



WIDA's XCPC Algorithm Template (I)

v8.7 2024.10.07
WIDA

目录

杂类	1
int128 库函数自定义	1
常用库函数重载	1
字符调整	2
二分算法	3
二分算法（整数域）	3
二分算法（实数域）	3
图与网络	5
强连通分量缩点（SCC）	5
割边与割边缩点（EBCC）	6
二分图最大权匹配（MaxAssignment 基于 KM）	7
一般图最大匹配（Graph 带花树算法）【久远】	9
TwoSat（2-Sat）	13
最大流（MaxFlow 新版）	13
费用流	15
费用流（MCFGGraph 旧版）	15
费用流（MinCostFlow 新版）	17
树链剖分（HLD）	18
数论、几何、多项式	21
快速幂	21
基姆拉尔森公式	21
欧拉筛	22
莫比乌斯函数筛（莫比乌斯反演）	23
扩展欧几里得（exgcd）	24
欧拉函数	24
欧拉函数（求解单个数的欧拉函数）	24
欧拉函数（求解全部数的欧拉函数）	25
组合数	25
组合数（小范围预处理，逆元+杨辉三角）	25
组合数（Comb, with. ModIntBase）	26
素数测试与因式分解（Miller-Rabin & Pollard-Rho）	27
平面几何	28
平面几何（Point）	28
平面几何（with. complex）	34
立体几何（Point）	35
静态凸包	36
静态凸包（with. Point, 新版）	36
静态凸包（with. complex）	37
多项式	38
多项式（Poly, 旧版）	38
多项式（Poly, with. MInt & MLong）	42
多项式乘法	49
生成函数	51
生成函数（q-int）	51
生成函数（q-Binomial）	51
生成函数（Binomial 任意模数二项式）	52

自适应辛普森法 (Simpson)	54
矩阵 (Matrix)	54
高斯消元法 (gaussian elimination) 【久远】	55
数据结构	57
树状数组 (Fenwick)	57
并查集	57
并查集 (DSU)	57
可撤销并查集 (DSU With Rollback)	58
线段树	59
线段树 (SegmentTree+Info 区间加+单点修改)	59
线段树 (SegmentTree 区间乘+单点加)	60
线段树 (SegmentTree+Info 初始赋值+单点修改+查找前驱后继)	61
线段树 (SegmentTree+Info+Merge 初始赋值+单点修改+区间合并)	63
懒标记线段树 (LazySegmentTree)	64
取模类	67
取模类 (Z 旧版)	67
取模类 (MLong & MInt 新版)	69
动态取模类 (ModIntBase)	72
状压 RMQ (RMQ)	74
Splay	75
其他平衡树	83
分数四则运算 (Frac)	88
线性基 (Basis)	89
高精度 (BigInt)	89
Link-Cut Tree	90
字符串	95
马拉车 (Manacher)	95
Z 函数	95
后缀数组	95
后缀数组 (SuffixArray 旧版)	95
后缀数组 (SA 及其应用 新版)	96
后缀自动机	98
后缀自动机 (SuffixAutomaton 旧版)	98
后缀自动机 (SAM 新版)	99
回文自动机 (PAM)	100
AC 自动机	101
AC 自动机 (AC 旧版)	101
AC 自动机 (AhoCorasick, with vector 新版)	102
AC 自动机 (AhoCorasick, with string 新版)	104
字符串哈希 (随机底模例题)	105
最长公共前缀 LCP (例题)	107
字典树 Trie	108
前缀函数 (KMP)	111

个人收集，仅供参考。如有需要，您可以通过以下渠道获取最新版本或与我取得联系

www.github.com/hh2048

WIDA, 2024.10.07

1 杂类

1.1 int128 库函数自定义

```

1  ostream &operator<<(ostream &os, i128 n) {
2      if (n == 0) {
3          return os << 0;
4      }
5      string s;
6      while (n > 0) {
7          s += char('0' + n % 10);
8          n /= 10;
9      }
10     reverse(s.begin(), s.end());
11     return os << s;
12 }
13 i128 toi128(const string &s) {
14     i128 n = 0;
15     for (auto c : s) {
16         n = n * 10 + (c - '0');
17     }
18     return n;
19 }
20 i128 sqrti128(i128 n) {
21     i128 lo = 0, hi = 1E16;
22     while (lo < hi) {
23         i128 x = (lo + hi + 1) / 2;
24         if (x * x <= n) {
25             lo = x;
26         } else {
27             hi = x - 1;
28         }
29     }
30     return lo;
31 }
32
33 i128 gcd(i128 a, i128 b) {
34     while (b) {
35         a %= b;
36         swap(a, b);
37     }
38     return a;
39 }

```

1.2 常用库函数重载

```

1  /** 上取整下取整 */
2  i64 ceilDiv(i64 n, i64 m) {
3      if (n >= 0) {
4          return (n + m - 1) / m;
5      } else {
6          return n / m;
7      }
8  }
9  i64 floorDiv(i64 n, i64 m) {
10     if (n >= 0) {
11         return n / m;
12     } else {
13         return (n - m + 1) / m;
14     }

```

```

15 }
16 /** 最大值赋值 */
17 template<class T>
18 void chmax(T &a, T b) {
19     if (a < b) {
20         a = b;
21     }
22 }
23 /** 最大公约数 */
24 i128 gcd(i128 a, i128 b) {
25     return b ? gcd(b, a % b) : a;
26 }
27 /** 精确开平方 */
28 i64 sqrt(i64 n) {
29     i64 s = sqrt(n);
30     while (s * s > n) {
31         s--;
32     }
33     while ((s + 1) * (s + 1) <= n) {
34         s++;
35     }
36     return s;
37 }
38 /** 精确开平方 */
39 i64 get(i64 n) {
40     i64 u = sqrt(2.0L * n);
41     while (u * (u + 1) / 2 < n) {
42         u++;
43     }
44     while (u * (u - 1) / 2 + 1 > n) {
45         u--;
46     }
47     return u;
48 }
49 /** 求 Log */
50 int logi(int a, int b) {
51     int t = 0;
52     i64 v = 1;
53     while (v < b) {
54         v *= a;
55         t++;
56     }
57     return t;
58 }
59 int llog(int a, int b) {
60     if (a <= b) {
61         int l = logi(a, b);
62         return (l == 0 ? 0 : __lg(2 * l - 1));
63     }
64     int l = logi(b, a + 1) - 1;
65     assert(l > 0);
66     return -__lg(l);
67 }

```

1.3 字符调整

```

1 /** 大小写转换、获取字母序 */
2 void rev(string &s) {
3     int l = s.size();
4     for (int i = 1; i < l; i += 2) {
5         if (isupper(s[i])) {
6             s[i] = tolower(s[i]);

```

```

7         } else {
8             s[i] = toupper(s[i]);
9         }
10    }
11 }
12
13 int get(char c) {
14     int x;
15     if (islower(c)) {
16         x = c - 'a';
17     } else {
18         x = 26 + c - 'A';
19     }
20     return x;
21 }

```

1.4 二分算法

1.4.1 二分算法（整数域）

```

1  /** 二分算法（整数域）：前驱 */
2  int lo = 1, hi = 1E9;
3  while (lo < hi) {
4      int m = (lo + hi + 1) / 2;
5      if (check(m)) {
6          lo = m;
7      } else {
8          hi = m - 1;
9      }
10 }
11 cout << lo << "\n";
12 /** 二分算法（整数域）：后继 */
13 int lo = 1, hi = n;
14 while (lo < hi) {
15     int m = (lo + hi) / 2;
16     if (check(m)) {
17         hi = m;
18     } else {
19         lo = m + 1;
20     }
21 }
22 cout << lo << "\n";

