10-Factor Experimental Design on Simulated Data

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1. Introduction

For this project, a secret model was created with ten independent variables and one dependent variable. The dependent variable is determined by an unknown function: $Y = f(a, b, ..., j) + \epsilon$. The value of each dependent variable, x_i, satisfies $-1 \leq x_i \leq 1$. To discern information about the model, we are tasked to create an experiment design with a number of run that each specify a particular setting of each dependent variable. Each run is then assigned a cost. A run where every variable is set to ± 1 costs 1 point, a run where each variable $x_i \in \{0, \pm 1\}$ costs 3 points, etc. If additional designs need to be run, points would be deducted. Finally, points would be awarded to correctly identifying the presence (or lack thereof) of each independent variable within the hidden model. This scenario calls for a **factor screening** expriment. Since the main factor settings will be ± 1, our starting point is a **two level factorial** design. With ten possible variables, a full 2^{10} design would require 1028 runs. This would cost over 1000 points and cost almost more than the total amount available on offer. Reducing this necessitates a **two-level fractional factorial** design, reducing the number of runs.

While a two-level fractional factorial (I will refer to it as **2FrF** going forward) design may allow us to detect linear or approximately-linear effects of factors and their interaction terms, with simply this design we are vulnerable to omitting significant factors that offer second-order (or even) effects. For example, if the unknown function $f(a, b, ...j)$ contains a factor x^2 or $cos(x)$, there exists a possibility that it will remain undetected in a 2FrF design. To this end, it would be useful to add **center runs** and **axial runs**. The combination of the two suggests a **central composite design** (**CCD**). Ideally, we would create a **small composite design** (**SCD**) for 10 factors, however I was only able to find a 9-factor SCD. Instead, I worked with a **hybrid design** by adding center and axis runs to a 2FrF design. Because of the points penalty for variable values that are not (-1), 0, or 1, these axial runs will be **face-centered**. This is a very important detail to note. The consequence of face-centered axial runs is that inaccuracy is introduced when modeling the coefficients corresponding to the a factor effect. However, this is a worthwhile trade off. Our priority is to *detect* the factor effects. Note also, however, that certain methodology (such as linear modeling) will be less accurate, create lower R^2 values, and have lower p-values requiring extra caution.

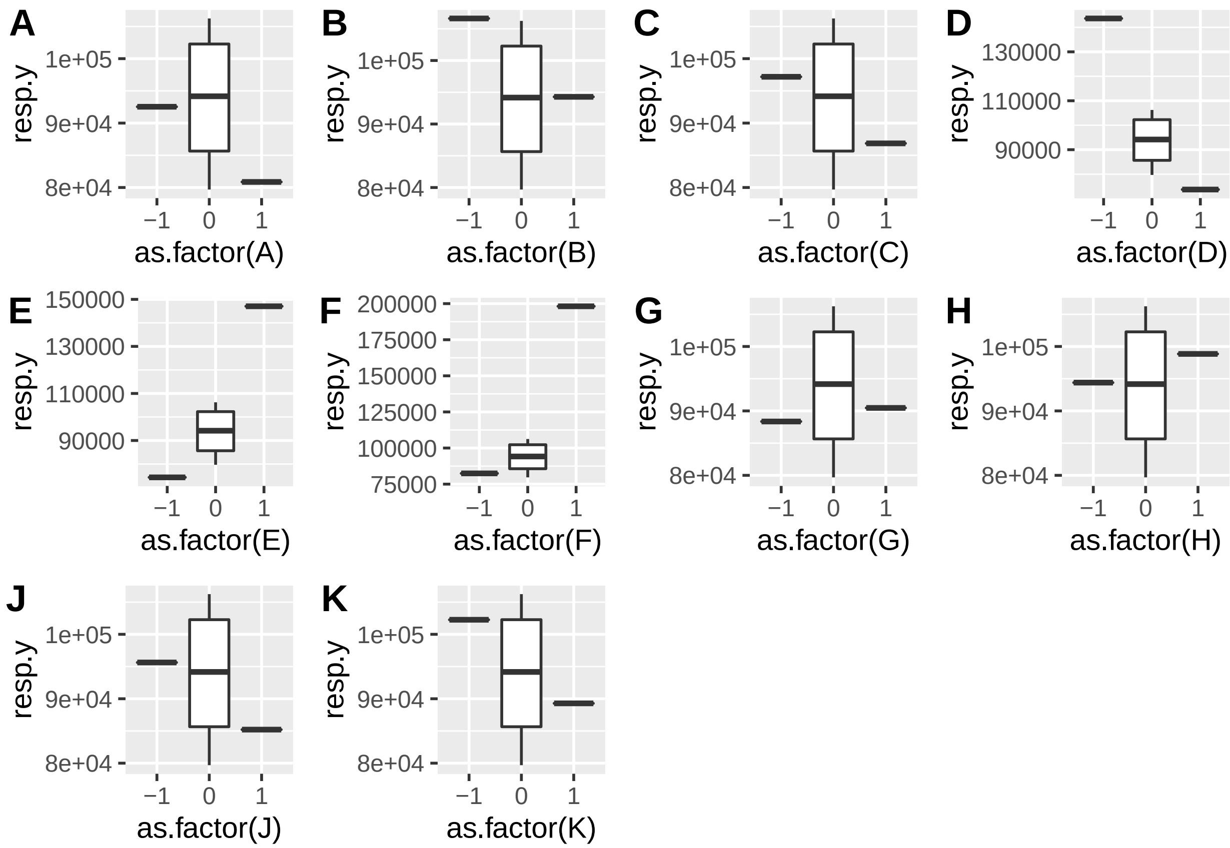
NOTE: FOR THE REMAINDER OF THIS PROJECT, I WILL REFER TO THE INDEPENDENT VARIABLES AS $A, B, C, D, E, F, G, H, J, K$, OMITTING I AS WAS USED IN THE ASSIGNMENT DOCUMENTS AS I IS USUALLY RESERVED FOR THE INTERCEPT EFFECT.

2. Methods

The design I decided upon was a hybrid design. A $2^{10-3}_{\mathbf{V}}$ design with the axial runs of a face-centered central composite design. This design was produced in R with the help of the `FrF2` and `DoE` packages. Analysis performed on them was aided by the `RcmdrPlugin.DoE` package. The design's generators are: $H = ABCDE, J = ABCFG, K = ABCDF$. The creation procedure is detailed in **Appendix 6.1** .The full design documentation (including matrix and attributes) is available in **Appendix 6.2**.

3. Analysis

The central and axial runs are incredibly valuable. In an axial run, all variables except one are set to 0. This means that all main and interaction effects are wiped with the exception of the main effect of the non-zero variable. My first step was to compare the effects of each individual variable on the function output compared with a baseline provided by the center runs. While there is a significant limitation to this (only one run exists for each variable at -1 and 1), if a difference is significant enough, it will mean the variable should be concluded. An additional benefit is that these axial runs bypass the problem of aliasing. However, because they are face-centered, we are likely to get inaccurate estimates for linear parameters.



A visual analysis of the means suggests that D, E, and F have significant main effects in the model. There is a possibility of outliers because of the limited sample size, but all of the effects are quite extreme. There is also the possibility that their effects are non-linear. F especially has a possibly exponential effect considering the asymmetry in effect on the mean from between it's two settings. Below are the p-values for the F-tests on each variable's fitted model.

```
fstats <- summary(lm(resp.y ~ D, data = design_centers))$fstatistic
1 - pf(fstats[1], fstats[2], fstats[3])
```

```
##      value
## 0.05591169
```

```
summary(lm(log(resp.y) ~ D, data = design_centers, na.action = na.omit))
```

```
##
## Call:
## lm.default(formula = log(resp.y) ~ D, data = design_centers,
##     na.action = na.omit)
```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.26318 -0.09899 -0.02553  0.05884  0.71727
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 11.47955    0.03806 301.594   <2e-16 ***
## D           -0.33353    0.13985  -2.385    0.025 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1978 on 25 degrees of freedom
## Multiple R-squared:  0.1853, Adjusted R-squared:  0.1528
## F-statistic: 5.688 on 1 and 25 DF,  p-value: 0.02498
```

```
fstats <- summary(lm(resp.y ~ E, data = design_centers))$fstatistic
1 - pf(fstats[1], fstats[2], fstats[3])
```

```
##      value
## 0.04609538
```

```
summary(lm(log(resp.y) ~ E, data = design_centers, na.action = na.omit))
```

```
##
## Call:
## lm.default(formula = log(resp.y) ~ E, data = design_centers,
##     na.action = na.omit)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.27157 -0.09899 -0.02553  0.06192  0.71727
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 11.47955    0.03786 303.178   <2e-16 ***
## E            0.34108    0.13912   2.452   0.0215 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1967 on 25 degrees of freedom
## Multiple R-squared:  0.1938, Adjusted R-squared:  0.1616
## F-statistic: 6.011 on 1 and 25 DF,  p-value: 0.02155
```

```
fstats <- summary(lm(resp.y ~ F, data = design_centers))$fstatistic
1 - pf(fstats[1], fstats[2], fstats[3])
```

```
##       value
## 0.000624253
```

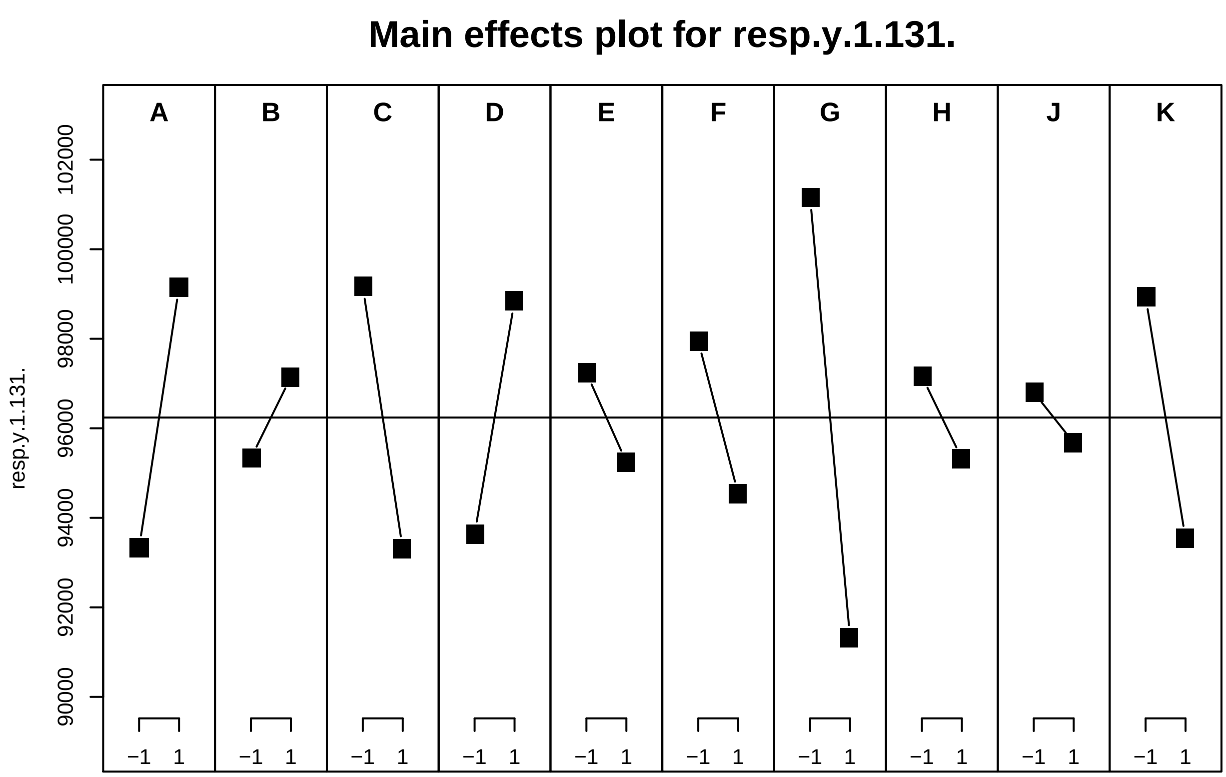
```
summary(lm(log(resp.y) ~ F, data = design_centers, na.action = na.omit))
```

```
##
## Call:
## lm.default(formula = log(resp.y) ~ F, data = design_centers,
##     na.action = na.omit)
##
```

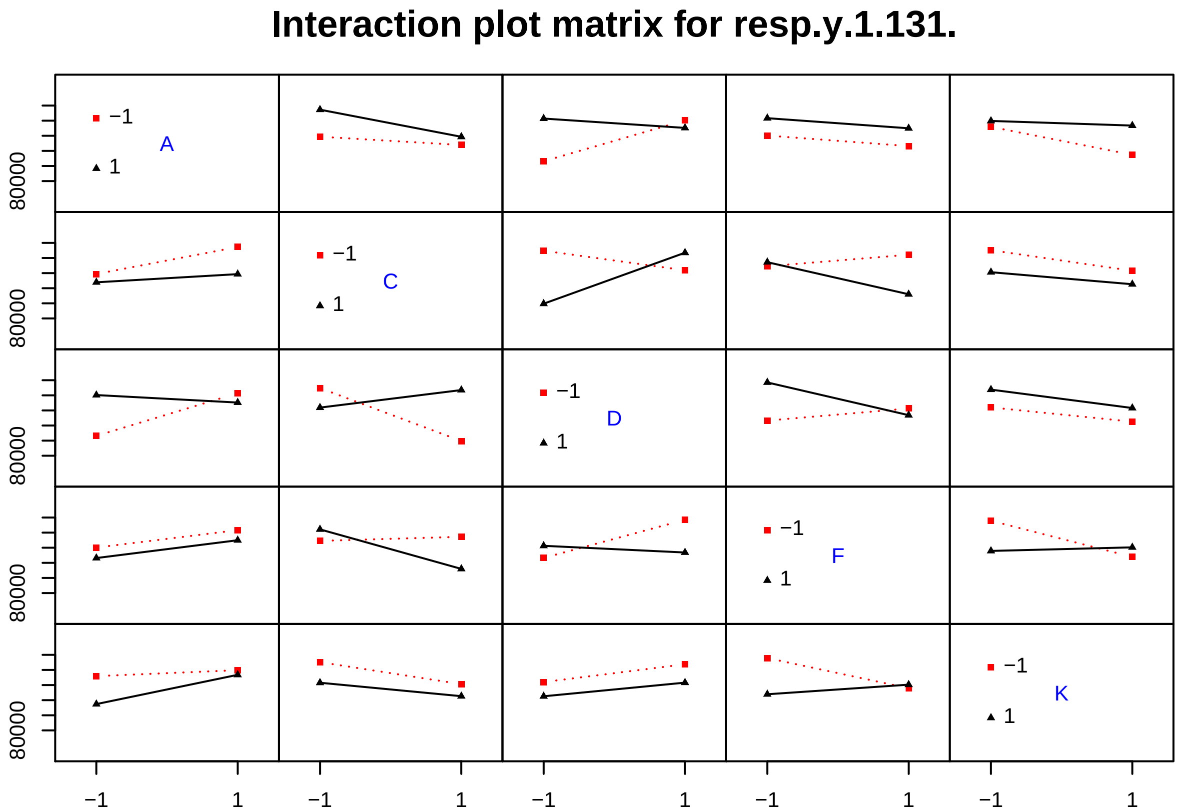
```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.27157 -0.09899 -0.02553  0.06192  0.41898
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 11.47955    0.03476 330.212  < 2e-16 ***
## F            0.43855    0.12773   3.433  0.00209 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1806 on 25 degrees of freedom
## Multiple R-squared:  0.3204, Adjusted R-squared:  0.2932
## F-statistic: 11.79 on 1 and 25 DF,  p-value: 0.002086
```

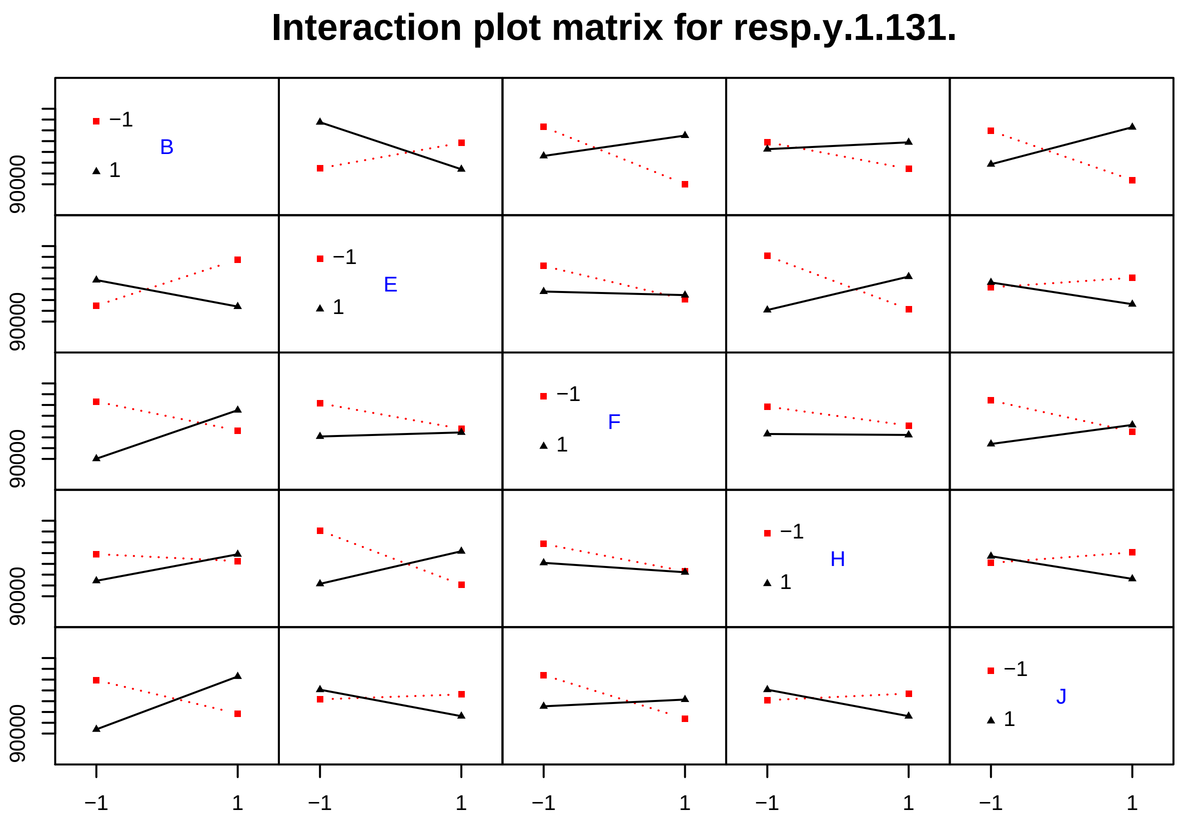
To test for potential transformations, a logarithmic transformation was applied to the responses and another model was fitted. This pushed D and E in a direction to suggests a likelier presence of their main effects, F got pushed closer to the rejection region. All three are likely significant and present within the model. If their effects are transformed, they may not show up when analyzing the full experimental results. However, because this is a factor screening experiment, it is safer to include an effect that can then be further trialed than to remove one that to reject one too early.

Next we analyze the experimental results as a whole. A good first step is to look at a main effects plot, identifying potential effects. From the below plot, we can see that the effects are $G, C, A, K, D, F, E, H, B, J$ (roughly in order from most to least significance).



There is already enough evidence to keep D, E, and F within the model, we should further investigate the remaining variables. The least significant variables in the main effects plot will only exist in the model if they have strong interactions with other variables. Since our design is resolution five, the lowest interaction term that can alias with a single interaction is a four-factor interaction. By scarcity of effects, we can conclude that if any variable has a weak main effect and weak two-factor interactions, it is unlikely to be present within the model.



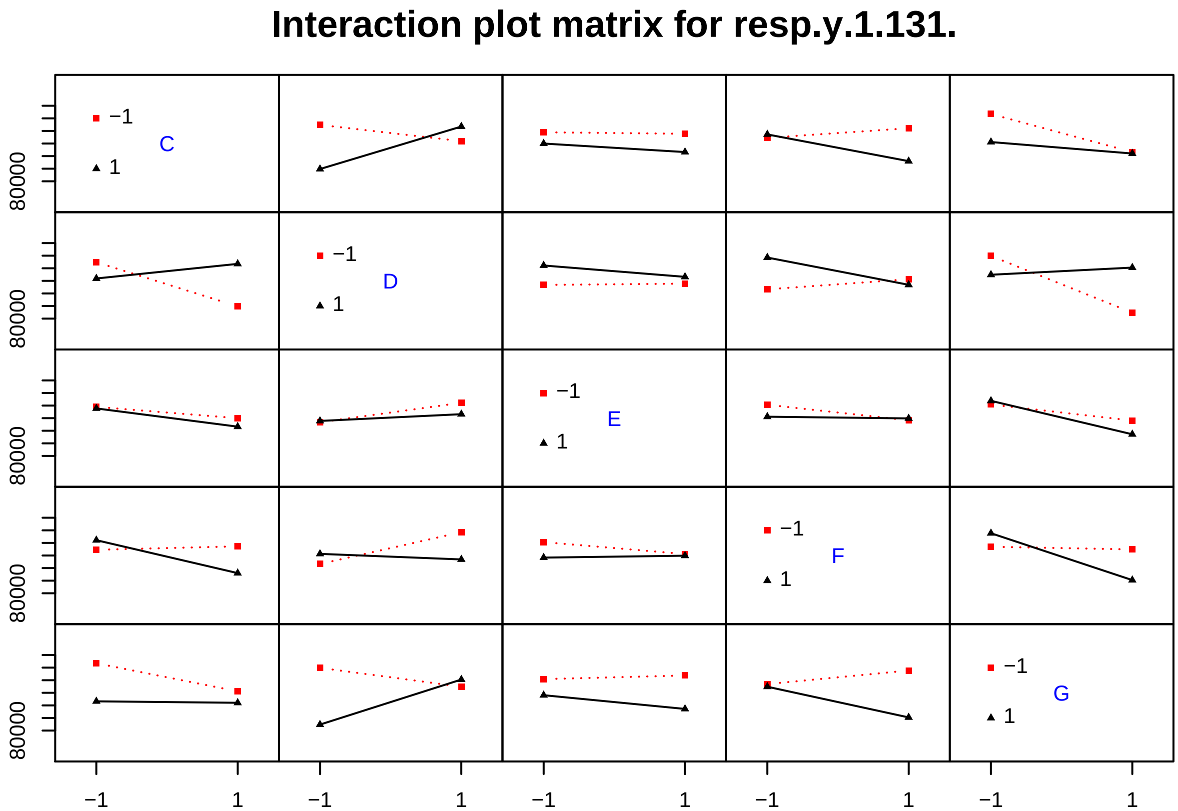



