证明：

$$l(\beta) = \sum_{i=1}^{n}(-y_i\beta^T\boldsymbol{x_i} + \log(1 + e^{\beta^T\boldsymbol{x_i}}))$$

$$\frac{\partial l}{\partial \beta} = \sum_{i=1}^{n}(-\boldsymbol{x_i}y_i + \frac{\boldsymbol{x_i}e^{\beta^T\boldsymbol{x_i}}}{1 + e^{\beta^T\boldsymbol{x_i}}}) = -X^Ty + X^T\mu = X^T(\mu - y)$$

$$\frac{\partial^2 l}{\partial \beta^2} = \frac{\partial(X^T(\mu - y))}{\partial \beta} = \frac{\sum_{i=1}^{n}\frac{\boldsymbol{x_i}}{1 + e^{-\beta^T\boldsymbol{x_i}}}}{\partial \beta} = \sum_{i=1}^{n}\boldsymbol{x_i}\frac{e^{-\beta^T\boldsymbol{x_i}}\boldsymbol{x_i}}{(1 + e^{-\beta^T\boldsymbol{x_i}})^2} = X^TSX$$

其中，

$$X = (\boldsymbol{x_1}, \ldots, \boldsymbol{x_n})^T$$

$$y = (y_1, \ldots, y_n)^T$$

$$\mu = (\mu_1, \ldots, \mu_n)^T$$

$$S = \boldsymbol{diag}(\mu_1(1 - \mu_1), \ldots, \mu_n(1 - \mu_n))$$

$$\mu_i = \frac{1}{1 + e^{-\beta^T\boldsymbol{x_i}}}, (i = 1, \ldots, n)$$