Glib Dolotov (U14734371) MA213 Spr 2019

Problem Set 5

Assigned on 03/04. Due on 03/11.

1. Textbook problem 5.18, "Time delays at a bus stop", page 238.

2. Textbook problem 5.40, "Casino gaming", page 251.

3. Textbook problem 5.48, "Rating employee performance", page 252.

4. Textbook problem 5.50, "Alcohol, threats, and electric shocks", page 252.

5. Textbook problem 5.62, page 258.

5.18 Time delays at a bus stop. A bus is scheduled to stop at a certain bus stop every half hour on the hour and the half hour. At the end of the day, buses still stop after every 30 minutes, but because delays often occur earlier in the day, the bus is never early and is likely to be late. The director of the bus line claims that the length of time a bus is late is uniformly distributed and the maximum time that a bus is late is 20 minutes.

a. If the director's claim is true, what is the expected number of minutes a bus will be late?

b. If the director's claim is true, what is the probability that the last bus on a given day will be more than 19 minutes late?

c. If you arrive at the bus stop at the end of a day at exactly half-past the hour and must wait more than 19 minutes for the bus, what would you conclude about the director's claim? Why?

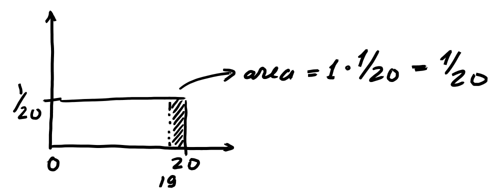
T = time (in min) by which a bus is late

$T \sim Unif(0, 20)$

a) $E[T] = \frac{20-0}{2} = \boxed{10 \text{ min}}$

b) $P(T > 19)$

$= \boxed{1/20}$



c) I can conclude that the director's claim is unlikely to be true as there is a less than 5% chance of this occurring.

5.40 Casino gaming. In *Chance* (Spring 2005), University of Denver statistician R. C. Hannum discussed casino gaming and the laws of probability. Casino games of pure chance (e.g., craps, roulette, baccarat, and keno) always yield a "house advantage." For example, in the game of double-zero roulette, the expected casino win percentage is 5.26% on bets made on whether the outcome will be either black or red. (This percentage implies that for every $5 bet on black or red, the casino will earn a net of about 25 cents.) It can be shown that in 100 roulette plays on black/red, the average casino win percentage is normally distributed with mean 5.26% and standard deviation 10%. Let x represent the average casino win percentage after 100 bets on black/red in double-zero roulette.

NW

a. Find $P(x > 0)$. (This is the probability that the casino wins money.)

b. Find $P(5 < x < 15)$.

c. Find $P(x < 1)$.

d. If you observed an average casino win percentage of -25% after 100 roulette bets on black/red, what would you conclude?

$$X \sim N(\mu = 5.26,\ \sigma = 10)$$

$$X^{st} = \frac{X-\mu}{\sigma} \sim Z$$

a) $P(X > 0)$

$$= P\left(\frac{X-5.26}{10} > \frac{-5.26}{10}\right)$$

$$= P(X^{st} > -0.526)$$

$$= 1 - P(X^{st} \le -0.526)$$

$$= 1 - \Phi(-0.526) = 1 - 0.299 = \boxed{0.701}$$

b) $P(5 < x < 15) = P\left(\frac{-0.26}{10} < X^{st} < \frac{9.74}{10}\right)$

$$= P(-0.026 < X^{st} < 0.974)$$

$$= 1 - P(X^{st} < -0.026 \text{ OR } X^{st} > 0.974)$$

↓by symmetry

$$= 1 - \left(P(X^{st} < -0.026) + P(X^{st} < -0.974)\right)$$

$$= 1 - \Phi(-0.026) - \Phi(-0.974) = 1 - 0.490 - 0.165$$

$$= \boxed{0.345}$$

c) $P(x < 1) = P\left(\frac{X-5.26}{10} < \frac{-4.26}{10}\right) = P(X^{st} < -0.426)$

$$= \Phi(-0.426) = \boxed{0.335}$$

d) $P(X < -10) = P(X^{st} < -1.526) = \Phi(-1.526) = 0.064$

$P(X < -15) = P(X^{st} < -2.026) = \Phi(-2.026) = 0.021$

$\Rightarrow$ I could conclude that something is wrong w/ the roulette table.