```

1.4.2 二分算法（实数域）

```

1  /** 二分算法（实数域） */
2  auto check = [&](double t) {
3      // write
4  };
5
6  double lo = 0;
7  double hi = 1E12;
8  while (hi - lo > max(1.0, lo) * eps) {
9      double x = (lo + hi) / 2;
10     if (check(x)) {
11         hi = x;
12     } else {
13         lo = x;
14     }
15 }
16

```

```

17 cout << lo << "\n";
18
19 /** 二分算法 (实数域) */
20 using i64 = long long;
21 using real = long double;
22
23 constexpr real eps = 1E-7;
24
25 auto get = [&](const auto &f) {
26     real lo = -1E4, hi = 1E4;
27     while (hi - lo > 3 * eps) {
28         real x1 = (lo + hi - eps) / 2;
29         real x2 = (lo + hi + eps) / 2;
30         if (f(x1) > f(x2)) {
31             lo = x1;
32         } else {
33             hi = x2;
34         }
35     }
36     return f((lo + hi) / 2);
37 };
38
39 cout << get([&](real px) {
40     return get([&](real py) {
41         // write
42     });
43 }) << "\n";

```

/END/

2 图与网络

2.1 强连通分量缩点 (SCC)

```

1 struct SCC {
2     int n;
3     vector<vector<int>> adj;
4     vector<int> stk;
5     vector<int> dfn, low, bel;
6     int cur, cnt;
7
8     SCC() {}
9     SCC(int n) {
10         init(n);
11     }
12
13     void init(int n) {
14         this->n = n;
15         adj.assign(n, {});
16         dfn.assign(n, -1);
17         low.resize(n);
18         bel.assign(n, -1);
19         stk.clear();
20         cur = cnt = 0;
21     }
22
23     void addEdge(int u, int v) {
24         adj[u].push_back(v);
25     }
26
27     void dfs(int x) {
28         dfn[x] = low[x] = cur++;
29         stk.push_back(x);
30
31         for (auto y : adj[x]) {
32             if (dfn[y] == -1) {
33                 dfs(y);
34                 low[x] = min(low[x], low[y]);
35             } else if (bel[y] == -1) {
36                 low[x] = min(low[x], dfn[y]);
37             }
38         }
39
40         if (dfn[x] == low[x]) {
41             int y;
42             do {
43                 y = stk.back();
44                 bel[y] = cnt;
45                 stk.pop_back();
46             } while (y != x);
47             cnt++;
48         }
49     }
50
51     vector<int> work() {
52         for (int i = 0; i < n; i++) {
53             if (dfn[i] == -1) {
54                 dfs(i);
55             }
56         }
57         return bel;
58     }

```


59 };

2.2 割边与割边缩点 (EBCC)

```

1  set<pair<int, int>> E;
2
3  struct EBCC {
4      int n;
5      vector<vector<int>> adj;
6      vector<int> stk;
7      vector<int> dfn, low, bel;
8      int cur, cnt;
9
10     EBCC() {}
11     EBCC(int n) {
12         init(n);
13     }
14
15     void init(int n) {
16         this->n = n;
17         adj.assign(n, {});
18         dfn.assign(n, -1);
19         low.resize(n);
20         bel.assign(n, -1);
21         stk.clear();
22         cur = cnt = 0;
23     }
24
25     void addEdge(int u, int v) {
26         adj[u].push_back(v);
27         adj[v].push_back(u);
28     }
29
30     void dfs(int x, int p) {
31         dfn[x] = low[x] = cur++;
32         stk.push_back(x);
33
34         for (auto y : adj[x]) {
35             if (y == p) {
36                 continue;
37             }
38             if (dfn[y] == -1) {
39                 E.emplace(x, y);
40                 dfs(y, x);
41                 low[x] = min(low[x], low[y]);
42             } else if (bel[y] == -1 && dfn[y] < dfn[x]) {
43                 E.emplace(x, y);
44                 low[x] = min(low[x], dfn[y]);
45             }
46         }
47
48         if (dfn[x] == low[x]) {
49             int y;
50             do {
51                 y = stk.back();
52                 bel[y] = cnt;
53                 stk.pop_back();
54             } while (y != x);
55             cnt++;
56         }
57     }
58

```

```

59     vector<int> work() {
60         dfs(0, -1);
61         return bel;
62     }
63
64     struct Graph {
65         int n;
66         vector<pair<int, int>> edges;
67         vector<int> siz;
68         vector<int> cnte;
69     };
70     Graph compress() {
71         Graph g;
72         g.n = cnt;
73         g.siz.resize(cnt);
74         g.cnte.resize(cnt);
75         for (int i = 0; i < n; i++) {
76             g.siz[bel[i]]++;
77             for (auto j : adj[i]) {
78                 if (bel[i] < bel[j]) {
79                     g.edges.emplace_back(bel[i], bel[j]);
80                 } else if (i < j) {
81                     g.cnte[bel[i]]++;
82                 }
83             }
84         }
85         return g;
86     }
87 };

```

2.3 二分图最大权匹配 (MaxAssignment 基于KM)

```

1  constexpr int inf = 1E7;
2  template<class T>
3  struct MaxAssignment {
4      public:
5          T solve(int nx, int ny, vector<vector<T>> a) {
6              assert(0 <= nx && nx <= ny);
7              assert(int(a.size()) == nx);
8              for (int i = 0; i < nx; ++i) {
9                  assert(int(a[i].size()) == ny);
10                 for (auto x : a[i])
11                     assert(x >= 0);
12             }
13
14             auto update = [&](int x) {
15                 for (int y = 0; y < ny; ++y) {
16                     if (lx[x] + ly[y] - a[x][y] < slack[y]) {
17                         slack[y] = lx[x] + ly[y] - a[x][y];
18                         slackx[y] = x;
19                     }
20                 }
21             };
22
23             costs.resize(nx + 1);
24             costs[0] = 0;
25             lx.assign(nx, numeric_limits<T>::max());
26             ly.assign(ny, 0);
27             xy.assign(nx, -1);
28             yx.assign(ny, -1);
29             slackx.resize(ny);
30             for (int cur = 0; cur < nx; ++cur) {

