# DEMOGRAPHIC GRAVITATION: EVIDENCE AND APPLICATIONS

John Q. Stewart
*Princeton University*

## *1. Agenda for Social Physics*

In the study of matter, physicists have two valid points of view, the macroscopic and the microscopic. The macroscopic view presents a gas as a continuous medium, and describes its physical condition in terms of such "field" quantities as density, pressure, temperature, gravitational potential. The microscopic view recognizes that a gas is made of a huge number of separate molecules. If observational attention could be focused on the rapid and intricate movement of a single molecule, bouncing against the others, there would be no hint to be gained, from its momentary velocity, momentum, or kinetic energy, as to the value of the temperature or pressure. And a physicist who insisted that the gas as a whole could be studied only by making a detailed point-to-point and instant-to-instant record of the paths of typical molecules would find his ill-chosen task impossibly laborious.

Fortunately for physics, the macroscopic approach was the commonsense one, and the early investigators—Boyle, Charles, Gay-Lussac—were able to establish the laws of gases. The situation with respect to "social physics" is reversed. If we liken people to molecules, the commonsense approach is the microscopic one—involving the study of mutual relations and reactions of individuals or of very small groups. Many experts in the social field nowadays are willing to adopt a scientific rather than a dogmatic attitude, attempting to study people as they really are rather than to set up and enforce an ideal geometry along some party line. But progress has been very slow because they always insist on knowing too much.

The late Sir James Jeans in a book about the kinetic theory of gases gave a spectacular illustration of the properties of large numbers. He pointed out that there are about $10^{44}$ molecules in the whole atmosphere of the earth, while a single breath of air contains something like $10^{22}$ molecules. If the assumption is made that by now the air once breathed by Julius Caesar is uniformly distributed throughout all the atmosphere, the probability is that every breath any one of us draws contains several molecules which were included in Caesar's last breath. If Robert Boyle had taken the attitude of many social scientists, he would not have been willing to measure the pressure and volume of a sample of air until an encyclopedic history of its molecules had been compiled. Boyle did not even know that air contained argon and helium but he found a very important law.

Merely verbal methods of description and analysis are all that are at the command of most students of society. These are powerless to formulate the quantitative averages which are required for the macroscopic approach to sociology. Continued insistence on examining people's purposes and motives only blocks the way to a science of society as a whole, just as similar sentimentality and animism in Aristotelian physics made that physics useless century after century.

On the other hand, mathematics, even mathematical statistics, unaccompanied by a feeling for physical or social reality is incompetent to create useful patterns of concepts for the organization of the suggested new discipline of social physics. To the pure mathematician all things are possible and he is ignorant of restrictions which must be accepted if mundane applications are quickly to be made. If the results of mathematical economics thus far have been disappointingly minor, the reason is that economists have not furnished mathematicians with adequately abstract and dynamic economic concepts for manipulation.

## 2. *Newtonian Gravitation*

Newton's law of gravitation may be expressed in any one of three different formulas. In the following presentation the name of each physical quantity is italicized when first introduced.—Suppose a particle of *mass* $M$ is at point A, at *distance* $d$ from a second particle of mass $m$ at point a. A *force* $F$ acts on each mass, attracting them together along the line joining them, and having the magnitude

$$F = \frac{GMm}{d^2}, \tag{1}$$

where $G$ is a universal constant, the *gravitational constant*. This was Newton's original statement.

The mutual *energy* of the two masses in the gravitational field is $E$, given by

$$E = \frac{GMm}{d}. \tag{2}$$

Finally, the gravitational *potential* $V_A$ which the mass $m$ produces at point A is

$$V_A = \frac{Gm}{d}; \tag{3a}$$

and the potential $V_a$ which $M$ produces at a is

$$V_a = \frac{Gm}{d}. \tag{3b}$$

The concept of potential was first introduced by Lagrange.

A comparison of (3a) and (3b) with (2) shows that

$$2E = MV_A + mV_a. \tag{4}$$

A reader familiar with elementary physics will note a divergence in the statement of equations (2), (3a), (3b). It is customary to assign negative signs to the three right-hand members, because the energy is identified with the mechanical work required to separate the two particles to an infinite distance. These considerations are not of significance in the present discussion.

When instead of only two particles we have many masses distributed through space, the above equations apply to each pair. The total potential at a point is the sum of the separate potentials. If the distribution of mass is confined to a plane surface, and if it may be regarded as continuous, the potential at any point C in the plane is

$$V_C = \int \frac{1}{r} \, DdS. \tag{5}$$

Here $D$ is the *surface density* of mass over the infinitesimal element of area $dS$; $r$ is the distance from that element to the point C; and the integration is extended to all areas of the plane where $D$ is not zero.

Thus if mass is distributed in a known manner over a plane, $D$ being known everywhere, equation (5) enables the potential to be computed at every point. The outcome of the computation can be presented on a map of the surface by the device of *contours of equipotential.* The familiar contours on a topographic map which represent altitude above sea level are precisely contours of equal gravitational potential.

It can be shown that the total resultant force of gravitation acting on unit mass at any point in the plane is directed at right angles to the equipotential contour there, and has the value

$$g = \frac{\partial V}{\partial n} \tag{6}$$

In this ratio of differentials, $n$ is the distance measured along the normal to the equipotential. Since the quantity $g$ is the force per unit mass, it is, by Newton's second law of motion, the *acceleration* of any mass produced by the gravitational field at the said points. However we shall use it only as the *gradient* of the potential—deferring the question of actual motion.

### 3. *The Formal Laws of Demographic Gravitation*

A great variety of evidence has been accumulated which shows that the above equations have applications to the average interrelations of people. To make them applicable it is only necessary to substitute $N$, the *number of people* involved, wherever the quantity mass appears in the original equations.

More strictly, we replace mass by $N\mu$, where $\mu$ is the *molecular weight* of the sort of person considered. It is necessary to have a standard of this weight. (Actually, *molecular mass* is the appropriate term, notwithstanding the customary physical usage). Accordingly, that of the "average American" is taken as unity. Presumably the molecular weight of an Australian aborigine, for example, is on this scale much less than one.

Thus equation (1) becomes

$$F = \frac{G(N_1\mu_1)\ (N_2\mu_2)}{d^2}\,, \tag{7}$$

when the $N_1$ people are at a distance d from the group $N_2$. The constant $G$ is left for future determination: a suitable choice of other units can reduce it to unity. $F$ is to be called the *demographic force*; what this means in terms of sociological data is left for examination further along. The physical straight-line distance often is what we use as $d$ although an illustration in section 9 applies to a case where $d$ in (7) must be otherwise interpreted

The demographic force of attraction between two groups $N_1$, $N_2$, of average Americans a distance $d$ apart, at points 1 and 2, respectively, is therefore

$$F = \frac{N_1N_2}{d^2} \tag{8}$$

and acts along the line joining them.

Their *demographic energy* by virtue of the force field is

$$E = \frac{GN_1N_2}{d}\,. \tag{9}$$

The *potentials of population* are

$$V_1 = \frac{GM_2}{d}; \; V_2 = \frac{GM_1}{d}. \tag{10}$$

Also

$$2E = N_1V_1 + N_2V_2. \tag{11}$$

Equation (5) is unchanged:

$$V = \int \frac{1}{r} \, DdS; \tag{12}$$

but V now is potential of population, while D is the *density of population* familiar to demographers. When $D$ is known over a plane the *equipotentials of population* can be computed.

Demography, strangely enough, has lacked a measure, like $V$, of the *influence of people at a distance.* Density cannot be such a measure, because it is a purely local quantity.

We shall see that demographic energy is proportional to economic wealth. (§ 8).

### 4. *Types of Application of Demographic Gravitation in Sociology*

W. J. Reilly's statement (1) (1929) of a "law of retail gravitation" seems to have been the first recognition of demographic gravitation. His expression is derivable from equation (1). It fitted his observations of the position of the point of equilibrium intermediate between two cities competing for the retail trade of the surrounding rural dwellers:

$$\frac{N_1}{d_1^2} = \frac{N_2}{d_2^2}, \tag{13}$$

$d_1$ being the distance from the city of population $N_1$ to the said point of balance, and $d_2$ being the distance from the city $N_2$.