5.48 Rating employee performance. Almost all companies utilize some type of year-end performance review for their employees. Human Resources (HR) at the University of Texas Health Science Center provides guidelines for supervisors rating their subordinates. For example, raters are advised to examine their ratings for a tendency to be either too lenient or too harsh. According to HR, "if you have this tendency, consider using a normal distribution—10% of employees (rated) exemplary, 20% distinguished, 40% competent, 20% marginal, and 10% unacceptable." Suppose you are rating an employee's performance on a scale of 1 (lowest) to 100 (highest). Also, assume the ratings follow a normal distribution with a mean of 50 and a standard deviation of 15.

a. What is the lowest rating you should give to an "exemplary" employee if you follow the University of Texas HR guidelines?

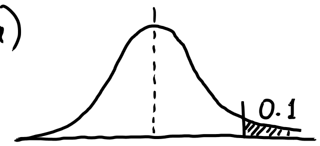
b. What is the lowest rating you should give to a "competent" employee if you follow the University of Texas HR guidelines?

$R =$ rating given

$R \sim N(\mu = 50, \sigma = 15)$

$R^{st} = \frac{R-50}{15} \sim Z$

a)

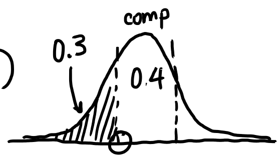


$R^{st} = \Phi^{-1}(0.9)$

$= 1.282 \Rightarrow R = 1.282 \cdot 15 + 50$

$= \boxed{69.22}$

b)



$R^{st} = \Phi^{-1}(0.3) = -0.524$

$\Rightarrow R = -0.524 \cdot 15 + 50 = \boxed{42.13}$

5.50 Alcohol, threats, and electric shocks. A group of Florida State University psychologists examined the effects of alcohol on the reactions of people to a threat (*Journal of Abnormal Psychology*, Vol. 107, 1998). After obtaining a specified blood alcohol level, the psychologists placed experimental subjects in a room and threatened them with electric shocks. Using sophisticated equipment to monitor the subjects' eye movements, the psychologists recorded the startle response (measured in milliseconds) of each subject. The mean and standard deviation of the startle responses were 37.9 and 12.4, respectively. Assume that the startle response x for a person with the specified blood alcohol level is approximately normally distributed.

a. Find the probability that x is between 40 and 50 milliseconds.

b. Find the probability that x is less than 30 milliseconds.

c. Give an interval for x centered around 37.9 milliseconds so that the probability that x falls in the interval is .95.

d. Ten percent of the experimental subjects have startle responses above what value?

X = time (in ms) of startle response

$X \sim N(\mu = 37.9,\ \sigma = 12.4)$

$X^{st} = \dfrac{X - 37.9}{12.4} \sim Z$ — pdf ϕ, cdf Φ

a) $P(40 < X < 50) = P\left(\dfrac{40-37.9}{12.4} < X^{st} < \dfrac{50-37.9}{12.4}\right)$

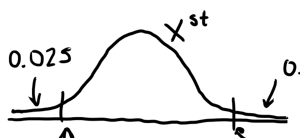
$= P(0.1694 < X^{st} < 0.9758)$

$= \Phi(0.9758) - \Phi(0.1694)$

$= 0.8354 - 0.5673 = \boxed{0.2681}$

b) $P(X < 30) = P\left(X^{st} < \dfrac{30-37.9}{12.4}\right) = P(X^{st} < -0.6371)$

$= \Phi(-0.6371) = \boxed{0.2620}$

c) 

$= \Phi^{-1}(0.975) = -\Phi^{-1}(0.025)$

$= \Phi^{-1}(0.025)$

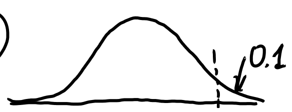
$\Phi^{-1}(0.025) = -1.960$

stdized interval $= (-1.960,\ 1.960)$

$\times 12.4$ $\quad$ $\times 12.4$

$+37.9$ $\quad$ $+37.9$

interval $= \Rightarrow \boxed{(13.596,\ 62.204)}$

d) 

$\Phi^{-1}(0.9) = 1.282$

$1.282 \times 12.4 + 37.9 = \boxed{53.791}$

5.62 Examine the sample data in the next column.

NW **a.** Construct a stem-and-leaf plot to assess whether the data are from an approximately normal distribution.

D L05062 **b.** Compute s for the sample data.

