Stochastic Optimization

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1 Variants of Stochastic Gradients Algorithms

consider problem:

$$\min_{w\in\mathbb{R}^d} \quad \frac{1}{n}\sum_{i=1}^{n} f_i(w) + \lambda\|w\|_1,$$

where $f_i(w) = \log(1+\exp(-y^i w^T x_i))$, and $\lambda > 0$. the prediction is:

$$p(y=1|x;w) = \frac{1}{1+\exp(-w^T x)},$$

$$p(y=-1|x;w) = \frac{1}{1+\exp(w^T x)}.$$

1.1 adagrad

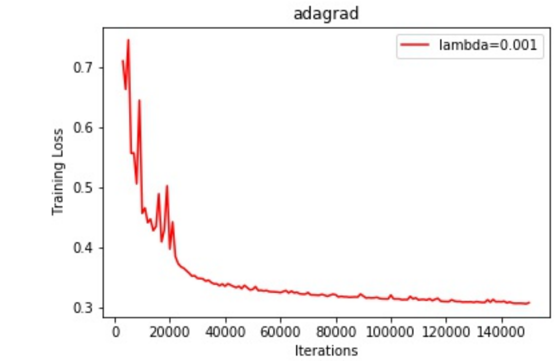
$$w_{t+1} = w_t - \frac{\eta}{\sqrt{G_t + \epsilon I}} \cdot g_t, (\epsilon = 10^{-8}).$$

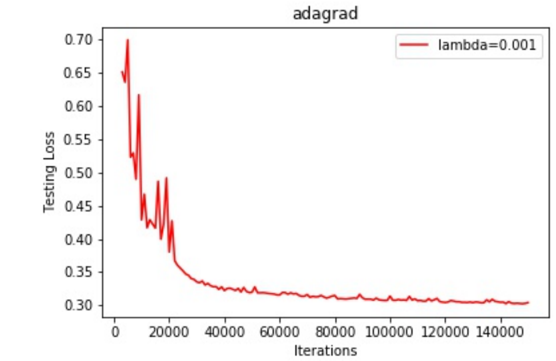
其中，G_t 是对角矩阵，对角元素 e_{ii} 为过去到当前第 i 个参数 θ_i 的梯度的平方和。

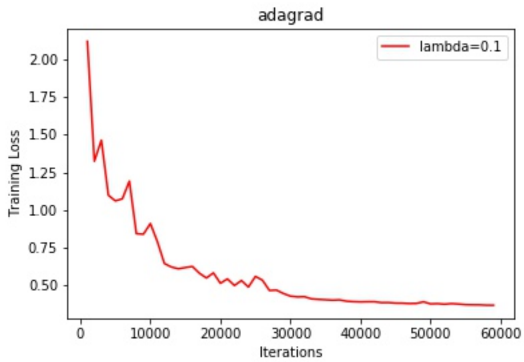
在训练过程中，首先取步长 0.003（$\lambda = 0.1 \ or \ 0.001$ 时）当训练误差累计超过 10 次下降不超过某个阈值时，我们将步长除以 10，继续迭代。