The concept of the potential of population was developed by Stewart (2), who published its application to the distribution of the residences of college undergraduates, etc.; and also maps of equipotentials for the United States and other areas.

G. K. Zipf (3) emphasized the usefulness of the formula $N_1 N_2/d$ as a determinant of various relations between pairs of cities—for example, the interchange of telephone calls.

There is no need further to recapitulate evidence already published, and we pass to a rather detailed study of potential of population as the determinant of a variety of demographic and economic quantities in the United States.

### 5. *Maps of Equipotentials in the United States, 1940*

Two maps of potential of population in the United States computed according to the Census of 1940 have been published (4). Table 0 lists the

TABLE 0
DATA FROM MAP OF U. S. POTENTIALS, 1940

| State or Section | Potential $V$, persons/mi. 0000 omitted | Gradient, $g$ Magnitude persons/mi.[2] | Gradient, $g$ Direction ("uphill" toward) |
|---|---|---|---|
| NJ (except NYC, Phila) | 59 | 3000 | var. |
| NH (S) | 35 | 2500 | S |
| Vt (S) | 34 | 2000 | S |
| Va (N) | 38 | 2000 | N |
| RI | 44 | 1750 | W |
| Conn | 51 | 1250 | SW |
| Mass | 45 | 1250 | SW |
| NY (except NYC) | 41 | 1000 | SSE |
| Me (S) | 30 | 1000 | SSW |
| Vt (N) | 26 | 1000 | S |
| Del | 45 | 800 | N |
| Md | 43 | 700 | NE |
| NH (N) | 25 | 700 | S |
| Me (central) | 25 | 700 | SSW |
| Mich | 32 | 500 | SSE |
| Mo | 28 | 450 | E |
| Pa | 45 | 450 | E |
| Va (S) | 34 | 380 | NNE |
| Ohio (W) | 40 | 350 | E |
| Ia (E) | 28 | 350 | E |
| Ark (E) | 27 | 350 | NE |
| Ind | 37 | 330 | ENE |
| WVa | 38 | 330 | NNE |
| Wis | 27 | 330 | SE |
| Ohio (E) | 42 | 330 | SE |
| Miss (N) | 26 | 310 | NNE |
| Me (N) | 17 | 290 | SSW |
| NC | 30 | 290 | NNW |
| SC | 27 | 290 | NNW |
| Ga | 26 | 280 | N |

Note: The arrangement is in order of decreasing gradient. One expects a fair degree of correlation between $g$ and $V$, in view of equations (8), (6) and (10). It is impossible in such a table to duplicate all the information on a large-scale map. The values of $V$ and $g$ are only approximate. The direction of $g$ may vary many degrees within a small area near a large city, and the value of $V$ likewise changes sharply there. The gradient indicates in magnitude and direction the maximum rate of increase of potential with distance in a neighborhood.

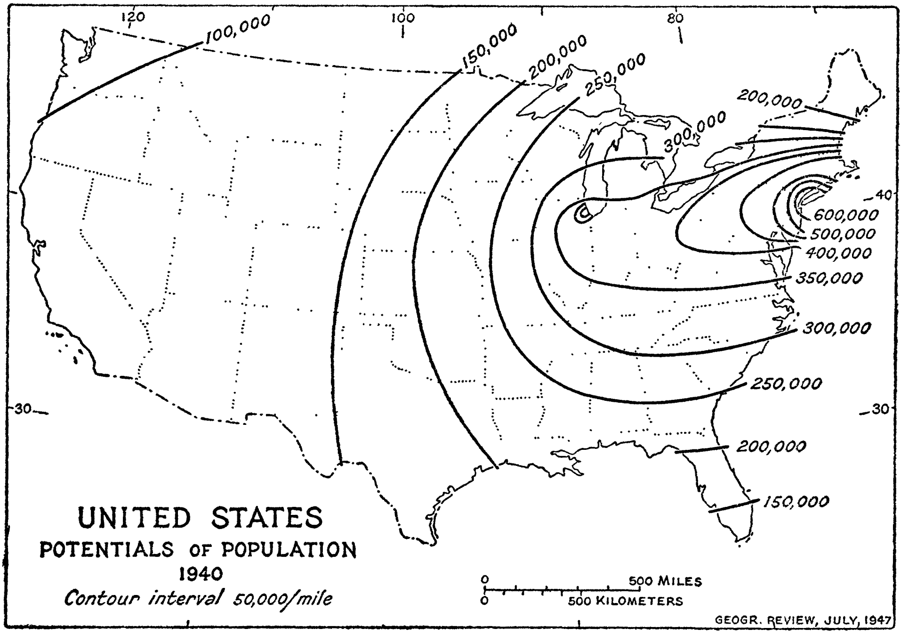
TABLE 0 (Continued)
DATA FROM MAP OF U. S. POTENTIALS, 1940

| State or Section | Potential $V$, persons/mi. 0000 omitted | Gradient, $g$ Magnitude persons/mi.$^2$ | Direction ("uphill" toward) |
|---|---|---|---|
| Ala (N) | 27 | 280 | NNE |
| Tenn | 31 | 250 | NNE |
| Minn | 21 | 250 | SE |
| Ky | 33 | 240 | NE |
| Ia (W) | 23 | 240 | ESE |
| Fla | 18 | 230 | NNW |
| NDak (SE) | 16 | 220 | ESE |
| Ala (S) | 24 | 210 | NNE |
| Ill (except Chicago) | 33 | 200 | E |
| SDak (SE) | 17 | 200 | ESE |
| Kans (E) | 22 | 190 | E |
| Ark (W) | 23 | 180 | NE |
| Tex (NE) | 20 | 180 | NE |
| La | 22 | 180 | NE |
| Miss (S) | 23 | 170 | NE |
| Okla | 21 | 170 | ENE |
| Neb | 19 | 170 | ESE |
| Kans (W) | 18 | 170 | E |
| SDak (NW) | 14 | 130 | ESE |
| Tex (SW) | 14 | 100 | ENE |
| Col | 15 | 100 | E |
| NM | 13 | 100 | ENE |
| Wy (E) | 13 | 100 | ESE |
| Mont (W) | 11 | 100 | E |
| NDak (NW) | 13 | 70 | ESE |
| Nev (W) | 10 | 70 | W |
| Nev (E) | 10 | 70 | E |
| Ida | 11 | 70 | E |
| U | 12 | 67 | ENE |
| Ariz | 11 | 57 | ENE |
| Wash | 10 | 40 | W |
| Calif (except cities and N) | 13 | 37 | WSW |
| Ore (except S) | 10 | 33 | WNW |
| Wy (W) | 12 | 33 | ESE |
| Mont (E) | 13 | 30 | ESE |
| Calif (N) | 9 | 20 | var. |
| Ore (S) | 9 | 20 | var. |

general characteristics of the second, more accurate one, and disagrees in a few minor respects with Figure 0, which is from the first computation.

The table gives the approximate average of the potential in every state or section of a state, together with the magnitude and direction of the corresponding gradient $g$ as defined by equation (6). For these maps the molecular weight $\mu$ has been taken as unity over the whole country.

Since potential is population divided by distance, it is expressed in units of *persons/per mile*. The gradient involves one more power of distance




in the denominator and therefore is in terms of the unit person/(mile squared.) To change a potential in persons/mile to the equivalent expression in persons/kilometer divide by 1.609, which is the number of kilometers in a mile. To change a gradient in persons/mile$^2$ to its equivalent in persons/kilometer$^2$ divide by $(1.609).^2$

The potential of population at a point may be regarded as a measure of the *proximity of people* to that point. In computing it we consider that every person makes a contribution which is less the farther away he lives. As we move from back-country rural areas toward a great city there is a rise in potential because of the concentration of people there. The gradient becomes steeper as the boundary of the city is approached. Inside the city the potential still continues to rise—all the way to the center, if the city is roughly circular. But nowhere does it reach values enormously greater than in rural areas nearby.