```

```

31     queue<int> que;
32     visx.assign(nx, false);
33     visy.assign(ny, false);
34     slack.assign(ny, numeric_limits<T>::max());
35     p.assign(nx, -1);
36
37     for (int x = 0; x < nx; ++x) {
38         if (xy[x] == -1) {
39             que.push(x);
40             visx[x] = true;
41             update(x);
42         }
43     }
44
45     int ex, ey;
46     bool found = false;
47     while (!found) {
48         while (!que.empty() && !found) {
49             auto x = que.front();
50             que.pop();
51             for (int y = 0; y < ny; ++y) {
52                 if (a[x][y] == lx[x] + ly[y] && !visy[y]) {
53                     if (yx[y] == -1) {
54                         ex = x;
55                         ey = y;
56                         found = true;
57                         break;
58                     }
59                     que.push(yx[y]);
60                     p[yx[y]] = x;
61                     visy[y] = visx[yx[y]] = true;
62                     update(yx[y]);
63                 }
64             }
65         }
66         if (found)
67             break;
68
69         T delta = numeric_limits<T>::max();
70         for (int y = 0; y < ny; ++y)
71             if (!visy[y])
72                 delta = min(delta, slack[y]);
73         for (int x = 0; x < nx; ++x)
74             if (visx[x])
75                 lx[x] -= delta;
76         for (int y = 0; y < ny; ++y) {
77             if (visy[y]) {
78                 ly[y] += delta;
79             } else {
80                 slack[y] -= delta;
81             }
82         }
83         for (int y = 0; y < ny; ++y) {
84             if (!visy[y] && slack[y] == 0) {
85                 if (yx[y] == -1) {
86                     ex = slackx[y];
87                     ey = y;
88                     found = true;
89                     break;
90                 }
91                 que.push(yx[y]);
92                 p[yx[y]] = slackx[y];
93                 visy[y] = visx[yx[y]] = true;
94                 update(yx[y]);

```

```

95         }
96     }
97 }
98
99     costs[cur + 1] = costs[cur];
100     for (int x = ex, y = ey, ty; x != -1; x = p[x], y = ty) {
101         costs[cur + 1] += a[x][y];
102         if (xy[x] != -1)
103             costs[cur + 1] -= a[x][xy[x]];
104         ty = xy[x];
105         xy[x] = y;
106         yx[y] = x;
107     }
108 }
109 return costs[nx];
110 }
111 vector<int> assignment() {
112     return xy;
113 }
114 pair<vector<T>, vector<T>> labels() {
115     return make_pair(lx, ly);
116 }
117 vector<T> weights() {
118     return costs;
119 }
120 private:
121     vector<T> lx, ly, slack, costs;
122     vector<int> xy, yx, p, slackx;
123     vector<bool> visx, visy;
124 };

```

2.4 一般图最大匹配 (Graph 带花树算法) 【久远】

```

1  /** 一般图最大匹配 (Graph 带花树算法)  */
2  struct Graph {
3      int n;
4      vector<vector<int>> e;
5      Graph(int n) : n(n), e(n) {}
6      void addEdge(int u, int v) {
7          e[u].push_back(v);
8          e[v].push_back(u);
9      }
10     vector<int> findMatching(int m, const auto &init) {
11         vector<int> match(n, -1), vis(n), link(n), f(n), dep(n);
12         for (auto [x, y] : init) {
13             match[x] = y;
14             match[y] = x;
15         }
16         // disjoint set union
17         auto find = [&](int u) {
18             while (f[u] != u)
19                 u = f[u] = f[f[u]];
20             return u;
21         };
22         auto lca = [&](int u, int v) {
23             u = find(u);
24             v = find(v);
25             while (u != v) {
26                 if (dep[u] < dep[v])
27                     swap(u, v);
28                 u = find(link[match[u]]);
29             }

```

```

30         return u;
31     };
32     queue<int> que;
33     auto blossom = [&](int u, int v, int p) {
34         while (find(u) != p) {
35             link[u] = v;
36             v = match[u];
37             if (vis[v] == 0) {
38                 vis[v] = 1;
39                 que.push(v);
40             }
41             f[u] = f[v] = p;
42             u = link[v];
43         }
44     };
45     // find an augmenting path starting from u and augment (if exist)
46     auto augment = [&](int u) {
47         while (!que.empty())
48             que.pop();
49         iota(f.begin(), f.end(), 0);
50         // vis = 0 corresponds to inner vertices, vis = 1 corresponds to outer
51         vertices
52         fill(vis.begin(), vis.end(), -1);
53         que.push(u);
54         vis[u] = 1;
55         dep[u] = 0;
56         int y = -1;
57         while (!que.empty()){
58             int u = que.front();
59             que.pop();
60             if (u >= m) {
61                 y = u;
62             }
63             for (auto v : e[u]) {
64                 if (vis[v] == -1) {
65                     vis[v] = 0;
66                     link[v] = u;
67                     dep[v] = dep[u] + 1;
68                     // found an augmenting path
69                     if (match[v] == -1) {
70                         for (int x = v, y = u, temp; y != -1; x = temp, y = x
71                         == -1 ? -1 : link[x]) {
72                             temp = match[y];
73                             match[x] = y;
74                             match[y] = x;
75                         }
76                         return;
77                     }
78                     vis[match[v]] = 1;
79                     dep[match[v]] = dep[u] + 2;
80                     que.push(match[v]);
81                 } else if (vis[v] == 1 && find(v) != find(u)) {
82                     // found a blossom
83                     int p = lca(u, v);
84                     blossom(u, v, p);
85                     blossom(v, u, p);
86                 }
87             }
88         }
89         if (y != -1) {
90             for (int x = -1, temp; y != -1; x = temp, y = x == -1 ? -1 :
link[x]) {
91                 temp = match[y];
92                 if (x != -1) {

```

```

91         match[x] = y;
92     }
93     match[y] = x;
94 }
95 }
96 };
97 for (int u = 0; u < m; ++u)
98     if (match[u] == -1)
99         augment(u);
100 return match;
101 }
102 };
103
104 /** 一般图最大匹配 (Graph 带花树算法) 【久远】 */
105 struct Graph {
106     int n;
107     vector<vector<int>> e;
108     Graph(int n) : n(n), e(n) {}
109     void addEdge(int u, int v) {
110         e[u].push_back(v);
111         e[v].push_back(u);
112     }
113     vector<int> findMatching() {
114         vector<int> match(n, -1), vis(n), link(n), f(n), dep(n);
115
116         // disjoint set union
117         auto find = [&](int u) {
118             while (f[u] != u)
119                 u = f[u] = f[f[u]];
120             return u;
121         };
122
123         auto lca = [&](int u, int v) {
124             u = find(u);
125             v = find(v);
126             while (u != v) {
127                 if (dep[u] < dep[v])
128                     swap(u, v);
129                 u = find(link[match[u]]);
130             }
131             return u;
132         };
133
134         queue<int> que;
135         auto blossom = [&](int u, int v, int p) {
136             while (find(u) != p) {
137                 link[u] = v;
138                 v = match[u];
139                 if (vis[v] == 0) {
140                     vis[v] = 1;
141                     que.push(v);
142                 }
143                 f[u] = f[v] = p;
144                 u = link[v];
145             }
146         };
147
148         // find an augmenting path starting from u and augment (if exist)
149         auto augment = [&](int u) {
150
151             while (!que.empty())
152                 que.pop();
153
154             iota(f.begin(), f.end(), 0);

