Vicissitudes of Cities Driven by Re-distributive Growth

Gezhi Xiu, Jianying Wang, Lei Dong and Yu Liu*
Institute of Remote Sensing and Geographic Information Systems (IRSGIS), Peking University
(Dated: April 28, 2020)

We propose a spatial growth model to address how cities emerge, grow, and especially, compete, over limited resource and space. The approach emphasizes on the evolutionary trajectory of cities, simultaneously determined by local (e.g., topography) and regional (e.g., industrial status) conditions, which can be attributed to the competition for redistributive resources in a given space. To model this spatial competition mechanism, our out-of-equilibrium growth model determines a fixed bound on global growth rates. We discuss two phases of urbanization predicted in our model: (a) free growth phase, and (b) resource constrained phase. Zipf's and Clark's laws in urban sciences are found in (a), indicating realistic urbanization process has not yet reached bottlenecks of resource; And when it reaches, (b) captures the inevitability of various urban diseases, e.g. urban shrinkage in developed cities and the spatial relocation of developments. Our approach sheds light on analyzing urbanization with early warnings of environmental capacity.

Introduction. How do cities co-evolve over limited resource and space? To improve the conceptional understanding of urbanization and landscape evolution under complex circumstances, we need to set up some spatial growth models, i.e., a collection of models that derive macroscopic dynamical state given rules for every individuals. Such models are powerful tools to understand urban growth dynamics [1–5]. Theoretically, these models fill the gap between macroscopic patterns, e.g. socio-economic scalars and material properties, and microscopic dynamics, e.g. individual level interaction patterns; Practically, they investigate how different growth factors contribute to city emergence, and how these dynamics lead to the observed scaling laws [6–9], fractality [10, 11], and city size distributions (i.e., Zipf's law) [12], giving quantitative references broadly through urban morphology and spatial structures for urban planers and policy makers [13]. These existing works have built natural paths in deriving macroscopic dynamical state from microscopic growth rules. Most of these approaches are based on the assumptions of homogeneous growth in Euclidean geometry. However, recent discoveries of complex spatial phenomena associated with realistic urban systems are better described using fractal or discrete geometry[3, 14, 15], to better consist with growth dynamics in disordered contexts and media. Individually, existing models cannot incorporate spatially heterogeneous social group structures, e.g., ages[16] and limited working chances. These call up a model with some simple mechanisms to include such information.

To capture the competitions, we develop a spatial growth model based on inconsistent space and memory-based growth dynamics. Here, new cities spontaneously emerge over free space, and a city is determined as the continuous surface that are developed by the same emergence. The spatial sprawl and the advance of urbanization are realized by the sequential settlement of new citizens. We claim that the location choice of the newly initiated citizens are adjacent to those limited total reserves of replacive *active population*, keeping spatial preferential attachment and competition all at once. We show that beyond the desolated growth of each city, competitions introduced by spatial specialization and resource limit result in the vicissitudes of cities and urban shrinkage. Our model combines two themes for many disciplines, including probability theory and ecology: The spatial preferential attachment mechanism [2], and the existence of environmental capacity under competition [17, 18].

On the first point, the *rich get richer* mechanism is well-observed in social systems, especially, human settlements are clustered hence cities [19]. Literature has discussed how urban features emerge from preferential attachment via interaction density [14, 20, 21], e.g., multiplicative or correlated percolation [3, 4, 15], spatial networks [2, 7, 19], and utility maximizing [22, 23]. Spatially, such mechanism leads to population clustering near every urban centers. Here, we take the spatial aspect from the idea of diffusion-limited aggregation [3, 4, 24], i.e., in each cities, new comers would settle near those active citizens.

On the second point, cities are systems with environmental capacity, that excessive employees do not guarantee the proportional urban output, considering finite technical demands and increasing infrastructural demand. In words, the marginal effect of urbanization leads to decreases in urban attractiveness [8, 25–28]. These are poorly reflected in the referred works, which are mostly base on certain equilibrium conditions or optimization aims [12]. However, cities are inter-competitive and out-of-equilibrium to be consistent under circumstances towards sustainability[14, 20, 21]. Thus we include rules for spatial exclusiveness and bounded growth rate. Here, as also shown in non-spatial contexts [29], our restrictive settings intensify the inter-city competitions for active population[30]. We later prove that our space-relevant model extends the previous non-spatial predictions for city size distributions. These settings also result in realistic urban phenomena like ranking turnovers[31], and urban shrinkage[32], that cannot be formulated by existing growth models.