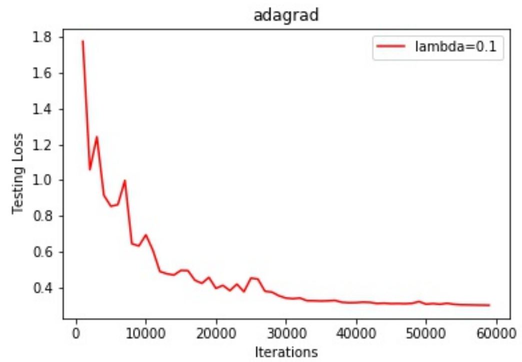
mnist 数据集上的结果：

λ	train_loss	test_loss
0.001	0.307	0.304
0.1	0.370	0.300



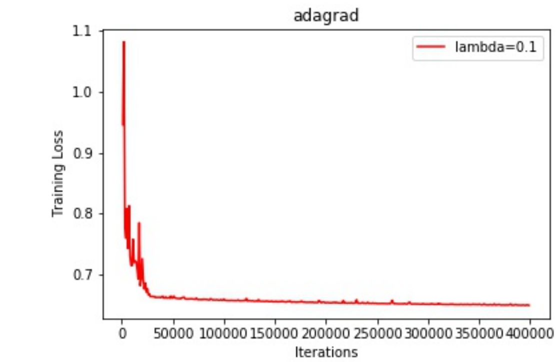


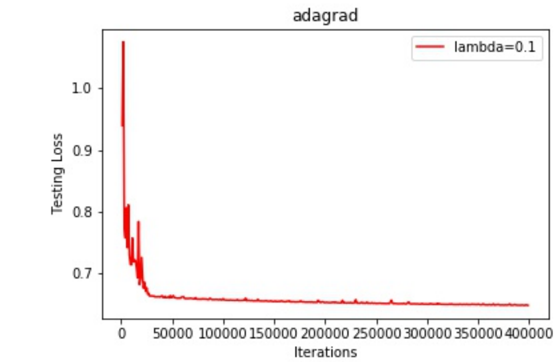


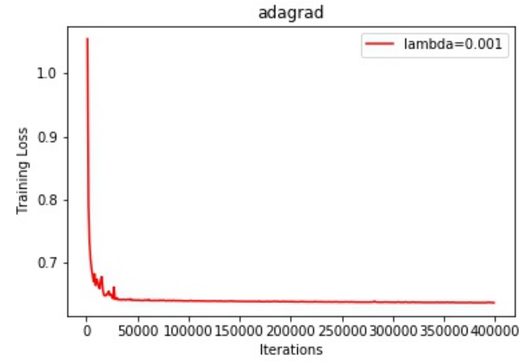


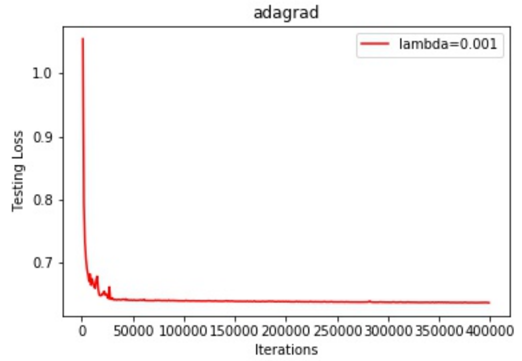
covertype 数据集上的结果：

λ	train_loss	test_loss
0.001	0.636	0.636
0.1	0.650	0.648







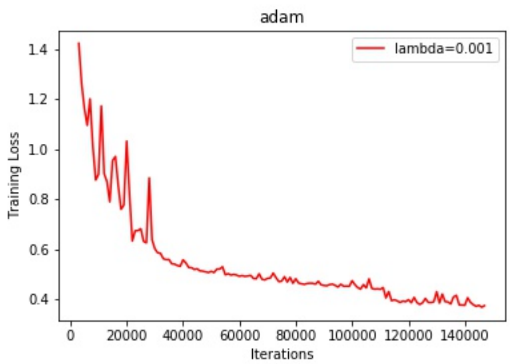


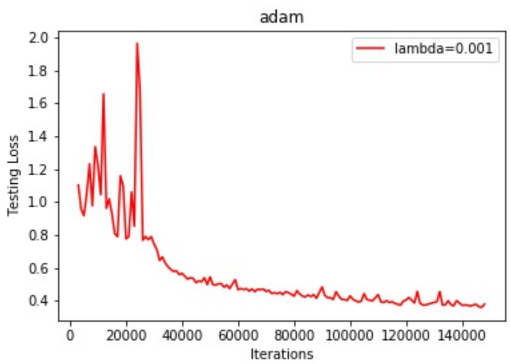
1.2 adam

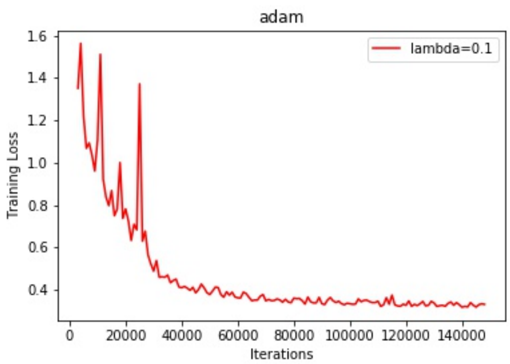
$$
\begin{aligned}
&m_t = \beta_1 m_{t-1} + (1-\beta_1) g_t \\
&v_t = \beta_2 v_{t-1} + (1-\beta_2) g_t^2 \\
&\hat{m}_t = \frac{m_t}{1-\beta_1^t} \\
&\hat{v}_t = \frac{v_t}{1-\beta_2^t} \\
