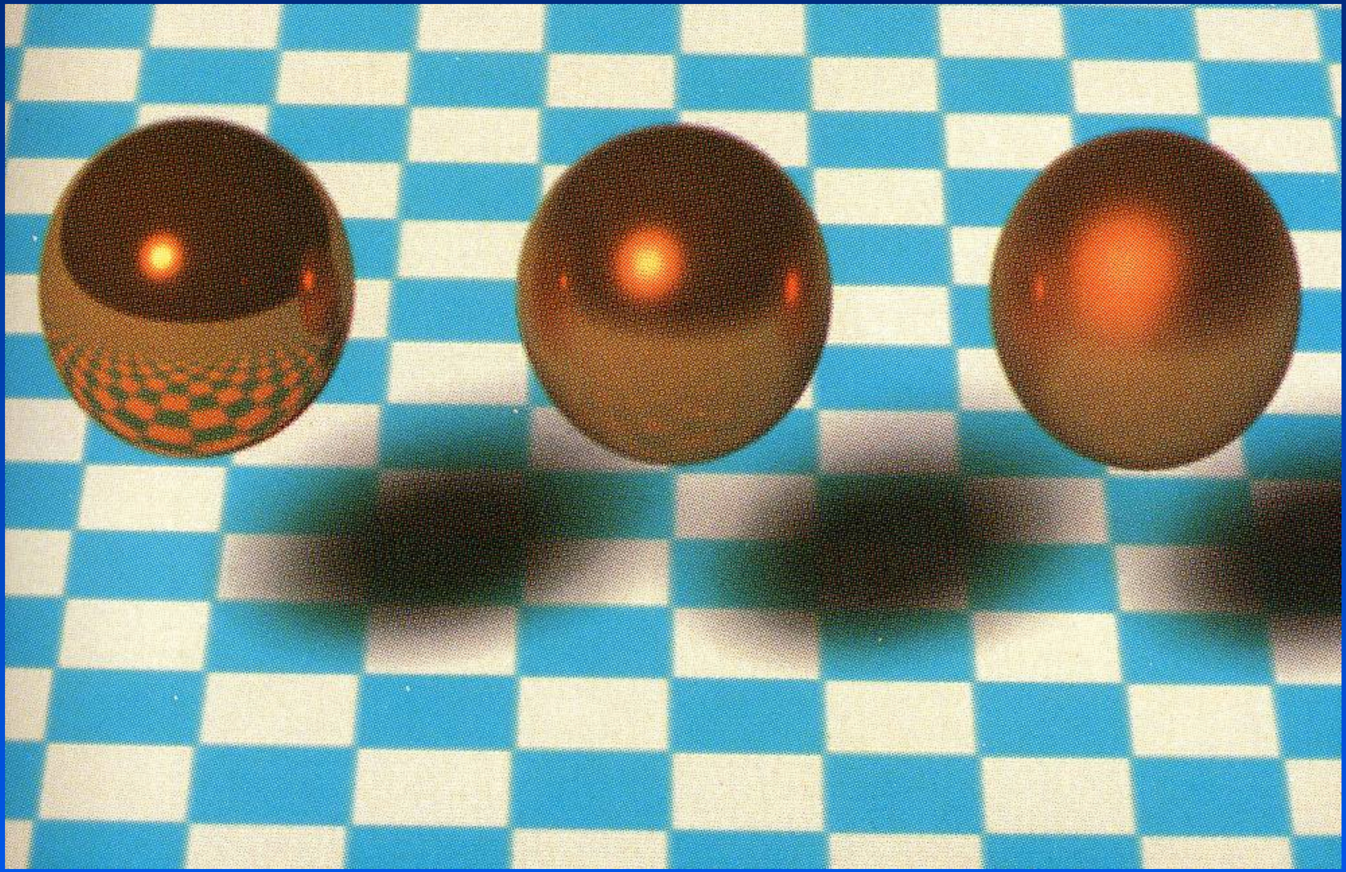
Illumination Models
Local reflection models



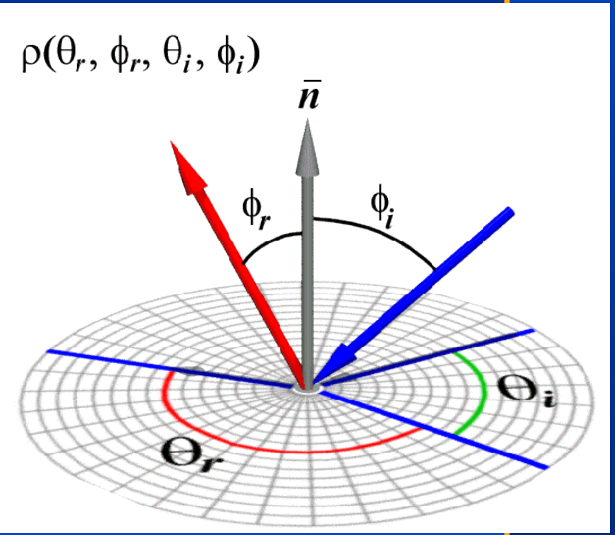
- Illumination model (reflection model)
  - Express factors determining surface “color” at a given point (intensity of reflected light for different frequencies)
- Shading model
  - Determines when illumination model is applied and arguments it receives
  - Some shading models invoke illumination model at each pixel (e.g. Phong shading)
  - Others invoke illumination models at selected points then interpolate (e.g. Gouraud shading)

Local illumination models

- Consider incoming light, surface and outgoing light only
- Local models used for most rendering applications
- Local models often used as a part of the global illumination models
  - E.g. ray tracing basically the local model recursively

Bidirectional Reflectance Distribution Function (BRDF)

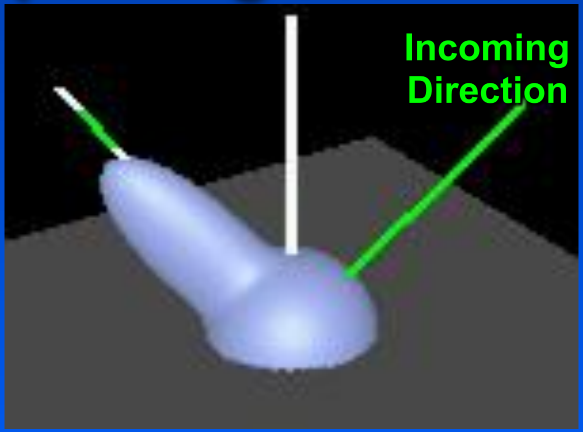
- Incoming energy $$E_i(\theta_i,\phi_i) = I_i(\theta_i,\phi_i)\cos\phi_i d\omega_i$$
  - dω: solid angle in which the energy is contained
  - Cosine term gives the amount of energy intercepted by the surface element

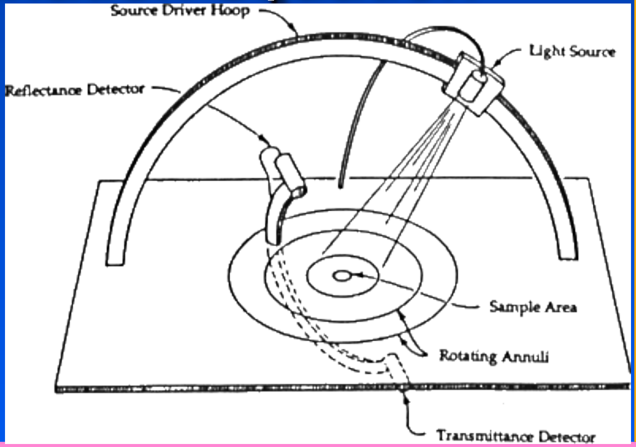


- BRDF

$$\rho_\lambda(\lambda,\theta_r,\varphi_r,\theta_i,\varphi_i) = \frac{I_{\lambda,r}(\lambda,\theta_r,\varphi_r)}{E_{\lambda,i}(\lambda,\theta_i,\varphi_i)}$$

- Bidirectional because $\rho_\lambda(\lambda,\theta_i,\phi_i,\theta_r,\phi_r) = \rho_\lambda(\lambda,\theta_r,\phi_r,\theta_i,\phi_i)$
- Can be measured for different material surfaces
- Completely local, physics-based reflectance model
- Can be used as part of global illumination equation
- Note frequency dependence





- BRDF can come from empirical model (like Phong), measured, or theoretical model
- Real models exhibit variations due to imperfections and anisotropy
- How to store the BRDF
  – Brute force table very large
  – Simple mathematical model (like cosine function in Phong)
  – Using parameterized model (like Gaussian)