Spatial Yule Model. Our model tells how cities emerge, grow, and compete over space. Its dynamics are mainly determined through three quantitative and spatial rules: 1) *Active citizen rule.* During urban growth process, we assume that only 'active' citizens attract new comers to a nearby place in their city, k and N_i are the number of cities and active citizens in the ith city, respectively. 2) *Memory kernel rule.* We take $\sum_i N_i \leq N^*$ ($N^* \gg 1$) as the satiation condition, i.e., when the total population exceeds N^*, a new comer would deactivate a random dweller who is previously active. This mechanism keeps up to N^* active citizens adding up in the whole region. 3) *Spatial growth rule.* We assume the studied area is an $L \times L$ 2-dimensional continuous space ($L \gg 1$) with grid of cells and periodic boundary conditions, i.e., the locations of citizens are continuous, but the boundaries of cities are discrete on cells. A new city is seeded randomly over the region as a Poisson point process [33], and survives if its cell is not taken; Every new citizen settles at a constant distance $r \leq 1$ and a random angle θ from its introducer. Once a cell c has held a citizen from the ith city, any citizens from another city j ($j \neq i$) cannot introduce new comers on cell c. Thus cities are identified through citizens' ancestral introducer and their geographical occupation of blocks. The spatial growth rule is parallel Yule's settings on modeling the distribution of species per genus [34]. So our model could be regarded as a spatial Yule model with constraints (SYM). A sketch for the SYM is shown in Fig 1.

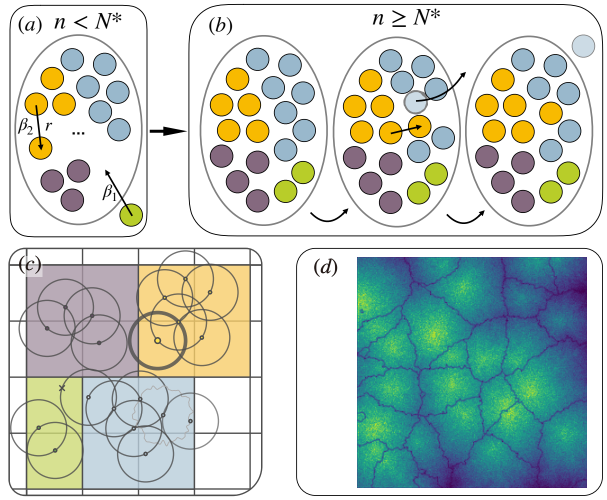

FIG. 1. **(a)** Status in the memory kernel at the free growth phase, i.e., the total population is less than N^*. Existing citizens introduce new dwellers at the rate β_2, while each existing city (noted by nodes in different colors) introduces new cities at the rate β_1. **(b)** When the memory kernel is fulfilled, every introduction of new city or citizen leads to an ejection of existing active citizen currently included in the memory kernel. **(c)** The spatial aspect is that, an offspring citizen's placement is at distance r from the ancestral dweller. Also, when the kernel is filled, a new yellow node ejects an existed blue node, or equivalently deprive her ability to introduce. **(d)** A simulated result for L, r, β equals to 256, 0.5, and 4, respectively. We choose $\beta = 4$ to avoid confusion of too many cities. This is parallel to a quarter of $2L \times 2L$ simulation with $\beta = 1$.

Based on these rules, we can define the model with a set of two-phase master equations. Specifically, we assume that the probability of a population increase in city i within the time interval $(t, t+dt)$ is $\beta_2 N_i dt$, where β_2 is the introduction rate of every active citizen; We also assume that new cities constantly emerge with a small rate $\beta_1 k dt$, proportional to the number of cities, where β_1 is the rate of new city generation. A city's generation is confirmed only if its location is at an empty cell. The master equation can be written as

$$\frac{\partial}{\partial t} N_i(t) = \delta_{N_i(t)} \cdot k\beta_1 + (1 - \delta_{N_i(t)}) \cdot N_i \beta_2, \qquad (1)$$

for the free growth phase, where urbanization is weakly dependent with space and resource N^*; and

$$\frac{\partial}{dt} N_i(t) = \beta_2 N_i(t) - \delta_{\{\sum N_i = N^*\}} \cdot (\beta_1 k(t) + (N^* - N_i(t))\beta_2) \qquad (2)$$

for the resource constraint phase, where the total resource N^* are partitioned, and new dwellers get resource only through redistribution.