```
LinearModel.1 <- lm(resp.y ~ (A + B + C + D + E + F + G + H + J + K)^2 +
                    I(A^2) + I(B^2) + I(C^2) + I(D^2) + I(E^2) + I(F^2) + I(G^2) + I(H^2) +
                    I(J^2) + I(K^2), data=design)
summary(aov(LinearModel.1))
```

```
##              Df    Sum Sq   Mean Sq F value Pr(>F)
## A             1 1.004e+09 1.004e+09   0.923 0.3392
## B             1 8.224e+07 8.224e+07   0.076 0.7840
## C             1 1.144e+09 1.144e+09   1.052 0.3078
## D             1 5.333e+08 5.333e+08   0.490 0.4856
## E             1 2.324e+07 2.324e+07   0.021 0.8841
## F             1 8.085e+07 8.085e+07   0.074 0.7857
## G             1 3.030e+09 3.030e+09   2.787 0.0986 .
## H             1 9.894e+07 9.894e+07   0.091 0.7636
## J             1 5.226e+07 5.226e+07   0.048 0.8270
## K             1 9.895e+08 9.895e+08   0.910 0.3427
## I(A^2)        1 3.513e+08 3.513e+08   0.323 0.5712
## I(B^2)        1 9.793e+07 9.793e+07   0.090 0.7648
## I(C^2)        1 1.531e+07 1.531e+07   0.014 0.9058
## I(D^2)        1 3.128e+08 3.128e+08   0.288 0.5931
## I(E^2)        1 2.674e+08 2.674e+08   0.246 0.6212
## I(F^2)        1 2.672e+09 2.672e+09   2.457 0.1206
## I(G^2)        1 5.239e+08 5.239e+08   0.482 0.4894
## I(H^2)        1 1.066e+08 1.066e+08   0.098 0.7550
## I(J^2)        1 3.025e+08 3.025e+08   0.278 0.5992
## I(K^2)        1 7.038e+07 7.038e+07   0.065 0.7998
```

```
## A:B          1 2.002e+07 2.002e+07   0.018 0.8924
## A:C          1 3.105e+08 3.105e+08   0.285 0.5945
## A:D          1 2.206e+09 2.206e+09   2.028 0.1579
## A:E          1 2.152e+08 2.152e+08   0.198 0.6575
## A:F          1 1.446e+05 1.446e+05   0.000 0.9908
## A:G          1 5.848e+05 5.848e+05   0.001 0.9816
## A:H          1 8.883e+07 8.883e+07   0.082 0.7757
## A:J          1 1.365e+09 1.365e+09   1.256 0.2655
## A:K          1 4.779e+08 4.779e+08   0.439 0.5091
## B:C          1 5.547e+06 5.547e+06   0.005 0.9432
## B:D          1 4.934e+07 4.934e+07   0.045 0.8318
## B:E          1 1.446e+09 1.446e+09   1.330 0.2519
## B:F          1 1.655e+09 1.655e+09   1.522 0.2206
## B:G          1 1.837e+09 1.837e+09   1.689 0.1971
## B:H          1 3.096e+08 3.096e+08   0.285 0.5950
## B:J          1 2.045e+09 2.045e+09   1.881 0.1737
## B:K          1 1.620e+08 1.620e+08   0.149 0.7004
## C:D          1 4.361e+09 4.361e+09   4.010 0.0483 *
## C:E          1 6.123e+07 6.123e+07   0.056 0.8130
## C:F          1 1.665e+09 1.665e+09   1.531 0.2192
## C:G          1 8.917e+08 8.917e+08   0.820 0.3676
## C:H          1 5.525e+07 5.525e+07   0.051 0.8222
## C:J          1 1.106e+09 1.106e+09   1.017 0.3159
## C:K          1 6.416e+07 6.416e+07   0.059 0.8086
## D:E          1 2.041e+08 2.041e+08   0.188 0.6659
## D:F          1 1.763e+09 1.763e+09   1.621 0.2063
## D:G          1 5.151e+09 5.151e+09   4.736 0.0322 *
## D:H          1 1.463e+07 1.463e+07   0.013 0.9079
## D:J          1 2.118e+07 2.118e+07   0.019 0.8893
## D:K          1 1.517e+07 1.517e+07   0.014 0.9063
## E:F          1 2.412e+08 2.412e+08   0.222 0.6388
## E:G          1 3.825e+08 3.825e+08   0.352 0.5546
## E:H          1 2.068e+09 2.068e+09   1.902 0.1714
## E:J          1 2.712e+08 2.712e+08   0.249 0.6188
## E:K          1 5.500e+06 5.500e+06   0.005 0.9435
## F:G          1 2.469e+09 2.469e+09   2.270 0.1354
## F:H          1 8.856e+07 8.856e+07   0.081 0.7760
## F:J          1 6.952e+08 6.952e+08   0.639 0.4261
## F:K          1 1.389e+09 1.389e+09   1.277 0.2615
## G:H          1 1.741e+07 1.741e+07   0.016 0.8996
## G:J          1 1.840e+09 1.840e+09   1.692 0.1967
## G:K          1 3.201e+08 3.201e+08   0.294 0.5888
## H:J          1 3.018e+08 3.018e+08   0.278 0.5996
## H:K          1 3.008e+08 3.008e+08   0.277 0.6002
## J:K          1 1.080e+08 1.080e+08   0.099 0.7534
## Residuals   89 9.678e+10 1.087e+09
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
IAPlot(design_2, abbrev=4, show.alias=FALSE, select=c(3,4,5,6,7))
```



Both our preliminary visual inspection and our preliminary linear model, which samples all main effects and two-factor interaction terms (including quadratic effects), suggest that A, B, E, F, H, J, K are unlikely to have two-factor interactions. Since A, B, H, J, K have no significant one- or two-factor effects, it is likely they are not present anywhere in the model. By running a stepwise variable selection, we fall onto the model `y ~ D + G + D:G`.

```
stepwise(LinearModel.1, direction='backward/forward', criterion='BIC', trace = 0)
```

```
##
## Direction:  backward/forward
## Criterion:  BIC
```

```
##
## Call:
## lm.default(formula = resp.y ~ D + G + D:G, data = design)
##
## Coefficients:
## (Intercept)            D            G          D:G
##       96754         2025        -4828         6343
```

Next, let's examine the potential of the C variable. While visually, it's main effect appears larger than that of most other variables, it is still statistically insignificant when quantitatively analyzed. However, it has an interaction term with D that is very close to significance. By fitting a linear model to the formula suggested by our stepwise procedure, `y ~ D + G + D:G`, we can add terms to it and conduct an F-test on the nested values to evaluate if specific terms should or shouldn't be included. This procedure, conducted below, suggests that $C : D$ is a wortwhile addition to the model. However, it does not suggest including E and F despite our previous evidence.

```r
LinearModel.2 <- lm(resp.y ~ D * G, data=design)
LinearModel.3 <- lm(resp.y ~ D*G + C:D, data=design)
LinearModel.4 <- lm(resp.y ~ D*G + C:D + E + F, data=design)

anova(LinearModel.2, LinearModel.3)
```

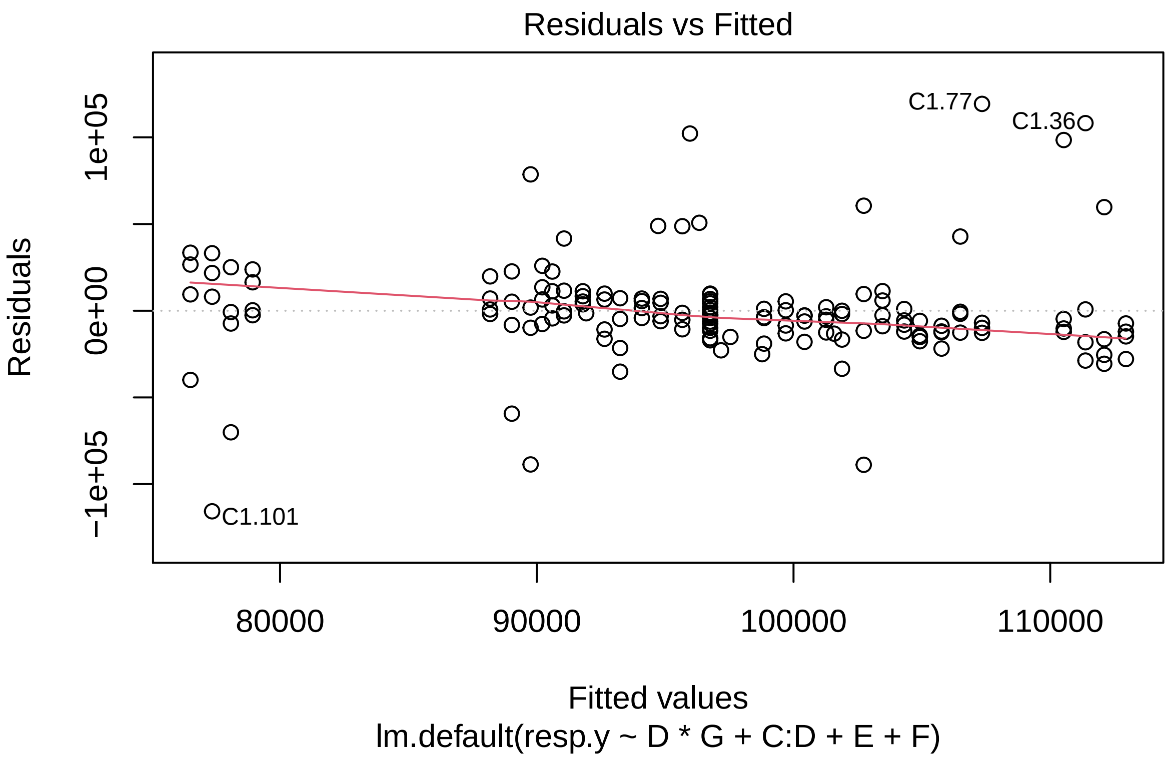
```
## Analysis of Variance Table
##
## Model 1: resp.y ~ D * G
## Model 2: resp.y ~ D * G + C:D
##   Res.Df        RSS Df Sum of Sq      F  Pr(>F)
## 1    151 1.3789e+11
## 2    150 1.3353e+11  1 4.361e+09 4.8989 0.02838 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

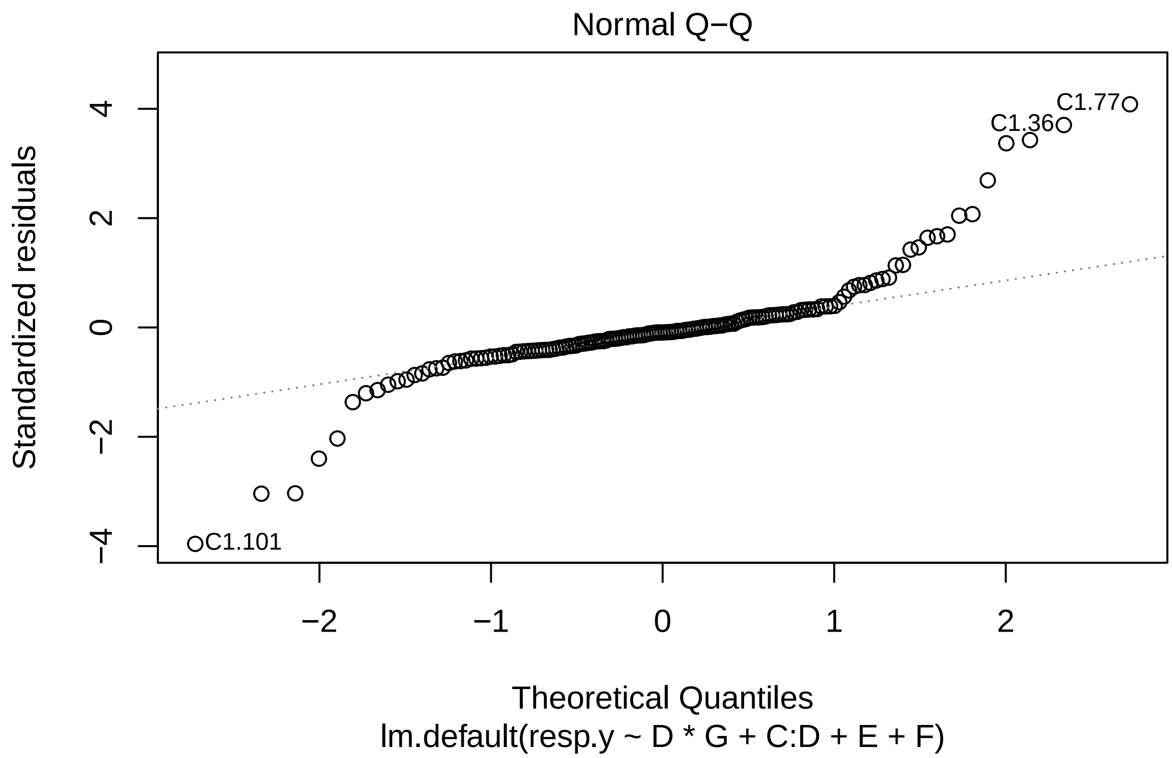
```r
anova(LinearModel.3, LinearModel.4)
```

