# The Growth Models for Science of Cities

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# Contents

# Overview

- the Zipf's law and its formation
  - Zipf's law without fine-tuning: static mesoscopic
  - stationary distribution of dynamical processes for the sizes of groups of individuals
    - mesoscopic: cities
    - microscopic: individuals
- Gibrat's law and Taylor's law

# Fractal Cities

- M. Batty and P. Longley, Fractal Cities: A Geometry of Form and Function (Academic, San Diego/London, 1994).
- Cellular automata have been used to model spatial structure of urban land use over time: Environ. Plan. A 25, 1175 (1993).
- The correlated percolation model

# the Zipf's law

The group size distribution.

- the number of employees in firms
- the distribution of family names
- the distribution of city sizes

$$P(n) \sim n^{-1-\gamma} \quad (1)$$

$$\gamma \simeq 1 \quad (2)$$

# Models that lead to the Zipf's law

Some up-to-date models are really fascinating.

- Models with latent variables can lead to Zipf's law without fine-tuning by mixing together narrow distributions with very different means.(PRL 113, 068102 (2014))
  - static systems (no time dependence)
- The stationary distribution of dynamical processes for the sizes of groups of individuals
  - mesoscopic models at the scale of the groups (e.g., cities) Am. J. Phys. 58, 267 (1990). Variance that is not Gaussian but exponential Gaussian leads to the Zipf's law.
  - microscopic models at the scale of the individuals (e.g., dwellers)

# Mesoscopic Models

- without the need to fine-tune their parameters to specific values
  - random multiplicative process, Am. J. Phys. 58, 267 (1990).*space independent*
  - Gibrat's law / proportionate random growth, Phys. Rev. E 57, 4811 (1998).
  - the interplay between intermittency and diffusion, Phys. Rev. E 58, 295 (1998).
- however they are coarse-grained descriptions of population dynamics and lack an explicit link to the underlying microscopic processes

# Microscopic Models

- Stochastic processes describing the events experienced by an individual, namely births, deaths, and migrations, that ultimately determine the change in the size of a population.
  - Yule's and Simon's models: rich-get-richer, Phil. Trans. R. Soc. Lond. B 213, 21 (1925), Biometrika 42, 425 (1955).
  - Cluster growth and aggregation, Phys. Rev. E 58, 7054 (1998).
  - Reaction diffusion models: explore the role of intermittency in creating spatial inhomogeneities in agreement with Zipf's law, Phys. Rev. Lett. 79, 523
  - Preferential migration to large aggregates, Phys. Rev. Lett. 88, 068301 (2002).
  - Spatial explicit preferential attachment: the probability that a city grows is essentially assumed to be proportional to the size of the city.
  - network growth with redirection, Phys. Rev. X 4, 011008 (2014).
- Only for fine-tuned parameters to get $\gamma = 1$.

# Radical Decay

- The correlated percolation model: an urban built environment is shaped by spatial correlations where the occupation probabilities of two sites are more similar the closer they are.

# Taylor's law

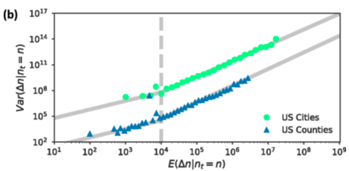



Figure: The variance of population in year $t+1$ conditioned to the population in year $t$ ($y$ axis) vs the average population in year $t+1$ conditioned to the population in year $t$ ($x$ axis) for cities (circles) and counties (triangles) in the United States during the period 1970–2010.

# My Current Work: Spatial Yule Model

How can we address more aspects of urban studies within a simpler model?

# Observations

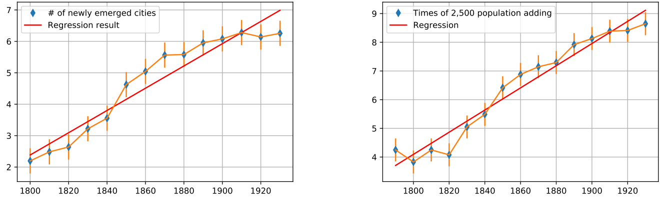
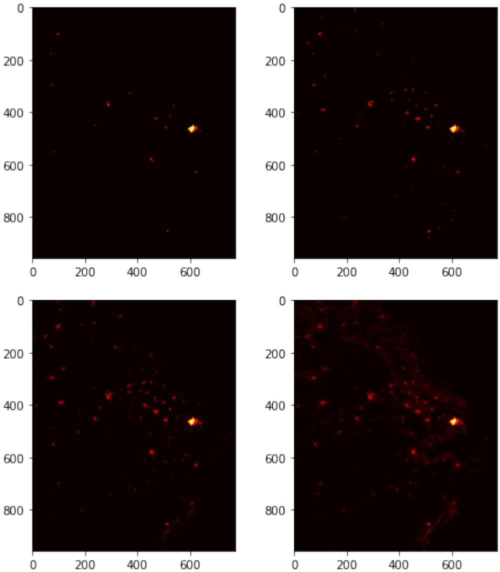
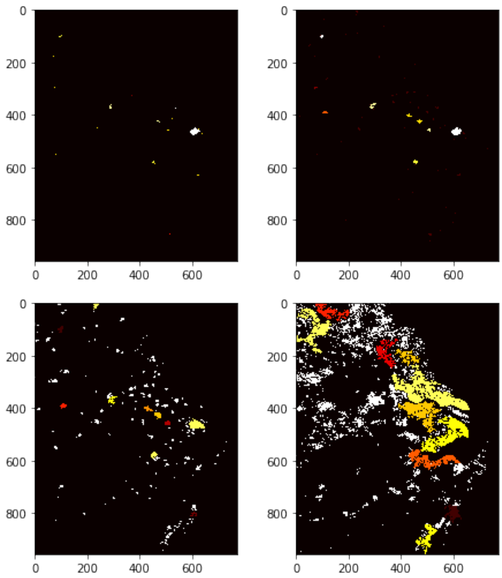



Figure: We take 2,500 as the study unit and get a) The emerging speed of cities in the United States, and b) the counts that 2,500 population are added to an existing city. Both slopes are around 0.04, $p < 0.0001$.

