per cent.

Prospects for the Future

The history of the population of north-east Scotland over the past 160 years is one of growth and decline, Although the peak for the region as a whole was in 1911. peripheral depopulation had been taking place for 50 years. The Orkney and Shetland Islands reached their peak in 1861, Moray County in 1881 and the remaining counties in 1891. The city of Aberdeen has continued to grow slowly until the present day, but may now also have passed its highest population (Fig. 3). The loss of population has clearly been caused chiefly by emigration, though in recent years low fertility, particularly in the city, has probably accentuated the effects.

In attempting to estimate the future direction of the regional population these historical features, as well as the purely statistical projections derived from recent data, and the current economic, social and political scene, must be taken into account. The limiting members of the family of projections presented in this report are almost certainly too wide and the actual trend may be expected to lie relatively close to the current estimate. The value of presenting a range of possible trends is two-fold. First, it

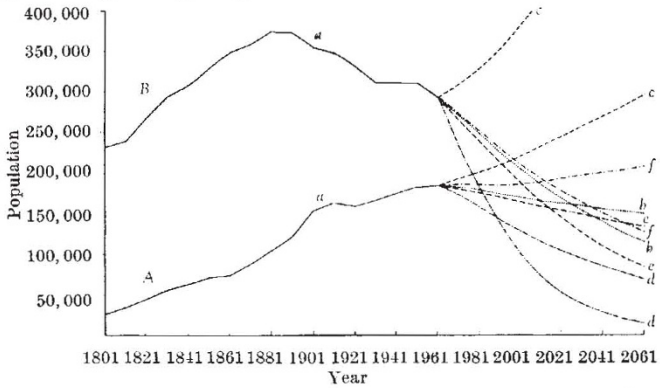

Fig. 3. North-east Scotland. Total populations of Aberdeen City and north-east counties. Censuses 1801 to 1961 and five projections to 2061. *a*, Census data; *b*, current estimate; *c*, zero net migration; *d*, double net migration; *e*, low fertility; *f*, high fertility. *A*, Aberdeen City; *B*, north-east counties.

emphasizes the conditional nature of statistical projections and places the burden of decision on the user rather than on the investigator. Second, it elucidates the effects of the quantifiable factors and so helps the user to evaluate the credibility of possible trends in the light of fuller information.

The kinds of question which must be asked in arriving at a practical range of values for the future population are whether, in present economic circumstances, the emigration pattern of the past 60 years is likely to continue or increase or decrease, whether the cultural mores on family size and obstetric policy on family limitation are likely to change, and whether any political action is envisaged which might materially affect the current trend, such as introduction of heavily manned industry. There is little doubt that even modest changes in emigration rates could markedly change the present course, whereas encouragement of high fertility would do little to slow the overall decline (unless the increase is very large as in the male city high fertility projection). Emigration is a process which feeds on itself, for it removes relatively more of the economically active and fertile and reduces the attractiveness of a community by lessening economic and cultural opportunity. The eventual outcome is ageing of the population and stagnation of its economic and social circumstances. Although the "current estimate" does not imply a radical change of this sort so much as a continuing rapid increase in the urban proportion, an increase in emigration of more than a quarter might drastically disturb the economic and social balance of the region as a whole.

This study was undertaken as part of the research programme at the Research Unit with the Department of Mental Health. I thank members of the unit for their help.

Appendix: Statistical Method Used for the Population Projections

The basic equation for the projection was

$$P_{i+1,\, j+10} = P_{i,\, j}\,(1 - d_i - e_i)$$

Where $P_{i,j}$ was the population in age group i, for projection year j

$i = 2, \ldots, 8$. $j = 0, 10, 20, \ldots$

d_i was the 10 year death rate for age group i, $i = 1, \ldots, 10$.

e_i was the 10 year net migration rate for age group i, $i = 1, \ldots, 10$.

It should be noted that, as the first age group 0–4 had only a 5 year span, half that of the other groups, special considerations had effect thus

$$P_{2,\, j+10} = P_{1,\, j}(1 - d_1 - e_1) + \tfrac{1}{2}Bj + (1 - d_0 - e_0) \times (1 - d_{\frac{1}{2}} - e_{\frac{1}{2}})$$
$$P_{1,\, j+10} = \tfrac{1}{2}Bj\,(1 - d_0 - e_0)$$

where Bj was the number of live births in the 10 year interval j to $j + 10$

d_0, e_0 were composite death and net migration rates which converted the births, which occur continuously throughout the interval, into the 0–4 cohort. This process is detailed below.