Thus every city is a local "peak" of potential, although Table 0 does not present sufficiently fine-grained data to show this. New York City is the major peak of the country, and in rural areas the general slope everywhere east of the Sierras is "downhill" away from New York. Even deep in New York City the potential does not exceed by more than a factor of roughly 4 that existing at a farm 50 miles away. But the potential at such a New Jersey or Connecticut farm exceeds that in a good-sized Middle-Western city, because of the low "base" of potential on which the city stands. That is to say, you are closer to people of the whole United States on a farm in Hunterdon County, N. J., than you are at the center of Omaha, Neb.

Every general contour of potential east of the Sierras closes around New York; all the other cities are local peaks on the general downhill slope away from the number one metropolis. The major structure of the U. S. potentials has changed in form remarkably little in 100 years; New York City was already the principal peak in 1840 (5). Meanwhile average potentials in the East have risen by a factor of roughly 7.

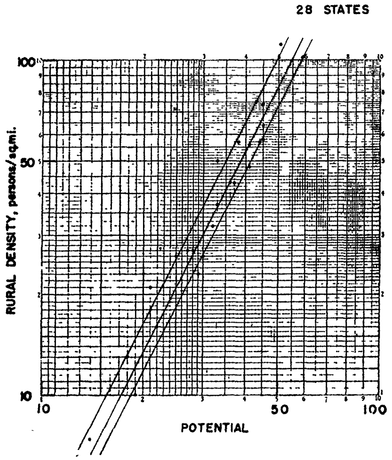
West of the Sierras there are two separate humps, each with its own closed contours around it. One hump centers in the region of Seattle, Washington, and the other in California. Each is as yet minor compared with the Eastern "massif", even if we allow for the rapid increase of West Coast population since 1940.

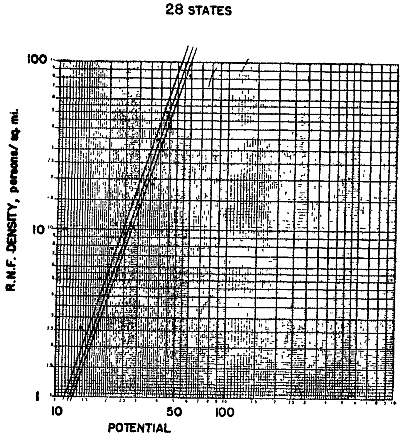
### 6. *Examination of Certain Averages of Demographic and Economic Statistics Along Equipotentials in Rural Areas*

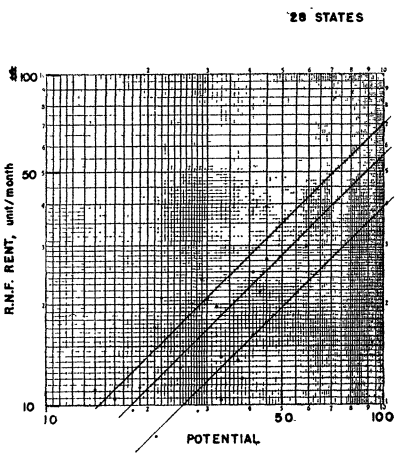
This section is presented principally by means of Tables 1-7 and the corresponding graphs. Because of the sharp local variations close to cities, the present study is confined to rural areas. Furthermore it is confined to 28 states from Texas to Maine, which we shall call the "Main Sequence." These states exhibit a considerable degree of statistical homogeneity. (See Table 1 for a list.)

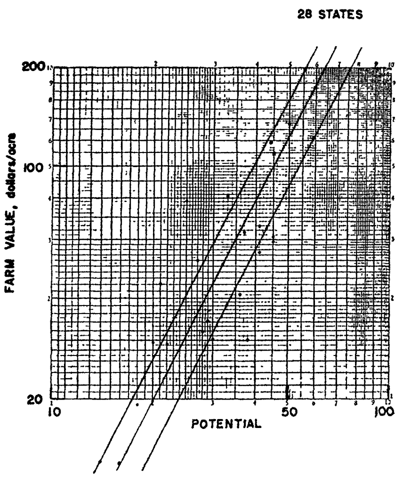
If the hypothesis is true that demographic gravitation is a fundamental social process, then a study of appropriate data should show a tendency to constant averages along any equipotential contour. This was confirmed in an early publication in the case of density of rural population (6), and later in the case of rents of rural non-farm dwelling units (7).

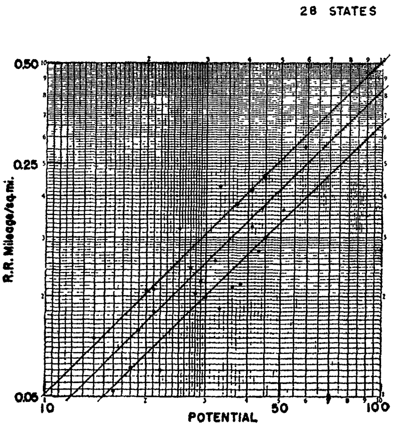
Tables and graphs here add confirmation also for the density of the rural non-farm population, the density of rural wage-earners in manufacturing, and for the value of farmland per acre. All of these quantities tend to be greater in states of high potential. When graphed on log-log paper against potential, they can be fitted by straight lines, which exhibit various degrees of steepness in their rise. All the data therefore are compatible with