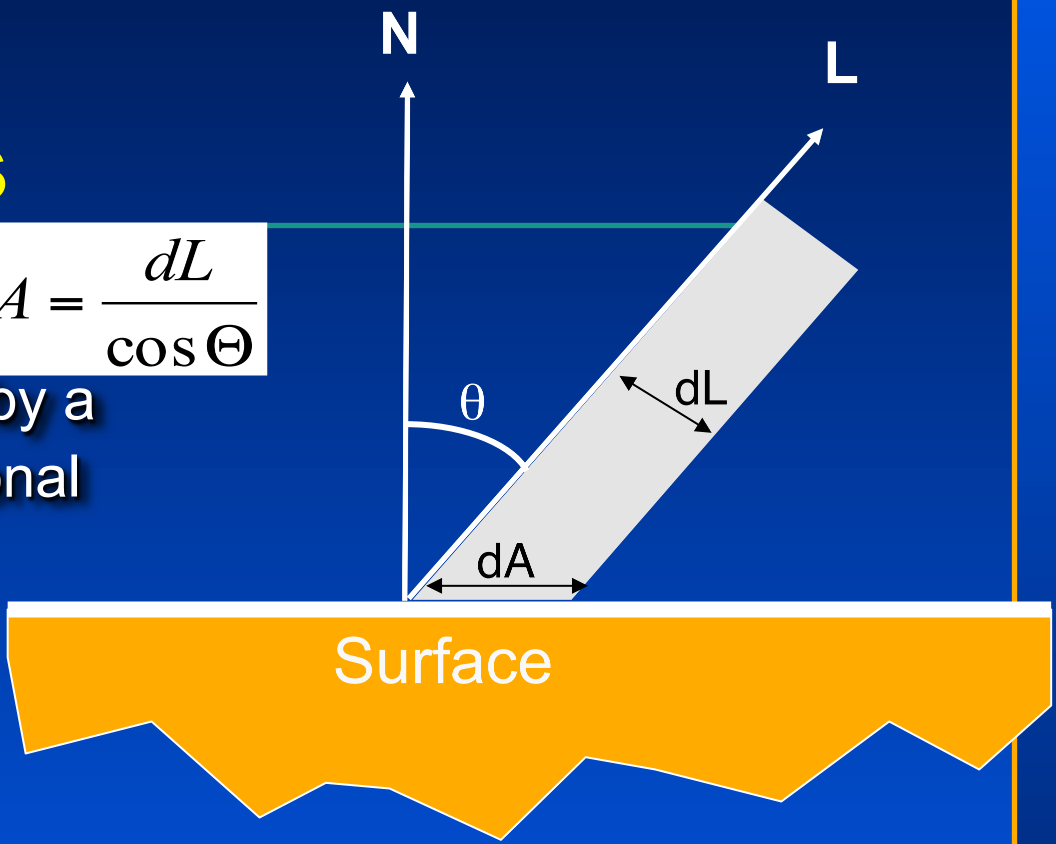
Phong illumination model (1975)

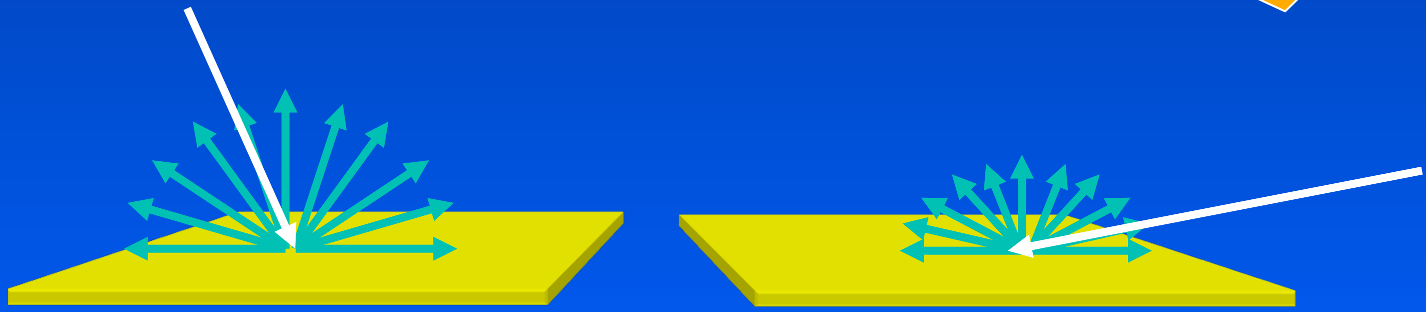
- An empirical model using
  - Lambert's cosine law for diffuse
  - Specular term
  - Ambient term to account for global illumination
- No dependence on wavelength

Lambert's Law
Diffuse Surfaces

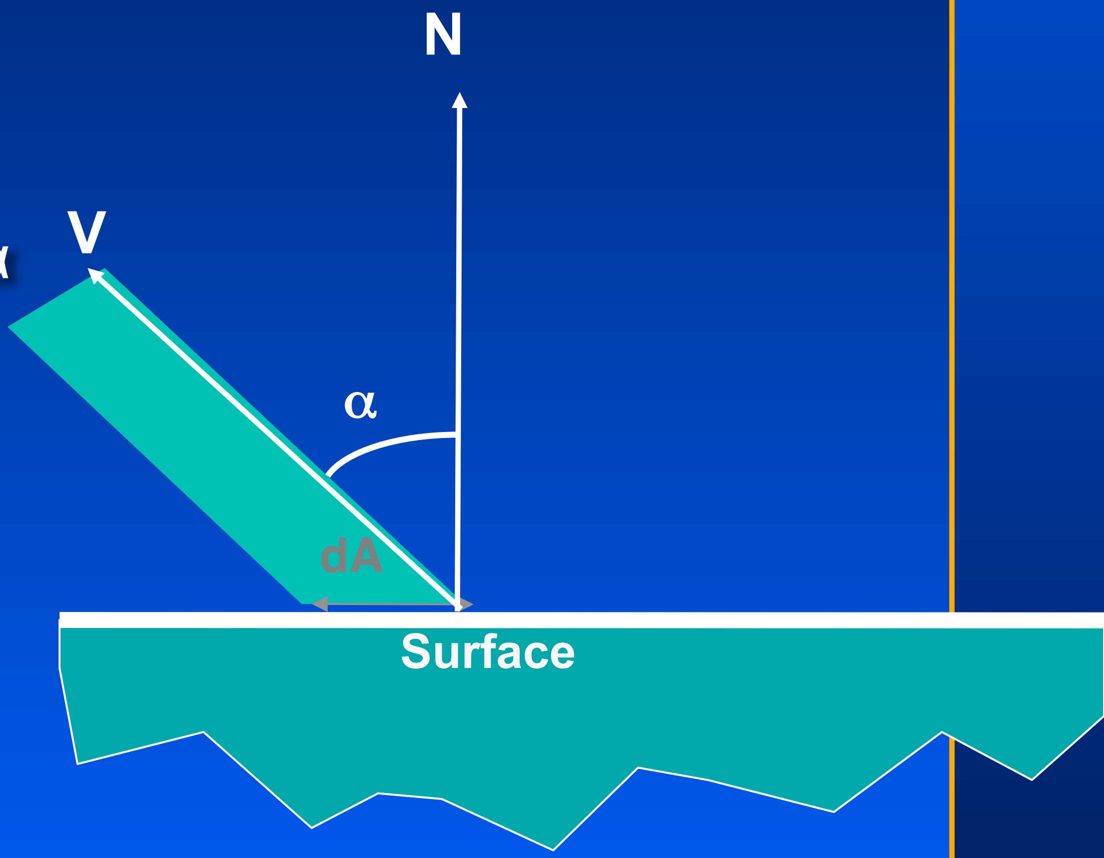
- Area subtended by dL: $dA = \dfrac{dL}{\cos\Theta}$

  $\therefore$ Amount of light received by a surface area dA is proportional to $\cos\theta$