To summarize, the SYM can be regulated with three tunable parameters: the individual exploration distance r, the size of the memory kernel N^*, and the ratio $\beta := \beta_2/\beta_1$. Though β_1 and β_2 have their actual meanings of generation speed, the model's dynamics and patterns are only determined by the relative growth rate β.

The model implies some simple assumptions. First, urban developments are density-driven. Literature has suggested that density-driven social ties and interactions comprise an important driver of the economies of scale [26, 35, 36]. In the SYM, we further assume only the density of attractive population is corresponding to urban developments. Such active part can be recognized as the total employed or productive people. Second, to make an analytic framework, the growth dynamics are set to be homogeneous. The choices of place of new comers are random; The rate of introduction and emergence is the same for every active citizen and every city. This diffusive setting of sequential settlements is also realistic urban growth [37].

In the numerical experiments, which is elaborated in *SI*, the truly worth-tuning parameters are three, β, r, and N^*. β contributes to the Zipf's coefficient and later defined turnover rate [38]; r contributes to the fractality of urban areas and the time to fill the whole space; N^* is the severeness of resource competition.

The free growth phase. SYM predicts the existence of three phases of regional growth of cities, distinguished by whether resource and space have been fully occupied: freely growth phase, economic constraint phase, and spatial constraint phase. We focus on the first two phases, which correspond with regional resource. Spatial constraint phase's evolution implies a fully urbanized area, which is unlikely seen in reality, we discuss the situation only in *SI*. In the freely growth phase, cities grow desolately, without being limited by resource and space. In this phase, SYM reformulates two important properties, stately (1) Zipf's law [39] for rank size distribution of cities' population, and (2) Clark's law for exponential decay of urban density[40].

The populations of cities typically decay proportionally to the inverse of their ranks [39]. This is referred as Zipf's law of cities' population sizes, i.e., the populations of cities distribute as a power of ranks, $f_r(r) \sim r^{-(1+\eta)}$. It is obvious that $N_i(t)$ has a geometric distribution [41], $P(N_i(t) = n) = e^{-\beta_1 t}(1 - \exp(-\beta_1 t))^{n-1}$. Combining which with the assumption that the number of cities will grow exponentially at rate β_1, if we randomly pick an existing city, the waiting time since its first appearance is exponential with parameter β_1. Thus the distribution of population of a random city is

$$f(n) = \frac{\Gamma(1 + 1/\beta)\Gamma(n)}{\beta\Gamma(n + 1 + 1/\beta)} \approx Cn^{-1-1/\beta}, \text{ as } n \to \infty, \quad (3)$$

where $\Gamma(\cdot)$ is Gamma function. This equation implies a Zipfian relationship with $n(\text{rank}) \sim rank^{-\beta}$. Noticing that β takes value from all positive real number in our model, we can derive arbitrary scaling behaviors by switching β. According to some studies [42], the power law dependence of population frequency is 2.03 ± 0.05 for the world, indicating that the average relative emerging rate of cities is around 1.

Varying β also leads to the consideration of different sizes of study area. A small (large) β interprets that the emergence of cities is fast (slow), corresponding to a large (smaller) study area. Thus varying β is parallel to investigate the spatial density of cities in an urban system. Some urban systems tend to form new cities to have sufficient infrastructures and less diversity of urban output[8] ($\beta > 1$) and some cities may go otherwise ($\beta < 1$). This value is actually a reflection of the intensity of regional population concentration in large cities. The experiments have confirmed our analytic results for free growth phase in SYM. A simulated validation for this result can be reflected in Fig. 2. Notably, when βs are large (> 2), the simulated Zipfian exponents are remarkably larger than their theoretical predictions. This is because the competition for space benefits small cities resulted from their higher density of edging cells, which is proved in *SI*. For large βs, however, of the same rank, the probability of successful emergence of new city decreases due to relatively larger area of existing cities. This exasperates the concentration of active population in large