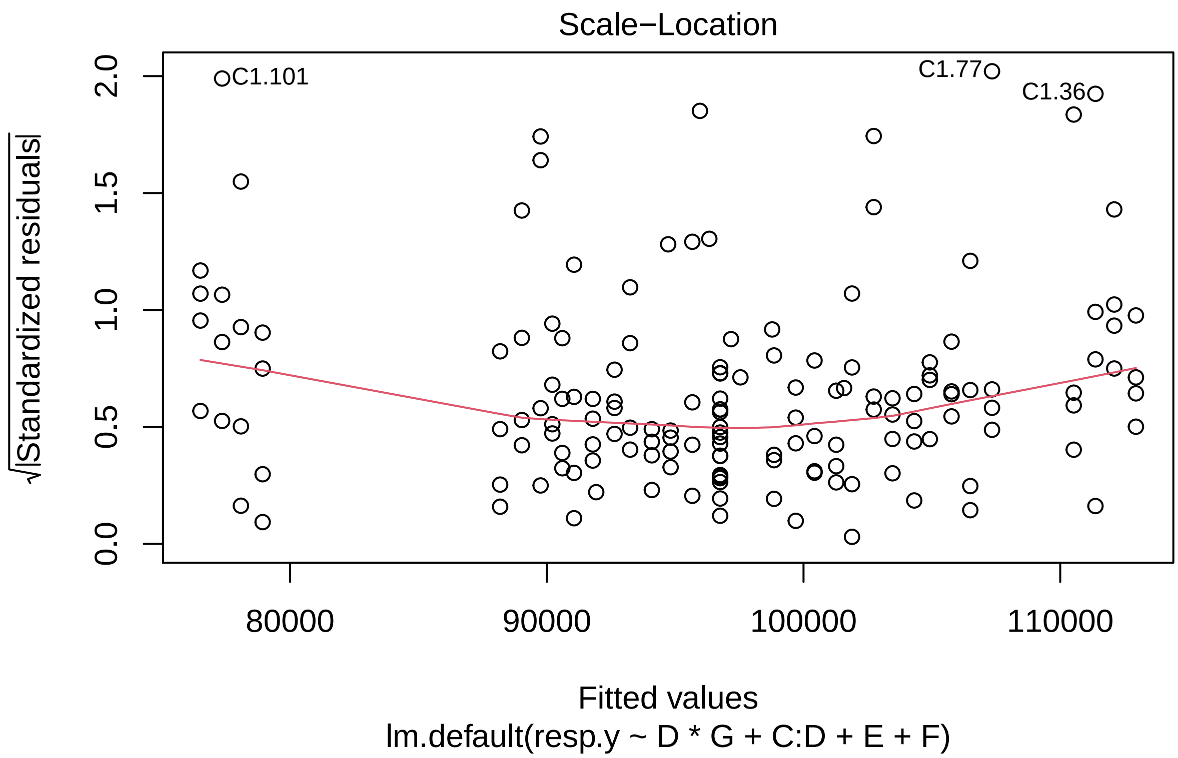
```
## Analysis of Variance Table
##
## Model 1: resp.y ~ D * G + C:D
## Model 2: resp.y ~ D * G + C:D + E + F
##   Res.Df        RSS Df Sum of Sq      F Pr(>F)
## 1    150 1.3353e+11
## 2    148 1.3343e+11  2 104089003 0.0577 0.9439
```

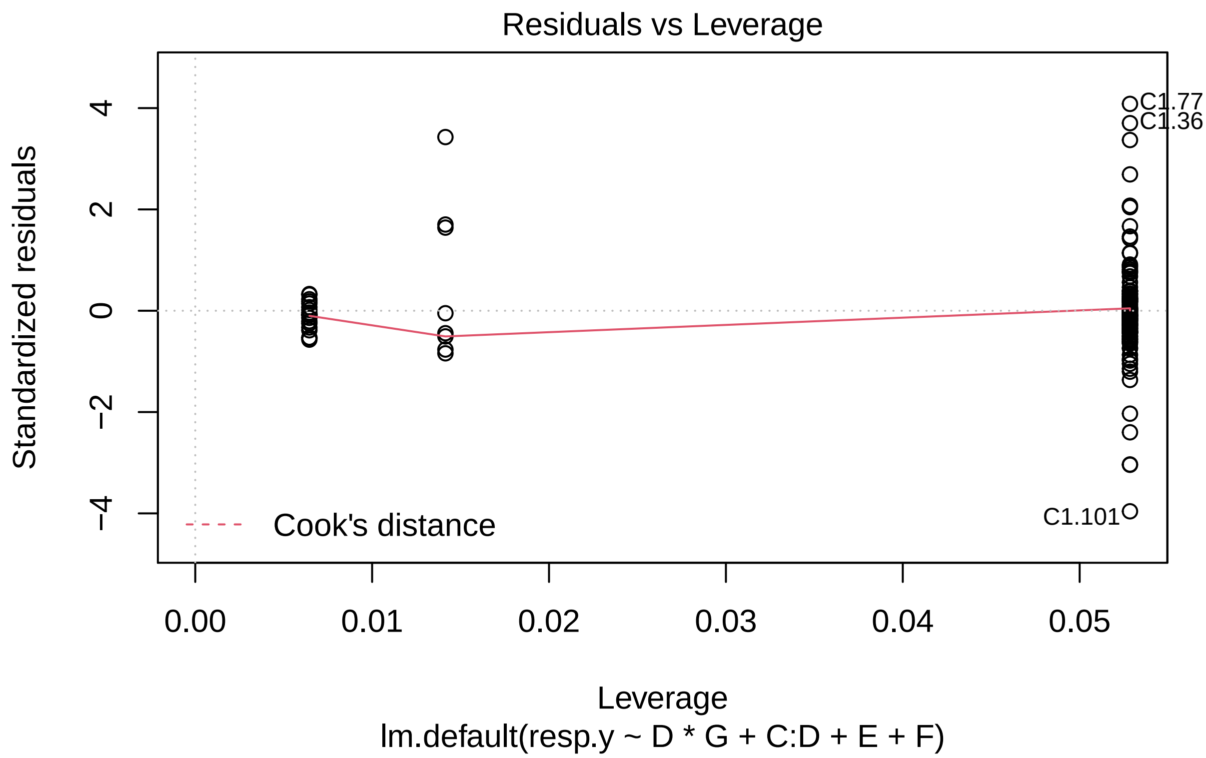
The metrics by which we would normally wish to evaluate our model suggest we are on perilous ground. However I will demonstrate later why chasing after our usual metrics can lead us into unsavory territory, especially given our design's priorities and limitations.

```r
#oldpar <- par(oma=c(0,0,3,0), mfrow=c(2,2))
plot(LinearModel.4)
```


Residuals vs Fitted
C1.77
C1.36
C1.101
1e+05
0e+00
−1e+05
Residuals
80000
90000
100000
110000
Fitted values
lm.default(resp.y ~ D * G + C:D + E + F)


Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm.default(resp.y ~ D * G + C:D + E + F)
C1.77
C1.36
C1.101
4
2
0
−2
−4
−2
−1
0
1
2

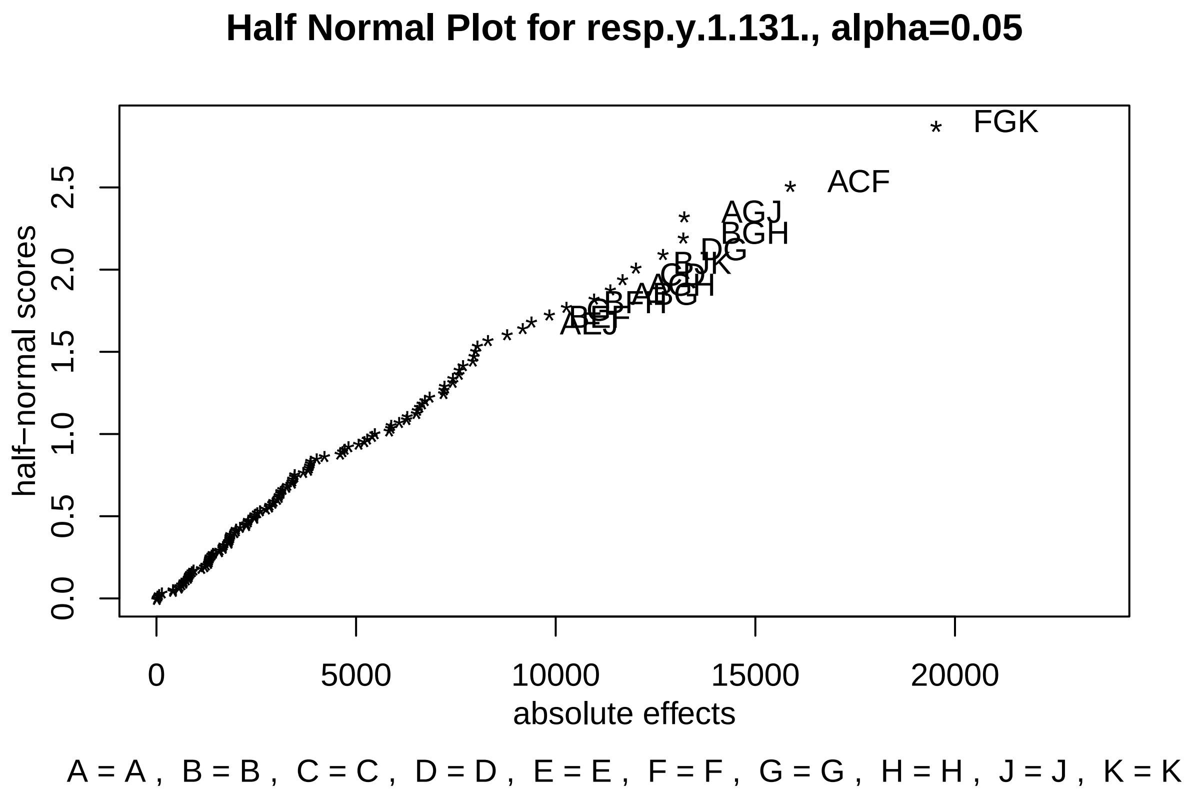

Scale−Location
C1.101
C1.77
C1.36
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
80000
90000
100000
110000
Fitted values
lm.default(resp.y ~ D * G + C:D + E + F)




```
#par(oldpar)
```

Next, I will examine the half-normal plot for potential three-factor interactions. By the resolution of my design, some three-factor interactions are aliased with some two-factor interactions. FGK, ACF, AGJ, BGH, and DG appear as significant outliers. DG is already present within our model. FGK includes both F and G which are also already present within our model. FGK aliases down to FG if K isn't present within the model. Similarly logic can be applied to ACF. AGJ and BGH are likely aliases of interactions of terms already included in our model. So, let us consider adding $F : C$ and $F : G$ to our model.

```
DanielPlot(design_2, code=TRUE, autolab=TRUE, alpha=0.05, half=TRUE,
           response="resp.y.1.131.")
```



Doing so and evaluating an F-test on the nested models shows that, although not definitively statistically significant, we may consider including them. However, the term *CDG* offers no improvement to our model.

```
LinearModel.5 <- lm(resp.y ~ D*G + C:D + E + F:(1 + C + G), data=design)
summary(LinearModel.5)
```

```
##
## Call:
## lm.default(formula = resp.y ~ D * G + C:D + E + F:(1 + C + G),
##     data = design)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -108474  -10830   -3136    7025  112094
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  96753.6     2382.9  40.604   <2e-16 ***
## D             2025.5     2601.9   0.778   0.4375
## G            -4828.1     2601.9  -1.856   0.0655 .
## E             -422.8     2601.9  -0.163   0.8711
## D:G           6343.4     2622.2   2.419   0.0168 *
## D:C           5837.0     2622.2   2.226   0.0275 *
## C:F          -3606.6     2622.2  -1.375   0.1711
## G:F          -4391.5     2622.2  -1.675   0.0961 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 29670 on 147 degrees of freedom
## Multiple R-squared:  0.1175, Adjusted R-squared:  0.07552
## F-statistic: 2.797 on 7 and 147 DF,  p-value: 0.009219
```

```
anova(LinearModel.3, LinearModel.5)
```

```
## Analysis of Variance Table
##
## Model 1: resp.y ~ D * G + C:D
## Model 2: resp.y ~ D * G + C:D + E + F:(1 + C + G)
##   Res.Df        RSS Df  Sum of Sq      F Pr(>F)
## 1    150 1.3353e+11
## 2    147 1.2937e+11  3 4156718477 1.5744 0.1981
```

```
#BJK alias CBDG -> CDG (assuming B isn't present)
LinearModel.6 <- lm(resp.y ~ D*G + C:D + E + F:(1 + C + G) + C:D:G,
                    data=design)
anova(LinearModel.5, LinearModel.6)
```

```
## Analysis of Variance Table
##
## Model 1: resp.y ~ D * G + C:D + E + F:(1 + C + G)
## Model 2: resp.y ~ D * G + C:D + E + F:(1 + C + G) + C:D:G
##   Res.Df        RSS Df Sum of Sq      F Pr(>F)
## 1    147 1.2937e+11
## 2    146 1.2927e+11  1 107968722 0.1219 0.7274
```

I would like to demonstrate the significance of the axial runs (despite their extra cost in the model). The below is an ANOVA table on the subset of runs that are exclusive to the two-factor fractional factorial portion of the utilized design. From this ANOVA table, we can find no evidence to suggest keeping E and F within our model despite significant evidence from the axial runs that they should both be factors within the model.

```
# Importance of axial runs (not center runs)
LinearModel.7 <- lm(resp.y.1.131. ~ (A + B + C + D + E + F + G + H + J +
                                     K)^2, data=design_2)
summary(aov(LinearModel.7))
```

```
##              Df    Sum Sq   Mean Sq F value Pr(>F)
## A             1 1.087e+09 1.087e+09   0.980 0.3253
## B             1 1.047e+08 1.047e+08   0.094 0.7595
## C             1 1.101e+09 1.101e+09   0.993 0.3223
## D             1 8.675e+08 8.675e+08   0.782 0.3792
## E             1 1.274e+08 1.274e+08   0.115 0.7356
## F             1 3.721e+08 3.721e+08   0.336 0.5641
## G             1 3.098e+09 3.098e+09   2.795 0.0988 .
## H             1 1.085e+08 1.085e+08   0.098 0.7552
## J             1 4.051e+07 4.051e+07   0.037 0.8489
## K             1 9.336e+08 9.336e+08   0.842 0.3618
## A:B           1 2.002e+07 2.002e+07   0.018 0.8935
## A:C           1 3.105e+08 3.105e+08   0.280 0.5983
## A:D           1 2.206e+09 2.206e+09   1.989 0.1625
## A:E           1 2.152e+08 2.152e+08   0.194 0.6608
## A:F           1 1.446e+05 1.446e+05   0.000 0.9909
## A:G           1 5.848e+05 5.848e+05   0.001 0.9817
## A:H           1 8.883e+07 8.883e+07   0.080 0.7779
```

```
## A:J           1 1.365e+09 1.365e+09   1.231 0.2707
## A:K           1 4.779e+08 4.779e+08   0.431 0.5135
## B:C           1 5.547e+06 5.547e+06   0.005 0.9438
## B:D           1 4.934e+07 4.934e+07   0.045 0.8335
## B:E           1 1.446e+09 1.446e+09   1.304 0.2570
## B:F           1 1.655e+09 1.655e+09   1.493 0.2256
## B:G           1 1.837e+09 1.837e+09   1.657 0.2020
## B:H           1 3.096e+08 3.096e+08   0.279 0.5988
## B:J           1 2.045e+09 2.045e+09   1.845 0.1785
## B:K           1 1.620e+08 1.620e+08   0.146 0.7033
## C:D           1 4.361e+09 4.361e+09   3.933 0.0510 .
## C:E           1 6.123e+07 6.123e+07   0.055 0.8149
## C:F           1 1.665e+09 1.665e+09   1.502 0.2243
## C:G           1 8.917e+08 8.917e+08   0.804 0.3727
## C:H           1 5.525e+07 5.525e+07   0.050 0.8240
## C:J           1 1.106e+09 1.106e+09   0.998 0.3211
## C:K           1 6.416e+07 6.416e+07   0.058 0.8106
## D:E           1 2.041e+08 2.041e+08   0.184 0.6692
## D:F           1 1.763e+09 1.763e+09   1.590 0.2113
## D:G           1 5.151e+09 5.151e+09   4.645 0.0343 *
## D:H           1 1.463e+07 1.463e+07   0.013 0.9089
## D:J           1 2.118e+07 2.118e+07   0.019 0.8904
## D:K           1 1.517e+07 1.517e+07   0.014 0.9072
## E:F           1 2.412e+08 2.412e+08   0.218 0.6422
## E:G           1 3.825e+08 3.825e+08   0.345 0.5587
## E:H           1 2.068e+09 2.068e+09   1.865 0.1761
## E:J           1 2.712e+08 2.712e+08   0.245 0.6224
## E:K           1 5.500e+06 5.500e+06   0.005 0.9440
## F:G           1 2.469e+09 2.469e+09   2.226 0.1399
## F:H           1 8.856e+07 8.856e+07   0.080 0.7782
## F:J           1 6.952e+08 6.952e+08   0.627 0.4310
## F:K           1 1.389e+09 1.389e+09   1.253 0.2666
## G:H           1 1.741e+07 1.741e+07   0.016 0.9006
## G:J           1 1.840e+09 1.840e+09   1.659 0.2017
## G:K           1 3.201e+08 3.201e+08   0.289 0.5926
## H:J           1 3.018e+08 3.018e+08   0.272 0.6034
## H:K           1 3.008e+08 3.008e+08   0.271 0.6040
## J:K           1 1.080e+08 1.080e+08   0.097 0.7559
## Residuals    75 8.316e+10 1.109e+09
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Lastly, I would like to demonstrate the dangers of chasing after metrics that are commonly used in **model selection** when running and experiment designed primarily for **factor screening**. Consider the below model `LinearModel.8`. It incorporates nine of the ten dependent variables and up to seven-factor interaction terms. However, it achieves a much higher correlation coefficient and an F-test between it and our working model `LinearModel.5` suggests the former to be superior. However, several issues present. Firstly, with such complex interaction terms, there is a significant risk that the true terms are hidden behind aliases. A seven-factor interaction term, for example, has aliases with lower-factor interaction terms on all levels. Ideally, we could work backwards from this model to sieve through the possible aliases and identify the most likely simplest aliases. However, I was unable to find software tools to help me accomplish this. This is one of the largest limitations of this project.