5.9	5.3	1.6	7.4	8.6	3.2	2.1
4.0	7.3	8.4	5.9	6.7	4.5	6.3
6.0	9.7	3.5	3.1	4.3	3.3	8.4
4.6	8.2	6.5	1.1	5.0	9.4	6.4

c. Find the values of Q_L and Q_U and the value of s from part **b** to assess whether the data come from an approximately normal distribution.

d. Generate a normal probability plot for the data, and use it to assess whether the data are approximately normal.

a)

```
> data <- c(5.9, 5.3, 1.6, 7.4, 8.6, 3.2, 2.1,
+           4.0, 7.3, 8.4, 5.9, 6.7, 4.5, 6.3,
+           6.0, 9.7, 3.5, 3.1, 4.3, 3.3, 8.4,
+           4.6, 8.2, 6.5, 1.1, 5.0, 9.4, 6.4)
> stem(data)

  The decimal point is at the |

  1 | 16
  2 | 1
  3 | 1235
  4 | 0356
  5 | 0399
  6 | 03457
  7 | 34
  8 | 2446
  9 | 47
```

b) $\boxed{s = 2.3525}$

hw05.R

```
# Problem 5)
# a)
data <- c(5.9, 5.3, 1.6, 7.4, 8.6, 3.2, 2.1,
          4.0, 7.3, 8.4, 5.9, 6.7, 4.5, 6.3,
          6.0, 9.7, 3.5, 3.1, 4.3, 3.3, 8.4,
          4.6, 8.2, 6.5, 1.1, 5.0, 9.4, 6.4)
stem(data)

# b)
sd(data)
n <- length(data)
sqrt(
  ( sum(data ^ 2) - sum(data)^2 / n ) / (n - 1)
)

# c)
summary(data)

# d)
library(ggplot2)
ggplot(data.frame(data), aes(x = data)) +
  geom_histogram(aes(y = ..density..), binwidth = 1) +
  geom_density() +
  stat_function(fun = function(x) dnorm(x, mean = mean(data), sd = sd(data)),
                color = "darkred",
                size = 1)
```

16:5 (Top Level) R Script

```
> # b)
> sd(data)
[1] 2.352537
> n <- length(data)
> sqrt(
+   ( sum(data ^ 2) - sum(data)^2 / n ) / (n - 1)
+ )
[1] 2.352537
```

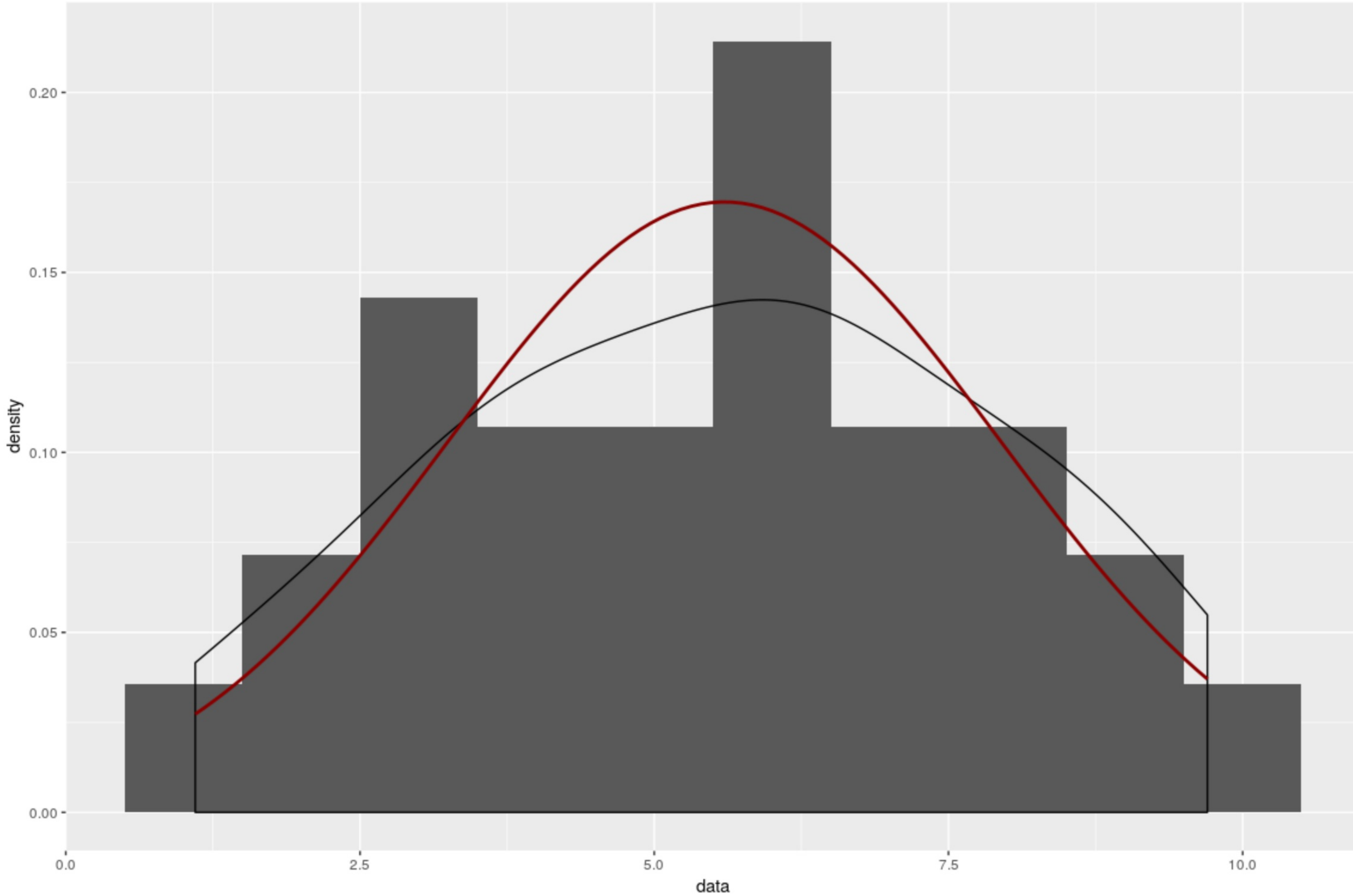
```
> # c)
> summary(data)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  1.100   3.875   5.900   5.596   7.325   9.700
```