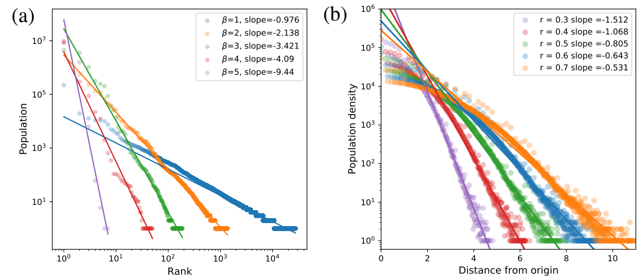

FIG. 2. **(a)** The distribution of population among cities. In the simulation we take $N^* = 10^5$ and alternate βs. The realistic Zipf's coefficient is reproduced when $\beta \approx 1$. The theoretical predictions of the slopes are $-\beta$, and is well approximated when βs are small. Larger β reduce the chance of latter city's emergence. Thus the spatial aspects of the SYM strengthen inequality. This result confirms that Zipf's law is valid for growing urban systems where all cities share the same rate to grow. From the other master equation we analyze that this observation vanishes if total growing force is finite. **(b)** The population distribution as a function of distance from a district's center for different step lengths r. The larger r stands for the larger degree of flatten of a city. The vertical axis is logarithmic processed, which represents that the spatial population density decays exponentially for each r.

cities.

SYM also revisits Clark's law in urban studies [40]. In SYM, population density evolves as two-dimensional diffusion within a city[43], where we can focus the density's growth on each axis from the oldest citizens of a city. Let (d) denote the active population density of locations at the distance d from a city center, and t_n as the time for the n'th citizen to generate, we have

$$\rho_{t_{n+1}}(d) = (\rho_{t_n}(d - r) + \rho_{t_n}(d + r))/2. \quad (4)$$

By re-scaling time as $\tau_n = t_n \cdot (k\beta_1 + N\beta_2)/T$, for a sufficient large T, this equation results in an exponential decay of density

$$\rho(d) \sim e^{-\alpha d}. \quad (5)$$

Details are presented in *SI*.

A direct implication of Clark's law is the competition strengths at urban edges, which also influence the local Zipfian exponents. From Clark's law, the population density is just a function of city's age and the distance from urban center. Specifically, the density at the edges is important since it determines the competition advantages for space. The population within an edging cell of city j is estimated by $e^{(T-T_j)} \int_d^{d+1} \rho(r)dr/(2\pi d)$, where T_j is the emerging time of city j. We also have the waiting time $T_{n+1} - T_n \sim 1/n$, and the total population approximation $e^{\beta_1+\beta_2}$, combining which we derive the density of edging cells if time and the urban radius are given. Since the attractiveness of large urban center is larger, the edging population of large cities is actually smaller than minor cities. We validate our prediction with simulations in Fig.2. In supplement, larger r will weaken the above prediction, since the settlement are more even, thus larger proportion of citizens live at edges. In reality, the metropolis areas over the world have very different densities. In SYM, it corresponds to the sprawl of a city with given population. It can also be taken as the area proportion for a city in the studied region. On the other hand, it also controls the spatial limit of cities given competitionless population.

The economic constraint phase. The multi-perspective coincidence between the exponents derived in our model and those in empirical evidences of population studies indicates that only two observation scales (the generation rates of city and citizen) lead to the behaviors of regional dynamics. This means that the actual urban growth has not yet reached the constrained cases. However, preventive measures are still necessary. Thus we bring a general constraining parameter N^* to further discuss the second phase of SYM, the economy constraint phase, i.e., the total population reaches N^*. Such setting is the abstract of many real-life rules set by global organizations such as the allowance of carbon emissions or sustainable development projects. In each city, a proportion of population are active. Here, $\sum_{i=1} N_i(t) = N^*$ for t that is sufficiently large. If in some period, the minor cities generate more offspring than major ones and the superiority of remaining population within the memory kernel changes, minor city will increase its ranking, as the growing rate for each city i is actually $N_i\beta_2$. As for the dynamics within memory kernel, in each city, N_i acts as a random walk with absorption wall 0, since no offspring will be expected if no nodes are left in the kernel. This result also works for single cell case within a city. Denote the population with cell j of city i as m_{ij}. According to [41], we use a result for branching process that a cell loses its vitality if the population goes downhill under a threshold

$$\rho_{threshold} = k/\beta. \tag{6}$$

This value shall be regarded as the sign for *urban shrinkage*, for the edging cells have lower density according to equation 5 thus have an exponentially higher probability to be languished. In other words, urban shrinkage shall be reasoned by limited systematic resources.