```r
# Ludicrous model, chasing R^2
LinearModel.8 <- lm(resp.y ~ G + C:D + D:G + E:H + B:J + G:B:J + C:G:E:H +
```

```
                      C:D:G:E:H + C:D:G:E:H:B:J, data=design)
summary(LinearModel.8)
```

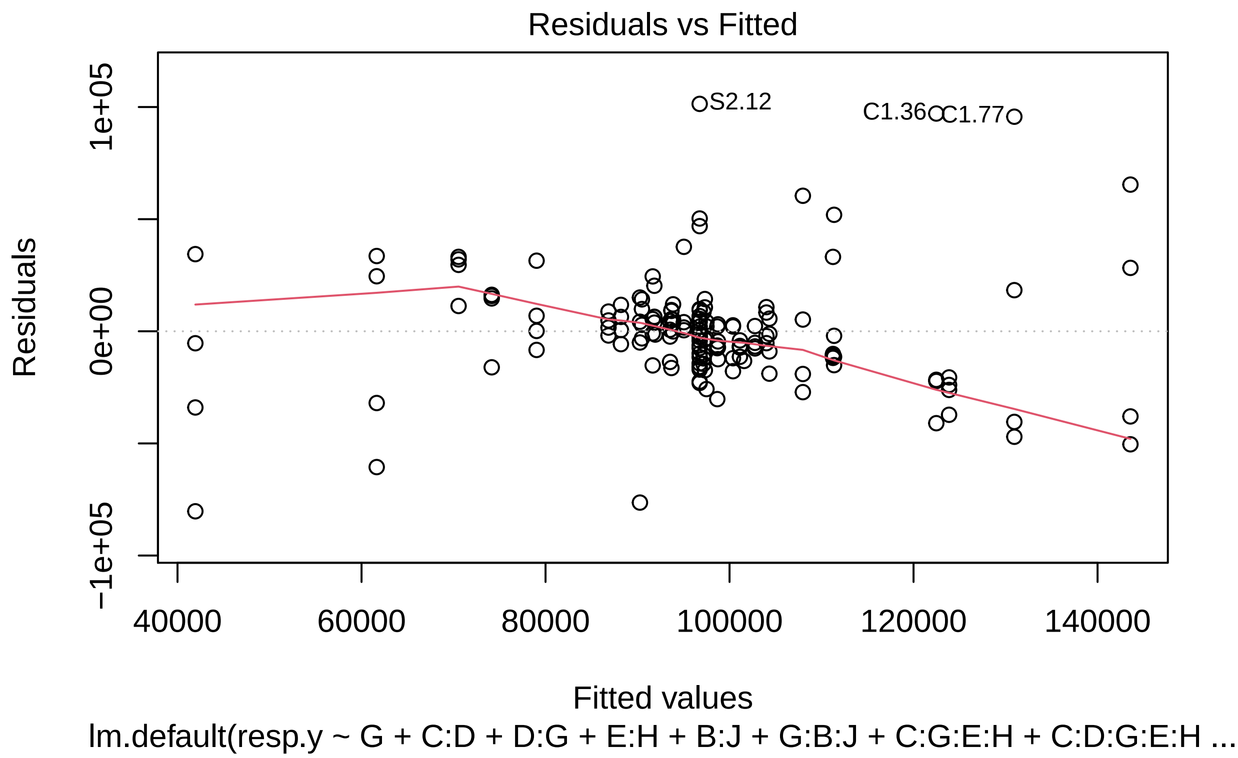
```
##
## Call:
## lm.default(formula = resp.y ~ G + C:D + D:G + E:H + B:J + G:B:J +
##     C:G:E:H + C:D:G:E:H + C:D:G:E:H:B:J, data = design)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -80267 -11932  -1152   8489 101404
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)       96754       2112  45.812  < 2e-16 ***
## G                 -4828       2306  -2.094 0.038034 *
## C:D                5837       2324   2.512 0.013116 *
## G:D                6343       2324   2.729 0.007130 **
## E:H                4019       2324   1.729 0.085860 .
## B:J                3997       2324   1.720 0.087570 .
## G:B:J              7938       2324   3.416 0.000826 ***
## G:C:E:H            9764       2324   4.201 4.62e-05 ***
## G:C:D:E:H         -5480       2324  -2.358 0.019710 *
## G:C:D:E:H:B:J      6610       2324   2.844 0.005099 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26290 on 145 degrees of freedom
## Multiple R-squared:  0.3162, Adjusted R-squared:  0.2738
## F-statistic:  7.45 on 9 and 145 DF,  p-value: 6.594e-09
```

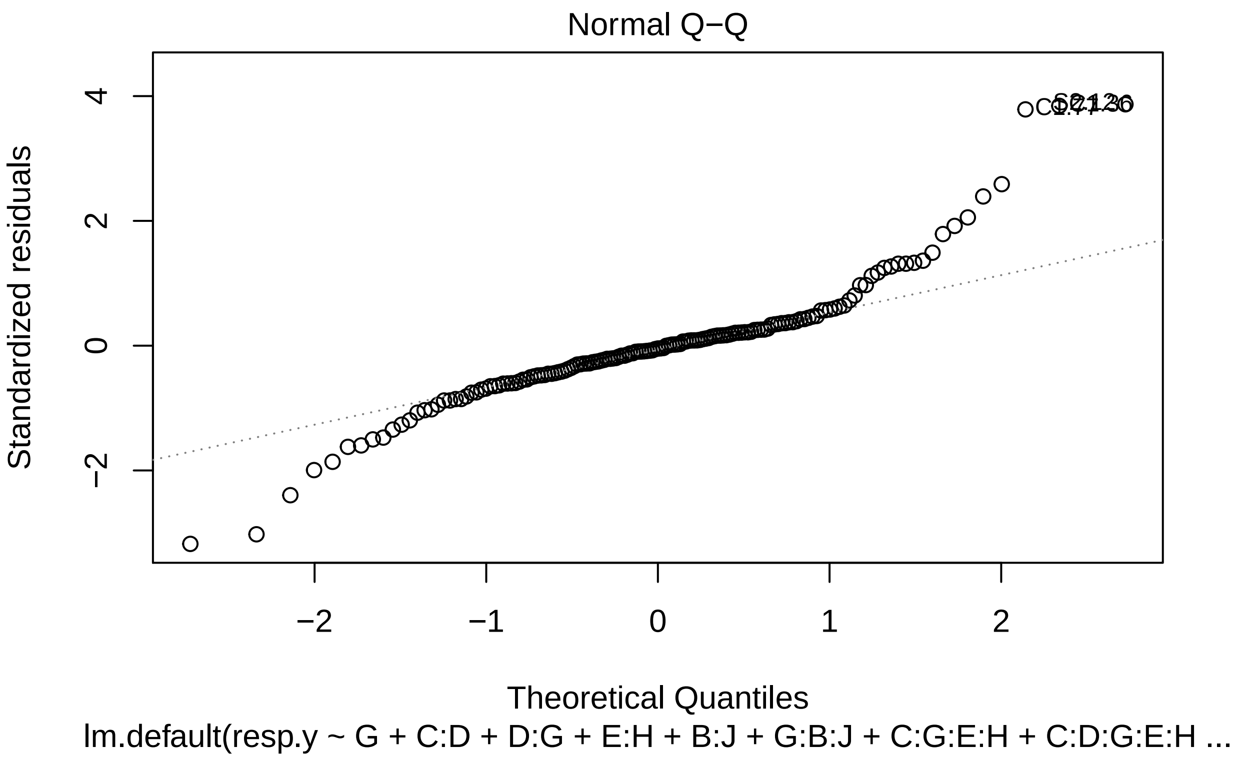
```
anova(LinearModel.5, LinearModel.8)
```

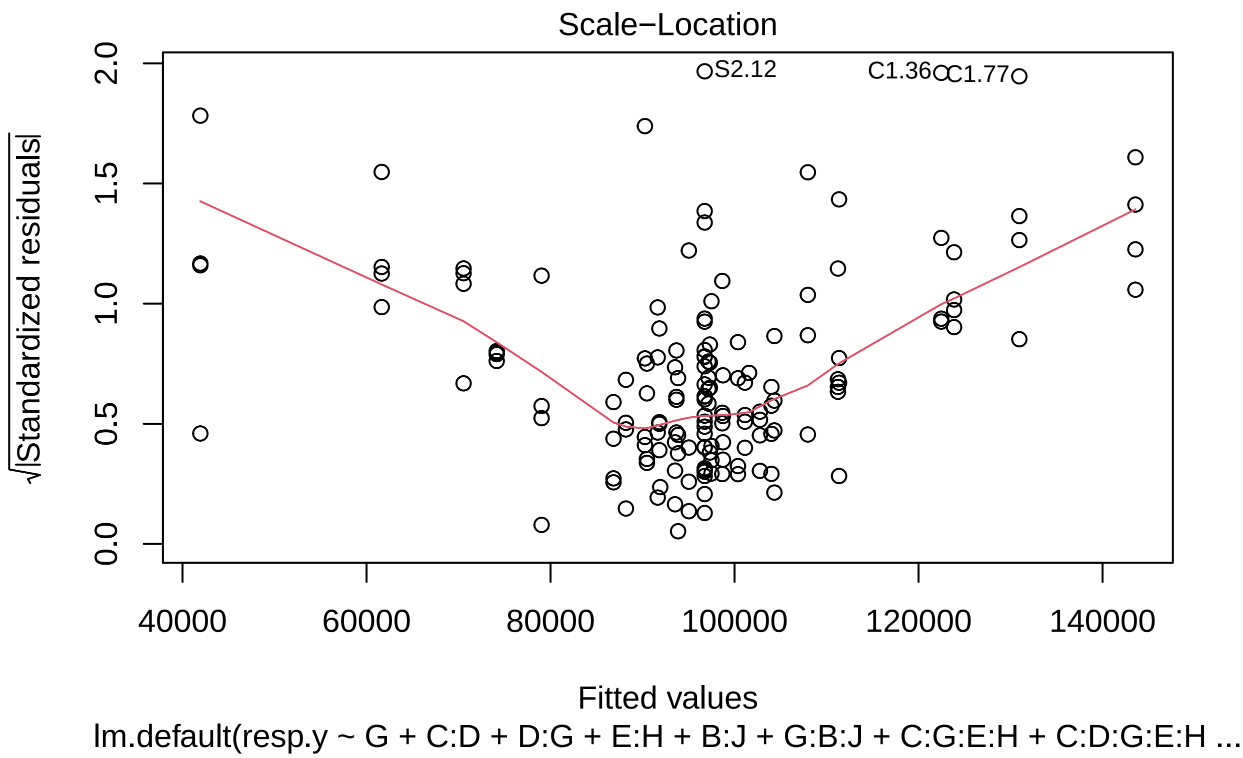
```
## Analysis of Variance Table
##
## Model 1: resp.y ~ D * G + C:D + E + F:(1 + C + G)
## Model 2: resp.y ~ G + C:D + D:G + E:H + B:J + G:B:J + C:G:E:H + C:D:G:E:H +
##     C:D:G:E:H:B:J
##   Res.Df        RSS Df  Sum of Sq      F    Pr(>F)
## 1    147 1.2937e+11
## 2    145 1.0025e+11  2 2.9126e+10 21.065 9.309e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

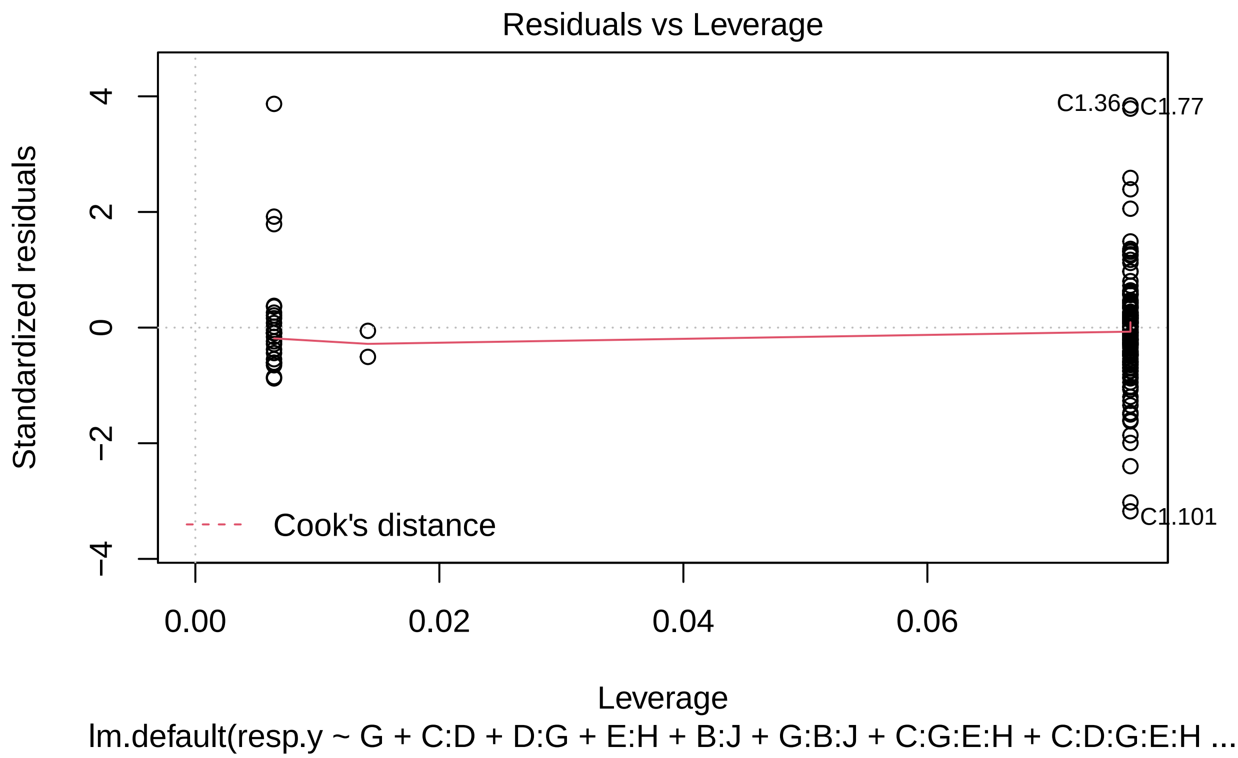
The risks posed by this sort of model selection also presents itself in diagnostic plots. Compare these plots to those of `LinearModel.5`. This model is possibly oversaturated and skewed.

```
plot(LinearModel.8)
```


Residuals vs Fitted
S2.12
C1.36
C1.77
Residuals
1e+05
0e+00
−1e+05
40000
60000
80000
100000
120000
140000
Fitted values
lm.default(resp.y ~ G + C:D + D:G + E:H + B:J + G:B:J + C:G:E:H + C:D:G:E:H ...


Normal Q−Q
Standardized residuals
4
2
0
−2
−2
−1
0
1
2
Theoretical Quantiles
lm.default(resp.y ~ G + C:D + D:G + E:H + B:J + G:B:J + C:G:E:H + C:D:G:E:H ...


Scale−Location
S2.12
C1.36
C1.77
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
40000
60000
80000
100000
120000
140000
Fitted values
lm.default(resp.y ~ G + C:D + D:G + E:H + B:J + G:B:J + C:G:E:H + C:D:G:E:H ...



4. Results

The results of the above analysis demonstrates that the C, D, E, F, G factors are likely present within the model. The remaining factors most likely aren't. The below table summarizes these results.