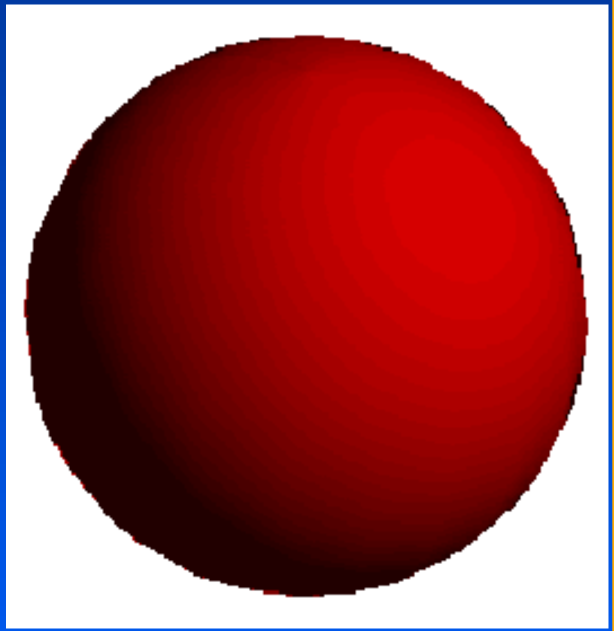


- As angle α between N and eyepoint increases, the amount of light emitted per unit area decreases by cos α but area seen by viewer increases by cos α
- ∴Reflected intensity independent of eyepoint



Diffuse term

- $I_{diffuse} = k_d\, I_{light}\, \mathbf{cos}\ \theta = k_d\, I_{light}\ \ (\boldsymbol{N} \bullet \boldsymbol{L})$
  - k_d diffuse reflectivity of the surface
  - I_{light} diffuse intensity of light
  - $\boldsymbol{N}$ surface normal at the point
  - $\boldsymbol{L}$ vector to the light



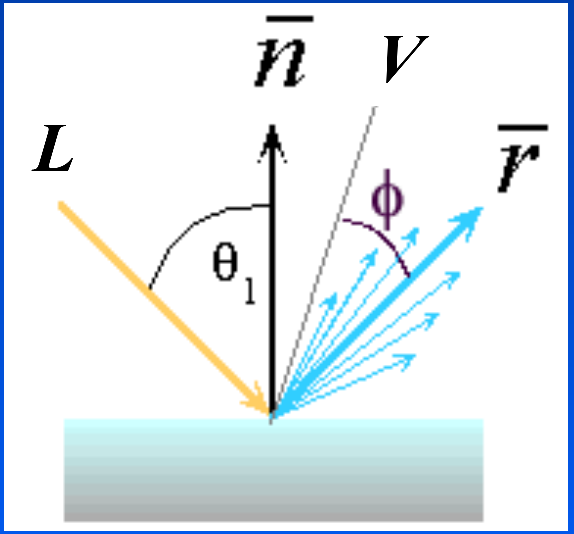
- Independent of where the camera is located
- Dependent on the direction to light
- Dependent on the surface normal at the point on the surface

Specular term

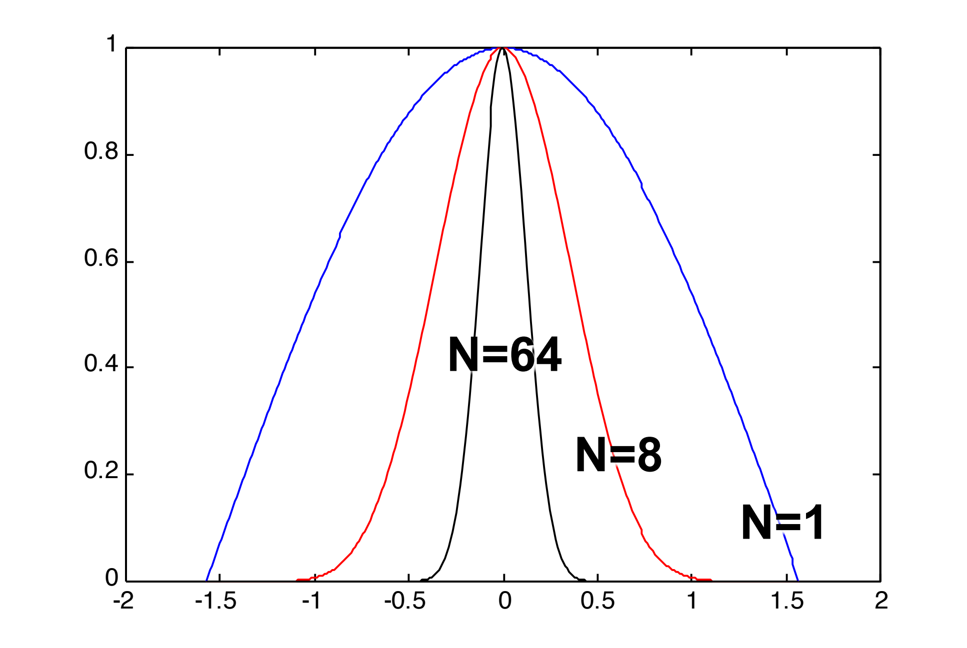
- Specular term given by cosine function to a shininess power as the shape of the falloff from the ideal direction

$$I_{specular} = k_s I_{light} (\cos\phi)^n = k_s I_{light} (V \cdot r)^n$$

  - r is the ideal reflective direction
  - V is the direction to the camera
  - N is the surface normal
  - L is the direction to the light

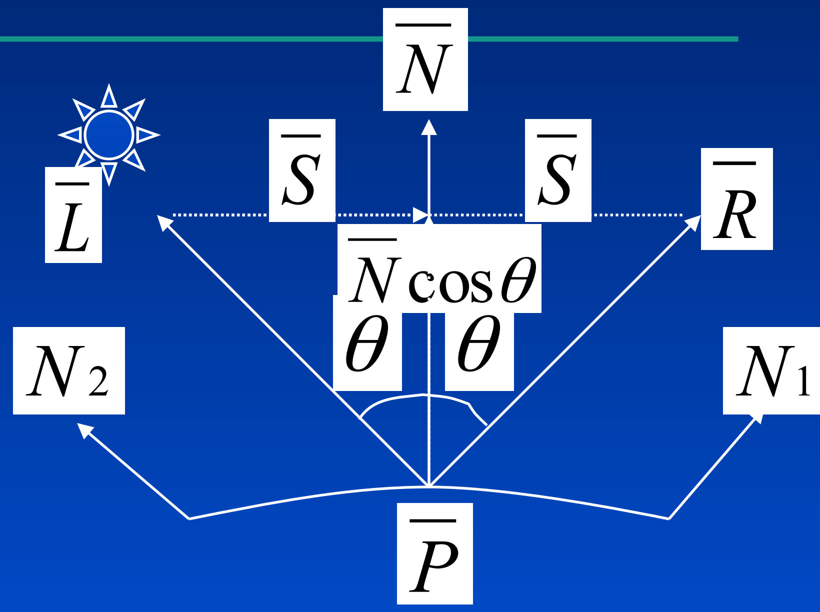


- Plot of $(\cos\phi)^n$ as a function of ϕ as n varies (n=1, 8, 64)
- As n gets large, function has a sharp peak near ϕ = 0



- To calculate R:

$$\overline{R} = \overline{N}\cos\theta + \overline{S}$$
$$\overline{S} = \overline{N}\cos\theta - \overline{L}$$
$$\therefore \overline{R} = 2\overline{N}\cos\theta - \overline{L}$$
$$= 2\overline{N}(\overline{N} \bullet \overline{L}) - \overline{L}$$

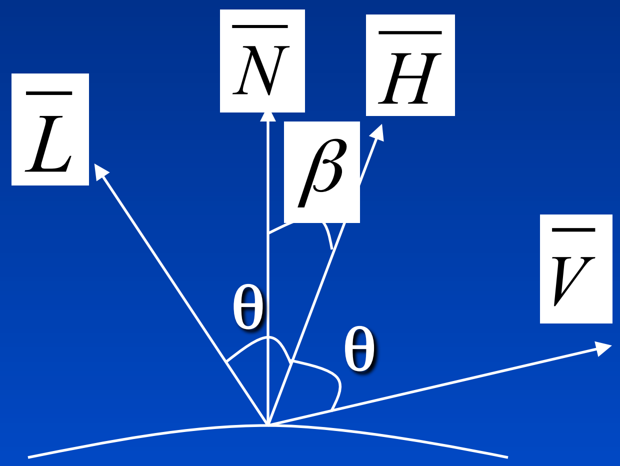


  - if light at infinity : L constant
  - if eye at infinity : V constant
  - if neither, inverse transform P from camera coordinate to world coordinate to determine L and/or V