$d_{\frac{1}{2}}$, $e_{\frac{1}{2}}$ were 5 year, that is, half interval, death and net migration rates for the 0–4 age group. They were required for those births, born in the first half interval, which first appeared in the 5–14 cohort.

Similarly the last age group, 85 years and over, had a special consideration with effect as

$$P_{10,\, j+10} = P_{10,\, j}(1 - d_{10} - e_{10}) + P_{9,\, j}(1 - d_9 - e_9)$$

that is, the last age group aged into itself as well as receiving the complement from the penultimate group.

Calculation of 10 year death rates Let the 1 year death rates for males aged 15–24 be r_1 and r_2 respectively. We wish to derive the 10 year death rate for males aged 15–24. The proportion of the 15–24 starting cohort remaining after 1 year will be $1 - r_1$, they will then be aged 16–25. The 1 year death rate for this group was not known explicitly, for calculations of these rates from available data was not possible. It can be closely approximated, however. The 16–25 group lies nine-tenths in the 15–24 range and one-tenth in the 25–34 range, hence interpolating linearly the 1 year rate for the 16–25 group is $\frac{9}{10}r_1 + \frac{1}{10}r_2$. The extension of this procedure for each 1 year step is straightforward. The final result, the proportion of the 15–24 starting cohort reaching 25–34, was thus given by $(1 - r_1)\,(1 - \frac{9}{10}r_1 - \frac{1}{10}r_2)\,(1 - \frac{8}{10}r_1 - \frac{2}{10}r_2) \ldots (1 - \frac{1}{10}r_1 - \frac{9}{10}r_2)$. The 10 year rate for 15–24 was then unity minus this value.

Only the 10 year death rates were calculated in this manner; 10 year migration rates were evaluated directly as described in the text.

Estimates of the death rate of births The individual age groups were considered as cohorts and their attrition over the years was studied. In the first case births, because they occur continuously throughout the interval between projection dates, cannot be treated as a cohort. They were first generated as the 0–4 cohort which consisted of all births born over the previous 5 years who were still alive at the end of this period. This cohort either appeared directly as the 0–4 age group if the births were in the second half of the projection interval, or, if the births were in the first half of the interval, they were subject to the further attrition of the 5 year death and migration rates of the 0–4 cohort and appeared in the projection within the 5–14 age group. The births in the second half of the projection interval can be arranged into five annual groups; those born 4 to 5 years before the next projection date; those born 3 to 4 years before the next projection date, etc. The "4 to 5 year" group will be completely exposed to the death rates of the first, second, third and fourth years of life and in varying extents to the rate for the fifth year. A first approximation to their exposure to this rate which was deemed sufficient for this study is to subject all of them to half of it. Thus the overall death rate for the "4 to 5 year" group was given by $1 - (1 - r_1)\,(1 - r_2)\,(1 - r_3)\,(1 - r_4)\,(1 - \frac{1}{2}r_5)$, where $r_1, \ldots, r_5$ are the respective 1 year rates for the first, . . . , fifth year of life. In like manner the "3 to 4 year" group has the rate $1 - (1 - r_1)\,(1 - r_2)\,(1 - r_3)\,(1 - \frac{1}{2}r_4)$. The other groups, apart from the 0–1 year group, have rates similarly defined. The rate for the "0–1 year" group is just r_1. This value was chosen rather than $\frac{1}{2}r_1$, because the feature of infant mortality is that around 80 per cent of deaths occur in the first 2 months of life and 70 per cent of these in the first week. Because all but a very small proportion of the births in the interval are subject to this high mortality any error caused by assuming that all births are subject to it is very small indeed. The imprecision that exists is towards having a slightly too high death rate.

Received May 20; revised June 11, 1968.

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Strontium-90 on the Earth's Surface

by

E. P. HARDY

Health and Safety Laboratory,
US Atomic Energy Commission,
New York, New York

M. W. MEYER
J. S. ALLEN
L. T. ALEXANDER*

US Department of Agriculture,
Soil Conservation Service,
Soil Survey Laboratory,
Beltsville, Maryland

Estimations of the amount of strontium-90 on the Earth's surface indicate that maximum fallout occurred late in 1966. Of the 13 MCi recorded at the beginning of 1967, 82 per cent of this was in the northern hemisphere.

NUCLEAR explosions in the atmosphere release radioactive fission products that eventually fall out on the Earth's surface. Most of these nuclides decay rapidly and are not measurable after a few years. Strontium-90 has a relatively long half-life (28 yr) and persists in the environment along with caesium-137. The strontium isotope is important in biological systems for it enters the food chain and is

* Present address: Resources for the Future, Inc., Washington, DC.