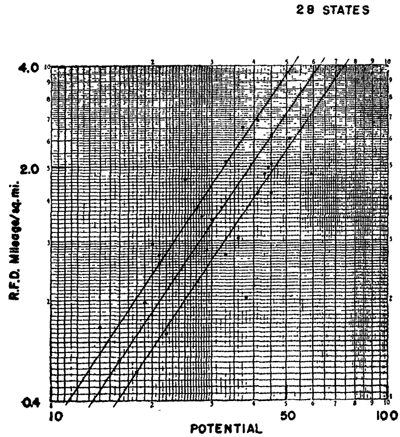

28 STATES
RURAL DENSITY, persons/sq.mi.
POTENTIAL
100
50
10
10
50
100

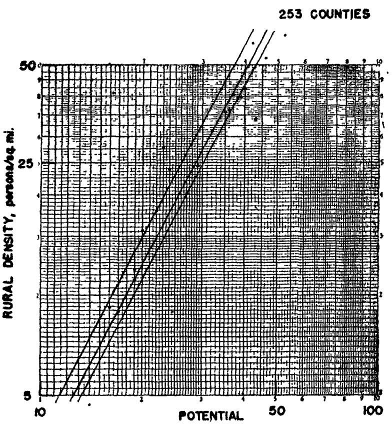

28 STATES
R.N.F. DENSITY, persons/ sq. mi.
POTENTIAL
100
10
1
10
50
100

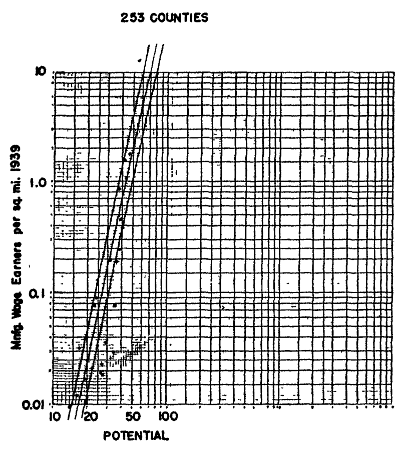

28 STATES
R.N.F. RENT, $/unit/month
POTENTIAL
100
50
10
10
50
100

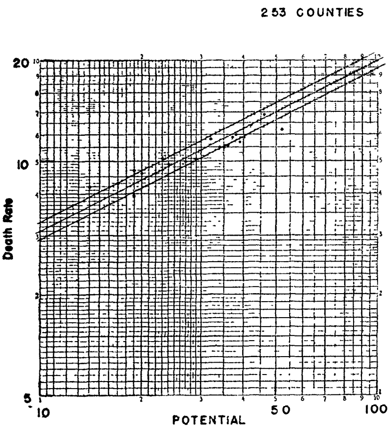

28 STATES
FARM VALUE, dollars/acre
POTENTIAL
200
100
20
10
50
100


28 STATES
R.R. Mileage/sq.mi.
0.50
0.25
0.05
10
50
100
POTENTIAL


28 STATES
R.F.D. Mileage/sq.mi.
4.0
2.0
0.4
10
50
100
POTENTIAL


253 COUNTIES
RURAL DENSITY, persons/sq.mi.
50
25
5
10
50
100
POTENTIAL


253 COUNTIES
Mnfg. Wage Earners per sq. mi. 1939
10
1.0
0.1
0.01
10
20
50
100
POTENTIAL



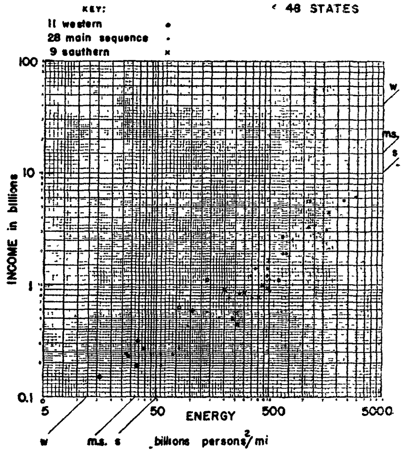




TABLE 1
VALUES OF POWERS OF THE POTENTIAL, V: 28 STATES 1940

| State | V | $V^{3/2}$ | $V^2$ | $V^3$ |
|---|---|---|---|---|
| NJ | 59 | 453.185 | 3481 | 2054. |
| Conn | 51 | 364.211 | 2601 | 1327. |
| Mass | 45 | 301.869 | 2025 | 911.3 |
| Penna | 45 | 301.869 | 2025 | 911.3 |
| Del | 45 | 301.869 | 2025 | 911.3 |
| RI | 44 | 291.861 | 1936 | 851.8 |
| Md | 43 | 281.968 | 1849 | 795.1 |
| NY | 41 | 262.527 | 1681 | 689.2 |
| O | 41 | 262.527 | 1681 | 689.2 |
| WVa | 38 | 234.247 | 1444 | 548.7 |
| Ind | 37 | 225.064 | 1369 | 506.5 |
| Va | 36 | 216.000 | 1296 | 466.6 |
| Ill | 33 | 189.572 | 1089 | 359.4 |
| Ky | 33 | 189.572 | 1089 | 359.4 |
| Mich | 32 | 181.021 | 1024 | 327.7 |
| Vt. | 30 | 164.316 | 900 | 270.0 |
| NH | 29 | 156.171 | 841 | 243.9 |
| Mo | 28 | 148.162 | 784 | 219.5 |
| Wis | 27 | 140.297 | 729 | 196.8 |
| Ia | 25 | 125.000 | 625 | 156.3 |
| Me | 22 | 103.189 | 484 | 106.5 |
| Minn | 21 | 96.235 | 441 | 92.61 |
| Okla | 21 | 96.235 | 441 | 80.00 |
| Kan | 20 | 89.442 | 400 | 80.00 |
| Neb | 19 | 82.819 | 361 | 68.59 |
| Tex | 18 | 76.367 | 324 | 58.32 |
| SDak | 16 | 64.000 | 256 | 40.96 |
| NDak | 14 | 52.384 | 196 | 27.44 |

Note: If the desired unit of potential is the person/mile, multiply the V column by $10^4$, the $V^{3/2}$ by $10^6$, the $V^2$ by $10^8$, and the $V^3$ by $10^{14}$. The potentials were approximately estimated for rural areas, and the various powers are computed with unnecessary accuracy.

formulas involving proportionality to various powers of the potential. Rural death rates increase only as $V^{1/2}$, while the density of wage earners in manufacturing increases as $V^5$. In computing the various sorts of rural density the area of the cities was assumed to be negligible with respect to the states or counties.

The rise in the death rate in eastern states is a phenomenon already noted by W. G. Bowerman (8), but his suggestion seems very insecure that it is causally related to an associated decrease in the density of animal population.

One interesting statistic studied (Table 4) is the ratio of the number of miles of railway line in a state to the area of the state. This ratio turns out to be proportional to the first power of the potential. It measures the accessibility of the land to the railroads. If the railroads were laid out on a square grid, of $k$ miles on a side, the ratio would equal $2/k$. For comparison, the grid of numbered streets and avenues in Manhattan exhibits a ratio of 27 miles of street per square mile of area. Information about the miles of railway is from Statistical Abstract of the United States (9).

Thus the higher the potential the greater the approximation to a straight-line communication between any two points. In low-potential regions what little rail traffic there is must often go by an indirect route.

Mathematical statisticians may object to the absence of least-square solutions to fit the assumed straight lines in the respective scatter-diagrams. The lines on each graph were drawn according to a slope which was chosen from the limited sequence 1/2, 1, 3/2, 2, 5/2, 3, . . . . . The central line in each graph divides the plotted points evenly into two halves. The two outer lines divide the points of each half into equal quarters. Thus the distance apart of the two outer lines gives a measure of the spread. If its ratio to the length of the assembly on the logarithmic paper is small the fit may be considered a strong one.

The reason for restricting the slopes to the sequence mentioned is a theoretical one which relates to considerations of the "human gas," not dealt with in this paper. (These considerations really apply to the slopes only of the graphs of population-density, rent, and farmland value.)

Tables 2 and 5 refer to a list of 253 "rural counties" which lie in the 28 states of the Main Sequence. These counties were selected for zero or relatively small urban populations. This was in order to use certain over-all statistics—such as the number of wage-earners in manufacturing—without having to worry about the local distortion of potentials which is produced

TABLE 2
VALUES OF POWERS OF THE POTENTIAL, V; 253 COUNTIES, 1940

| Median | $V$ | $V^{1/2}$ | $V^2$ | $V^5$ |
|---|---|---|---|---|
| a | 52 | 7.21 | 2704 | 3802. |
| b | 46 | 6.78 | 2116 | 2060. |
| c | 43 | 6.56 | 1849 | 1470. |
| d | 42 | 6.48 | 1764 | 1307. |
| e | 40 | 6.32 | 1600 | 1024. |
| f | 39 | 6.24 | 1521 | 902.2 |
| g | 38 | 6.16 | 1444 | 792.4 |
| h | 37 | 6.08 | 1369 | 693.4 |
| i | 36 | 6.00 | 1296 | 604.7 |
| j | 35 | 5.92 | 1225 | 525.2 |
| k | 34 | 5.83 | 1156 | 454.4 |
| l | 32 | 5.66 | 1024 | 335.5 |
| m | 29 | 5.39 | 841 | 205.1 |
| n | 27 | 5.20 | 729 | 143.5 |
| o | 24 | 4.90 | 576 | 79.63 |
| p | 23 | 4.80 | 529 | 64.36 |
| q | 22 | 4.69 | 484 | 51.54 |
| r | 20 | 4.47 | 400 | 32.00 |
| s | 19 | 4.36 | 361 | 24.76 |
| t | 19 | 4.36 | 361 | 24.76 |
| u | 17 | 4.12 | 289 | 14.20 |
| v | 16 | 4.00 | 256 | 10.49 |
| w | 14 | 3.74 | 196 | 5.378 |

Note: If the unit of potential is to be the person/mile, multiply the respective columns by 10 raised to the following powers: 4, 2, 8, 25. Tables 1 and 2 are for reference with respect to the computation of values of quantities in later tables.

by the influence of nearby cities. Such distortions cannot be allowed for on a map without great labor.

Since the total population of the 253 counties is much less than that of the 28 states, it is gratifying to find agreement within 12 per cent, for the coefficient of $V^2$ in the empirical rule for the density of rural population. For the 28 states (Table 3) we have the rural density averaging 0.0336 $V^2$, as compared with 0.0301 $V^2$ for the counties. (Of course the third figure is meaningless.)

Results for the 253 counties are smoothed by the device of plotting 23 medians, a, b, c, . . . . w, each of which represents 11 counties. With so small a sample of population as a single county, large fluctuations from the average would be expected if each were plotted.