Variable	Model
A	Out
B	Out
C	In
D	In
E	In
F	In
G	In
H	Out
J (I)	Out
K (J)	Out

ANOVA tables of other considered models were shown in the **Analysis** section. The final model I propose is `Y ~ D + G + E + F + D:G + C:D + C:F + F:G`.

```
LinearModel.5
```

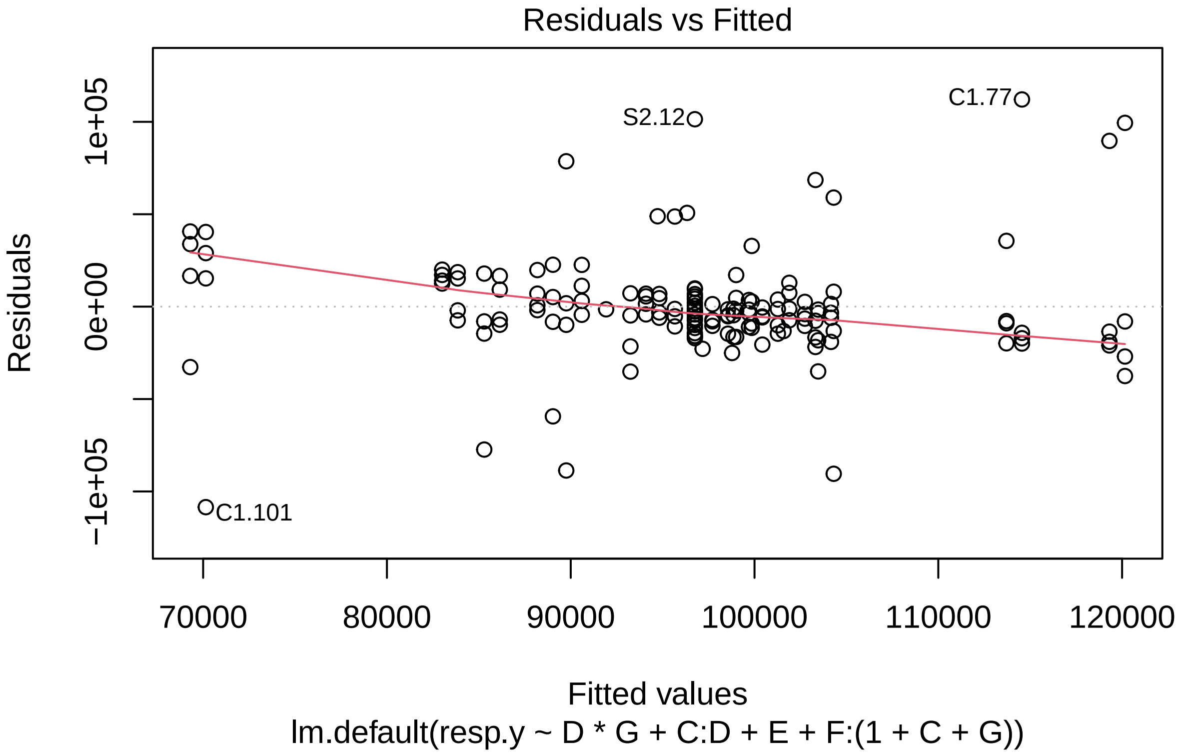
```
##
## Call:
## lm.default(formula = resp.y ~ D * G + C:D + E + F:(1 + C + G),
```

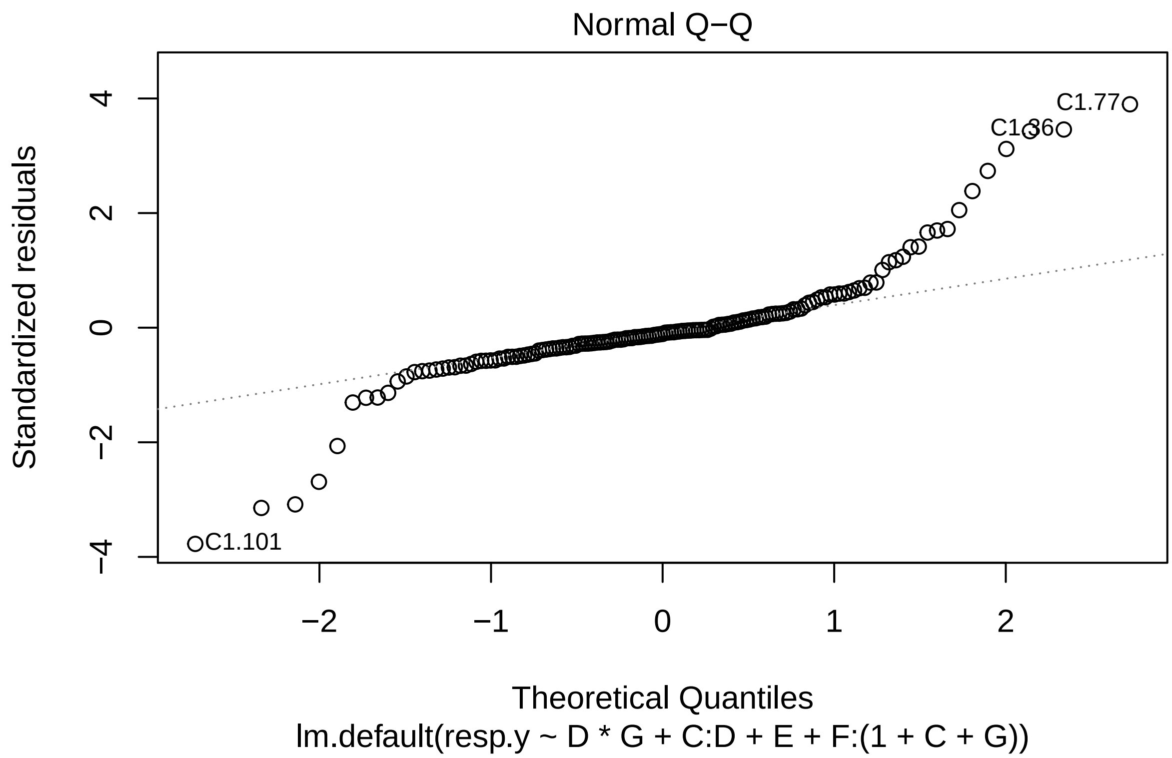
```
##     data = design)
##
## Coefficients:
## (Intercept)            D            G            E          D:G          D:C
##     96753.6       2025.5      -4828.1       -422.8       6343.4       5837.0
##         C:F          G:F
##     -3606.6      -4391.5
```

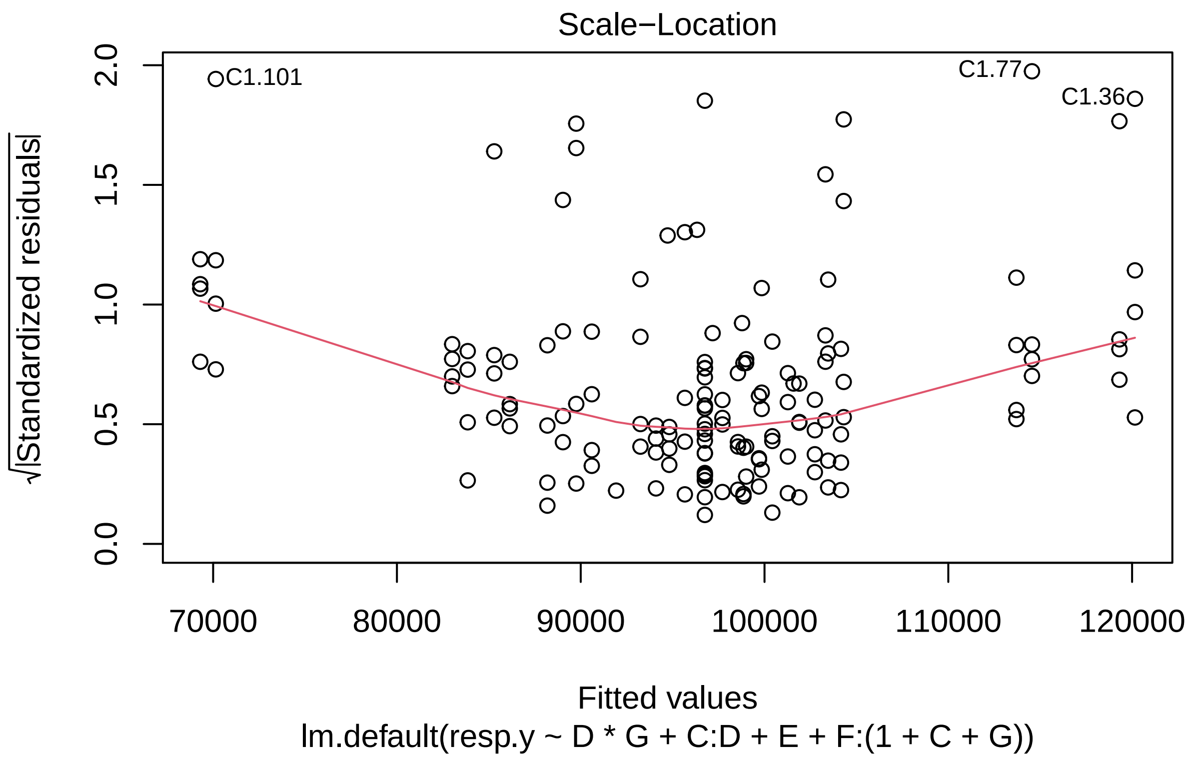
```r
summary(anova(LinearModel.5))
```

```
##        Df            Sum Sq             Mean Sq             F value
##  Min.   :  1.00   Min.   :2.324e+07   Min.   :2.324e+07   Min.   :0.02641
##  1st Qu.:  1.00   1st Qu.:1.382e+09   1st Qu.:7.934e+08   1st Qu.:1.24890
##  Median :  1.00   Median :2.749e+09   Median :2.067e+09   Median :2.80486
##  Mean   : 19.25   Mean   :1.833e+10   Mean   :2.264e+09   Mean   :2.79713
##  3rd Qu.:  1.00   3rd Qu.:4.558e+09   3rd Qu.:3.363e+09   3rd Qu.:4.19925
##  Max.   :147.00   Max.   :1.294e+11   Max.   :5.151e+09   Max.   :5.85234
##                                                           NA's   :1
##      Pr(>F)
##  Min.   :0.01678
##  1st Qu.:0.04652
##  Median :0.09611
##  Mean   :0.24081
##  3rd Qu.:0.30432
##  Max.   :0.87113
##  NA's   :1
```

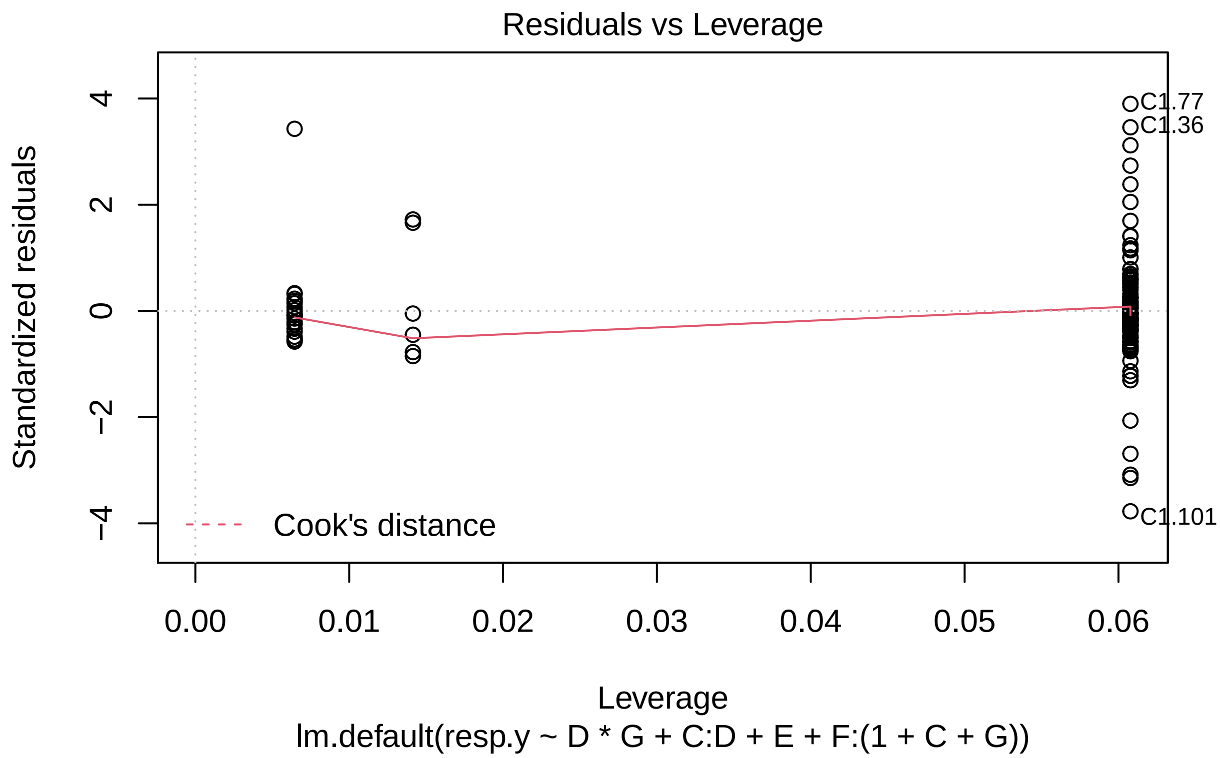
```r
plot(LinearModel.5)
```


Residuals vs Fitted
Residuals
1e+05
0e+00
−1e+05
S2.12
C1.77
C1.101
70000
80000
90000
100000
110000
120000
Fitted values
lm.default(resp.y ~ D * G + C:D + E + F:(1 + C + G))


Normal Q−Q
Standardized residuals
4
2
0
−2
−4
C1.77
C1.36
C1.101
−2
−1
0
1
2
Theoretical Quantiles
lm.default(resp.y ~ D * G + C:D + E + F:(1 + C + G))



Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
70000
80000
90000
100000
110000
120000
Fitted values
lm.default(resp.y ~ D * G + C:D + E + F:(1 + C + G))
C1.101
C1.77
C1.36



5. Conclusions and Discussion

This experiment had many limitations. Firstly, full modeling (as opposed to identifying presence) of factor effects is difficult. The design was made to be non-rotatable to minimize the cost of the experiment which reduced modeling accuracy. Identifying factor effect nature (linear, polynomial, exponential, etc.) was another limitation. Ideally, we'd like to identify transformations when developing a statistical model. **Dispersion effects** were also not analyzed. I believe they are outside scope of the problem, but dispersion effects are likely to occur and real life and must also be modeled. Improvements that can be made: **fold-over** design, **sequential experimentation**, and With lower costs a 3^k design may be considered. A **rotatable** central composite design may be useful, as well as finding / creating a SCD for 10 factors.

6. Appendix

6.1 Design Creation

```
design_centers <- design[c("C1.129","C1.130","C1.131"), 2:12]
design_centers <- rbind(design_centers, design[132:155, 2:12])

cs <- c("C1.129","C1.130","C1.131")

all_0 <- design_centers$A == 0 & design_centers$B == 0 &
  design_centers$C == 0 & design_centers$D == 0 &
  design_centers$E == 0 & design_centers$F == 0 &
  design_centers$G == 0 & design_centers$H == 0 &
  design_centers$J == 0 & design_centers$K == 0

As <- xor(design_centers$A != 0, all_0)
Bs <- xor(design_centers$B != 0, all_0)
Cs <- xor(design_centers$C != 0, all_0)
Ds <- xor(design_centers$D != 0, all_0)
Es <- xor(design_centers$E != 0, all_0)
Fs <- xor(design_centers$F != 0, all_0)
Gs <- xor(design_centers$G != 0, all_0)
Hs <- xor(design_centers$H != 0, all_0)
Js <- xor(design_centers$J != 0, all_0)
Ks <- xor(design_centers$K != 0, all_0)
```