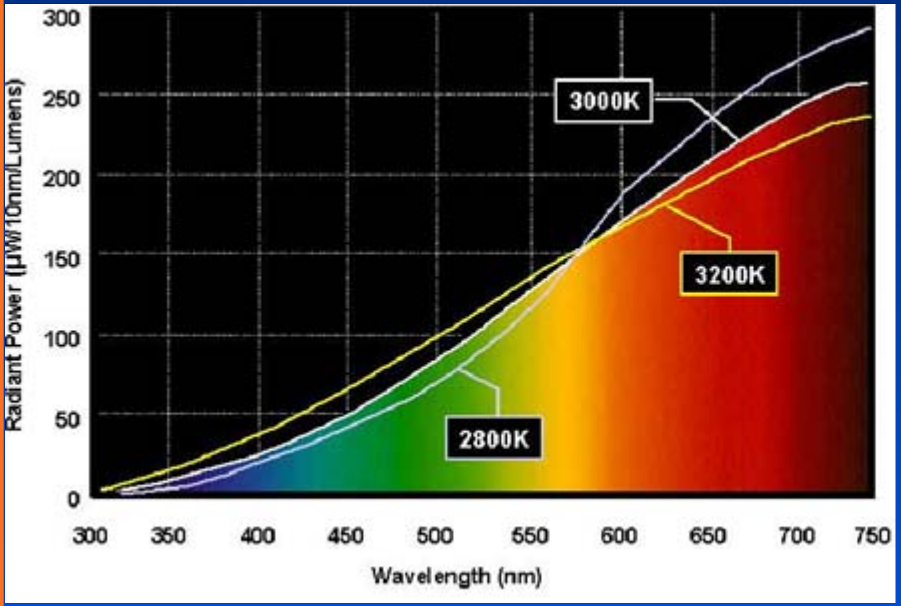
Alternate formulation Phong illumination

- $$\overline{H} = \frac{(\overline{L} + \overline{V})}{|\,\overline{L} + \overline{V}\,|}$$
- If β =0, brightest
- Smooth falloff $\propto (\overline{N} \bullet \overline{H})^n$
- When Light & Viewer at ∞,

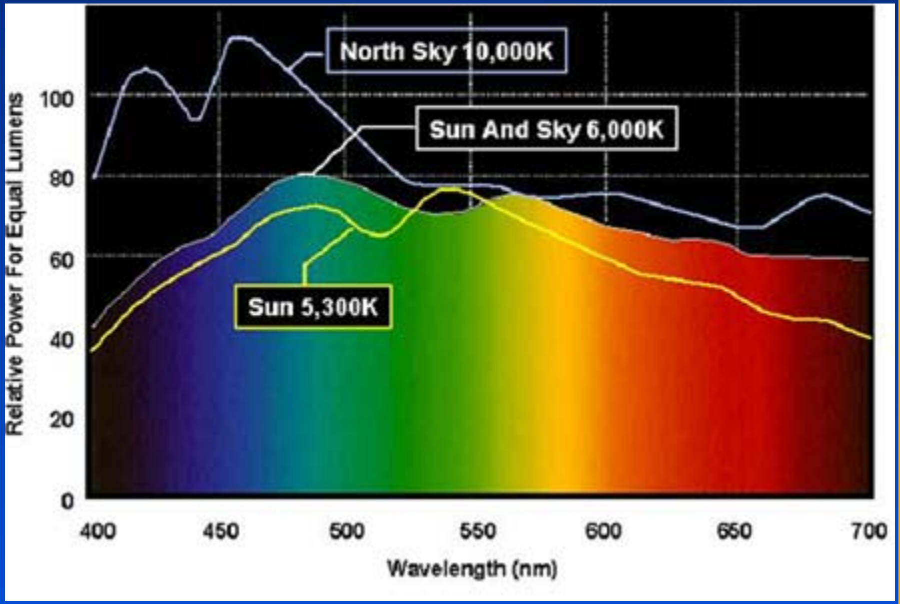
  H constant over the scene
- β≠ϕ(ϕ is angle between *R* and *V*)



Wavelength dependence

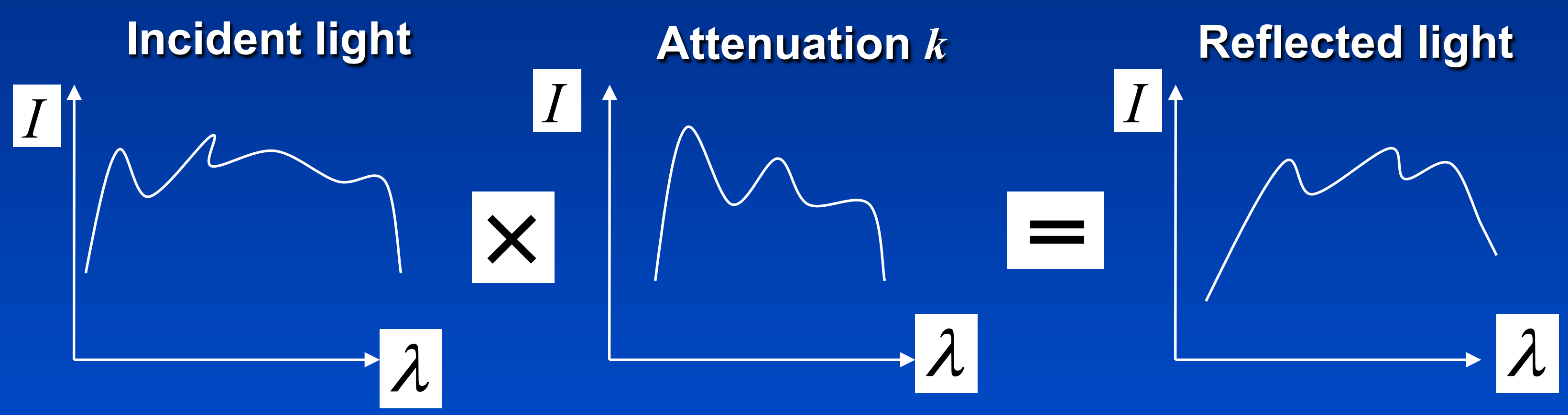


Incandescent



GE

Daylight



Incident light
I
λ
$\times$
Attenuation k
I
λ
$=$
Reflected light
I
λ

- To handle wavelength dependence more accurately:

1. Sample I_{in} at several λ
2. Sample k at several λ
3. Calculate several samples of I_{out}
4. Convert spectral distribution to RGB for monitor using tristimulus theory

- By sampling at 3 points
  - Assuming certain conversion function (wrong)
  - Undersampling in frequency space (aliasing problems)
- Reflectance model still ad hoc

Physics-Based Specular Model
Blinn (1977), Cook-Torrance (1982)

- Surface composed of mirror like microfacets
  - Normals distributed using some distribution function (e.g. Gaussian or Beckman)
  - Take into consideration self shadowing and masking of microfacets
  - Grazing incident angle give specular peak (paper example)
- Fresnel term (from classical wave theory EM radiation)
  - Wave length dependent reflection
  - Metal, non-plastic look – color of reflection not color of light
- Although more accurate, not used in favor of simpler Phong model

$$I_{\lambda,r}(\lambda,\theta_i) = I_{\lambda,a} k_a(\lambda) + I_{\lambda,i} d\omega (k_d R_d(\lambda)(L \bullet N) + k_s R_s(\lambda,\theta_i))$$

- Ambient term same as before
- Diffuse essentially the same as before
  - $d\omega$ is solid angle of light source
- BRDF sum of diffuse and specular components (attempt at energy conservation):

$$R = k_s R_s + k_d R_d \qquad k_s + k_d = 1$$

Specular term

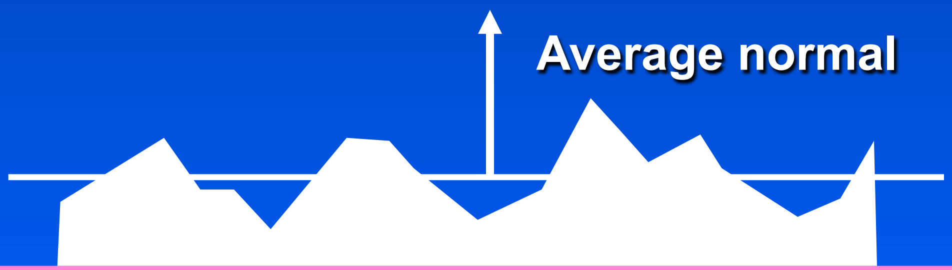
$$R_s(\lambda, \theta_i) = \frac{DG\rho'_\lambda(\lambda, \theta_i)}{\pi N \bullet V}$$