The kernel mechanism also plays a role at the cross-city scale: The preference of larger cities is easier to fail in a system with the memory kernel. The competition for active citizens in SYM receives more than pure birth settings because the sum of active population is given as N^*. In other words, SYM system doesn't consider natural growth. To test this interpretation, we analyze the turnover rate, defined as the average frequency of time steps in a realization that the second largest city surpasses the largest in active population. We conduct numerical experiments, and receive power law dependence of the frequency on simulating steps, shown in Fig.3.

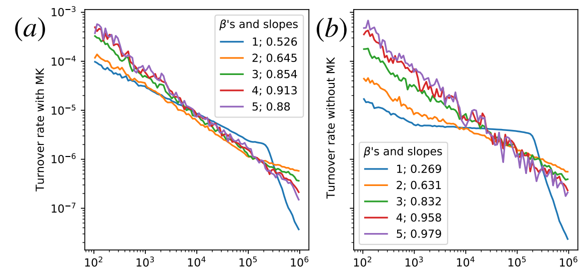

FIG. 3. The change rate statistics with **(a)** and without **(b)** a memory kernel. The kernel keeps turning over more often. With same β's, a kernel-based SYM's decay in turnover rate is smaller. These results validate our prediction that with finite resource, advantages are more likely to be kept.

Moreover, the switching is more likely to happen *with* a memory kernel, i.e., turnover rate decay slower in probability if the system has constraints in resource. It is also a clear result since a growing society (a society without a memory kernel) suffer less from inter-specific competition.

The last property of SYM is the fractality of urban envelop, stately, the length of urban edges vary with the used measurement. Inspired by multi-player interaction in fractal financial market[44], we interpret that fractal urban boundary is driven by the competition for land at cities' edges. In SYM, the uncertain competition for space lies in parameter r. A larger r indicates larger randomness and brings an extra advantage for minor cities, resulting in a larger fractal dimension. We apply the box counting technique to calculate the fractal dimension of urban envelops, and receive an stable output of $d_f = 1.2 \pm 0.05$ with $r = 0.5$, similar to empirical results[36]. We also find larger d_f's for greater r. These results validate our hypothesis that fractal edges coexist with spatial competition. Also, this result also confirms that SYM replicates an urban system.

Discussion. In recent years, clearly we have witnessed the assumption of individual based models (IBMs) is deficient for multi-phased urban growth dynamics. The simplest approach is to modify the classical dynamical systems, adjusting the growth potentials via empirical derived geographical heterogeneity in the IBMs. This is not satisfactory since it confounds attempts to make extrapolations beyond the particular situations. An alternative, given the basis of economic complexity and the central place theory, is to derive causes of diversity, in which measures are developed abstracting the aggregated factors of production, and are predictive of future growth [45]. It bridges a macroscopic dual to the IBMs. However, such an approach does not reach an end itself, for the economic complexity is even harder to monitor.

In this *Letter*, we have attempted to bridge the gap between IBMs and macroscopic descriptions for urban systems. The introduced memory kernel is a *global* condition clarifying how the resources are shared by all cities: the matching investment from the cities to introduce more new citizens. The recipient at different scales of this mechanism exhibits fruitful characteristics: Microscopically, it is a realization of system-conditioned spatial preferential attachment; In the intermediate level of cities, it introduces citizens' age structures and the accumulation of random shocks. The stationary age can be calculated as the average time for a new city to emerge is $(\beta_2 N^* + \beta_1 k)^{-1}$, which equals to the average losing age of the whole kernel. This result gives an instruction of the length of workable age in a given social urban system, elaborated in *SI*. The global trends predicted in our model is more random, allowing us to explained the vicissitude of cities, and the changes in spatial structures in the post-urbanization phase. Despite the model's simplicity, it is still capable of reformulating major aspects of empirical observations of urban population, including Zipfian ranking of cities by population size and Clark's law for urban population density. It also demonstrated the emergence of fractal urban edges. The generality of SYM can easily be embedded by spatial heterogeneous geographical circumstances, by adjusting the growing rate on each cell towards the product $m_{ij}\beta_2$, where m_{ij} is the local characteristic.