Numerous additional statistics were examined for the counties. The suicide rate increases as the first power of the potential, while the birth rate declines and the median age increases. The excess of births over deaths decreases. The reader must remember that these are results for *rural*

TABLE 3
CORRELATIONS FOR 28 STATES, 1940

| State | V | Rural Density Obs. | Rural Density Comp. | RNF Density Obs. | RNF Density Comp. | RNF Rent Obs. | RNF Rent Comp. |
|---|---|---|---|---|---|---|---|
| NJ | 59 | 102. | 117 | 84.0 | 115.4 | 29.98 | 32.80 |
| Conn | 51 | 112. | 87 | 92.8 | 74.6 | 35.74 | 28.36 |
| Mass | 45 | 58. | 68 | 45.9 | 51.2 | 39.07 | 25.02 |
| Penna | 45 | 74. | 68 | 53.4 | 51.2 | 17.60 | 25.02 |
| Del. | 45 | 64. | 68 | 41.1 | 51.2 | 27.17 | 25.02 |
| RI | 44 | 57. | 65 | 47.2 | 47.9 | 24.96 | 24.46 |
| Md | 43 | 75. | 62 | 50.2 | 44.7 | 21.83 | 23.91 |
| NY | 41 | 48. | 56 | 33.4 | 38.7 | 24.95 | 22.80 |
| O | 41 | 56. | 56 | 29.7 | 38.7 | 16.17 | 22.80 |
| WVa | 38 | 57. | 49 | 34.8 | 30.8 | 11.93 | 21.13 |
| Ind | 37 | 43. | 46 | 20.1 | 28.5 | 13.62 | 20.57 |
| Va | 36 | 43. | 44 | 18.8 | 26.2 | 14.08 | 20.02 |
| Ill | 33 | 37. | 37 | 20.0 | 20.2 | 13.85 | 18.35 |
| Ky | 33 | 50. | 37 | 18.4 | 20.2 | 10.29 | 18.35 |
| Mich | 32 | 32. | 34 | 16.5 | 18.4 | 19.59 | 17.80 |
| Vt | 30 | 26. | 30 | 14.2 | 15.2 | 20.67 | 16.68 |
| NH | 29 | 23. | 28 | 16.4 | 13.7 | 23.72 | 16.12 |
| Mo | 28 | 27. | 26 | 10.2 | 12.3 | 9.60 | 15.67 |
| Wis | 27 | 27. | 24 | 10.7 | 11.1 | 20.00 | 15.01 |
| Ia | 25 | 26. | 21 | 9.5 | 8.78 | 12.97 | 13.90 |
| Me | 22 | 16. | 16 | 10.8 | 5.99 | 20.77 | 12.23 |
| Minn | 21 | 18. | 15 | 7.7 | 5.20 | 17.07 | 11.68 |
| Okla | 21 | 21. | 15 | 5.4 | 4.50 | 8.13 | 11.68 |
| Kan | 20 | 13. | 13 | 6.2 | 4.50 | 11.11 | 11.12 |
| Neb | 19 | 10. | 12 | 4.0 | 3.85 | 11.30 | 10.56 |
| Tex | 18 | 13. | 11 | 5.1 | 3.28 | 9.74 | 10.01 |
| SDak | 16 | 6.3 | 9 | 2.3 | 2.30 | 10.37 | 8.90 |
| NDak | 14 | 7.3 | 7 | 2.6 | 1.54 | 11.17 | 7.78 |

Note: Computed rural density, persons per square mile is 0.0336 $V^2$; R.N.F. Density is 0.000562 $V^3$. Observed values are from the U. S. Census. R.N.F. Rent is computed as 0.556 V, in dollars per month per dwelling unit.

*areas,* of density everywhere less than roughly 100 persons/mile.$^2$ In cities the densities are all above about 2,000 persons/mile,$^2$ rising to more than 250,000 in the densest census tract in Manhattan. The fall of the birth rate in cities is notorious, but the present study shows that most of the fall from the backwoods is evident in the high potential rural districts. Therefore population density is less suitable than potential as a determinant of vital statistics.

## 7. *Discussion of These Results*

These results make it evident that an intricate network of mutual correlations exists in the 28 states. If quantities *b, c, d,* . . . . are functions of *a,* then *b* is also a function of *c,* and again of *d,* and so on. This makes it impossible to assert on merely statistical grounds that potential is the primary variable in the situations examined. Indeed later study may show that the *temperature* of the "human gas" may be of still greater significance. It is likely to be proportional to the first power of the potential.

TABLE 4
ADDITIONAL CORRELATIONS FOR 28 STATES, 1940

| State | V | Farm Values Obs. | Farm Values Comp. | RR Mi/sq.mi. Obs. | RR Mi/sq.mi. Comp. | RFD Mi/sq.mi. Obs. | RFD Mi/sq.mi. Comp. |
|---|---|---|---|---|---|---|---|
| NJ | 59 | 121.54 | 174.05 | .280 | .246 | 1.179 | 2.343 |
| Conn | 51 | 135.41 | 130.05 | .181 | .213 | 1.502 | 1.883 |
| Mass | 45 | 109.40 | 101.25 | .227 | .188 | 1.047 | 1.561 |
| Penna | 45 | 59.22 | 101.25 | .229 | .188 | 1.272 | 1.561 |
| Del | 45 | 61.30 | 101.25 | .149 | .188 | 1.543 | 1.561 |
| RI | 44 | 118.67 | 96.80 | .183 | .183 | 1.235 | 1.509 |
| Md | 43 | 65.27 | 92.45 | .138 | .179 | 1.180 | 1.458 |
| NY | 41 | 55.16 | 84.05 | .161 | .171 | 1.037 | 1.357 |
| O | 41 | 65.91 | 84.05 | .208 | .171 | 1.706 | 1.357 |
| WVa | 38 | 30.29 | 72.20 | .159 | .158 | .503 | 1.211 |
| Ind | 37 | 63.20 | 68.45 | .190 | .154 | 1.638 | 1.164 |
| Va | 36 | 41.05 | 64.80 | .107 | .150 | .770 | 1.117 |
| Ill | 33 | 81.76 | 54.45 | .214 | .138 | 1.299 | .9801 |
| Ky | 33 | 38.26 | 54.45 | .092 | .138 | .682 | .9801 |
| Mich | 32 | 50.59 | 51.20 | .128 | .133 | .947 | .9359 |
| Vt | 30 | 30.30 | 45.00 | .099 | .125 | .870 | .8495 |
| NH | 29 | 34.38 | 42.05 | .111 | .121 | .703 | .8074 |
| Mo | 28 | 31.87 | 39.20 | .102 | .117 | .886 | .7660 |
| Wis | 27 | 51.96 | 36.45 | .121 | .113 | .936 | .7253 |
| Ia | 25 | 78.79 | 31.25 | .160 | .104 | 1.147 | .6463 |
| Me | 22 | 29.38 | 24.20 | .060 | .092 | .397 | .5335 |
| Minn | 21 | 44.26 | 22.05 | .105 | .088 | .682 | .4975 |
| Okla | 21 | 23.88 | 22.05 | .090 | .088 | .640 | .4975 |
| Kan | 20 | 29.51 | 20.00 | .104 | .083 | .733 | .4624 |
| Neb | 19 | 24.03 | 18.05 | .079 | .079 | .490 | .4282 |
| Tex | 18 | 18.81 | 16.20 | .062 | .075 | .301 | .3948 |
| SDak | 16 | 12.80 | 12.80 | .052 | .067 | .330 | .3309 |
| NDak | 14 | 12.92 | 9.80 | .075 | .058 | .416 | .2709 |

Note: Computed values per acre of farmland are 0.050 $V^2$, in dollars, if the unit of V is 10,000. Observed values are from the U. S. Census. Miles of railroad by states, and R.F.D. miles are from Statistical Abstract of the United States; the formulas are, respectively, 0.00413 V and 0.00517 $V^{3/2}$.