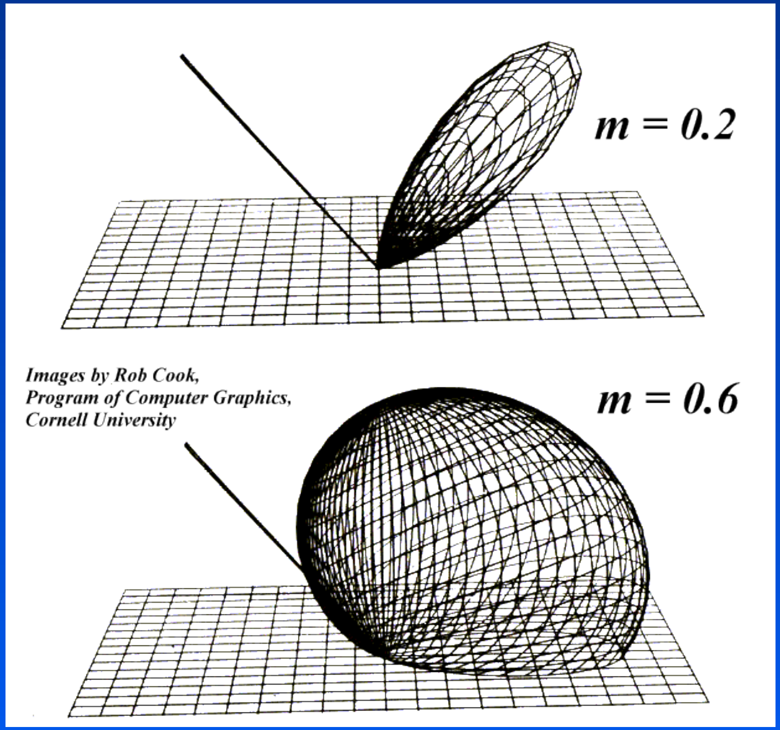
- $N \cdot V$: as the angle between N and V increases, more of the area is seen by viewer
  - Specular peak at grazing angle
- D is Beckmann distribution
- G is masking/shadowing term
- $\rho'_\lambda(\lambda, \theta_i)$ is Fresnel term

Beckmann distribution

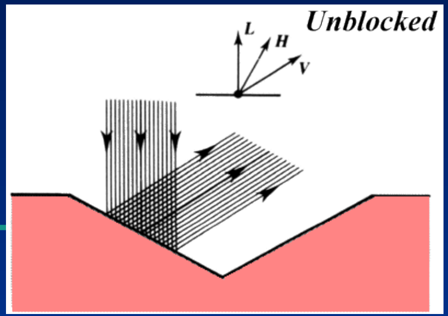
- Fractional area of microfacets oriented at angle α to average normal of surface, m is RMS slope of microfacets
  - Very close to Gaussian

$$D = \frac{1}{4m^2 \cos^4 \alpha} e^{\left(-\frac{\tan \alpha}{m}\right)^2}$$





Masking/Shadowing term



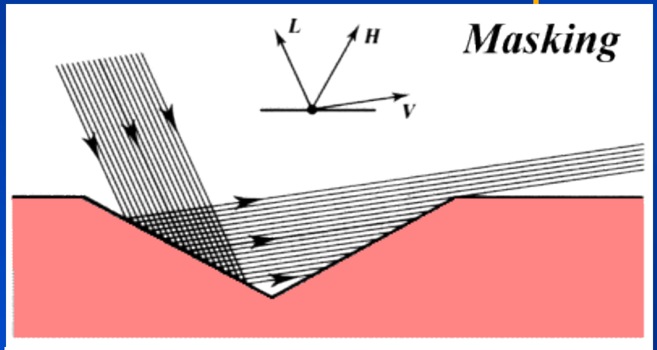
- G: maximum of 1 (no masking or shadowing)
- Energy that is not masked or shadowed contribute to specular term
- Can be shown that masking and shadowing terms are:

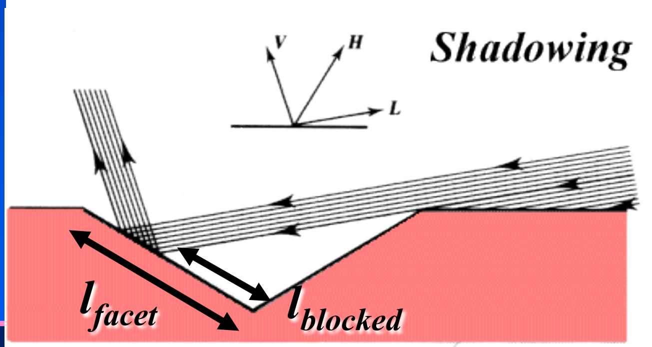
$$G = 1 - \frac{l_{blocked}}{l_{facet}}$$

$$G_{masking} = \frac{2(\bar{n}\cdot\bar{h})(\bar{n}\cdot\bar{v})}{\bar{v}\cdot\bar{h}}$$

$$G_{shadowing} = \frac{2(\bar{n}\cdot\bar{h})(\bar{n}\cdot\bar{l})}{\bar{v}\cdot\bar{h}}$$

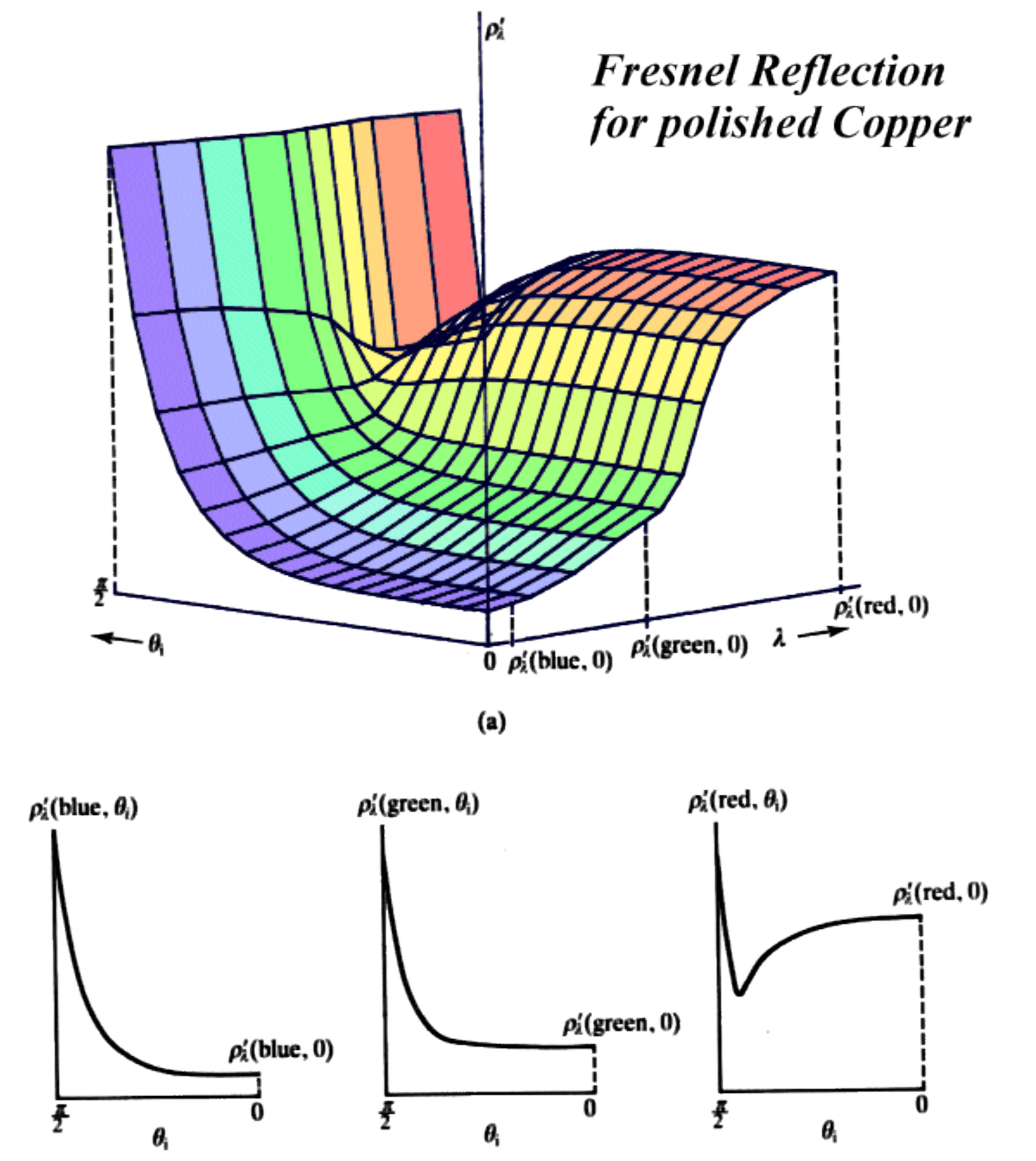
$$G = \min\{1, G_{masking}, G_{shadowing}\}$$