6.2 Design

```
print(design)
```

```
##        Block.ccd  A  B  C  D  E  F  G  H  J  K     resp.y
## C1.129         1  0  0  0  0  0  0  0  0  0  0  94162.434
## C1.102         1  1 -1  1 -1 -1  1  1 -1 -1  1  85430.443
## C1.30          1  1 -1  1  1  1 -1 -1 -1 -1  1 114789.788
## C1.82          1  1 -1 -1 -1  1 -1  1 -1 -1 -1 168404.075
## C1.95          1 -1  1  1  1  1 -1  1 -1  1  1  93864.418
## C1.31          1 -1  1  1  1  1 -1 -1 -1 -1  1 100804.436
## C1.39          1 -1  1  1 -1 -1  1 -1 -1  1  1  91181.800
## C1.16          1  1  1  1  1 -1 -1 -1 -1  1 -1 105319.684
## C1.53          1 -1 -1  1 -1  1  1 -1 -1 -1 -1  95109.129
## C1.18          1  1 -1 -1 -1  1 -1 -1 -1  1 -1  95674.325
## C1.88          1  1  1  1 -1  1 -1  1 -1 -1  1   7964.088
## C1.15          1 -1  1  1  1 -1 -1 -1  1 -1  1  92309.467
## C1.75          1 -1  1 -1  1 -1 -1  1 -1 -1  1  84964.605
## C1.50          1  1 -1 -1 -1  1  1 -1 -1 -1  1 105787.870
## C1.59          1 -1  1 -1  1  1  1 -1  1 -1 -1  96727.560
## C1.103         1 -1  1  1 -1 -1  1  1 -1 -1  1  99113.416
## C1.64          1  1  1  1  1  1  1 -1  1 -1  1  99986.205
## C1.60          1  1  1 -1  1  1  1 -1 -1  1  1 103747.233
## C1.111         1 -1  1  1  1 -1  1  1  1 -1 -1  93352.191
## C1.12          1  1  1 -1  1 -1 -1 -1  1 -1 -1  76430.851
## C1.17          1 -1 -1 -1 -1  1 -1 -1  1 -1  1  81494.616
## C1.77          1 -1 -1  1  1 -1 -1  1 -1 -1 -1 226646.146
## C1.47          1 -1  1  1  1 -1  1 -1  1  1 -1 112367.609
```

```
## C1.104           1  1  1  1 -1 -1  1  1  1  1 -1 110516.131
## C1.76            1  1  1 -1  1 -1 -1  1  1  1 -1 144420.676
## C1.113           1 -1 -1 -1 -1  1  1  1  1 -1 -1  86303.755
## C1.79            1 -1  1  1  1 -1 -1  1  1  1  1  94593.098
## C1.14            1  1 -1  1  1 -1 -1 -1  1 -1  1  98709.237
## C1.19            1 -1  1 -1 -1  1 -1 -1 -1  1 -1  86662.415
## C1.128           1  1  1  1  1  1  1  1  1  1  1  90553.555
## C1.8             1  1  1  1 -1 -1 -1 -1  1  1  1  96021.619
## C1.108           1  1  1 -1  1 -1  1  1  1 -1  1  89902.874
## C1.125           1 -1 -1  1  1  1  1  1  1  1  1  99059.651
## C1.57            1 -1 -1 -1  1  1  1 -1 -1  1  1 116120.847
## C1.112           1  1  1  1  1 -1  1  1 -1  1  1  83919.177
## C1.34            1  1 -1 -1 -1 -1  1 -1  1 -1  1  82615.131
## C1.44            1  1  1 -1  1 -1  1 -1  1  1  1 102608.606
## C1.56            1  1  1  1 -1  1  1 -1 -1 -1 -1  99934.242
## C1.100           1  1  1 -1 -1 -1  1  1 -1 -1 -1  29662.972
## C1.43            1 -1  1 -1  1 -1  1 -1 -1 -1 -1 132699.236
## C1.7             1 -1  1  1 -1 -1 -1 -1 -1 -1 -1  88741.567
## C1.27            1 -1  1 -1  1  1 -1 -1  1  1  1 103021.241
## C1.58            1  1 -1 -1  1  1  1 -1  1 -1 -1  82569.428
## C1.55            1 -1  1  1 -1  1  1 -1  1  1  1  79884.555
## C1.40            1  1  1  1 -1 -1  1 -1  1 -1 -1  86638.669
## C1.6             1  1 -1  1 -1 -1 -1 -1 -1 -1 -1  98046.813
## C1.46            1  1 -1  1  1 -1  1 -1  1  1 -1  91143.263
## C1.68            1  1  1 -1 -1 -1 -1  1 -1  1  1 101842.398
## C1.126           1  1 -1  1  1  1  1  1 -1 -1 -1  89751.308
## C1.51            1 -1  1 -1 -1  1  1 -1 -1 -1  1  98313.993
## C1.26            1  1 -1 -1  1  1 -1 -1  1  1  1  95502.066
## C1.116           1  1  1 -1 -1  1  1  1  1 -1 -1  95214.509
## C1.45            1 -1 -1  1  1 -1  1 -1 -1 -1  1  13873.347
## C1.120           1  1  1  1 -1  1  1  1 -1  1 -1  85945.034
## C1.35            1 -1  1 -1 -1 -1  1 -1  1 -1  1 112136.092
## C1.87            1 -1  1  1 -1  1 -1  1  1  1 -1 103181.283
## C1.3             1 -1  1 -1 -1 -1 -1 -1  1  1 -1  85075.300
## C1.54            1  1 -1  1 -1  1  1 -1  1  1  1  94614.184
## C1.127           1 -1  1  1  1  1  1  1 -1 -1 -1  87317.877
## C1.99            1 -1  1 -1 -1 -1  1  1  1  1  1  80831.781
## C1.23            1 -1  1  1 -1  1 -1 -1  1 -1 -1  82462.249
## C1.114           1  1 -1 -1 -1  1  1  1 -1  1  1 107984.822
## C1.110           1  1 -1  1  1 -1  1  1  1 -1 -1  97085.162
## C1.1             1 -1 -1 -1 -1 -1 -1 -1 -1 -1  1 100846.001
## C1.80            1  1  1  1  1 -1 -1  1 -1 -1 -1  97449.680
## C1.130           1  0  0  0  0  0  0  0  0  0  0 103535.347
## C1.124           1  1  1 -1  1  1  1  1 -1 -1  1  58087.345
## C1.109           1 -1 -1  1  1 -1  1  1 -1  1  1  93794.126
## C1.86            1  1 -1  1 -1  1 -1  1  1  1 -1  77307.856
## C1.24            1  1  1  1 -1  1 -1 -1 -1  1  1  97606.967
## C1.11            1 -1  1 -1  1 -1 -1 -1 -1  1  1 102499.323
## C1.107           1 -1  1 -1  1 -1  1  1 -1  1 -1  99658.902
## C1.115           1 -1  1 -1 -1  1  1  1 -1  1  1  88919.933
## C1.33            1 -1 -1 -1 -1 -1  1 -1 -1  1 -1  93185.520
## C1.97            1 -1 -1 -1 -1 -1  1  1 -1 -1 -1  94222.527
## C1.96            1  1  1  1  1  1 -1  1  1 -1 -1 105889.966
## C1.42            1  1 -1 -1  1 -1  1 -1 -1 -1 -1  88367.429
```

```
## C1.48           1  1  1  1  1 -1  1 -1 -1 -1  1 163305.110
## C1.13           1 -1 -1  1  1 -1 -1 -1 -1  1 -1  96258.288
## C1.118          1  1 -1  1 -1  1  1  1  1 -1  1 109966.571
## C1.41           1 -1 -1 -1  1 -1  1 -1  1  1  1  90709.220
## C1.78           1  1 -1  1  1 -1 -1  1  1  1  1 100392.014
## C1.9            1 -1 -1 -1  1 -1 -1 -1  1 -1 -1  81829.286
## C1.72           1  1  1  1 -1 -1 -1  1  1 -1  1  76338.298
## C1.70           1  1 -1  1 -1 -1 -1  1 -1  1 -1 102775.417
## C1.106          1  1 -1 -1  1 -1  1  1 -1  1 -1  95640.559
## C1.21           1 -1 -1  1 -1  1 -1 -1 -1  1  1  97722.043
## C1.65           1 -1 -1 -1 -1 -1 -1  1 -1  1  1  86187.382
## C1.94           1  1 -1  1  1  1 -1  1 -1  1  1 104711.034
## C1.69           1 -1 -1  1 -1 -1 -1  1  1 -1  1  79184.857
## C1.61           1 -1 -1  1  1  1  1 -1  1 -1  1  85257.873
## C1.71           1 -1  1  1 -1 -1 -1  1 -1  1 -1  95347.517
## C1.66           1  1 -1 -1 -1 -1 -1  1  1 -1 -1 113205.611
## C1.117          1 -1 -1  1 -1  1  1  1 -1  1 -1  36601.612
## C1.122          1  1 -1 -1  1  1  1  1  1  1 -1  88500.360
## C1.36           1  1  1 -1 -1 -1  1 -1 -1  1 -1 219584.156
## C1.101          1 -1 -1  1 -1 -1  1  1  1  1 -1 -38330.113
## C1.28           1  1  1 -1  1  1 -1 -1 -1 -1 -1  97079.587
## C1.22           1  1 -1  1 -1  1 -1 -1  1 -1 -1  94218.992
## C1.20           1  1  1 -1 -1  1 -1 -1  1 -1  1 171852.401
## C1.90           1  1 -1 -1  1  1 -1  1  1 -1  1 101682.224
## C1.93           1 -1 -1  1  1  1 -1  1  1 -1 -1 149288.073
## C1.81           1 -1 -1 -1 -1  1 -1  1  1  1  1   1099.033
## C1.91           1 -1  1 -1  1  1 -1  1  1 -1  1  91683.539
## C1.29           1 -1 -1  1  1  1 -1 -1  1  1 -1 109344.586
## C1.123          1 -1  1 -1  1  1  1  1  1  1 -1  71726.271
## C1.119          1 -1  1  1 -1  1  1  1  1 -1  1 103145.467
## C1.63           1 -1  1  1  1  1  1 -1 -1  1 -1 101862.642
## C1.121          1 -1 -1 -1  1  1  1  1 -1 -1  1 100456.861
## C1.2            1  1 -1 -1 -1 -1 -1 -1  1  1 -1  98139.545
## C1.25           1 -1 -1 -1  1  1 -1 -1 -1 -1 -1 100165.042
## C1.84           1  1  1 -1 -1  1 -1  1  1  1  1  91582.757
## C1.62           1  1 -1  1  1  1  1 -1 -1  1 -1  68414.445
## C1.10           1  1 -1 -1  1 -1 -1 -1 -1  1  1  99097.288
## C1.37           1 -1 -1  1 -1 -1  1 -1  1 -1 -1 105099.346
## C1.52           1  1  1 -1 -1  1  1 -1  1  1 -1 100279.993
## C1.89           1 -1 -1 -1  1  1 -1  1 -1  1 -1  88796.367
## C1.98           1  1 -1 -1 -1 -1  1  1  1  1  1 111703.672
## C1.74           1  1 -1 -1  1 -1 -1  1 -1 -1  1  94434.997
## C1.85           1 -1 -1  1 -1  1 -1  1 -1 -1  1  70707.961
## C1.92           1  1  1 -1  1  1 -1  1 -1  1 -1  99382.176
## C1.105          1 -1 -1 -1  1 -1  1  1  1 -1  1 101115.650
## C1.67           1 -1  1 -1 -1 -1 -1  1  1 -1 -1  93660.576
## C1.32           1  1  1  1  1  1 -1 -1  1  1 -1  94525.042
## C1.38           1  1 -1  1 -1 -1  1 -1 -1  1  1  99980.520
## C1.83           1 -1  1 -1 -1  1 -1  1 -1 -1 -1  79924.220
## C1.5            1 -1 -1  1 -1 -1 -1 -1  1  1  1 103294.958
## C1.49           1 -1 -1 -1 -1  1  1 -1  1  1 -1 208988.648
## C1.4            1  1  1 -1 -1 -1 -1 -1 -1 -1  1 105614.053
## C1.73           1 -1 -1 -1  1 -1 -1  1  1  1 -1  90420.589
## C1.131          1  0  0  0  0  0  0  0  0  0  0  80801.145
```

```
## S2.14          2  0  0  0  0  0  0  1  0  0  0  90467.237
## S2.2           2  1  0  0  0  0  0  0  0  0  0  80866.497
## S2.23          2  0  0  0  0  0  0  0  0  0  0 101005.709
## S2.16          2  0  0  0  0  0  0  0  1  0  0  98846.582
## S2.12          2  0  0  0  0  0  1  0  0  0  0 198157.287
## S2.10          2  0  0  0  0  1  0  0  0  0  0 147050.845
## S2.7           2  0  0  0 -1  0  0  0  0  0  0 143636.681
## S2.19          2  0  0  0  0  0  0  0  0  0 -1 102259.860
## S2.24          2  0  0  0  0  0  0  0  0  0  0  79687.058
## S2.17          2  0  0  0  0  0  0  0  0 -1  0  95628.330
## S2.21          2  0  0  0  0  0  0  0  0  0  0 106239.812
## S2.3           2  0 -1  0  0  0  0  0  0  0  0 106621.398
## S2.4           2  0  1  0  0  0  0  0  0  0  0  94279.241
## S2.13          2  0  0  0  0  0  0 -1  0  0  0  88357.682
## S2.9           2  0  0  0  0 -1  0  0  0  0  0  74337.010
## S2.22          2  0  0  0  0  0  0  0  0  0  0  90513.420
## S2.20          2  0  0  0  0  0  0  0  0  0  1  89284.997
## S2.18          2  0  0  0  0  0  0  0  0  1  0  85209.962
## S2.15          2  0  0  0  0  0  0  0 -1  0  0  94391.556
## S2.8           2  0  0  0  1  0  0  0  0  0  0  73716.290
## S2.1           2 -1  0  0  0  0  0  0  0  0  0  92533.426
## S2.5           2  0  0 -1  0  0  0  0  0  0  0  97186.295
## S2.6           2  0  0  1  0  0  0  0  0  0  0  86852.827
## S2.11          2  0  0  0  0  0 -1  0  0  0  0  82431.671
## class=design, type= ccd
```

```
attributes(design)
```

```
## $row.names
##   [1] "C1.129" "C1.102" "C1.30"  "C1.82"  "C1.95"  "C1.31"  "C1.39"  "C1.16"
##   [9] "C1.53"  "C1.18"  "C1.88"  "C1.15"  "C1.75"  "C1.50"  "C1.59"  "C1.103"
##  [17] "C1.64"  "C1.60"  "C1.111" "C1.12"  "C1.17"  "C1.77"  "C1.47"  "C1.104"
##  [25] "C1.76"  "C1.113" "C1.79"  "C1.14"  "C1.19"  "C1.128" "C1.8"   "C1.108"
##  [33] "C1.125" "C1.57"  "C1.112" "C1.34"  "C1.44"  "C1.56"  "C1.100" "C1.43"
##  [41] "C1.7"   "C1.27"  "C1.58"  "C1.55"  "C1.40"  "C1.6"   "C1.46"  "C1.68"
##  [49] "C1.126" "C1.51"  "C1.26"  "C1.116" "C1.45"  "C1.120" "C1.35"  "C1.87"
##  [57] "C1.3"   "C1.54"  "C1.127" "C1.99"  "C1.23"  "C1.114" "C1.110" "C1.1"
##  [65] "C1.80"  "C1.130" "C1.124" "C1.109" "C1.86"  "C1.24"  "C1.11"  "C1.107"
##  [73] "C1.115" "C1.33"  "C1.97"  "C1.96"  "C1.42"  "C1.48"  "C1.13"  "C1.118"
##  [81] "C1.41"  "C1.78"  "C1.9"   "C1.72"  "C1.70"  "C1.106" "C1.21"  "C1.65"
##  [89] "C1.94"  "C1.69"  "C1.61"  "C1.71"  "C1.66"  "C1.117" "C1.122" "C1.36"
##  [97] "C1.101" "C1.28"  "C1.22"  "C1.20"  "C1.90"  "C1.93"  "C1.81"  "C1.91"
## [105] "C1.29"  "C1.123" "C1.119" "C1.63"  "C1.121" "C1.2"   "C1.25"  "C1.84"
## [113] "C1.62"  "C1.10"  "C1.37"  "C1.52"  "C1.89"  "C1.98"  "C1.74"  "C1.85"
## [121] "C1.92"  "C1.105" "C1.67"  "C1.32"  "C1.38"  "C1.83"  "C1.5"   "C1.49"
## [129] "C1.4"   "C1.73"  "C1.131" "S2.14"  "S2.2"   "S2.23"  "S2.16"  "S2.12"
## [137] "S2.10"  "S2.7"   "S2.19"  "S2.24"  "S2.17"  "S2.21"  "S2.3"   "S2.4"
## [145] "S2.13"  "S2.9"   "S2.22"  "S2.20"  "S2.18"  "S2.15"  "S2.8"   "S2.1"
## [153] "S2.5"   "S2.6"   "S2.11"
##
## $desnum
##        Block.ccd         A         B         C         D         E         F
## C1.129         0  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## C1.102         0  1.000000 -1.000000  1.000000 -1.000000 -1.000000  1.000000
## C1.30          0  1.000000 -1.000000  1.000000  1.000000  1.000000 -1.000000
```

```
## C1.82          0  1.000000 -1.000000 -1.000000 -1.000000  1.000000 -1.000000
## C1.95          0 -1.000000  1.000000  1.000000  1.000000  1.000000 -1.000000
## C1.31          0 -1.000000  1.000000  1.000000  1.000000  1.000000 -1.000000
## C1.39          0 -1.000000  1.000000  1.000000 -1.000000 -1.000000  1.000000
## C1.16          0  1.000000  1.000000  1.000000  1.000000 -1.000000 -1.000000
## C1.53          0 -1.000000 -1.000000  1.000000 -1.000000  1.000000  1.000000
## C1.18          0  1.000000 -1.000000 -1.000000 -1.000000  1.000000 -1.000000
## C1.88          0  1.000000  1.000000  1.000000 -1.000000  1.000000 -1.000000
## C1.15          0 -1.000000  1.000000  1.000000  1.000000 -1.000000 -1.000000
## C1.75          0 -1.000000  1.000000 -1.000000  1.000000 -1.000000 -1.000000
## C1.50          0  1.000000 -1.000000 -1.000000 -1.000000  1.000000  1.000000
## C1.59          0 -1.000000  1.000000 -1.000000  1.000000  1.000000  1.000000
## C1.103         0 -1.000000  1.000000  1.000000 -1.000000 -1.000000  1.000000
## C1.64          0  1.000000  1.000000  1.000000  1.000000  1.000000  1.000000
## C1.60          0  1.000000  1.000000 -1.000000  1.000000  1.000000  1.000000
## C1.111         0 -1.000000  1.000000  1.000000  1.000000 -1.000000  1.000000
## C1.12          0  1.000000  1.000000 -1.000000  1.000000 -1.000000 -1.000000
## C1.17          0 -1.000000 -1.000000 -1.000000 -1.000000  1.000000 -1.000000
## C1.77          0 -1.000000 -1.000000  1.000000  1.000000 -1.000000 -1.000000
## C1.47          0 -1.000000  1.000000  1.000000  1.000000 -1.000000  1.000000
## C1.104         0  1.000000  1.000000  1.000000 -1.000000 -1.000000  1.000000
## C1.76          0  1.000000  1.000000 -1.000000  1.000000 -1.000000 -1.000000
## C1.113         0 -1.000000 -1.000000 -1.000000 -1.000000  1.000000  1.000000
## C1.79          0 -1.000000  1.000000  1.000000  1.000000 -1.000000 -1.000000
## C1.14          0  1.000000 -1.000000  1.000000  1.000000 -1.000000 -1.000000
## C1.19          0 -1.000000  1.000000 -1.000000 -1.000000  1.000000 -1.000000
## C1.128         0  1.000000  1.000000  1.000000  1.000000  1.000000  1.000000
## C1.8           0  1.000000  1.000000  1.000000 -1.000000 -1.000000 -1.000000
## C1.108         0  1.000000  1.000000 -1.000000  1.000000 -1.000000  1.000000
## C1.125         0 -1.000000 -1.000000  1.000000  1.000000  1.000000  1.000000
## C1.57          0 -1.000000 -1.000000 -1.000000  1.000000  1.000000  1.000000
## C1.112         0  1.000000  1.000000  1.000000  1.000000 -1.000000  1.000000
## C1.34          0  1.000000 -1.000000 -1.000000 -1.000000 -1.000000  1.000000
## C1.44          0  1.000000  1.000000 -1.000000  1.000000 -1.000000  1.000000
## C1.56          0  1.000000  1.000000  1.000000 -1.000000  1.000000  1.000000
## C1.100         0  1.000000  1.000000 -1.000000 -1.000000 -1.000000  1.000000
## C1.43          0 -1.000000  1.000000 -1.000000  1.000000 -1.000000  1.000000
## C1.7           0 -1.000000  1.000000  1.000000 -1.000000 -1.000000 -1.000000
## C1.27          0 -1.000000  1.000000 -1.000000  1.000000  1.000000 -1.000000
## C1.58          0  1.000000 -1.000000 -1.000000  1.000000  1.000000  1.000000
## C1.55          0 -1.000000  1.000000  1.000000 -1.000000  1.000000  1.000000
## C1.40          0  1.000000  1.000000  1.000000 -1.000000 -1.000000  1.000000
## C1.6           0  1.000000 -1.000000  1.000000 -1.000000 -1.000000 -1.000000
## C1.46          0  1.000000 -1.000000  1.000000  1.000000 -1.000000  1.000000
## C1.68          0  1.000000  1.000000 -1.000000 -1.000000 -1.000000 -1.000000
## C1.126         0  1.000000 -1.000000  1.000000  1.000000  1.000000  1.000000
## C1.51          0 -1.000000  1.000000 -1.000000 -1.000000  1.000000  1.000000
## C1.26          0  1.000000 -1.000000 -1.000000  1.000000  1.000000 -1.000000
## C1.116         0  1.000000  1.000000 -1.000000 -1.000000  1.000000  1.000000
## C1.45          0 -1.000000 -1.000000  1.000000  1.000000 -1.000000  1.000000
## C1.120         0  1.000000  1.000000  1.000000 -1.000000  1.000000  1.000000
## C1.35          0 -1.000000  1.000000 -1.000000 -1.000000 -1.000000  1.000000
## C1.87          0 -1.000000  1.000000  1.000000 -1.000000  1.000000 -1.000000
## C1.3           0 -1.000000  1.000000 -1.000000 -1.000000 -1.000000 -1.000000
```