Although our results are not all analytically proved, it is an essential step to strip out the power of urban dynamics. The model is non-commuting, but the community structure is naturally embedded. For further consideration, we can extend the model by adding links as the volume of exploration and preferential return between cities[46]. The model can further be extended with multi-dimensional memory kernel, allowing one citizen to be introduced if different factors[47] (i.e., the existing citizens in different dimensions of kernel) agree to allow her in the system.

* liuyu@urban.pku.edu.cn

[1] G. F. Frasco, J. Sun, H. D. Rozenfeld, and D. ben Avraham, Spatially distributed social complex networks, Phys. Rev. X **4**, 011008 (2014).

[2] R. Li, L. Dong, J. Zhang, X. Wang, W. X. Wang, Z. Di, and H. E. Stanley, Simple spatial scaling rules behind complex cities, Nature Communications **8**, 1841 (2017).

[3] H. A. Makse, S. Havlin, and H. E. Stanley, Modelling urban growth patterns, Nature **377**, 608 (1995).

[4] D. Rybski, A. G. C. Ros, and J. P. Kropp, Distance-weighted city growth, Physical Review E **87**, 042114 (2013).

[5] M. Nanda and R. Durrett, Spatial evolutionary games with weak selection, Proceedings of the National Academy of Sciences **114**, 6046 (2017).

[6] L. M. Bettencourt, J. Lobo, D. Helbing, C. K"uhnert, and G. B. West, Growth, innovation, scaling, and the pace of life in cities, Proceedings of the national academy of sciences **104**, 7301 (2007).

[7] L. M. Bettencourt, The origins of scaling in cities, science **340**, 1438 (2013).

[8] M. Batty, The size, scale, and shape of cities, science **319**, 769 (2008).

[9] D. Rybski, E. Arcaute, and M. Batty, Urban scaling laws, Environment and Planning B: Urban Analytics and City Science **46**, 1605 (2019).

[10] M. Batty and P. A. Longley, *Fractal cities: a geometry of form and function* (Academic press, 1994).

[11] M. Batty, *Cities and complexity: understanding cities with cellular automata, agent-based models, and fractals* (The MIT press, 2007).

[12] G. K. Zipf, *Human behavior and the principle of least effort.* (addison-wesley press, 1949).

[13] A. Anas, R. Arnott, and K. A. Small, Urban spatial structure, Journal of economic literature **36**, 1426 (1998).

[14] R. Louf and M. Barthelemy, How congestion shapes cities: from mobility patterns to scaling, Scientific reports **4**, 1 (2014).

[15] H. A. Makse, J. S. Andrade, M. Batty, S. Havlin, and H. E. Stanley, Modeling urban growth patterns with correlated percolation, Phys. Rev. E **58**, 7054 (1998).

[16] C. D. Greenman and T. Chou, Kinetic theory of age-structured stochastic birth-death processes, Phys. Rev. E **93**, 012112 (2016).

[17] S. Gude, E. Pinçe, K. M. Taute, A.-B. Seinen, T. S. Shimizu, and S. J. Tans, Bacterial coexistence driven by motility and spatial competition, Nature **578**, 588 (2020).

[18] W. Liu, J. Cremer, D. Li, T. Hwa, and C. Liu, An evolutionarily stable strategy to colonize spatially extended habitats, Nature **575**, 664 (2019).

[19] M. Marsili and Y.-C. Zhang, Interacting individuals leading to zipf's law, Physical review letters **80**, 2741 (1998).

[20] S. Çolak, A. Lima, and M. C. Gonz'alez, Understanding congested travel in urban areas, Nature communications **7**, 1 (2016).

[21] M. Fujita, Spatial patterns of urban growth: Optimum and market, Journal of Urban Economics **3**, 209 (1976).

[22] A. Ghosh, A. Chatterjee, A. S. Chakrabarti, and B. K. Chakrabarti, Zipf's law in city size from a resource utilization model, Phys. Rev. E **90**, 042815 (2014).