Fresnel term

- $\rho'_\lambda(\lambda, \theta_i)$
- Electromagnetic propagation of light
- A function of angle of incidence, wavelength, and index of refraction (characteristic of material)
- No color change as angle approach $\pi/2$ (grazing angle)

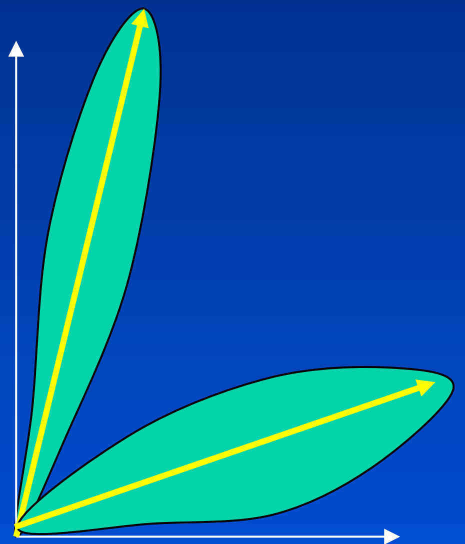


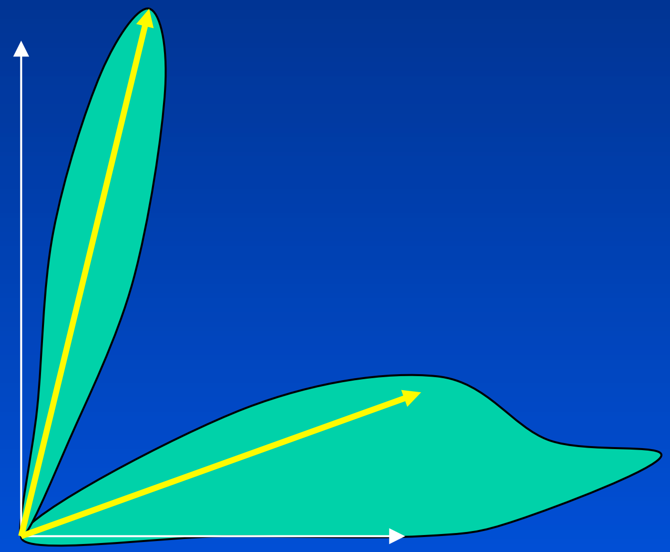
Limitations with Cook-Torrance Model

- Artificial separation into diffuse, specular
- Only local illumination, global hacked using ambient term as before
- Microfacet model only cross-sectional 2-D
- No anisotropy
  - Certain materials like cloth and "brushed" metal exhibit preferred reflection direction
  - Can be modeled by pre-computing the BRDF for different L direction
- Does not handle dirty, oxidized surfaces
- No polarization
- No sub-surface effects (e.g. important for skin)

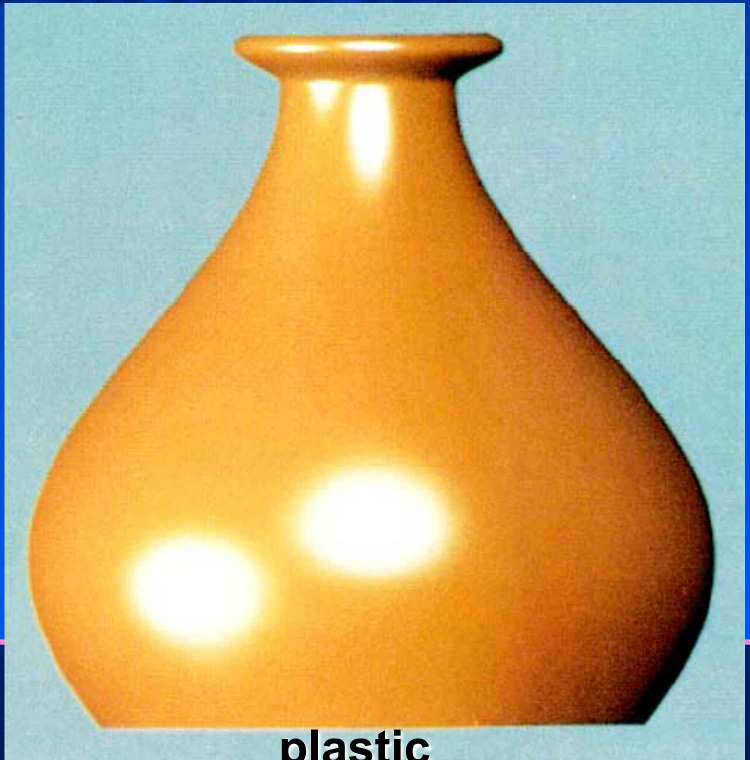
Gains with Cook-Torrance Model

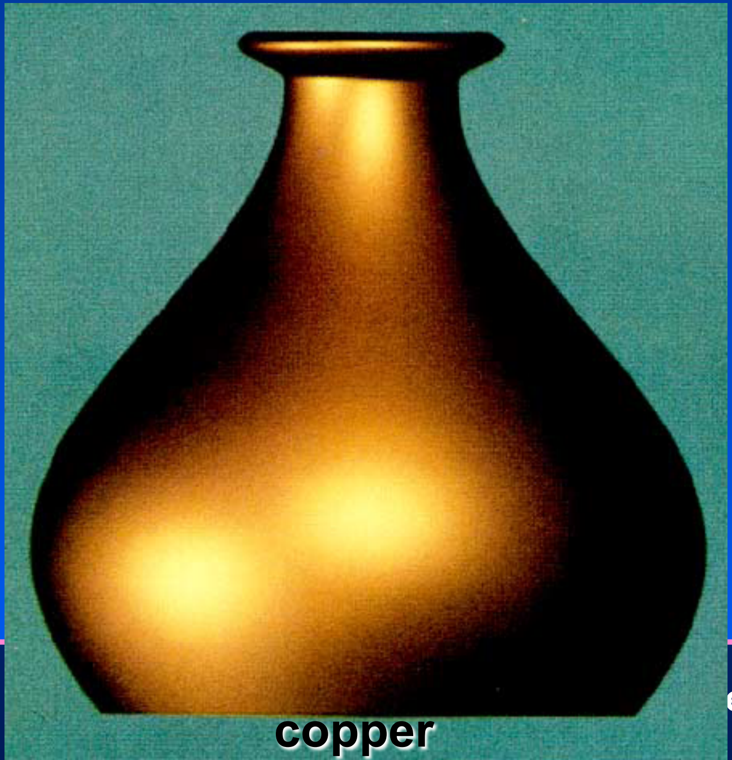
- More accurate specular peak at grazing angle





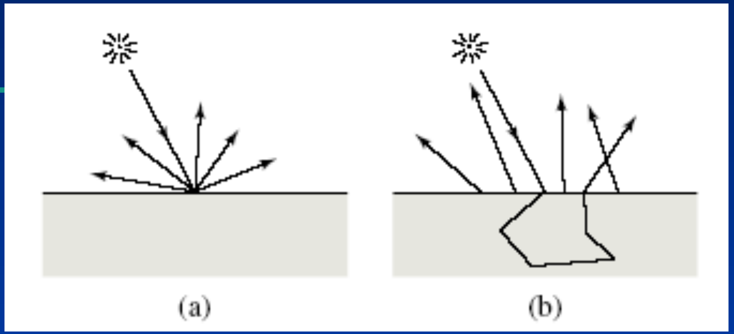
- More accurate specular color shift in metals
- Plastic, no color shift: sub-surface scatter different color from surface scatter color