```
## C1.54          0  1.000000 -1.000000  1.000000 -1.000000  1.000000  1.000000
## C1.127         0 -1.000000  1.000000  1.000000  1.000000  1.000000  1.000000
## C1.99          0 -1.000000  1.000000 -1.000000 -1.000000 -1.000000  1.000000
## C1.23          0 -1.000000  1.000000  1.000000 -1.000000  1.000000 -1.000000
## C1.114         0  1.000000 -1.000000 -1.000000 -1.000000  1.000000  1.000000
## C1.110         0  1.000000 -1.000000  1.000000  1.000000 -1.000000  1.000000
## C1.1           0 -1.000000 -1.000000 -1.000000 -1.000000 -1.000000 -1.000000
## C1.80          0  1.000000  1.000000  1.000000  1.000000 -1.000000 -1.000000
## C1.130         0  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## C1.124         0  1.000000  1.000000 -1.000000  1.000000  1.000000  1.000000
## C1.109         0 -1.000000 -1.000000  1.000000  1.000000 -1.000000  1.000000
## C1.86          0  1.000000 -1.000000  1.000000 -1.000000  1.000000 -1.000000
## C1.24          0  1.000000  1.000000  1.000000 -1.000000  1.000000 -1.000000
## C1.11          0 -1.000000  1.000000 -1.000000  1.000000 -1.000000 -1.000000
## C1.107         0 -1.000000  1.000000 -1.000000  1.000000 -1.000000  1.000000
## C1.115         0 -1.000000  1.000000 -1.000000 -1.000000  1.000000  1.000000
## C1.33          0 -1.000000 -1.000000 -1.000000 -1.000000 -1.000000  1.000000
## C1.97          0 -1.000000 -1.000000 -1.000000 -1.000000 -1.000000  1.000000
## C1.96          0  1.000000  1.000000  1.000000  1.000000  1.000000 -1.000000
## C1.42          0  1.000000 -1.000000 -1.000000  1.000000 -1.000000  1.000000
## C1.48          0  1.000000  1.000000  1.000000  1.000000 -1.000000  1.000000
## C1.13          0 -1.000000 -1.000000  1.000000  1.000000 -1.000000 -1.000000
## C1.118         0  1.000000 -1.000000  1.000000 -1.000000  1.000000  1.000000
## C1.41          0 -1.000000 -1.000000 -1.000000  1.000000 -1.000000  1.000000
## C1.78          0  1.000000 -1.000000  1.000000  1.000000 -1.000000 -1.000000
## C1.9           0 -1.000000 -1.000000 -1.000000  1.000000 -1.000000 -1.000000
## C1.72          0  1.000000  1.000000  1.000000 -1.000000 -1.000000 -1.000000
## C1.70          0  1.000000 -1.000000  1.000000 -1.000000 -1.000000 -1.000000
## C1.106         0  1.000000 -1.000000 -1.000000  1.000000 -1.000000  1.000000
## C1.21          0 -1.000000 -1.000000  1.000000 -1.000000  1.000000 -1.000000
## C1.65          0 -1.000000 -1.000000 -1.000000 -1.000000 -1.000000 -1.000000
## C1.94          0  1.000000 -1.000000  1.000000  1.000000  1.000000 -1.000000
## C1.69          0 -1.000000 -1.000000  1.000000 -1.000000 -1.000000 -1.000000
## C1.61          0 -1.000000 -1.000000  1.000000  1.000000  1.000000  1.000000
## C1.71          0 -1.000000  1.000000  1.000000 -1.000000 -1.000000 -1.000000
## C1.66          0  1.000000 -1.000000 -1.000000 -1.000000 -1.000000 -1.000000
## C1.117         0 -1.000000 -1.000000  1.000000 -1.000000  1.000000  1.000000
## C1.122         0  1.000000 -1.000000 -1.000000  1.000000  1.000000  1.000000
## C1.36          0  1.000000  1.000000 -1.000000 -1.000000 -1.000000  1.000000
## C1.101         0 -1.000000 -1.000000  1.000000 -1.000000 -1.000000  1.000000
## C1.28          0  1.000000  1.000000 -1.000000  1.000000  1.000000 -1.000000
## C1.22          0  1.000000 -1.000000  1.000000 -1.000000  1.000000 -1.000000
## C1.20          0  1.000000  1.000000 -1.000000 -1.000000  1.000000 -1.000000
## C1.90          0  1.000000 -1.000000 -1.000000  1.000000  1.000000 -1.000000
## C1.93          0 -1.000000 -1.000000  1.000000  1.000000  1.000000 -1.000000
## C1.81          0 -1.000000 -1.000000 -1.000000 -1.000000  1.000000 -1.000000
## C1.91          0 -1.000000  1.000000 -1.000000  1.000000  1.000000 -1.000000
## C1.29          0 -1.000000 -1.000000  1.000000  1.000000  1.000000 -1.000000
## C1.123         0 -1.000000  1.000000 -1.000000  1.000000  1.000000  1.000000
## C1.119         0 -1.000000  1.000000  1.000000 -1.000000  1.000000  1.000000
## C1.63          0 -1.000000  1.000000  1.000000  1.000000  1.000000  1.000000
## C1.121         0 -1.000000 -1.000000 -1.000000  1.000000  1.000000  1.000000
## C1.2           0  1.000000 -1.000000 -1.000000 -1.000000 -1.000000 -1.000000
## C1.25          0 -1.000000 -1.000000 -1.000000  1.000000  1.000000 -1.000000
```

```
## C1.84          0  1.000000  1.000000 -1.000000 -1.000000  1.000000 -1.000000
## C1.62          0  1.000000 -1.000000  1.000000  1.000000  1.000000  1.000000
## C1.10          0  1.000000 -1.000000 -1.000000  1.000000 -1.000000 -1.000000
## C1.37          0 -1.000000 -1.000000  1.000000 -1.000000 -1.000000  1.000000
## C1.52          0  1.000000  1.000000 -1.000000 -1.000000  1.000000  1.000000
## C1.89          0 -1.000000 -1.000000 -1.000000  1.000000  1.000000 -1.000000
## C1.98          0  1.000000 -1.000000 -1.000000 -1.000000 -1.000000  1.000000
## C1.74          0  1.000000 -1.000000 -1.000000  1.000000 -1.000000 -1.000000
## C1.85          0 -1.000000 -1.000000  1.000000 -1.000000  1.000000 -1.000000
## C1.92          0  1.000000  1.000000 -1.000000  1.000000  1.000000 -1.000000
## C1.105         0 -1.000000 -1.000000 -1.000000  1.000000 -1.000000  1.000000
## C1.67          0 -1.000000  1.000000 -1.000000 -1.000000 -1.000000 -1.000000
## C1.32          0  1.000000  1.000000  1.000000  1.000000  1.000000 -1.000000
## C1.38          0  1.000000 -1.000000  1.000000 -1.000000 -1.000000  1.000000
## C1.83          0 -1.000000  1.000000 -1.000000 -1.000000  1.000000 -1.000000
## C1.5           0 -1.000000 -1.000000  1.000000 -1.000000 -1.000000 -1.000000
## C1.49          0 -1.000000 -1.000000 -1.000000 -1.000000  1.000000  1.000000
## C1.4           0  1.000000  1.000000 -1.000000 -1.000000 -1.000000 -1.000000
## C1.73          0 -1.000000 -1.000000 -1.000000  1.000000 -1.000000 -1.000000
## C1.131         0  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.14          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.2           1  3.424207  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.23          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.16          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.12          1  0.000000  0.000000  0.000000  0.000000  0.000000  3.424207
## S2.10          1  0.000000  0.000000  0.000000  0.000000  3.424207  0.000000
## S2.7           1  0.000000  0.000000  0.000000 -3.424207  0.000000  0.000000
## S2.19          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.24          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.17          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.21          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.3           1  0.000000 -3.424207  0.000000  0.000000  0.000000  0.000000
## S2.4           1  0.000000  3.424207  0.000000  0.000000  0.000000  0.000000
## S2.13          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.9           1  0.000000  0.000000  0.000000  0.000000 -3.424207  0.000000
## S2.22          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.20          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.18          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.15          1  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.8           1  0.000000  0.000000  0.000000  3.424207  0.000000  0.000000
## S2.1           1 -3.424207  0.000000  0.000000  0.000000  0.000000  0.000000
## S2.5           1  0.000000  0.000000 -3.424207  0.000000  0.000000  0.000000
## S2.6           1  0.000000  0.000000  3.424207  0.000000  0.000000  0.000000
## S2.11          1  0.000000  0.000000  0.000000  0.000000  0.000000 -3.424207
##                G         H         J         K     resp.y
## C1.129  0.000000  0.000000  0.000000  0.000000  94162.434
## C1.102  1.000000 -1.000000 -1.000000  1.000000  85430.443
## C1.30  -1.000000 -1.000000 -1.000000  1.000000 114789.788
## C1.82   1.000000 -1.000000 -1.000000 -1.000000 168404.075
## C1.95   1.000000 -1.000000  1.000000  1.000000  93864.418
## C1.31  -1.000000 -1.000000 -1.000000  1.000000 100804.436
## C1.39  -1.000000 -1.000000  1.000000  1.000000  91181.800
## C1.16  -1.000000 -1.000000  1.000000 -1.000000 105319.684
## C1.53  -1.000000 -1.000000 -1.000000 -1.000000  95109.129
```

```
## C1.18  -1.000000 -1.000000  1.000000 -1.000000  95674.325
## C1.88   1.000000 -1.000000 -1.000000  1.000000   7964.088
## C1.15  -1.000000  1.000000 -1.000000  1.000000  92309.467
## C1.75   1.000000 -1.000000 -1.000000  1.000000  84964.605
## C1.50  -1.000000 -1.000000 -1.000000  1.000000 105787.870
## C1.59  -1.000000  1.000000 -1.000000 -1.000000  96727.560
## C1.103  1.000000 -1.000000 -1.000000  1.000000  99113.416
## C1.64  -1.000000  1.000000 -1.000000  1.000000  99986.205
## C1.60  -1.000000 -1.000000  1.000000  1.000000 103747.233
## C1.111  1.000000  1.000000 -1.000000 -1.000000  93352.191
## C1.12  -1.000000  1.000000 -1.000000 -1.000000  76430.851
## C1.17  -1.000000  1.000000 -1.000000  1.000000  81494.616
## C1.77   1.000000 -1.000000 -1.000000 -1.000000 226646.146
## C1.47  -1.000000  1.000000  1.000000 -1.000000 112367.609
## C1.104  1.000000  1.000000  1.000000 -1.000000 110516.131
## C1.76   1.000000  1.000000  1.000000 -1.000000 144420.676
## C1.113  1.000000  1.000000 -1.000000 -1.000000  86303.755
## C1.79   1.000000  1.000000  1.000000  1.000000  94593.098
## C1.14  -1.000000  1.000000 -1.000000  1.000000  98709.237
## C1.19  -1.000000 -1.000000  1.000000 -1.000000  86662.415
## C1.128  1.000000  1.000000  1.000000  1.000000  90553.555
## C1.8   -1.000000  1.000000  1.000000  1.000000  96021.619
## C1.108  1.000000  1.000000 -1.000000  1.000000  89902.874
## C1.125  1.000000  1.000000  1.000000  1.000000  99059.651
## C1.57  -1.000000 -1.000000  1.000000  1.000000 116120.847
## C1.112  1.000000 -1.000000  1.000000  1.000000  83919.177
## C1.34  -1.000000  1.000000 -1.000000  1.000000  82615.131
## C1.44  -1.000000  1.000000  1.000000  1.000000 102608.606
## C1.56  -1.000000 -1.000000 -1.000000 -1.000000  99934.242
## C1.100  1.000000 -1.000000 -1.000000 -1.000000  29662.972
## C1.43  -1.000000 -1.000000 -1.000000 -1.000000 132699.236
## C1.7   -1.000000 -1.000000 -1.000000 -1.000000  88741.567
## C1.27  -1.000000  1.000000  1.000000  1.000000 103021.241
## C1.58  -1.000000  1.000000 -1.000000 -1.000000  82569.428
## C1.55  -1.000000  1.000000  1.000000  1.000000  79884.555
## C1.40  -1.000000  1.000000 -1.000000 -1.000000  86638.669
## C1.6   -1.000000 -1.000000 -1.000000 -1.000000  98046.813
## C1.46  -1.000000  1.000000  1.000000 -1.000000  91143.263
## C1.68   1.000000 -1.000000  1.000000  1.000000 101842.398
## C1.126  1.000000 -1.000000 -1.000000 -1.000000  89751.308
## C1.51  -1.000000 -1.000000 -1.000000  1.000000  98313.993
## C1.26  -1.000000  1.000000  1.000000  1.000000  95502.066
## C1.116  1.000000  1.000000 -1.000000 -1.000000  95214.509
## C1.45  -1.000000 -1.000000 -1.000000  1.000000  13873.347
## C1.120  1.000000 -1.000000  1.000000 -1.000000  85945.034
## C1.35  -1.000000  1.000000 -1.000000  1.000000 112136.092
## C1.87   1.000000  1.000000  1.000000 -1.000000 103181.283
## C1.3   -1.000000  1.000000  1.000000 -1.000000  85075.300
## C1.54  -1.000000  1.000000  1.000000  1.000000  94614.184
## C1.127  1.000000 -1.000000 -1.000000 -1.000000  87317.877
## C1.99   1.000000  1.000000  1.000000  1.000000  80831.781
## C1.23  -1.000000  1.000000 -1.000000 -1.000000  82462.249
## C1.114  1.000000 -1.000000  1.000000  1.000000 107984.822
## C1.110  1.000000  1.000000 -1.000000 -1.000000  97085.162
```

```
## C1.1   -1.000000 -1.000000 -1.000000  1.000000 100846.001
## C1.80   1.000000 -1.000000 -1.000000 -1.000000  97449.680
## C1.130  0.000000  0.000000  0.000000  0.000000 103535.347
## C1.124  1.000000 -1.000000 -1.000000  1.000000  58087.345
## C1.109  1.000000 -1.000000  1.000000  1.000000  93794.126
## C1.86   1.000000  1.000000  1.000000 -1.000000  77307.856
## C1.24  -1.000000 -1.000000  1.000000  1.000000  97606.967
## C1.11  -1.000000 -1.000000  1.000000  1.000000 102499.323
## C1.107  1.000000 -1.000000  1.000000 -1.000000  99658.902
## C1.115  1.000000 -1.000000  1.000000  1.000000  88919.933
## C1.33  -1.000000 -1.000000  1.000000 -1.000000  93185.520
## C1.97   1.000000 -1.000000 -1.000000 -1.000000  94222.527
## C1.96   1.000000  1.000000 -1.000000 -1.000000 105889.966
## C1.42  -1.000000 -1.000000 -1.000000 -1.000000  88367.429
## C1.48  -1.000000 -1.000000 -1.000000  1.000000 163305.110
## C1.13  -1.000000 -1.000000  1.000000 -1.000000  96258.288
## C1.118  1.000000  1.000000 -1.000000  1.000000 109966.571
## C1.41  -1.000000  1.000000  1.000000  1.000000  90709.220
## C1.78   1.000000  1.000000  1.000000  1.000000 100392.014
## C1.9   -1.000000  1.000000 -1.000000 -1.000000  81829.286
## C1.72   1.000000  1.000000 -1.000000  1.000000  76338.298
## C1.70   1.000000 -1.000000  1.000000 -1.000000 102775.417
## C1.106  1.000000 -1.000000  1.000000 -1.000000  95640.559
## C1.21  -1.000000 -1.000000  1.000000  1.000000  97722.043
## C1.65   1.000000 -1.000000  1.000000  1.000000  86187.382
## C1.94   1.000000 -1.000000  1.000000  1.000000 104711.034
## C1.69   1.000000  1.000000 -1.000000  1.000000  79184.857
## C1.61  -1.000000  1.000000 -1.000000  1.000000  85257.873
## C1.71   1.000000 -1.000000  1.000000 -1.000000  95347.517
## C1.66   1.000000  1.000000 -1.000000 -1.000000 113205.611
## C1.117  1.000000 -1.000000  1.000000 -1.000000  36601.612
## C1.122  1.000000  1.000000  1.000000 -1.000000  88500.360
## C1.36  -1.000000 -1.000000  1.000000 -1.000000 219584.156
## C1.101  1.000000  1.000000  1.000000 -1.000000 -38330.113
## C1.28  -1.000000 -1.000000 -1.000000 -1.000000  97079.587
## C1.22  -1.000000  1.000000 -1.000000 -1.000000  94218.992
## C1.20  -1.000000  1.000000 -1.000000  1.000000 171852.401
## C1.90   1.000000  1.000000 -1.000000  1.000000 101682.224
## C1.93   1.000000  1.000000 -1.000000 -1.000000 149288.073
## C1.81   1.000000  1.000000  1.000000  1.000000   1099.033
## C1.91   1.000000  1.000000 -1.000000  1.000000  91683.539
## C1.29  -1.000000  1.000000  1.000000 -1.000000 109344.586
## C1.123  1.000000  1.000000  1.000000 -1.000000  71726.271
## C1.119  1.000000  1.000000 -1.000000  1.000000 103145.467
## C1.63  -1.000000 -1.000000  1.000000 -1.000000 101862.642
## C1.121  1.000000 -1.000000 -1.000000  1.000000 100456.861
## C1.2   -1.000000  1.000000  1.000000 -1.000000  98139.545
## C1.25  -1.000000 -1.000000 -1.000000 -1.000000 100165.042
## C1.84   1.000000  1.000000  1.000000  1.000000  91582.757
## C1.62  -1.000000 -1.000000  1.000000 -1.000000  68414.445
## C1.10  -1.000000 -1.000000  1.000000  1.000000  99097.288
## C1.37  -1.000000  1.000000 -1.000000 -1.000000 105099.346
## C1.52  -1.000000  1.000000  1.000000 -1.000000 100279.993
## C1.89   1.000000 -1.000000  1.000000 -1.000000  88796.367
```