&w_{t+1} = w_t - \frac{\eta}{\sqrt{(\hat{v}_t)}+\epsilon}\hat{m}_t \\
&\beta_1 = 0.9, \beta_2 = 0.999, \epsilon = 10^{-8}
\end{aligned}
$$

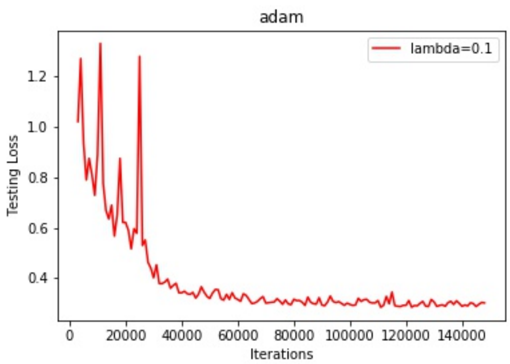
mnist 数据集上的结果：在训练过程中，首先取步长 0.0008（$\lambda = 0.1$），0.001（$\lambda = 0.001$）当训练误差累计超过 10 次下降不超过某个阈值时，我们将步长除以 10，继续迭代。

λ	train_loss	test_loss
0.001	0.358	0.360
0.1	0.332	0.301



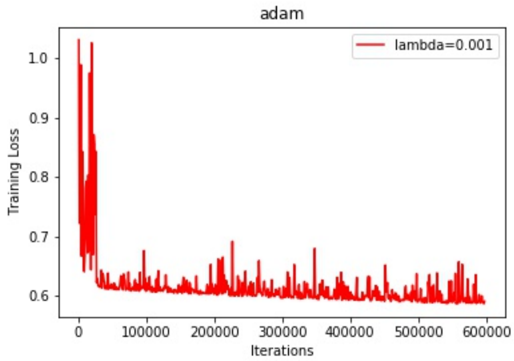


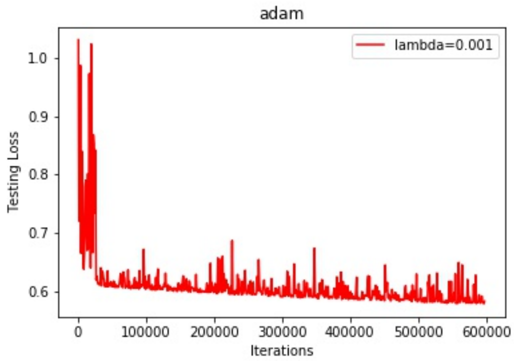


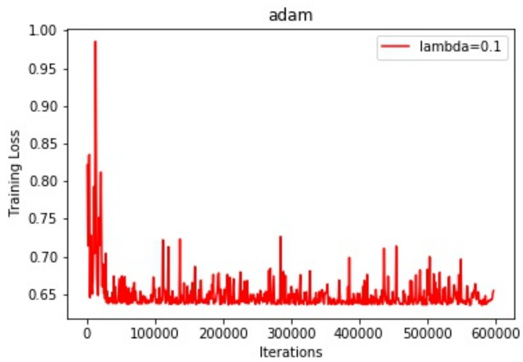


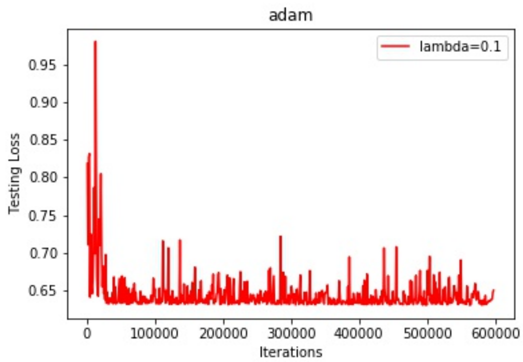
covertype 数据集上的结果：

λ	train_loss	test_loss
0.001	0.586	0.579
0.1	0.636	0.631









1.3 SVRG

ref: https://papers.nips.cc/paper/4937-accelerating-stochastic-gradient-descent-using-predictive-varianc
pdf