```

```

155
156 // vis = 0 corresponds to inner vertices, vis = 1 corresponds to outer
vertices
157 fill(vis.begin(), vis.end(), -1);
158
159 que.push(u);
160 vis[u] = 1;
161 dep[u] = 0;
162
163 while (!que.empty()){
164     int u = que.front();
165     que.pop();
166     for (auto v : e[u]) {
167         if (vis[v] == -1) {
168
169             vis[v] = 0;
170             link[v] = u;
171             dep[v] = dep[u] + 1;
172
173             // found an augmenting path
174             if (match[v] == -1) {
175                 for (int x = v, y = u, temp; y != -1; x = temp, y = x
== -1 ? -1 : link[x]) {
176                     temp = match[y];
177                     match[x] = y;
178                     match[y] = x;
179                 }
180                 return;
181             }
182
183             vis[match[v]] = 1;
184             dep[match[v]] = dep[u] + 2;
185             que.push(match[v]);
186
187         } else if (vis[v] == 1 && find(v) != find(u)) {
188             // found a blossom
189             int p = lca(u, v);
190             blossom(u, v, p);
191             blossom(v, u, p);
192         }
193     }
194 }
195
196 };
197
198 // find a maximal matching greedily (decrease constant)
199 auto greedy = [&]() {
200     for (int u = 0; u < n; ++u) {
201         if (match[u] != -1)
202             continue;
203         for (auto v : e[u]) {
204             if (match[v] == -1) {
205                 match[u] = v;
206                 match[v] = u;
207                 break;
208             }
209         }
210     }
211 };
212
213 greedy();
214
215 for (int u = 0; u < n; ++u)
216     if (match[u] == -1)

```

```

217         augment(u);
218
219     return match;
220 }
221 };

```

2.5 TwoSat (2-Sat)

```

1  struct TwoSat {
2      int n;
3      vector<vector<int>> e;
4      vector<bool> ans;
5      TwoSat(int n) : n(n), e(2 * n), ans(n) {}
6      void addClause(int u, bool f, int v, bool g) {
7          e[2 * u + !f].push_back(2 * v + g);
8          e[2 * v + !g].push_back(2 * u + f);
9      }
10     bool satisfiable() {
11         vector<int> id(2 * n, -1), dfn(2 * n, -1), low(2 * n, -1);
12         vector<int> stk;
13         int now = 0, cnt = 0;
14         function<void(int)> tarjan = [&](int u) {
15             stk.push_back(u);
16             dfn[u] = low[u] = now++;
17             for (auto v : e[u]) {
18                 if (dfn[v] == -1) {
19                     tarjan(v);
20                     low[u] = min(low[u], low[v]);
21                 } else if (id[v] == -1) {
22                     low[u] = min(low[u], dfn[v]);
23                 }
24             }
25             if (dfn[u] == low[u]) {
26                 int v;
27                 do {
28                     v = stk.back();
29                     stk.pop_back();
30                     id[v] = cnt;
31                 } while (v != u);
32                 ++cnt;
33             }
34         };
35         for (int i = 0; i < 2 * n; ++i) if (dfn[i] == -1) tarjan(i);
36         for (int i = 0; i < n; ++i) {
37             if (id[2 * i] == id[2 * i + 1]) return false;
38             ans[i] = id[2 * i] > id[2 * i + 1];
39         }
40         return true;
41     }
42     vector<bool> answer() { return ans; }
43 };

```

2.6 最大流 (MaxFlow 新版)

```

1  constexpr int inf = 1E9;
2  template<class T>
3  struct MaxFlow {
4      struct _Edge {
5          int to;
6          T cap;
7          _Edge(int to, T cap) : to(to), cap(cap) {}

```



```

8     };
9
10    int n;
11    vector<_Edge> e;
12    vector<vector<int>> g;
13    vector<int> cur, h;
14
15    MaxFlow() {}
16    MaxFlow(int n) {
17        init(n);
18    }
19
20    void init(int n) {
21        this->n = n;
22        e.clear();
23        g.assign(n, {});
24        cur.resize(n);
25        h.resize(n);
26    }
27
28    bool bfs(int s, int t) {
29        h.assign(n, -1);
30        queue<int> que;
31        h[s] = 0;
32        que.push(s);
33        while (!que.empty()) {
34            const int u = que.front();
35            que.pop();
36            for (int i : g[u]) {
37                auto [v, c] = e[i];
38                if (c > 0 && h[v] == -1) {
39                    h[v] = h[u] + 1;
40                    if (v == t) {
41                        return true;
42                    }
43                    que.push(v);
44                }
45            }
46        }
47        return false;
48    }
49
50    T dfs(int u, int t, T f) {
51        if (u == t) {
52            return f;
53        }
54        auto r = f;
55        for (int &i = cur[u]; i < int(g[u].size()); ++i) {
56            const int j = g[u][i];
57            auto [v, c] = e[j];
58            if (c > 0 && h[v] == h[u] + 1) {
59                auto a = dfs(v, t, min(r, c));
60                e[j].cap -= a;
61                e[j ^ 1].cap += a;
62                r -= a;
63                if (r == 0) {
64                    return f;
65                }
66            }
67        }
68        return f - r;
69    }
70    void addEdge(int u, int v, T c) {
71        g[u].push_back(e.size());

```

```

72     e.emplace_back(v, c);
73     g[v].push_back(e.size());
74     e.emplace_back(u, 0);
75 }
76 T flow(int s, int t) {
77     T ans = 0;
78     while (bfs(s, t)) {
79         cur.assign(n, 0);
80         ans += dfs(s, t, numeric_limits<T>::max());
81     }
82     return ans;
83 }
84
85 vector<bool> minCut() {
86     vector<bool> c(n);
87     for (int i = 0; i < n; i++) {
88         c[i] = (h[i] != -1);
89     }
90     return c;
91 }
92
93 struct Edge {
94     int from;
95     int to;
96     T cap;
97     T flow;
98 };
99 vector<Edge> edges() {
100     vector<Edge> a;
101     for (int i = 0; i < e.size(); i += 2) {
102         Edge x;
103         x.from = e[i + 1].to;
104         x.to = e[i].to;
105         x.cap = e[i].cap + e[i + 1].cap;
106         x.flow = e[i + 1].cap;
107         a.push_back(x);
108     }
109     return a;
110 }
111 };

```

2.7 费用流

2.7.1 费用流 (MCFGGraph 旧版)

```

1  /** 费用流 (MCFGGraph 旧版)
2   *   下方为最小费用**最大流**模板，如需求解最小费用**可行流**，需要去除建边限制
3   */
4  struct MCFGGraph {
5      struct Edge {
6          int v, c, f;
7          Edge(int v, int c, int f) : v(v), c(c), f(f) {}
8      };
9      const int n;
10     vector<Edge> e;
11     vector<vector<int>> g;
12     vector<i64> h, dis;
13     vector<int> pre;
14     bool dijkstra(int s, int t) {
15         dis.assign(n, numeric_limits<i64>::max());
16         pre.assign(n, -1);

```

```

17     priority_queue<pair<i64, int>, vector<pair<i64, int>>, greater<pair<i64,
18     int>>> que;
19     dis[s] = 0;
20     que.emplace(0, s);
21     while (!que.empty()) {
22         i64 d = que.top().first;
23         int u = que.top().second;
24         que.pop();
25         if (dis[u] < d) continue;
26         for (int i : g[u]) {
27             int v = e[i].v;
28             int c = e[i].c;
29             int f = e[i].f;
30             if (c > 0 && dis[v] > d + h[u] - h[v] + f) {
31                 dis[v] = d + h[u] - h[v] + f;
32                 pre[v] = i;
33                 que.emplace(dis[v], v);
34             }
35         }
36     }
37     return dis[t] != numeric_limits<i64>::max();
38 }
39 MCFGGraph(int n) : n(n), g(n) {}
40 void addEdge(int u, int v, int c, int f) {
41     // if (f < 0) {
42     g[u].push_back(e.size());
43     e.emplace_back(v, 0, f);
44     g[v].push_back(e.size());
45     e.emplace_back(u, c, -f);
46     // } else {
47     //     g[u].push_back(e.size());
48     //     e.emplace_back(v, c, f);
49     //     g[v].push_back(e.size());
50     //     e.emplace_back(u, 0, -f);
51     // }
52 }
53 pair<int, i64> flow(int s, int t) {
54     int flow = 0;
55     i64 cost = 0;
56     h.assign(n, 0);
57     while (dijkstra(s, t)) {
58         for (int i = 0; i < n; ++i) h[i] += dis[i];
59         int aug = numeric_limits<int>::max();
60         for (int i = t; i != s; i = e[pre[i] ^ 1].v) aug = min(aug,
61         e[pre[i]].c);
62         for (int i = t; i != s; i = e[pre[i] ^ 1].v) {
63             e[pre[i]].c -= aug;
64             e[pre[i] ^ 1].c += aug;
65         }
66         flow += aug;
67         cost += i64(aug) * h[t];
68     }
69     return make_pair(flow, cost);
70 }
71 };