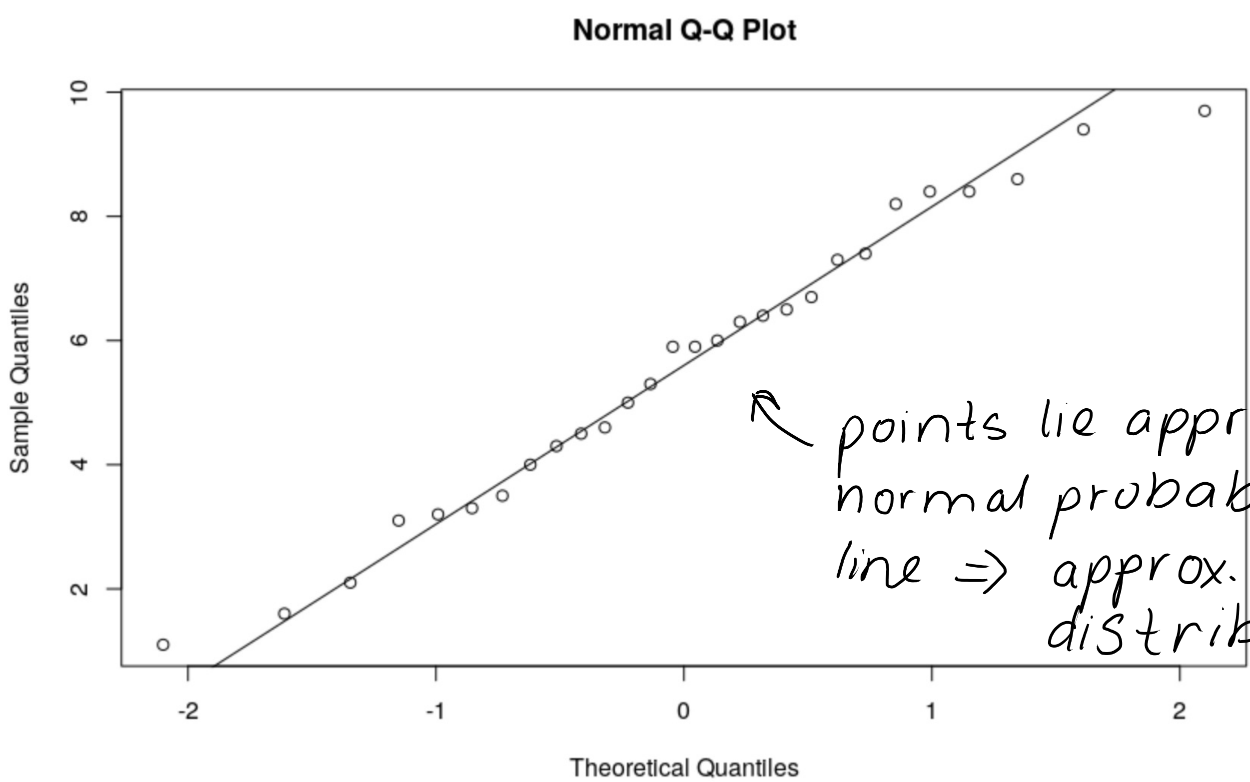
c) $Q_L = 3.875$
$Q_u = 7.325$

$$\frac{IQR}{s} = \frac{7.325 - 3.875}{2.3525} = 1.48$$

$1.48 \approx 1.3$

$\Rightarrow$ approx. normally dist'd





points lie approx. on the normal probability plot line ⇒ approx. normally distributed