```
## C1.98    1.000000  1.000000  1.000000  1.000000 111703.672
## C1.74    1.000000 -1.000000 -1.000000  1.000000  94434.997
## C1.85    1.000000 -1.000000 -1.000000  1.000000  70707.961
## C1.92    1.000000 -1.000000  1.000000 -1.000000  99382.176
## C1.105   1.000000  1.000000 -1.000000  1.000000 101115.650
## C1.67    1.000000  1.000000 -1.000000 -1.000000  93660.576
## C1.32   -1.000000  1.000000  1.000000 -1.000000  94525.042
## C1.38   -1.000000 -1.000000  1.000000  1.000000  99980.520
## C1.83    1.000000 -1.000000 -1.000000 -1.000000  79924.220
## C1.5    -1.000000  1.000000  1.000000  1.000000 103294.958
## C1.49   -1.000000  1.000000  1.000000 -1.000000 208988.648
## C1.4    -1.000000 -1.000000 -1.000000  1.000000 105614.053
## C1.73    1.000000  1.000000  1.000000 -1.000000  90420.589
## C1.131   0.000000  0.000000  0.000000  0.000000  80801.145
## S2.14    3.424207  0.000000  0.000000  0.000000  90467.237
## S2.2     0.000000  0.000000  0.000000  0.000000  80866.497
## S2.23    0.000000  0.000000  0.000000  0.000000 101005.709
## S2.16    0.000000  3.424207  0.000000  0.000000  98846.582
## S2.12    0.000000  0.000000  0.000000  0.000000 198157.287
## S2.10    0.000000  0.000000  0.000000  0.000000 147050.845
## S2.7     0.000000  0.000000  0.000000  0.000000 143636.681
## S2.19    0.000000  0.000000  0.000000 -3.424207 102259.860
## S2.24    0.000000  0.000000  0.000000  0.000000  79687.058
## S2.17    0.000000  0.000000 -3.424207  0.000000  95628.330
## S2.21    0.000000  0.000000  0.000000  0.000000 106239.812
## S2.3     0.000000  0.000000  0.000000  0.000000 106621.398
## S2.4     0.000000  0.000000  0.000000  0.000000  94279.241
## S2.13   -3.424207  0.000000  0.000000  0.000000  88357.682
## S2.9     0.000000  0.000000  0.000000  0.000000  74337.010
## S2.22    0.000000  0.000000  0.000000  0.000000  90513.420
## S2.20    0.000000  0.000000  0.000000  3.424207  89284.997
## S2.18    0.000000  0.000000  3.424207  0.000000  85209.962
## S2.15    0.000000 -3.424207  0.000000  0.000000  94391.556
## S2.8     0.000000  0.000000  0.000000  0.000000  73716.290
## S2.1     0.000000  0.000000  0.000000  0.000000  92533.426
## S2.5     0.000000  0.000000  0.000000  0.000000  97186.295
## S2.6     0.000000  0.000000  0.000000  0.000000  86852.827
## S2.11    0.000000  0.000000  0.000000  0.000000  82431.671
##
## $run.order
##     run.no.in.std.order run.no run.no.std.rp
## 1                C1.129      1        C1.129
## 2                C1.102      2        C1.102
## 3                 C1.30      3         C1.30
## 4                 C1.82      4         C1.82
## 5                 C1.95      5         C1.95
## 6                 C1.31      6         C1.31
## 7                 C1.39      7         C1.39
## 8                 C1.16      8         C1.16
## 9                 C1.53      9         C1.53
## 10                C1.18     10         C1.18
## 11                C1.88     11         C1.88
## 12                C1.15     12         C1.15
## 13                C1.75     13         C1.75
```

```
## 14                   C1.50    14         C1.50
## 15                   C1.59    15         C1.59
## 16                  C1.103    16        C1.103
## 17                   C1.64    17         C1.64
## 18                   C1.60    18         C1.60
## 19                  C1.111    19        C1.111
## 20                   C1.12    20         C1.12
## 21                   C1.17    21         C1.17
## 22                   C1.77    22         C1.77
## 23                   C1.47    23         C1.47
## 24                  C1.104    24        C1.104
## 25                   C1.76    25         C1.76
## 26                  C1.113    26        C1.113
## 27                   C1.79    27         C1.79
## 28                   C1.14    28         C1.14
## 29                   C1.19    29         C1.19
## 30                  C1.128    30        C1.128
## 31                    C1.8    31          C1.8
## 32                  C1.108    32        C1.108
## 33                  C1.125    33        C1.125
## 34                   C1.57    34         C1.57
## 35                  C1.112    35        C1.112
## 36                   C1.34    36         C1.34
## 37                   C1.44    37         C1.44
## 38                   C1.56    38         C1.56
## 39                  C1.100    39        C1.100
## 40                   C1.43    40         C1.43
## 41                    C1.7    41          C1.7
## 42                   C1.27    42         C1.27
## 43                   C1.58    43         C1.58
## 44                   C1.55    44         C1.55
## 45                   C1.40    45         C1.40
## 46                    C1.6    46          C1.6
## 47                   C1.46    47         C1.46
## 48                   C1.68    48         C1.68
## 49                  C1.126    49        C1.126
## 50                   C1.51    50         C1.51
## 51                   C1.26    51         C1.26
## 52                  C1.116    52        C1.116
## 53                   C1.45    53         C1.45
## 54                  C1.120    54        C1.120
## 55                   C1.35    55         C1.35
## 56                   C1.87    56         C1.87
## 57                    C1.3    57          C1.3
## 58                   C1.54    58         C1.54
## 59                  C1.127    59        C1.127
## 60                   C1.99    60         C1.99
## 61                   C1.23    61         C1.23
## 62                  C1.114    62        C1.114
## 63                  C1.110    63        C1.110
## 64                    C1.1    64          C1.1
## 65                   C1.80    65         C1.80
## 66                  C1.130    66        C1.130
## 67                  C1.124    67        C1.124
```

```
## 68                 C1.109     68          C1.109
## 69                  C1.86     69           C1.86
## 70                  C1.24     70           C1.24
## 71                  C1.11     71           C1.11
## 72                 C1.107     72          C1.107
## 73                 C1.115     73          C1.115
## 74                  C1.33     74           C1.33
## 75                  C1.97     75           C1.97
## 76                  C1.96     76           C1.96
## 77                  C1.42     77           C1.42
## 78                  C1.48     78           C1.48
## 79                  C1.13     79           C1.13
## 80                 C1.118     80          C1.118
## 81                  C1.41     81           C1.41
## 82                  C1.78     82           C1.78
## 83                   C1.9     83            C1.9
## 84                  C1.72     84           C1.72
## 85                  C1.70     85           C1.70
## 86                 C1.106     86          C1.106
## 87                  C1.21     87           C1.21
## 88                  C1.65     88           C1.65
## 89                  C1.94     89           C1.94
## 90                  C1.69     90           C1.69
## 91                  C1.61     91           C1.61
## 92                  C1.71     92           C1.71
## 93                  C1.66     93           C1.66
## 94                 C1.117     94          C1.117
## 95                 C1.122     95          C1.122
## 96                  C1.36     96           C1.36
## 97                 C1.101     97          C1.101
## 98                  C1.28     98           C1.28
## 99                  C1.22     99           C1.22
## 100                 C1.20    100           C1.20
## 101                 C1.90    101           C1.90
## 102                 C1.93    102           C1.93
## 103                 C1.81    103           C1.81
## 104                 C1.91    104           C1.91
## 105                 C1.29    105           C1.29
## 106                C1.123    106          C1.123
## 107                C1.119    107          C1.119
## 108                 C1.63    108           C1.63
## 109                C1.121    109          C1.121
## 110                  C1.2    110            C1.2
## 111                 C1.25    111           C1.25
## 112                 C1.84    112           C1.84
## 113                 C1.62    113           C1.62
## 114                 C1.10    114           C1.10
## 115                 C1.37    115           C1.37
## 116                 C1.52    116           C1.52
## 117                 C1.89    117           C1.89
## 118                 C1.98    118           C1.98
## 119                 C1.74    119           C1.74
## 120                 C1.85    120           C1.85
## 121                 C1.92    121           C1.92
```

```
## 122               C1.105    122          C1.105
## 123                C1.67    123           C1.67
## 124                C1.32    124           C1.32
## 125                C1.38    125           C1.38
## 126                C1.83    126           C1.83
## 127                 C1.5    127            C1.5
## 128                C1.49    128           C1.49
## 129                 C1.4    129            C1.4
## 130                C1.73    130           C1.73
## 131               C1.131    131          C1.131
## 132                S2.14    132           S2.14
## 133                 S2.2    133            S2.2
## 134                S2.23    134           S2.23
## 135                S2.16    135           S2.16
## 136                S2.12    136           S2.12
## 137                S2.10    137           S2.10
## 138                 S2.7    138            S2.7
## 139                S2.19    139           S2.19
## 140                S2.24    140           S2.24
## 141                S2.17    141           S2.17
## 142                S2.21    142           S2.21
## 143                 S2.3    143            S2.3
## 144                 S2.4    144            S2.4
## 145                S2.13    145           S2.13
## 146                 S2.9    146            S2.9
## 147                S2.22    147           S2.22
## 148                S2.20    148           S2.20
## 149                S2.18    149           S2.18
## 150                S2.15    150           S2.15
## 151                 S2.8    151            S2.8
## 152                 S2.1    152            S2.1
## 153                 S2.5    153            S2.5
## 154                 S2.6    154            S2.6
## 155                S2.11    155           S2.11
##
## $design.info
## $design.info$type
## [1] "ccd"
##
## $design.info$nruns
## [1] 155
##
## $design.info$nfactors
## [1] 10
##
## $design.info$factor.names
## $design.info$factor.names$A
## [1] -1  1
##
## $design.info$factor.names$B
## [1] -1  1
##
## $design.info$factor.names$C
## [1] -1  1
```

```
##
## $design.info$factor.names$D
## [1] -1  1
##
## $design.info$factor.names$E
## [1] -1  1
##
## $design.info$factor.names$F
## [1] -1  1
##
## $design.info$factor.names$G
## [1] -1  1
##
## $design.info$factor.names$H
## [1] -1  1
##
## $design.info$factor.names$J
## [1] -1  1
##
## $design.info$factor.names$K
## [1] -1  1
##
##
## $design.info$catlg.name
## [1] "catlg"
##
## $design.info$catlg.entry
## Design:  10-3.1
##    128  runs,  10  factors,
##    Resolution  V
##    Generating columns:  31 103 43
##    WLP (3plus):  0 0 3 3 ,  45  clear 2fis
##
## $design.info$aliased
## $design.info$aliased$legend
##  [1] "A=A" "B=B" "C=C" "D=D" "E=E" "F=F" "G=G" "H=H" "J=J" "K=K"
##
## $design.info$aliased[[2]]
## [1] "no aliasing among main effects and 2fis"
##
##
## $design.info$FrF2.version
## [1] "2.2-2"
##
## $design.info$replications
## [1] 1
##
## $design.info$repeat.only
## [1] FALSE
##
## $design.info$randomize
## [1] TRUE
##
## $design.info$seed
```

```
## NULL
##
## $design.info$creator
## $design.info$creator[[1]]
## FrF2(nfactors = 10, resolution = 5, ncenter = 3)
##
## $design.info$creator[[2]]
## ccd.augment(design)
##
##
## $design.info$quantitative
##    A    B    C    D    E    F    G    H    J    K
## TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE
##
## $design.info$ncube
## [1] 128
##
## $design.info$ncenter
## [1] 3 4
##
## $design.info$coding
## $design.info$coding$x1
## x1 ~ (A - 0)/1
##
## $design.info$coding$x2
## x2 ~ (B - 0)/1
##
## $design.info$coding$x3
## x3 ~ (C - 0)/1
##
## $design.info$coding$x4
## x4 ~ (D - 0)/1
##
## $design.info$coding$x5
## x5 ~ (E - 0)/1
##
## $design.info$coding$x6
## x6 ~ (F - 0)/1
##
## $design.info$coding$x7
## x7 ~ (G - 0)/1
##
## $design.info$coding$x8
## x8 ~ (H - 0)/1
##
## $design.info$coding$x9
## x9 ~ (J - 0)/1
##
## $design.info$coding$x10
## x10 ~ (K - 0)/1
##
##
## $design.info$block.name
## [1] "Block.ccd"
```

```
##
## $design.info$cube.gen
## $design.info$cube.gen[[1]]
## x8 ~ x1 * x2 * x3 * x4 * x5
## <environment: 0x0000000026d771d0>
##
## $design.info$cube.gen[[2]]
## x9 ~ x1 * x2 * x3 * x6 * x7
## <environment: 0x0000000026d83550>
##
## $design.info$cube.gen[[3]]
## x10 ~ x1 * x2 * x4 * x6
## <environment: 0x0000000026d8baa0>
##
##
## $design.info$nstar
## [1] 20
##
## $design.info$response.names
## [1] "resp.y"
##
##
## $names
##  [1] "Block.ccd" "A"         "B"         "C"         "D"         "E"
##  [7] "F"         "G"         "H"         "J"         "K"         "resp.y"
##
## $class
## [1] "design"     "data.frame"
```

```
summary(design)
```

```
## Multi-step-call:
## [[1]]
## FrF2(nfactors = 10, resolution = 5, ncenter = 3)
##
## [[2]]
## ccd.augment(design)
##
##
## Experimental design of type  ccd
## 155  runs
##
## Factor settings (cube):
##    A  B  C  D  E  F  G  H  J  K
## 1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1
## 2  1  1  1  1  1  1  1  1  1  1
##
## Numbers of cube and star points:
## Cube Star
##  128   20
## Numbers of center points:
## Cube Star
##    3    4
##
## Responses:
```

```
## [1] resp.y
```