```

2.7.2 费用流 (MinCostFlow 新版)

```

1  template<class T>
2  struct MinCostFlow {
3      struct _Edge {
4          int to;
5          T cap;
6          T cost;
7          _Edge(int to_, T cap_, T cost_) : to(to_), cap(cap_), cost(cost_) {}
8      };
9      int n;
10     vector<_Edge> e;
11     vector<vector<int>>> g;
12     vector<T> h, dis;
13     vector<int> pre;
14     bool dijkstra(int s, int t) {
15         dis.assign(n, numeric_limits<T>::max());
16         pre.assign(n, -1);
17         priority_queue<pair<T, int>, vector<pair<T, int>>, greater<pair<T, int>>>
que;
18         dis[s] = 0;
19         que.emplace(0, s);
20         while (!que.empty()) {
21             T d = que.top().first;
22             int u = que.top().second;
23             que.pop();
24             if (dis[u] != d) {
25                 continue;
26             }
27             for (int i : g[u]) {
28                 int v = e[i].to;
29                 T cap = e[i].cap;
30                 T cost = e[i].cost;
31                 if (cap > 0 && dis[v] > d + h[u] - h[v] + cost) {
32                     dis[v] = d + h[u] - h[v] + cost;
33                     pre[v] = i;
34                     que.emplace(dis[v], v);
35                 }
36             }
37         }
38         return dis[t] != numeric_limits<T>::max();
39     }
40     MinCostFlow() {}
41     MinCostFlow(int n_) {
42         init(n_);
43     }
44     void init(int n_) {
45         n = n_;
46         e.clear();
47         g.assign(n, {});
48     }
49     void addEdge(int u, int v, T cap, T cost) {
50         g[u].push_back(e.size());
51         e.emplace_back(v, cap, cost);
52         g[v].push_back(e.size());
53         e.emplace_back(u, 0, -cost);
54     }
55     pair<T, T> flow(int s, int t) {
56         T flow = 0;
57         T cost = 0;
58         h.assign(n, 0);
59         while (dijkstra(s, t)) {
60             for (int i = 0; i < n; ++i) {

```

```

61         h[i] += dis[i];
62     }
63     T aug = numeric_limits<int>::max();
64     for (int i = t; i != s; i = e[pre[i] ^ 1].to) {
65         aug = min(aug, e[pre[i]].cap);
66     }
67     for (int i = t; i != s; i = e[pre[i] ^ 1].to) {
68         e[pre[i]].cap -= aug;
69         e[pre[i] ^ 1].cap += aug;
70     }
71     flow += aug;
72     cost += aug * h[t];
73 }
74 return make_pair(flow, cost);
75 }
76 struct Edge {
77     int from;
78     int to;
79     T cap;
80     T cost;
81     T flow;
82 };
83 vector<Edge> edges() {
84     vector<Edge> a;
85     for (int i = 0; i < e.size(); i += 2) {
86         Edge x;
87         x.from = e[i + 1].to;
88         x.to = e[i].to;
89         x.cap = e[i].cap + e[i + 1].cap;
90         x.cost = e[i].cost;
91         x.flow = e[i + 1].cap;
92         a.push_back(x);
93     }
94     return a;
95 }
96 };

```

2.8 树链剖分 (HLD)

```

1  struct HLD {
2      int n;
3      vector<int> siz, top, dep, parent, in, out, seq;
4      vector<vector<int>> adj;
5      int cur;
6
7      HLD() {}
8      HLD(int n) {
9          init(n);
10     }
11     void init(int n) {
12         this->n = n;
13         siz.resize(n);
14         top.resize(n);
15         dep.resize(n);
16         parent.resize(n);
17         in.resize(n);
18         out.resize(n);
19         seq.resize(n);
20         cur = 0;
21         adj.assign(n, {});
22     }
23     void addEdge(int u, int v) {

```

```

24     adj[u].push_back(v);
25     adj[v].push_back(u);
26 }
27 void work(int root = 0) {
28     top[root] = root;
29     dep[root] = 0;
30     parent[root] = -1;
31     dfs1(root);
32     dfs2(root);
33 }
34 void dfs1(int u) {
35     if (parent[u] != -1) {
36         adj[u].erase(find(adj[u].begin(), adj[u].end(), parent[u]));
37     }
38
39     siz[u] = 1;
40     for (auto &v : adj[u]) {
41         parent[v] = u;
42         dep[v] = dep[u] + 1;
43         dfs1(v);
44         siz[u] += siz[v];
45         if (siz[v] > siz[adj[u][0]]) {
46             swap(v, adj[u][0]);
47         }
48     }
49 }
50 void dfs2(int u) {
51     in[u] = cur++;
52     seq[in[u]] = u;
53     for (auto v : adj[u]) {
54         top[v] = v == adj[u][0] ? top[u] : v;
55         dfs2(v);
56     }
57     out[u] = cur;
58 }
59 int lca(int u, int v) {
60     while (top[u] != top[v]) {
61         if (dep[top[u]] > dep[top[v]]) {
62             u = parent[top[u]];
63         } else {
64             v = parent[top[v]];
65         }
66     }
67     return dep[u] < dep[v] ? u : v;
68 }
69
70 int dist(int u, int v) {
71     return dep[u] + dep[v] - 2 * dep[lca(u, v)];
72 }
73
74 int jump(int u, int k) {
75     if (dep[u] < k) {
76         return -1;
77     }
78
79     int d = dep[u] - k;
80
81     while (dep[top[u]] > d) {
82         u = parent[top[u]];
83     }
84
85     return seq[in[u] - dep[u] + d];
86 }
87

```

```

88     bool isAncestor(int u, int v) {
89         return in[u] <= in[v] && in[v] < out[u];
90     }
91
92     int rootedParent(int u, int v) {
93         swap(u, v);
94         if (u == v) {
95             return u;
96         }
97         if (!isAncestor(u, v)) {
98             return parent[u];
99         }
100         auto it = upper_bound(adj[u].begin(), adj[u].end(), v, [&](int x, int y) {
101             return in[x] < in[y];
102         }) - 1;
103         return *it;
104     }
105
106     int rootedSize(int u, int v) {
107         if (u == v) {
108             return n;
109         }
110         if (!isAncestor(v, u)) {
111             return siz[v];
112         }
113         return n - siz[rootedParent(u, v)];
114     }
115
116     int rootedLca(int a, int b, int c) {
117         return lca(a, b) ^ lca(b, c) ^ lca(c, a);
118     }
119 };