However it cannot be denied that the equipotential contours give a systematic picture of our national differentials. Even though the picture is valid only on the average, it ought to be presented in elementary geography courses, as well as in treatises devoted to advanced national planning.

The fact is very important that rural areas have a sociological structure which in the first rough approximation centers on New York City—and has so centered for 100 years. The Census Bureau's time-honored "center of population," now in Indiana, is a socially meaningless, politically misleading mathematical whimsey.

Obviously this study of the Main Sequence should be extended to the remaining 20 states. So far this has been published only for the rural densities (10). The study of incomes presented in § 8 is a contribution to the same problem, although it does not deal specifically with the rural populations.

Another needed extension is to statistics for the cities in each of the three territories, namely the "main sequence" of 28 states, the Deep South, and the 11 states of the Far West. The smaller cities in the 28 states are distributed in conformity with the equipotentials: the number of cities 2500-5000, for example, per 10,000 square miles varies about as $V^{5/2}$. But the largest cities cannot be expected to follow a regular spatial pattern.

A good deal of data have already been accumulated for the cities, but inclusion here would unduly lengthen this paper. Annual urban taxes

TABLE 5
CORRELATIONS WITH POTENTIAL FOR 253 COUNTIES

| Median | V | Rural Density Obs. | Rural Density Comp. | Density of Wage Earners Obs. | Density of Wage Earners Comp. | Death Rate Obs. | Death Rate Comp. |
|---|---|---|---|---|---|---|---|
| a | 52 | 61.7 | 81.4 | 11.26 | 2.832 | 12.3 | 14.2 |
| b | 46 | 52.5 | 63.7 | 1.79 | 1.535 | 13.8 | 13.3 |
| c | 43 | 30.4 | 55.7 | 1.051 | 1.095 | 13.7 | 12.9 |
| d | 42 | 57.8 | 53.1 | 1.52 | 0.9737 | 12.8 | 12.7 |
| e | 40 | 42.2 | 48.2 | 0.38 | 0.7629 | 11.8 | 12.4 |
| f | 39 | 39.9 | 45.8 | 0.46 | 0.6721 | 11.3 | 12.3 |
| g | 38 | 39.5 | 43.5 | .289 | 0.5903 | 12.0 | 12.1 |
| h | 37 | 40.5 | 41.2 | .85 | 0.5166 | 11.7 | 12.0 |
| i | 36 | 36.3 | 39.0 | .19 | 0.4505 | 11.2 | 11.8 |
| j | 35 | 35.6 | 36.9 | .077 | 0.3913 | 11.0 | 11.6 |
| k | 34 | 42.4 | 34.8 | .029 | 0.3385 | 10.9 | 11.4 |
| l | 32 | 31.1 | 30.8 | .200 | 0.2499 | 11.5 | 11.1 |
| m | 29 | 24.5 | 25.3 | .036 | 0.1528 | 10.2 | 10.6 |
| n | 27 | 20.2 | 21.9 | .023 | .1069 | 10.0 | 10.2 |
| o | 24 | 23.8 | 17.3 | .09 | .0593 | 9.8 | 9.6 |
| p | 23 | 27.7 | 15.9 | .077 | .0479 | 10.1 | 9.4 |
| q | 22 | 17.9 | 14.6 | .021 | .0384 | 9.1 | 9.2 |
| r | 20 | 15.6 | 12.0 | .025 | .0238 | 9.1 | 8.8 |
| s | 19 | 11.9 | 10.9 | .039 | .0184 | 9.3 | 8.5 |
| t | 19 | 11.1 | 10.9 | .017 | .0184 | 7.9 | 8.5 |
| u | 17 | 8.9 | 8.7 | .032 | .0106 | 8.5 | 8.1 |
| v | 16 | 9.2 | 7.7 | .012 | .00782 | 8.4 | 8.0 |
| w | 14 | 4.7 | 5.9 | .0097 | .00401 | 8.4 | 7.4 |

Note: The formula of rural density, 1940, is $0.0301\ V^2$; density of wage earners in manufacturing industries is $7.45 \times 10^{-9}\ V^5$; death rate is $1.97\ V^{1/2}$. Observed values from U. S. Census, the death rate being the average of the two years, 1939-40.

per capita, for example, increase with the average total potential in a city (i.e., the rural base potential plus the city's own contribution). The suicide rate continues to increase with increasing potential in cities, but less rapidly than in rural areas.

### *8. The Relation of Demographic Energy to Income*

The first suggestion that demographic energy is related to economic wealth came from the the fact that rents and land values tend to be dependent on the potential of population. When a potential map of the United States has been made for any census, the total demographic energy of the whole country, $E_t$, can be obtained from the formula

$$2E_t = \sum N_i V_i, \tag{14}$$

The summation is taken over the United States: $N_i$ is the population of state or county, and $V_i$ is its average potential there. This equation is a consequence of equation (11).

Furthermore the assumption is plausible that

$$2E_i = N_i V_i \tag{15}$$

where $E_i$ is the demographic energy of the i-th state or county. Equation (15) is not an inevitable consequence of (14) and requires empirical justification.

Table 6 lists the demographic energies so computed for each of the 48 states in 1940, together with their average potentials, state by state, as estimated from the recently computed map (which is a refinement of the one shown in Figure 0.) In each state an additional correction was applied to $V_i$ to take roughly into account the extra contributions made by the urban concentrations to their own potentials.

These corrections were based on the empirical formula which relates the area of a city in 1940 to its population (11)

$$A = P^{3/4}/357, \tag{16}$$

A being the area in square miles and P the population.

If a city is assumed to be roughly circular the radius computed from this area for a particular city, when divided into the city's population, gives roughly the potential at the city limits. The average potential inside the city may be twice as great as this. Full discussion is reserved for a later publication.

TABLE 6
DISTRIBUTION AMONG THE 48 STATES OF INCOME TO INDIVIDUALS, 1940, COMPARED WITH THE DEMOGRAPHIC ENERGIES

| | State | Average Potential | Demographic Energy (billions of units) | Income (billions of cents) Obs. | Comp. |
|---|---|---|---|---|---|
| (1) | NY | 98.7 | 6,660 | 1,183 | 2,000 |
| | Penna | 57.2 | 2,830 | 623 | 850 |
| | Ill | 57.1 | 2,250 | 574 | 680 |
| | O | 48.0 | 1,650 | 445 | 500 |
| | NJ | 86.4 | 1,620 | 314 | 490 |
| | Mich | 46.6 | 1,225 | 343 | 370 |
| | Mass | 50.5 | 1,090 | 331 | 330 |
| | Ind | 41.3 | 707 | 186 | 210 |
| | Tex | 20.8 | 667 | 265 | 200 |
| | Mo | 33.6 | 636 | 191 | 190 |
| | Ky | 36.4 | 519 | 88 | 160 |
| | Va | 36.8 | 492 | 113 | 150 |
| | Wis | 30.9 | 486 | 162 | 150 |
| | Conn | 56.1 | 479 | 142 | 140 |
| | Md | 52.0 | 473 | 122 | 140 |
| | Minn | 26.8 | 374 | 142 | 112 |
| | WVa | 36.9 | 351 | 76 | 105 |
| | Ia | 26.7 | 339 | 123 | 102 |
| | Okla | 21.5 | 270 | 83 | 81 |
| | Kan | 24.4 | 220 | 76 | 66 |
| | RI | 50.5 | 180 | 51 | 54 |
| | Neb | 24.9 | 141 | 57 | 42 |
| | Me | 22.8 | 97 | 43 | 29 |
| | NH | 33.3 | 82 | 27 | 25 |
| | Del | 48.0 | 71 | 24 | 21 |
| | SDak | 17.3 | 56 | 24 | 18 |
| | Vt. | 30.4 | 55 | 19 | 17 |
| | NDak | 14.4 | 46 | 24 | 14 |
| (2) | NC | 32.4 | 578 | 113 | 116 |
| | Tenn | 32.8 | 478 | 93 | 96 |
| | Ga | 28.0 | 437 | 99 | 88 |
| | Ala | 26.6 | 405 | 76 | 81 |
| | La | 25.4 | 300 | 85 | 60 |
| | Miss | 23.8 | 260 | 44 | 52 |
| | SC | 26.8 | 255 | 55 | 51 |
| | Ark | 24.6 | 240 | 49 | 48 |
| | Fla | 21.2 | 201 | 90 | 40 |
| (3) | Calif | 31.6 | 1,090 | 561 | 905 |
| | Wash | 15.9 | 138 | 110 | 145 |
| | Col | 18.6 | 105 | 59 | 87 |
| | Ore | 14.4 | 78 | 64 | 65 |
| | Utah | 14.5 | 40 | 27 | 33 |
| | Mont | 12.3 | 34 | 32 | 28 |
| | NM | 12.7 | 34 | 19 | 28 |
| | Ida | 10.9 | 28 | 23 | 23 |
| | Ariz | 11.0 | 27 | 24 | 22 |
| | Wy | 12.9 | 16 | 15 | 13 |
| | Nev | 10.0 | 6 | 9 | 5 |