[23] R. Axtell, R. Florida, *et al.*, *Emergent cities: a microeconomic explanation for Zipf's law*, Tech. Rep. (Society for Computational Economics, 2001).

[24] J. M. Kleinberg, Navigation in a small world, Nature **406**, 845 (2000).

[25] R. Atkinson, Urban governance and competitiveness: Improving 'urban attractiveness', in *Regieren* (Springer, 2012) pp. 297–312.

[26] F. Girardin, A. Vaccari, A. Gerber, A. Biderman, and C. Ratti, Quantifying urban attractiveness from the distribution and density of digital footprints, International Journal of Spatial Data Infrastructures Research (2009).

[27] A. Gomez-Lievano, O. Patterson-Lomba, and R. Hausmann, Explaining the prevalence, scaling and variance of urban phenomena, Nature Energy , 1 (2018).

[28] T. M. Parris and R. W. Kates, Characterizing a sustainability transition: Goals, targets, trends, and driving forces, Proceedings of the National Academy of Sciences **100**, 8068 (2003).

[29] Y.-C. Zhang, Quasispecies evolution of finite populations,

Phys. Rev. E **55**, R3817 (1997).

[30] M. Batty, Urban studies: Diverse cities, successful cities, Nature Human Behaviour **1**, 1 (2017).

[31] X. Gabaix and Y. M. Ioannides, The evolution of city size distributions, in *Handbook of regional and urban economics*, Vol. 4 (Elsevier, 2004) pp. 2341–2378.

[32] A. Haase, D. Rink, K. Grossmann, M. Bernt, and V. Mykhnenko, Conceptualizing urban shrinkage, Environment and Planning A **46**, 1519 (2014).

[33] R. E. Miles, On the homogeneous planar poisson point process, Mathematical Biosciences **6**, 85 (1970).

[34] G. U. Yule, A mathematical theory of evolution, based on the conclusions of dr. willis, fr s, Philosophical transactions of the Royal Society of London. Series B, containing papers of a biological character **213**, 21 (1925).

[35] W. Pan, G. Ghoshal, C. Krumme, M. Cebrian, and A. Pentland, Urban characteristics attributable to density-driven tie formation, Nature communications **4**, 1961 (2013).

[36] M. Batty and K. Sik Kim, Form follows function: reformulating urban population density functions, Urban studies **29**, 1043 (1992).

[37] R. Pastor-Satorras, C. Castellano, P. Van Mieghem, and A. Vespignani, Epidemic processes in complex networks, Rev. Mod. Phys. **87**, 925 (2015).

[38] N. Rooney, K. McCann, G. Gellner, and J. C. Moore, Structural asymmetry and the stability of diverse food webs, Nature **442**, 265 (2006).

[39] X. Gabaix, Zipf's law for cities: An explanation, Quarterly Journal of Economics **114**, 739 (1999).

[40] C. Clark, Urban population densities, Journal of the Royal Statistical Society **114**, 490 (1951).

[41] R. Durrett and R. Durrett, *Essentials of stochastic processes*, Vol. 1 (Springer, 1999).

[42] D. H. Zanette and S. C. Manrubia, Role of intermittency in urban development: A model of large-scale city formation, Phys. Rev. Lett. **79**, 523 (1997).

[43] N. F. Britton, Spatial structures and periodic travelling waves in an integro-differential reaction-diffusion population model, SIAM Journal on Applied Mathematics **50**, 1663 (1990).

[44] B. J. West and S. Picozzi, Fractional langevin model of memory in financial time series, Phys. Rev. E **65**, 037106 (2002).

[45] C. A. Hidalgo and R. Hausmann, The building blocks of economic complexity, Proceedings of the National Academy of Sciences **106**, 10570 (2009), https://www.pnas.org/content/106/26/10570.full.pdf.

[46] J. Wang, L. Dong, X. Cheng, W. Yang, and Y. Liu, An extended exploration and preferential return model for human mobility simulation at individual and collective levels, Physica A: Statistical Mechanics and its Applications **534**, 121921 (2019).

[47] C. K. Tokita and C. E. Tarnita, Social influence and interaction bias can drive emergent behavioural specialization and modular social networks across systems, Journal of the Royal Society Interface **17**, 20190564 (2020).