```

/END/

3 数论、几何、多项式

3.1 快速幂

```

1  /** 快速幂 - 普通版 */
2  int power(int a, i64 b, int p) {
3      int res = 1;
4      for (; b; b /= 2, a = 1LL * a * a % p) {
5          if (b % 2) {
6              res = 1LL * res * a % p;
7          }
8      }
9      return res;
10 }
11 /** 快速幂 - 手写乘法 */
12 i64 mul(i64 a, i64 b, i64 p) {
13     i64 c = a * b - i64(1.0L * a * b / p) * p;
14     c %= p;
15     if (c < 0) {
16         c += p;
17     }
18     return c;
19 }
20 i64 power(i64 a, i64 b, i64 p) {
21     i64 res = 1;
22     for (; b; b /= 2, a = mul(a, a, p)) {
23         if (b % 2) {
24             res = mul(res, a, p);
25         }
26     }
27     return res;
28 }

```

3.2 基姆拉尔森公式

```

1  const int d[] = {31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};
2
3  bool isLeap(int y) {
4      return y % 400 == 0 || (y % 4 == 0 && y % 100 != 0);
5  }
6
7  int daysInMonth(int y, int m) {
8      return d[m - 1] + (isLeap(y) && m == 2);
9  }
10
11 int getDay(int y, int m, int d) {
12     int ans = 0;
13     for (int i = 1970; i < y; i++) {
14         ans += 365 + isLeap(i);
15     }
16     for (int i = 1; i < m; i++) {
17         ans += daysInMonth(y, i);
18     }
19     ans += d;
20     return (ans + 2) % 7 + 1;
21 }

```


3.3 欧拉筛

```

1  /** 欧拉筛 **/
2  vector<int> minp, primes;
3
4  void sieve(int n) {
5      minp.assign(n + 1, 0);
6      primes.clear();
7
8      for (int i = 2; i <= n; i++) {
9          if (minp[i] == 0) {
10             minp[i] = i;
11             primes.push_back(i);
12         }
13
14         for (auto p : primes) {
15             if (i * p > n) {
16                 break;
17             }
18             minp[i * p] = p;
19             if (p == minp[i]) {
20                 break;
21             }
22         }
23     }
24 }
25
26 bool isprime(int n) {
27     return minp[n] == n;
28 }
29
30 /** 欧拉筛 **/
31 void sieve(int n) {
32     minp.assign(n + 1, 0);
33     phi.assign(n + 1, 0);
34     primes.clear();
35
36     for (int i = 2; i <= n; i++) {
37         if (minp[i] == 0) {
38             minp[i] = i;
39             phi[i] = i - 1;
40             primes.push_back(i);
41         }
42
43         for (auto p : primes) {
44             if (i * p > n) {
45                 break;
46             }
47             minp[i * p] = p;
48             if (p == minp[i]) {
49                 phi[i * p] = phi[i] * p;
50                 break;
51             }
52             phi[i * p] = phi[i] * (p - 1);
53         }
54     }
55     for (int i = 2; i <= n; i++) {
56         phi[i] += phi[i - 1];
57     }
58 }

```

3.4 莫比乌斯函数筛（莫比乌斯反演）

```

1 unordered_map<int, Z> fMu;
2
3 vector<int> minp, primes, phi, mu;
4 vector<i64> sph;
5
6 void sieve(int n) {
7     minp.assign(n + 1, 0);
8     phi.assign(n + 1, 0);
9     sph.assign(n + 1, 0);
10    mu.assign(n + 1, 0);
11    primes.clear();
12    phi[1] = 1;
13    mu[1] = 1;
14
15    for (int i = 2; i <= n; i++) {
16        if (minp[i] == 0) {
17            minp[i] = i;
18            phi[i] = i - 1;
19            mu[i] = -1;
20            primes.push_back(i);
21        }
22
23        for (auto p : primes) {
24            if (i * p > n) {
25                break;
26            }
27            minp[i * p] = p;
28            if (p == minp[i]) {
29                phi[i * p] = phi[i] * p;
30                break;
31            }
32            phi[i * p] = phi[i] * (p - 1);
33            mu[i * p] = -mu[i];
34        }
35    }
36
37    for (int i = 1; i <= n; i++) {
38        sph[i] = sph[i - 1] + phi[i];
39        mu[i] += mu[i - 1];
40    }
41 }
42
43 Z sumMu(int n) {
44     if (n <= N) {
45         return mu[n];
46     }
47     if (fMu.count(n)) {
48         return fMu[n];
49     }
50     if (n == 0) {
51         return 0;
52     }
53     Z ans = 1;
54     for (int l = 2, r; l <= n; l = r + 1) {
55         r = n / (n / l);
56         ans -= (r - l + 1) * sumMu(n / l);
57     }
58     return ans;
59 }

```

3.5 扩展欧几里得 (exgcd)

```

1  /** 扩展欧几里得 (exgcd)  */
2  i64 exgcd(i64 a, i64 b, i64 &x, i64 &y) {
3      if (b == 0) {
4          x = 1;
5          y = 0;
6          return a;
7      }
8      i64 g = exgcd(b, a % b, y, x);
9      y -= a / b * x;
10     return g;
11 }
12 pair<i64, i64> sol(i64 a, i64 b, i64 m) { // ax + b = 0 (mod m)
13     assert(m > 0);
14     b *= -1;
15     i64 x, y;
16     i64 g = exgcd(a, m, x, y);
17     if (g < 0) {
18         g *= -1;
19         x *= -1;
20         y *= -1;
21     }
22     if (b % g != 0) {
23         return {-1, -1};
24     }
25     x = x * (b / g) % (m / g);
26     if (x < 0) {
27         x += m / g;
28     }
29     return {x, m / g};
30 }
31
32 /** 扩展欧几里得 (exgcd)  */
33 array<i64, 3> exgcd(i64 a, i64 b) {
34     if (!b) {
35         return {a, 1, 0};
36     }
37     auto [g, x, y] = exgcd(b, a % b);
38     return {g, y, x - a / b * y};
39 }

```

3.6 欧拉函数

3.6.1 欧拉函数 (求解单个数的欧拉函数)

```

1  int phi(int n) {
2      int res = n;
3      for (int i = 2; i * i <= n; i++) {
4          if (n % i == 0) {
5              while (n % i == 0) {
6                  n /= i;
7              }
8              res = res / i * (i - 1);
9          }
10     }
11     if (n > 1) {
12         res = res / n * (n - 1);
13     }
14     return res;
15 }

```

3.6.2 欧拉函数（求解全部数的欧拉函数）

```

1  constexpr int N = 1E7;
2  constexpr int P = 1000003;
3
4  bool isprime[N + 1];
5  int phi[N + 1];
6  vector<int> primes;
7
8  fill(isprime + 2, isprime + N + 1, true);
9  phi[1] = 1;
10 for (int i = 2; i <= N; i++) {
11     if (isprime[i]) {
12         primes.push_back(i);
13         phi[i] = i - 1;
14     }
15     for (auto p : primes) {
16         if (i * p > N) {
17             break;
18         }
19         isprime[i * p] = false;
20         if (i % p == 0) {
21             phi[i * p] = phi[i] * p;
22             break;
23         }
24         phi[i * p] = phi[i] * (p - 1);
25     }
26 }