ref: https://arxiv.org/pdf/1403.4699.pdf proxSVRG 算法描述：

Algorithm 2: SVRG method

Input: $\tilde{x}_0$, α, m

for $e = 1 : E$ **do**

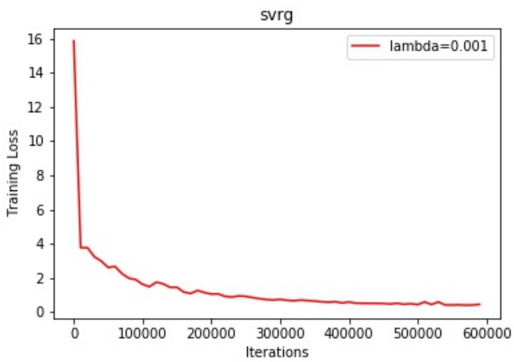
- $y \leftarrow \tilde{x}_{e-1}$, $x_1 \leftarrow \tilde{x}_{e-1}$.
- $g \leftarrow \nabla f(y)$ (full gradient)
- **for** $k = 1 : m$ **do**
  - pick $s_k \in \{1, \cdots, n\}$ uniformly at random.
  - $v_k = \nabla f_{s_k}(x_k) - \nabla f_{s_k}(y) + \nabla f(y)$
  - $x_{k+1} = x_k - \alpha v_k$
- **end for**
- $\boxed{\tilde{x}_e \leftarrow \frac{1}{m} \sum_{k=1}^{m} x_k}$

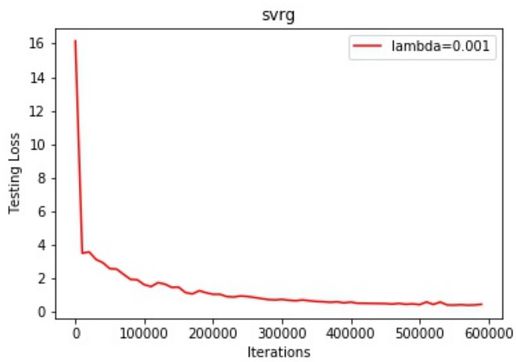
end for

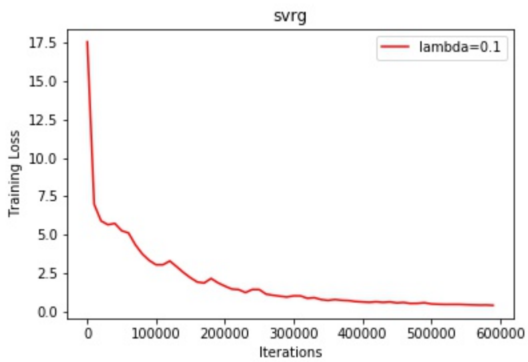
在算法迭代中，我们先做几步 SGD，然后再进行 SVRG 更新，当内层循环结束后，将内层循环得到的变量进行平均。

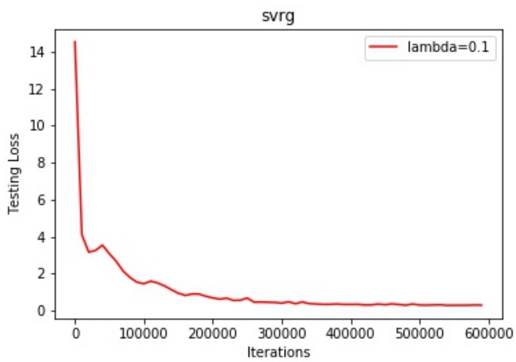
mnist 数据集上的结果：

λ	train_loss	test_loss
0.001	0.397	0.402
0.1	0.405	0.274









1.4 SVRG-BB

ref: 《BarzilaiBorwein Step Size for Stochastic Gradient Descent》NIPS2016

http://lanl.arxiv.org/pdf/1605.04131v2

ref: https://github.com/tanconghui/Stochastic_BB

算法描述：

Algorithm 2 SVRG with BB step size (SVRG-BB)

