

Memory Matters for Citeis

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Fast urbanization process leads to unforeseen dilemmas. Stately the secondary phase of urban growth, rather than naturally attracting people elsewhere, exhibits restricted dynamics for internal conflicts. Here, we propose a model to reproduce the statistical features of growing urban systems, such as population density within a city (Clark's law), fractality, and cities' rank size distributions (Zipf's law). We additionally add a *memory kernel* to account for the development ceiling of the region. We show that cities exhibit transitions in various ways such as the shuffling of city ranks and urban shrinking, suggesting the overall resource as the upper bound for later stage of regional growth of urbanization.

Urban growth dynamics has always drawn dramatic attention in the past few decades. This problem lies in two parts, population settlement and economic growth, both of which changes spatially over time. Traditional studies of spatial economics have attempted to construct this phenomenon under equilibrium models of regional urban systems[1]. These models base their idea on agglomeration economies to explain why urban features tend to gather. However, the dynamics of shifts between urban equilibriums (say, polycentric transitions of cities), is poorly interpreted throughout these models. First, the initial states of urban structure are mostly isotropic, that the present state of cities may be only one possible state of urban equilibriums. Moreover, urban constructions, especially urban sprawling and urban shrinkage, show that cities are out-of equilibrium systems with unsynchronized growth between infrastructure and population distribution. Second, these models mostly address city agglomeration's benefits by the interaction between pairs of individuals. This brings difficulties in understanding the hierarchy and functional divisions that can be seen in nearly all cities. Yet, separating the growth dynamics of infrastructures and population has not yet been taken serious attention in existing models, though some work in complex networks has realized that preferential attach-

ment has hysteresis effect in building contacts. Lastly, throughout the abundant data describing cities, existing models cannot make quantitative predictions with the more information added. We present in this Letter a stochastic growing model of urban systems, which relies on the assumption that the agglomeration effects of infrastructures and population are separated, which can cause the urban core to shift over time.

While our interest lies in the spatial distribution of population, there are other feathers of urban growth for which parallels with economical development over geographical space can be drawn, for mathematical methods applied. For example, the rank-frequency distribution of urban population[2], can be explained using a novel form of the Yule process, first introduced to explain the distribution of the number of species

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