```

3.7 组合数

3.7.1 组合数（小范围预处理，逆元+杨辉三角）

```

1  constexpr int P = 1000000007;
2  constexpr int L = 10000;
3
4  int fac[L + 1], invfac[L + 1];
5  int sumbinom[L + 1][7];
6
7  int binom(int n, int m) {
8      if (n < m || m < 0) {
9          return 0;
10     }
11     return 1LL * fac[n] * invfac[m] % P * invfac[n - m] % P;
12 }
13
14 int power(int a, int b) {
15     int res = 1;
16     for (; b /= 2, a = 1LL * a * a % P) {
17         if (b % 2) {
18             res = 1LL * res * a % P;
19         }
20     }
21     return res;
22 }
23
24 int main() {
25     fac[0] = 1;
26     for (int i = 1; i <= L; i++) {
27         fac[i] = 1LL * fac[i - 1] * i % P;
28     }
29     invfac[L] = power(fac[L], P - 2);

```

```

30     for (int i = L; i; i--) {
31         invfac[i - 1] = 1LL * invfac[i] * i % P;
32     }
33
34     sumbinom[0][0] = 1;
35     for (int i = 1; i <= L; i++) {
36         for (int j = 0; j < 7; j++) {
37             sumbinom[i][j] = (sumbinom[i - 1][j] + sumbinom[i - 1][(j + 6) % 7]) %
P;
38         }
39     }
40 }

```

3.7.2 组合数 (Comb, with. ModIntBase)

```

1  struct Comb {
2      int n;
3      vector<Z> _fac;
4      vector<Z> _invfac;
5      vector<Z> _inv;
6
7      Comb() : n{0}, _fac{1}, _invfac{1}, _inv{0} {}
8      Comb(int n) : Comb() {
9          init(n);
10     }
11
12     void init(int m) {
13         if (m <= n) return;
14         _fac.resize(m + 1);
15         _invfac.resize(m + 1);
16         _inv.resize(m + 1);
17
18         for (int i = n + 1; i <= m; i++) {
19             _fac[i] = _fac[i - 1] * i;
20         }
21         _invfac[m] = _fac[m].inv();
22         for (int i = m; i > n; i--) {
23             _invfac[i - 1] = _invfac[i] * i;
24             _inv[i] = _invfac[i] * _fac[i - 1];
25         }
26         n = m;
27     }
28
29     Z fac(int m) {
30         if (m > n) init(2 * m);
31         return _fac[m];
32     }
33     Z invfac(int m) {
34         if (m > n) init(2 * m);
35         return _invfac[m];
36     }
37     Z inv(int m) {
38         if (m > n) init(2 * m);
39         return _inv[m];
40     }
41     Z binom(int n, int m) {
42         if (n < m || m < 0) return 0;
43         return fac(n) * invfac(m) * invfac(n - m);
44     }
45 } comb;

```

3.8 素数测试与因式分解 (Miller-Rabin & Pollard-Rho)

```

1  i64 mul(i64 a, i64 b, i64 m) {
2      return static_cast<__int128>(a) * b % m;
3  }
4  i64 power(i64 a, i64 b, i64 m) {
5      i64 res = 1 % m;
6      for (; b >= 1, a = mul(a, a, m))
7          if (b & 1)
8              res = mul(res, a, m);
9      return res;
10 }
11 bool isprime(i64 n) {
12     if (n < 2)
13         return false;
14     static constexpr int A[] = {2, 3, 5, 7, 11, 13, 17, 19, 23};
15     int s = __builtin_ctzll(n - 1);
16     i64 d = (n - 1) >> s;
17     for (auto a : A) {
18         if (a == n)
19             return true;
20         i64 x = power(a, d, n);
21         if (x == 1 || x == n - 1)
22             continue;
23         bool ok = false;
24         for (int i = 0; i < s - 1; ++i) {
25             x = mul(x, x, n);
26             if (x == n - 1) {
27                 ok = true;
28                 break;
29             }
30         }
31         if (!ok)
32             return false;
33     }
34     return true;
35 }
36 vector<i64> factorize(i64 n) {
37     vector<i64> p;
38     function<void(i64)> f = [&](i64 n) {
39         if (n <= 10000) {
40             for (int i = 2; i * i <= n; ++i)
41                 for (; n % i == 0; n /= i)
42                     p.push_back(i);
43             if (n > 1)
44                 p.push_back(n);
45             return;
46         }
47         if (isprime(n)) {
48             p.push_back(n);
49             return;
50         }
51         auto g = [&](i64 x) {
52             return (mul(x, x, n) + 1) % n;
53         };
54         i64 x0 = 2;
55         while (true) {
56             i64 x = x0;
57             i64 y = x0;
58             i64 d = 1;
59             i64 power = 1, lam = 0;
60             i64 v = 1;
61             while (d == 1) {

```

```

62         y = g(y);
63         ++lam;
64         v = mul(v, abs(x - y), n);
65         if (lam % 127 == 0) {
66             d = gcd(v, n);
67             v = 1;
68         }
69         if (power == lam) {
70             x = y;
71             power *= 2;
72             lam = 0;
73             d = gcd(v, n);
74             v = 1;
75         }
76     }
77     if (d != n) {
78         f(d);
79         f(n / d);
80         return;
81     }
82     ++x0;
83 }
84 };
85 f(n);
86 sort(p.begin(), p.end());
87 return p;
88 }

```

3.9 平面几何

3.9.1 平面几何 (Point)

```

1  template<class T>
2  struct Point {
3      T x;
4      T y;
5      Point(const T &x_ = 0, const T &y_ = 0) : x(x_), y(y_) {}
6
7      template<class U>
8      operator Point<U>() {
9          return Point<U>(U(x), U(y));
10     }
11     Point &operator+=(const Point &p) & {
12         x += p.x;
13         y += p.y;
14         return *this;
15     }
16     Point &operator-=(const Point &p) & {
17         x -= p.x;
18         y -= p.y;
19         return *this;
20     }
21     Point &operator*=(const T &v) & {
22         x *= v;
23         y *= v;
24         return *this;
25     }
26     Point &operator/=(const T &v) & {
27         x /= v;
28         y /= v;
29         return *this;
30     }

```

```

31     Point operator-() const {
32         return Point(-x, -y);
33     }
34     friend Point operator+(Point a, const Point &b) {
35         return a += b;
36     }
37     friend Point operator-(Point a, const Point &b) {
38         return a -= b;
39     }
40     friend Point operator*(Point a, const T &b) {
41         return a *= b;
42     }
43     friend Point operator/(Point a, const T &b) {
44         return a /= b;
45     }
46     friend Point operator*(const T &a, Point b) {
47         return b *= a;
48     }
49     friend bool operator==(const Point &a, const Point &b) {
50         return a.x == b.x && a.y == b.y;
51     }
52     friend istream &operator>>(istream &is, Point &p) {
53         return is >> p.x >> p.y;
54     }
55     friend ostream &operator<<(ostream &os, const Point &p) {
56         return os << "(" << p.x << ", " << p.y << ")";
57     }
58 };
59
60 template<class T>
61 struct Line {
62     Point<T> a;
63     Point<T> b;
64     Line(const Point<T> &a_ = Point<T>(), const Point<T> &b_ = Point<T>()) : a(a_),
65     b(b_) {}
66 };
67
68 template<class T>
69 T dot(const Point<T> &a, const Point<T> &b) {
70     return a.x * b.x + a.y * b.y;
71 }
72
73 template<class T>
74 T cross(const Point<T> &a, const Point<T> &b) {
75     return a.x * b.y - a.y * b.x;
76 }
77
78 template<class T>
79 T square(const Point<T> &p) {
80     return dot(p, p);
81 }
82
83 template<class T>
84 double length(const Point<T> &p) {
85     return sqrt(square(p));
86 }
87
88 template<class T>
89 double length(const Line<T> &l) {
90     return length(l.a - l.b);
91 }
92
93 template<class T>
94 Point<T> normalize(const Point<T> &p) {