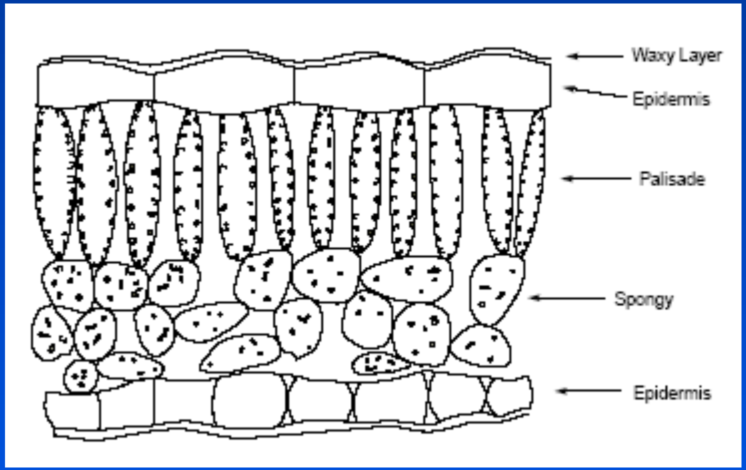

plastic


copper

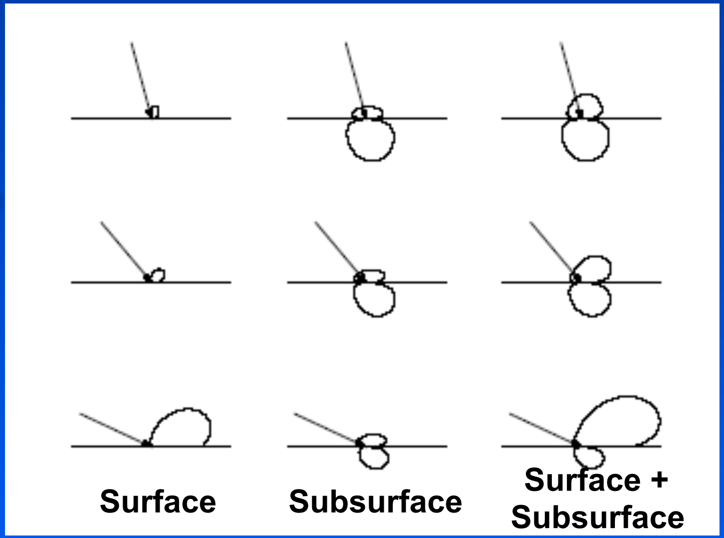
Subsurface scattering

- Scattering due to subsurface particles and different layers
- Seen in skin, leaf, etc.
- Result in Bidirectional Surface Scattering Reflection Distribution Function (BSSRDF)





- Reflection increases with material thickness
- Scattering can be backward, isotropic, forward
- Shape of BRDF lobe more flat
- Color bleeding
- Diffusion of light across shadow boundaries



BSSRDF

- In addition to incoming and outgoing direction, include incoming point x_i and outgoing point x_r

$$\rho_\lambda(\lambda, \theta_r, \phi_r, x_r, \theta_i, \phi_i, x_i)$$

- BRDF is simplification of BSSRDF assuming $x_i = x_r$

- Problem: Given an outgoing direction and position, how to determine the integration of all incoming direction and position…no longer a local model
- Material may not be isotropic
- Basically a “volume rendering the participating media” problem
  - E.g. by Monte-Carlo methods (more on this later)
  - Assume isotropic media

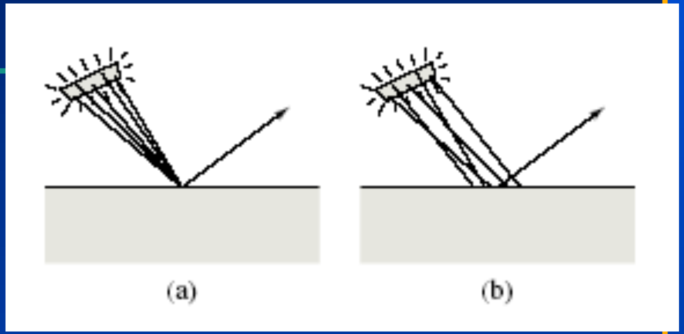


Photo of backlit grape and hand


BSSRDF
Skim milk
BSSRDF
Whole milk
Traditional BRDF
Paint
RENDERED USING DALI - HENRIK WANN JENSEN 2001


HENRIK WANN JENSEN - 2002


RENDERED BY HENRIK WANN JENSEN - 2001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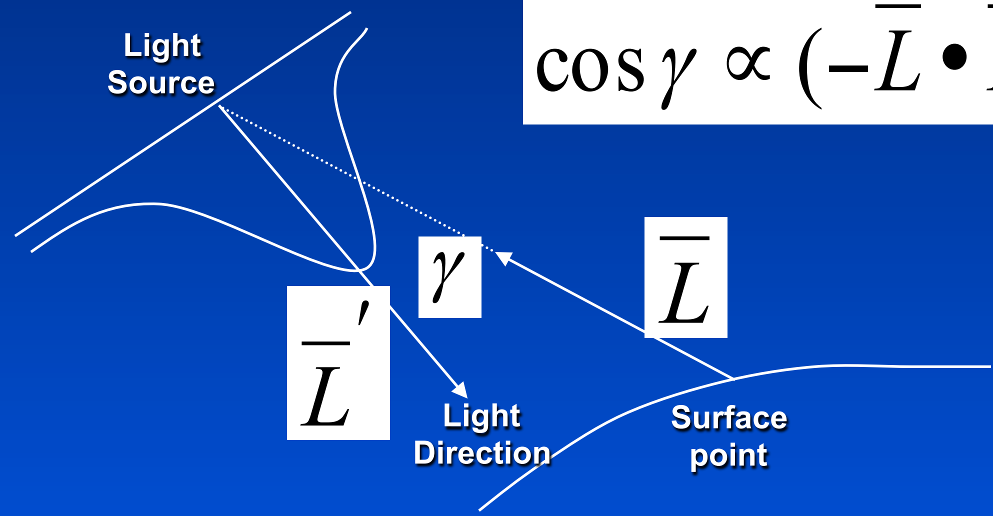
Backlit BSSRDF