Parameters: update frequency m, initial point $\tilde{x}_0$, initial step size η_0 (only used in the first epoch)

for $k = 0, 1, \cdots$ **do**

  $g_k = \frac{1}{n}\sum_{i=1}^{n} \nabla f_i(\tilde{x}_k)$

  if $k > 0$ **then**

$$\eta_k = \frac{1}{m} \cdot \|\tilde{x}_k - \tilde{x}_{k-1}\|_2^2 / (\tilde{x}_k - \tilde{x}_{k-1})^\top (g_k - g_{k-1}) \tag{3.2}$$

  end if

  $x_0 = \tilde{x}_k$

  for $t = 0, \cdots, m-1$ **do**

    Randomly pick $i_t \in \{1, \ldots, n\}$

    $x_{t+1} = x_t - \eta_k(\nabla f_{i_t}(x_t) - \nabla f_{i_t}(\tilde{x}_k) + g_k)$

  end for

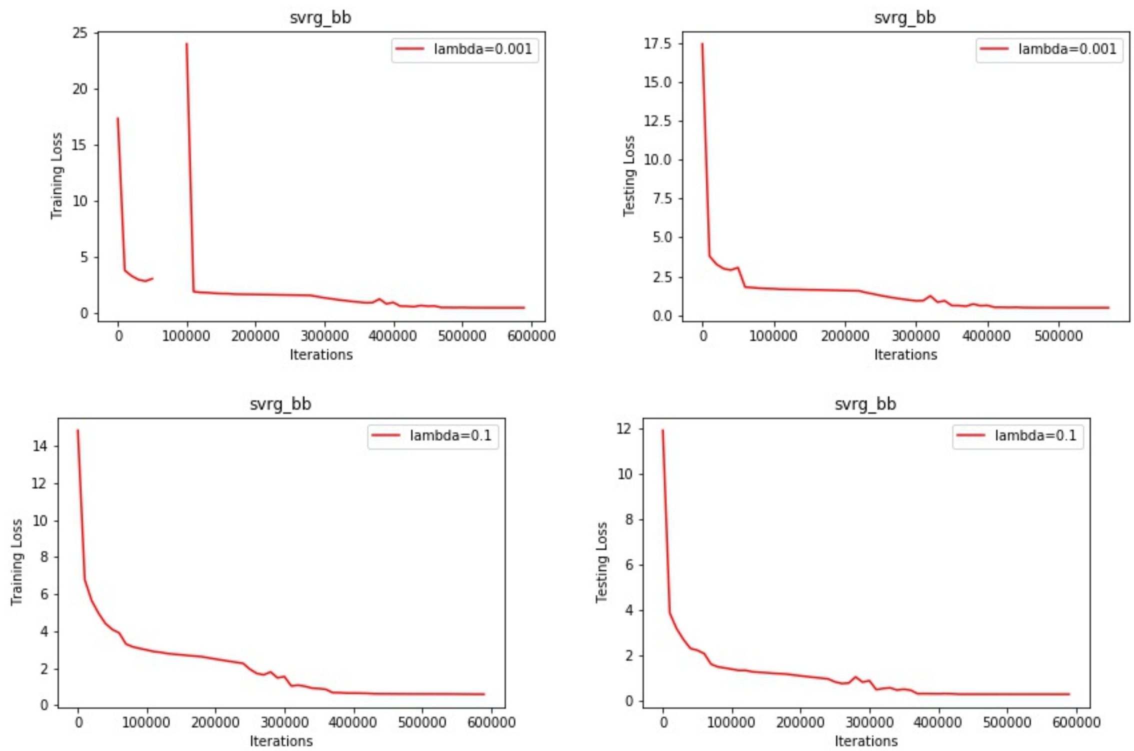
  $\tilde{x}_{k+1} = x_m$

end for

在算法迭代中，我们先做几步 SGD，然后再进行 SVRGBB 更新。

mnist 数据集上的结果：

λ	train_loss	test_loss
0.001	0.470	0.476
0.1	0.594	0.292



1.5 FTRL: follow the regularized leader

Reference:

1. 《Online Learning and Online Convex Optimization1》section 2.2,2.3

URL: http://www.cs.huji.ac.il/~shais/papers/OLsurvey.pdf

3. 原始论文：

URL: https://www.eecs.tufts.edu/~dsculley/papers/ad-click-prediction.pdf

2. from 美团点评技术团队

URL: https://tech.meituan.com/online-learning.html

4. 其他：

URL: http://www.datakit.cn/blog/2016/05/11/ftrl.html

Algorithm 1 Per-Coordinate FTRL-Proximal with L_1 and L_2 Regularization for Logistic Regression

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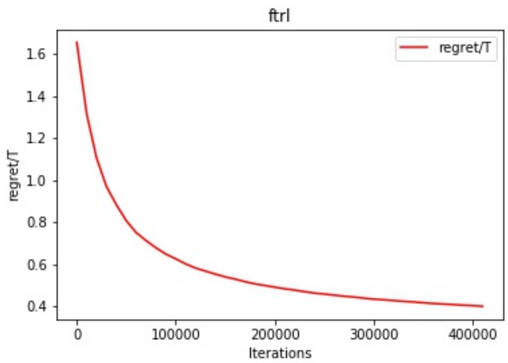
 # With per-coordinate learning rates of Eq. (2).
Input: parameters α, β, λ_1, λ_2
$(\forall i \in \{1, \ldots, d\})$, initialize $z_i = 0$ and $n_i = 0$
for $t = 1$ **to** T **do**
  Receive feature vector $\mathbf{x}_t$ and let $I = \{i \mid x_i \neq 0\}$
  For $i \in I$ compute

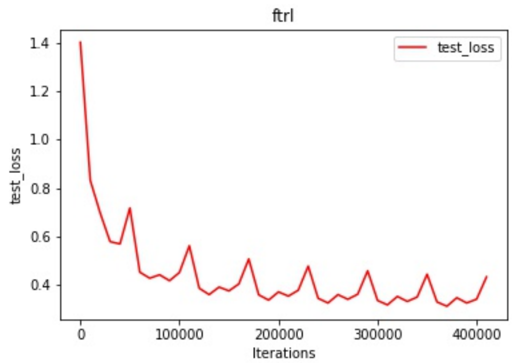
$$w_{t,i} = \begin{cases} 0 & \text{if } |z_i| \le \lambda_1 \\ -\left(\frac{\beta+\sqrt{n_i}}{\alpha} + \lambda_2\right)^{-1}(z_i - \operatorname{sgn}(z_i)\lambda_1) & \text{otherwise.} \end{cases}$$

  Predict $p_t = \sigma(\mathbf{x}_t \cdot \mathbf{w})$ using the $w_{t,i}$ computed above
  Observe label $y_t \in \{0, 1\}$
  for all $i \in I$ **do**
    $g_i = (p_t - y_t)x_i$ *#gradient of loss w.r.t.* w_i
    $\sigma_i = \frac{1}{\alpha}\left(\sqrt{n_i + g_i^2} - \sqrt{n_i}\right)$ *#equals* $\frac{1}{\eta_{t,i}} - \frac{1}{\eta_{t-1,i}}$
    $z_i \leftarrow z_i + g_i - \sigma_i w_{t,i}$
    $n_i \leftarrow n_i + g_i^2$
  end for
end for

以上算法是针对 0，1 分类问题，对于 1，-1 分类，需稍加改动。

由于在线算法测评方式不太一样，我们采用 regret/T，和 test_loss 两种指标。





2 future work

由于 mnist dataset 的样本向量比较稀疏，所以可以考虑稀疏矩阵运算将其加速，我了解到 xlearn 这个大规模稀疏数据机器学习库（https://github.com/aksnzhy/xlearn）实现了 SGD，AdaGrad, FTRL 等，可以借鉴。