# Population based spreading



# Population based spreading



# Formation Philosophy

- Scaling of inner- intra-city (area, population, infrastructure)
  - Gradient descent to the sum of two-dimensional normal distributions for a optimal locating problem lead to some scaling behavior.
- Interacting cities lead to Zipf's law and fractality of urban boundary.
- Asymptotic behaviors (standard cause of scaling) is far from reality.

# Rules



The background is on a $L \times L$ grid space. The growing mechanism states as follows:

1. Growth rate per capita $\beta_2$, and the growth rate for # of cities $\beta_1$.
   - ignoring the correspondence to real-life time scale, the actual effective parameter the relative $\beta := \beta_2/\beta_1$.
2. The distance a ball lands a meta-population nearby as a constant, $r$, towards a random direction $\theta$.
3. The *productive* balls are limited, with no more than $N^*$.

# Points

1. possessiveness
2. homogeneity
3. cut-off
   - comparing to logistic?

- The settings provide a border line between free growth (Scaling laws) and constrained growth (vicissitude phenomena).

# Results

- the Clark's law of urban population density
- the Zipf's law of city population sizes
- vicissitude
- fractality

# the Clark's law

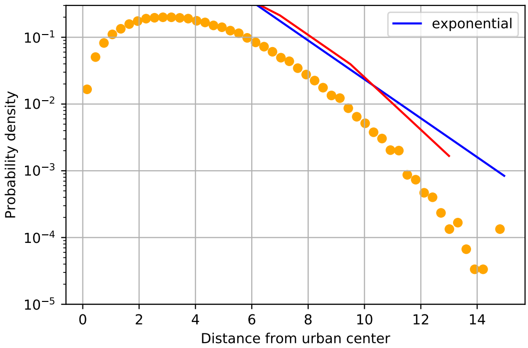
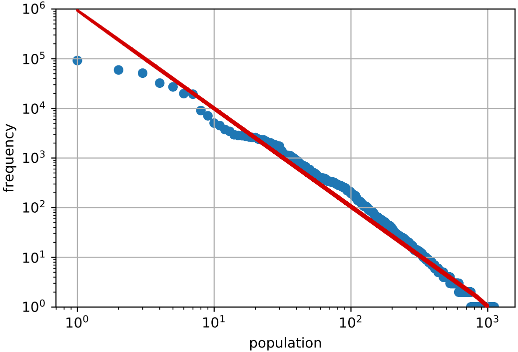



Figure: The popualtion density as a function of distance from urban center.

# the Zipf's law




$$P(n) \sim n^{-2}$$

# the Zipf's law

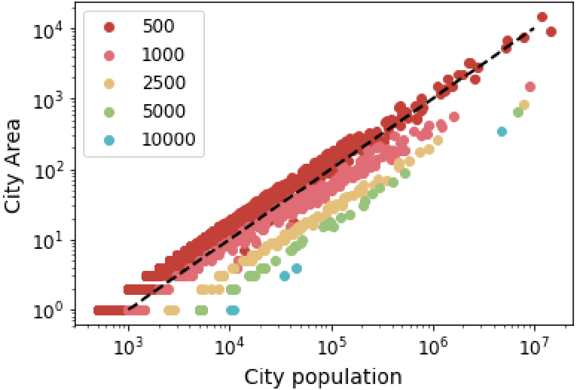


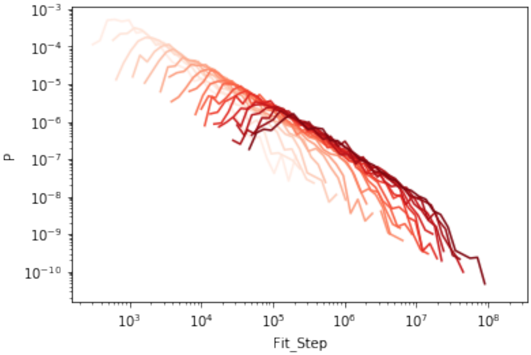

Figure: Population versus area

# vicissitude

- Blockwise

$$\rho_{threshold} = k\beta_1/(2\beta_2) + N^*/2 \qquad (3)$$

- Citywise



# fractality

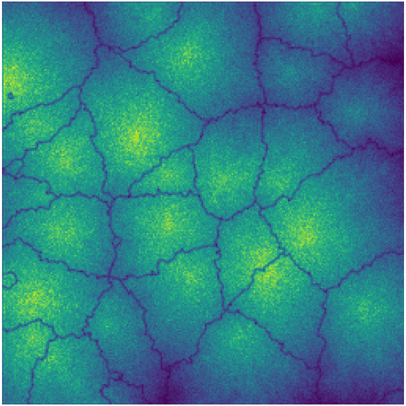


Figure: Fractality is driven by the probabilistic competition for edging space.

# Topics about *what is a city*?

Owuor Otieno, Mark . (2017, October 23). What is the Difference Between a City and a Town? Retrieved from https://www.worldatlas.com/articles/what-is-the-difference-between-a-city-and-a-town.html