Note: The computed incomes in cents per year are found by multiplying the demographic energy (units of persons$^2$/mi.) by (1) 0.83, (2) 0.30, (3) 0.20, respectively. The potentials listed in this table are higher than in the previous tables because here the computations of the local "city peaks" are included. The observed incomes are from a tabulation by the Department of Commerce.

Figure 6-1 shows that the states divide into three groups as regards the proportionality of income to demographic energy: namely, the Main Sequence of 28 states, the Far West, including 11 states, and the Deep South, which strictly speaking comprises 8 states, because the ninth, Florida, in many respects, belongs with the Far West.

We have, letting $E_i$ represent the energy in persons$^2$/mile, $I_i$ the income in cents/year of a state (1940):

$$\begin{aligned} 28 \text{ states: } \quad I_i &= 0.30\ E_i; \\ 11 \text{ states: } \quad I_i &= 0.83\ E_i; \\ 8 \text{ states: } \quad I_i &= 0.20\ E_i. \end{aligned}$$

One way of introducing into the theory the differences which appear in these three empirical formulas is to assume that the "molecular weight" of people in the Far West is about double that of people in the Main Sequence; while the molecular weight of negroes in the Deep South averages only 1/3.

It probably is significant that the study (12) of residences of undergraduates who attended Princeton, Yale, Harvard, and M.I.T. during the 1930's showed about the same discrepancies for the West. Princeton had 2.34 times as many students from the 11 states as are computed by the potential formula if the latter is calibrated to fit the 28 states.

If further investigation shows that differences in molecular weight really exist, it will be necessary to correct the potential map accordingly—lowering potentials a little in the South and raising them along the West Coast. The changes would be by factors less than the suggested deviations from unity of the weights, because the influence of the Main Sequence is important everywhere. Then all the tables in this paper, especially Table 6, would require slight revisions, based on the new map.

Since consideration of demographic energy suffices to provide an interesting spatial treatment of incomes in the United States, the obvious suggestion is to examine the relation of the two in time. The result is presented in Table 7, and shows a remarkable proportionality for 120 years.

Evidently the raw American dollar of income has been endowed with a sort of statistical inertia which has kept it stable, on the average, in ratio to the demographic unit of wealth, the person$^2$/mile. One can advance reasons why this might be so.

The long-term rise in per capita income (and in prices) is exhibited as a consequence of the increase in population and of the consequent increase in the number of demographic units represented by any assigned economic good—such as an acre of farmland.

TABLE 7
NATIONAL INCOME AS A FUNCTION OF DEMOGRAPHIC ENERGY, 1829-1939

| Year | Energy persons$^2$/mi. | Income cents/yr. | Income/Energy |
|---|---|---|---|
| 1829 | $3.6 \times 10^{11}$ | $1.0 \times 10^{11}$ | 0.28 |
| 1839 | 6.1 | 1.6 | 0.26 |
| 1849 | 10.7 | 2.4 | 0.22 |
| 1859 | 20. | 4.3 | 0.22 |
| 1869 | 31. | 6.8 | 0.22 |
| 1879 | 47. | 7.2 | 0.15 |
| 1889 | 70. | 10.7 | 0.15 |
| 1899 | 102. | 15. | 0.15 |
| 1909 | 149. | 26. | 0.17 |
| 1919 | 192. | 63. | 0.33 |
| 1929 | 256. | 79. | 0.31 |
| 1931 | 264. | 60. | 0.23 |
| 1933 | 269. | 45. | 0.17 |
| 1935 | 275. | 57. | 0.21 |
| 1937 | 280. | 69. | 0.25 |
| 1939 | 285 | 69. | 0.24 |
| 1940 | 288. | 75. | 0.26 |

Note: The incomes are in current dollars, uncorrected for price changes, and are from a tabulation by the Conference Board (*The Economic Almanac,* 1948, p. 352.) The influence of the business cycle is evident, especially when data for a series of consecutive years are examined. In booms the dollar quotation of the demographic unit rises, while it declines in deflations. The 1947 income of about 200 billion dollars gives an income/energy ratio of 0.67 cent per demographic unit, which is the most extreme inflation in the whole record. The demographic unit, the persons$^2$/ mile, is the contribution to the national wealth made by a pair of average Americans who live one mile apart.

The ups and downs of the business cycle have been accompanied by a rise in the dollar quotation of the demographic unit during the boom phases, followed by an equal fall during the busts. We find the United States now in the most extreme inflation of the 128-year record. Economists can point to the aggressiveness of the labor unions, to the farm-parity law, and to our departure from the gold standard as factors which, even without further enormous "defense" spending, may have broken the old anchorage to the demographic unit. Borrowing, lending, and insurance cannot work well if the dollar ceases to represent an objective value.

### 9. *Additional Specific Applications and Suggestions*

The Cost Ascertainment Report of the postal service gave the length of the average haul of non-local mail in the United States in 1931 as 444 miles. This compares closely with the result of a computation which assumes that the number of letters $N_1$ people write in a given time to $N_2$ people is $kN_1N_2/r$, $r$ being the distance between them; $k$ is supposed to be a constant throughout the country. In the notation of calculus, the number of letters written by dq people who live in a certain place in the United States is $dq\int k\,dp/r$, where the integral is taken for all the people in the country. And the total number of letters written is

$$L = \int dq \int k\,dp/r = 2kE. \tag{17}$$

Here

$$2E = \int dq \int dp/r, \tag{18}$$

where the integrals are taken throughout the country; E is the demographic energy, since $\int dp/r$ is the whole potential acting on the element of population dq.

The total miles traveled by all the letters written is

$$T = \int dq \int (k\ dp/r) r = kP^2, \tag{19}$$

where

$$P = \int dp;$$

so P is the total population of the country.

Evidently the average length of haul is

$$h = \frac{P^2}{2E}\ . \tag{20}$$

In 1930 *P* was 124 million people, and *2E,* without including the extra contribution of the city peaks, can be estimated as 38,000 billion $\text{persons}^2/\text{mile}$. Then (20) gives $h = 430$ miles, in satisfactory agreement with the observed value of 444 miles.

It is worth noting that since the Census of 1790 the computed value of *h* has increased less than common opinion about the expansion of the country would suggest. In 1790 this average distance of people from people in the United States was roughly 200 miles.

If it is wished to include the practically zero length of haul of local mail in the average for 1930, we add in the computation a considerable positive correction to E, which represents the extra energy of the urban concentrations. Equations (17) - (20) may be expected to apply to a variety of person-to-person interchanges resembling mail. Further testing would be in order.

The concepts of potential and energy are applicable for solving readily, to a first rough approximation, such a question as, Would a bridge over a river at a given point, or a certain highway tunnel through a long mountain ridge, be economically justified? In these cases, the straight-line physical distance is not the one to use in computing the initial potentials, because the topographical barrier forces communications to take roundabout routes. Examination of the map would show what distances it is reasonable to use in equation (10) before and after the new facility is constructed. The expected increase in potential in each area is multiplied into the size of the population affected; the sum of the products is twice the increase in wealth in demographic units.