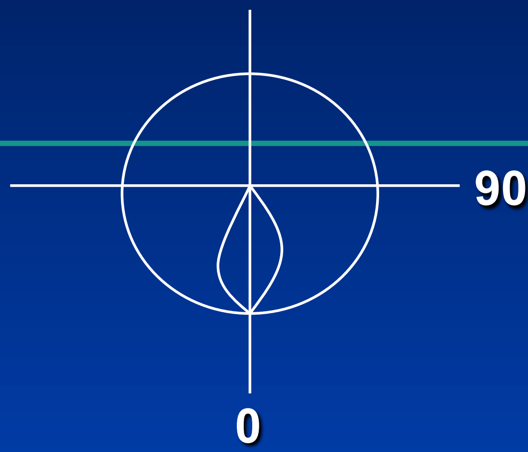
Directional Light Source

$$\cos\gamma \propto (-\overline{L} \bullet \overline{L}')$$



- $$I_{p\lambda} = I_{L'\lambda}(-\overline{L} \bullet \overline{L}')^p$$

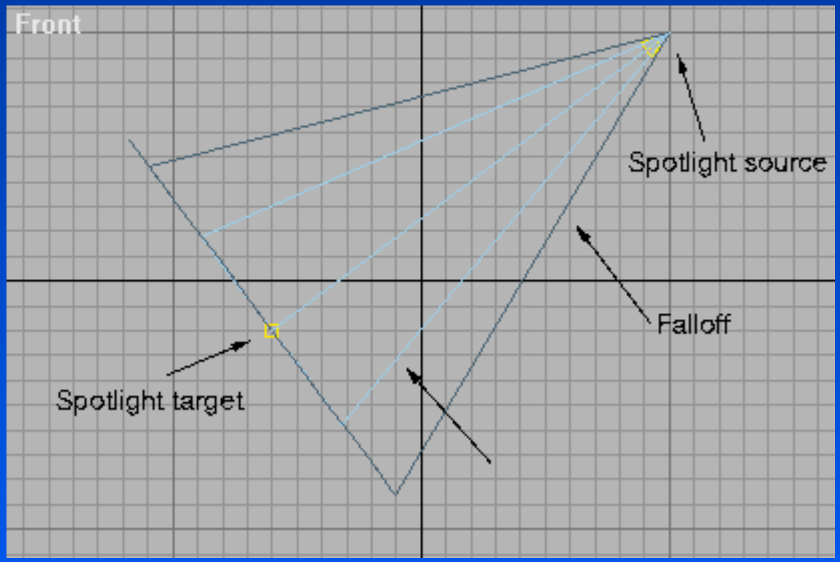
- Looking at equal $I_{p\lambda}$ vs γ

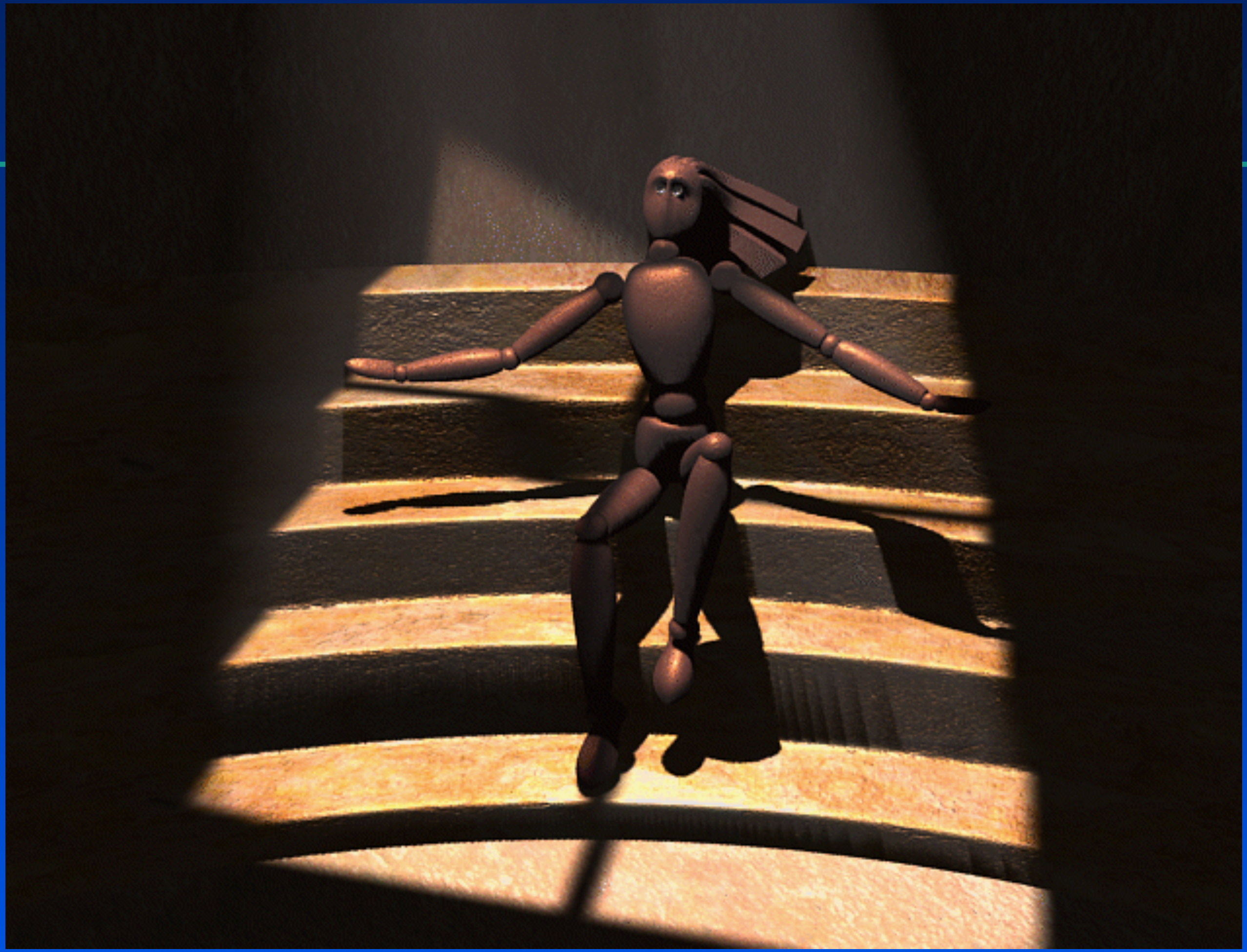


- Can also restrict range of $I_{p\lambda} = 0$ for $\gamma > \delta$

  (spot light)

- L now varies in the scene





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Multiple Light Sources

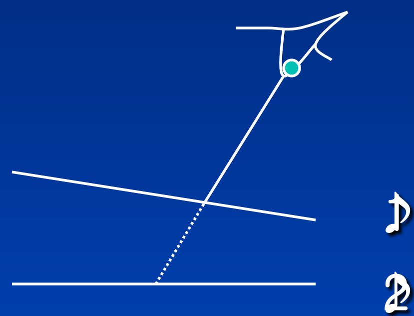
$$I_\lambda = I_{a\lambda} k_a O_{d\lambda} + \sum_i f_{att} I_{p\lambda i} [k_d O_{d\lambda} (\overline{N} \bullet \overline{L_i}) + k_s O_{s\lambda} (\overline{N} \bullet \overline{H})^n]$$

- Clamp I_λ to max or normalize
- Must map to dynamic range of imaging system

Non-refractive transparency

- Partially transparent polygon

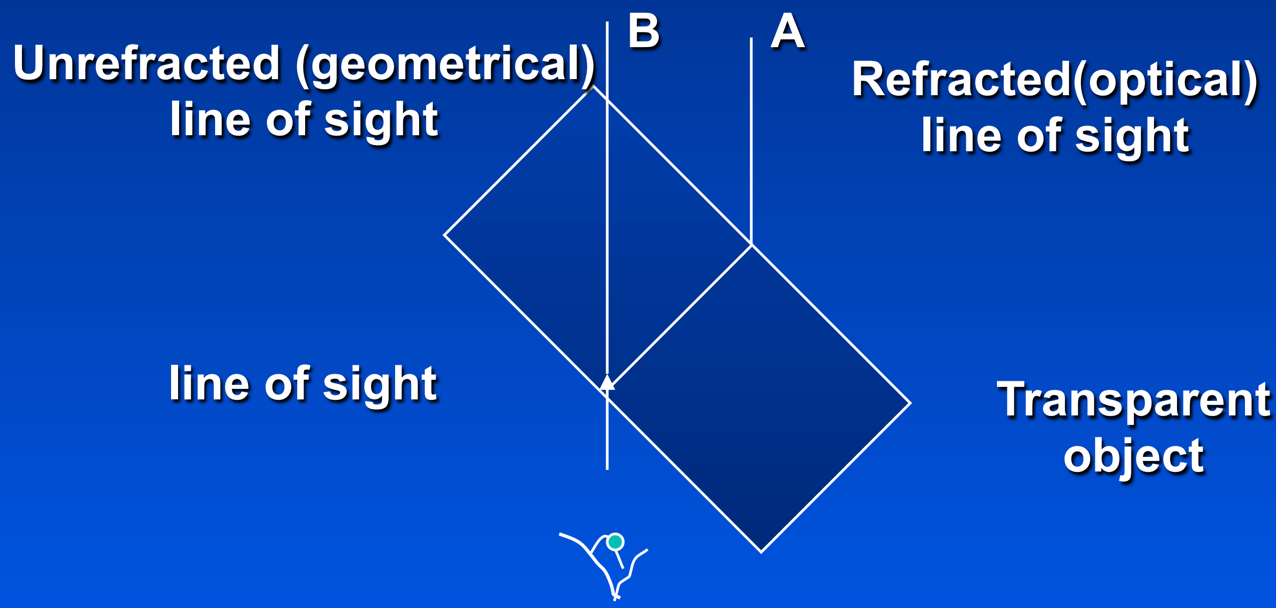
$$I_\lambda = (1 - k_{t1}) I_{\lambda 1} + k_{t1} I_{\lambda 2}$$



  - k_{t1} transmittance of polygon 1
  - $I_{\lambda 1}$ intensity calculated for polygon 1
  - $I_{\lambda 2}$ intensity calculated for polygon 2
  - Assumption that polygon 1 does not reduce light reaching polygon 2
  - If more semi-transparent polygons (say polygon 3) above, combine this result by a weighted sum using transmittance/ opacity of polygon 3

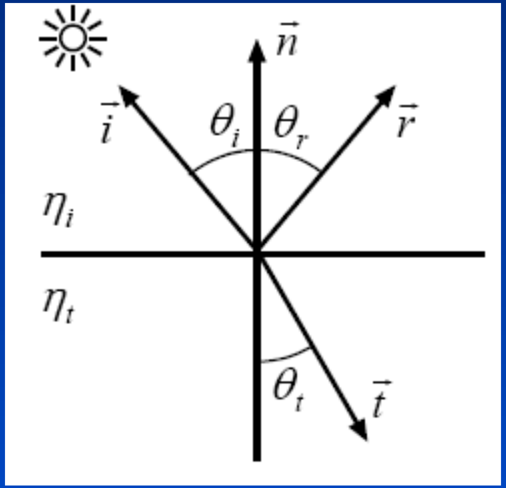
Refractive transparency♪

- Usually by ray-tracing (to come later)



- Snell's law:

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\eta_{i\lambda}}{\eta_{t\lambda}}$$



$\eta_{i\lambda}$ and $\eta_{t\lambda}$ are indices of refraction of the two media

- Use trigonometric identity

$$\sin^2\theta + \cos^2\theta = 1$$

- Snell's law for cosines

$$\cos^2\theta_t = 1 - \frac{\eta_i^2(1-\cos^2\theta_i)}{\eta_t^2}$$

- Transmission vector

$$T = \left(\frac{\eta_i}{\eta_t}\cos\theta_i - \cos\theta_t\right)N - \frac{\eta_i}{\eta_t}L$$

Next: Anti-aliasing and Texture mapping