For example suppose two cities of 10,000 and 15,000 people, respectively, lie on opposite banks of a river a mile wide which has neither a bridge nor a ferry for a hundred miles. If the average distance apart of the people in the two cities is 2 miles, the gain in demographic energy provided by the bridge, from the improvement in local connectivity only, will be

$$G = (10{,}000) \quad (15{,}000)/2 \quad \text{persons}^2/\text{mile},$$

assuming unit molecular weights. This is 75 million demographic units. At the inflated 1947 monetary quotation (see footnote, Table 7) of about 2/3 cent/year, this gain, after the new equilibrium became established, might produce an increase of 500,000 dollars (1947 variety) in local incomes per year.

Consequently an expenditure of perhaps 1,000,000 dollars to build the bridge would seem justified. However, the officials would know that only a fraction of the 500,000 dollars per year would operate to maintain an increased value of the real estate in their two cities. The remainder would escape their taxation.

With respect to the relation of real estate value to demographic energy in small cities, the residential rental data compiled in the 1940 Census are important. A quick survey shows that the rent per month per dwelling unit in cities of the 2,500 - 5,000 size class was about five dollars greater than the rural non-farm rent in the respective counties. Allowing 4 persons to a dwelling unit, and an average population of 3,500, we find that such an urban concentration supported an increased rental of about 50,000 dollars per year, as compared with what the 3,500 people would have paid at the rural non-farm rate of the neighborhood.

Equation (16) suggests that the radius of a city of 3,500 might be 2/3 mile. The potential of population at the edge would be nearly that which would result if all the people lived at the central point; namely, 5,000 persons/mile; and the average potential within the city might be twice this. Since, by equation (14), energy is half the product of population by average potential, the extra demographic energy associated with the formation of the little city is, roughly, half 10,000 times 3,500, or 18 million persons$^2$/mile. At the 1940 quotation of 0.26 cent/year, this energy corresponds to an increase of income of 47,000 dollars a year above what the same people would have been expected to have if they lived as rural nonfarm population in the same general vicinity of the United States and were not concentrated in their city. Evidently, all of this extra income would be needed to meet the urban increase in residential rent.

This result does not seem reasonable, because only a fraction of income normally is spent for rent. Doubtless when people live close enough together to form a city each exerts on his immediate neighbors a demographic force of *cohesion,* which adds to that of demographic gravitation and is associated with additional economic value. In order to use the same sewer and water lines, etc., people must live very close together. If they spread out a little their mutual energy of gravitation does not lessen much, but the required extensions of graded streets, curbs, sidewalks, gas mains, electric wires, postmen's routes, and the like, are costly. These urban improvements must be a factor in the increase of urban rent.

When gas molecules come close enough, their cohesion binds them together and they form a liquid. Investigation shows that the rural population is analogous to a gas. The suggestion that a city may be looked upon as a drop of liquid is worth following in future work.

This paper has outlined a good deal of material which points to the validity of the energy and potential equations of demographic gravitation. Work also has been done which illustrates the applicability of the third form of the law of demographic gravitation, namely, equation (6) for the gradient (slope) of potential. East of the Sierra Mountains, as Table 0 indicates, every city except New York itself is a local peak on the general downhill slope of potential of population away from New York. On topographic contour maps a mountain peak is shown by closed contours of potential (elevation) around it. Likewise, closed contours of potential of population extend over a sizeable area around a large city, and over a small area around a small one. The outermost such closed contour marks the limit of the region where land values would be expected to increase toward the city, and where the city in other ways may be exercising the dominant influence.

From the mathematics of potential theory it is not hard to show that this area is roughly equal to $P/g$, where $P$ is the city's population, and $g$ is the general slope of potential in that section of the country. Where $g$ is especially large a small city or a village has a very small area of influence. Indeed if $P$ is small enough in ratio to the gradient, $g$, the village has no zone of its own at all, and we expect it then to show signs of being torn apart by the "tidal" action of the rest of the nation—like a planet too close to the sun.

Within the limits of a great metropolitan district, the gradient is especially large, and it is fairly large in all the sections listed near the beginning of Table 0. It is a fact, which the reader may verify by inspection of such sections, that the smaller settlements often tend to be ragged in outline and to lack the individual identity and regular form which they assume where $g$ is small. Space is lacking here to give a full discussion, and work is being continued; but this is another confirmation of the existence of demographic gravitation and of the authenticity of the equations presented in this discussion.

The studies of Section 6 also are being continued. One objective is to draw up a list of "normal" counties, for which the statistics fall close to the central lines of the different graphs. "Hot" counties, of superior-to-average economic conditions, and "cold" ones lower than average, are also of interest.

In Europe, rural densities in the 1930's resembled those in the United States in varying as the square of the potential of population (13). The lack of homogeneity would make some of the statistics represented in Section 6 difficult to set up in Europe. If we assume a molecular weight of unity, equal to that of the average American, the demographic energy of Europe works out as about 10 times that of the United States. A comparison with American activity and productivity indicates that Europe in the 1930's excelled the United States by a factor of less than 2.

There are many quantitative peacetime statistics which show that the magnitude of the measurable objective influences of Europe and other continents on the United States is a small fraction of the magnitude of the relations of our own citizens with one another. For example, the value of imports per year from all the world has been only about 6 per cent of our national income, over a long period. Distance alone does not account for so small an interchange; our isolation is greater than the width of the oceans would suggest.

There is of course plenty of evidence that the stringent national frontiers and cultural differences weaken the mutual relationships of Europeans among themselves. In 1938 the Netherlands post office handled 320 million letters of internal origin and destination, but the foreign correspondence amounted to only 67 million (14). With 8 million Dutchmen in Europe's 500 million people, the international correspondence would have greatly exceeded the domestic if equations (17) - (20) applied as they do in the United States.

We might take this sort of thing as evidence that frontiers can produce the equivalent of a large increase in all the international distances, reducing the energies and potentials accordingly. Again more study is required. It might be mentioned that over a certain period 10 times fewer Canadians and 25 times fewer Mexicans came as undergraduates to Princeton University than would have been expected from the potentials of population. But against this must be set in Europe the fact that no marked distortion in the relation of rural density to potential seems to be produced by the frontiers.

No doubt certain relations are sensitive to cultural differences while others are not. In a first study we can afford to omit the sensitive ones. And also we are justified in concentrating upon relatively simple and undisturbed conditions within the United States. Thorough understanding of these must precede the refinements necessary to incorporate into the theory all the diversifications of a region so rich in and so vexed with demographic parameters as Europe.

As a working hypothesis, demographic energy may be interpreted as the *number of human relations per unit time*. We may think of a relation between a pair of people not as a static thing, like a string connecting them, but as an impulse which happens, like a wave along the string. The *accumulating total* of such happenings is the integral of the energy with respect to the time, and would seem to resemble *action* in physics.

Relations of people to one another require more than people to support them: they require also natural resources and technological facilities. How these two essentials are to be represented in formulas is not yet clear. They may have something to do with the molecular weight, which is the *demographic mass per capita*.

In physics mass is looked upon as interconvertible with energy. Mass is proportional to the *constitutional energy* of matter. This was first foreshadowed in equations published by J. J. Thompson about 1880, and then vaguely revealed in the late 90's in phenomena of spontaneous radioactivity.

In 1906 Einstein made the connection explicit, with his famous equation: energy equals mass times the square of the speed of light. But more than thirty years elapsed before uranium fission was discovered, and it is only months since a cyclotron transformed energy into mesons.

It may be that sociologists face problems resembling those which engage physics's newest phases. If the maximation of demographic energy or mass is set as a proper social goal, then what part must the freedom of the individual play? Can the sum of human relations represented by $(N_1\mu_1)\ (N_2\mu_2)/d$ be increased if instead of being permitted to occur spontaneously between pairs of persons the relations are required to be channelled through a chain of social command? Can an individual by taking thought add a unit to his demographic stature? Can society do this for him? Were the prophets right in insisting that every increase comes as a gift of spiritual grace from a Power above mankind?

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