```



```

94     return p / length(p);
95 }
96
97 template<class T>
98 bool parallel(const Line<T> &l1, const Line<T> &l2) {
99     return cross(l1.b - l1.a, l2.b - l2.a) == 0;
100 }
101
102 template<class T>
103 double distance(const Point<T> &a, const Point<T> &b) {
104     return length(a - b);
105 }
106
107 template<class T>
108 double distancePL(const Point<T> &p, const Line<T> &l) {
109     return abs(cross(l.a - l.b, l.a - p)) / length(l);
110 }
111
112 template<class T>
113 double distancePS(const Point<T> &p, const Line<T> &l) {
114     if (dot(p - l.a, l.b - l.a) < 0) {
115         return distance(p, l.a);
116     }
117     if (dot(p - l.b, l.a - l.b) < 0) {
118         return distance(p, l.b);
119     }
120     return distancePL(p, l);
121 }
122
123 template<class T>
124 Point<T> rotate(const Point<T> &a) {
125     return Point(-a.y, a.x);
126 }
127
128 template<class T>
129 int sgn(const Point<T> &a) {
130     return a.y > 0 || (a.y == 0 && a.x > 0) ? 1 : -1;
131 }
132
133 template<class T>
134 bool pointOnLineLeft(const Point<T> &p, const Line<T> &l) {
135     return cross(l.b - l.a, p - l.a) > 0;
136 }
137
138 template<class T>
139 Point<T> lineIntersection(const Line<T> &l1, const Line<T> &l2) {
140     return l1.a + (l1.b - l1.a) * (cross(l2.b - l2.a, l1.a - l2.a) / cross(l2.b -
141     l2.a, l1.a - l1.b));
142 }
143
144 template<class T>
145 bool pointOnSegment(const Point<T> &p, const Line<T> &l) {
146     return cross(p - l.a, l.b - l.a) == 0 && min(l.a.x, l.b.x) <= p.x && p.x <=
147     max(l.a.x, l.b.x)
148     && min(l.a.y, l.b.y) <= p.y && p.y <= max(l.a.y, l.b.y);
149 }
150
151 template<class T>
152 bool pointInPolygon(const Point<T> &a, const vector<Point<T>> &p) {
153     int n = p.size();
154     for (int i = 0; i < n; i++) {
155         if (pointOnSegment(a, Line(p[i], p[(i + 1) % n]))) {
156             return true;
157         }
158     }
159 }

```

```

156     }
157
158     int t = 0;
159     for (int i = 0; i < n; i++) {
160         auto u = p[i];
161         auto v = p[(i + 1) % n];
162         if (u.x < a.x && v.x >= a.x && pointOnLineLeft(a, Line(v, u))) {
163             t ^= 1;
164         }
165         if (u.x >= a.x && v.x < a.x && pointOnLineLeft(a, Line(u, v))) {
166             t ^= 1;
167         }
168     }
169
170     return t == 1;
171 }
172
173 // 0 : not intersect
174 // 1 : strictly intersect
175 // 2 : overlap
176 // 3 : intersect at endpoint
177 template<class T>
178 tuple<int, Point<T>, Point<T>> segmentIntersection(const Line<T> &l1, const Line<T>
&l2) {
179     if (max(l1.a.x, l1.b.x) < min(l2.a.x, l2.b.x)) {
180         return {0, Point<T>(), Point<T>()};
181     }
182     if (min(l1.a.x, l1.b.x) > max(l2.a.x, l2.b.x)) {
183         return {0, Point<T>(), Point<T>()};
184     }
185     if (max(l1.a.y, l1.b.y) < min(l2.a.y, l2.b.y)) {
186         return {0, Point<T>(), Point<T>()};
187     }
188     if (min(l1.a.y, l1.b.y) > max(l2.a.y, l2.b.y)) {
189         return {0, Point<T>(), Point<T>()};
190     }
191     if (cross(l1.b - l1.a, l2.b - l2.a) == 0) {
192         if (cross(l1.b - l1.a, l2.a - l1.a) != 0) {
193             return {0, Point<T>(), Point<T>()};
194         } else {
195             auto maxx1 = max(l1.a.x, l1.b.x);
196             auto minx1 = min(l1.a.x, l1.b.x);
197             auto maxy1 = max(l1.a.y, l1.b.y);
198             auto miny1 = min(l1.a.y, l1.b.y);
199             auto maxx2 = max(l2.a.x, l2.b.x);
200             auto minx2 = min(l2.a.x, l2.b.x);
201             auto maxy2 = max(l2.a.y, l2.b.y);
202             auto miny2 = min(l2.a.y, l2.b.y);
203             Point<T> p1(max(minx1, minx2), max(miny1, miny2));
204             Point<T> p2(min(maxx1, maxx2), min(maxy1, maxy2));
205             if (!pointOnSegment(p1, l1)) {
206                 swap(p1.y, p2.y);
207             }
208             if (p1 == p2) {
209                 return {3, p1, p2};
210             } else {
211                 return {2, p1, p2};
212             }
213         }
214     }
215     auto cp1 = cross(l2.a - l1.a, l2.b - l1.a);
216     auto cp2 = cross(l2.a - l1.b, l2.b - l1.b);
217     auto cp3 = cross(l1.a - l2.a, l1.b - l2.a);
218     auto cp4 = cross(l1.a - l2.b, l1.b - l2.b);

```

```

219
220     if ((cp1 > 0 && cp2 > 0) || (cp1 < 0 && cp2 < 0) || (cp3 > 0 && cp4 > 0) ||
221         (cp3 < 0 && cp4 < 0)) {
222         return {0, Point<T>(), Point<T>()};
223     }
224     Point p = lineIntersection(l1, l2);
225     if (cp1 != 0 && cp2 != 0 && cp3 != 0 && cp4 != 0) {
226         return {1, p, p};
227     } else {
228         return {3, p, p};
229     }
230 }
231
232 template<class T>
233 double distanceSS(const Line<T> &l1, const Line<T> &l2) {
234     if (get<0>(segmentIntersection(l1, l2)) != 0) {
235         return 0.0;
236     }
237     return min({distancePS(l1.a, l2), distancePS(l1.b, l2), distancePS(l2.a, l1),
238         distancePS(l2.b, l1)});
239 }
240
241 template<class T>
242 bool segmentInPolygon(const Line<T> &l, const vector<Point<T>> &p) {
243     int n = p.size();
244     if (!pointInPolygon(l.a, p)) {
245         return false;
246     }
247     if (!pointInPolygon(l.b, p)) {
248         return false;
249     }
250     for (int i = 0; i < n; i++) {
251         auto u = p[i];
252         auto v = p[(i + 1) % n];
253         auto w = p[(i + 2) % n];
254         auto [t, p1, p2] = segmentIntersection(l, Line(u, v));
255
256         if (t == 1) {
257             return false;
258         }
259         if (t == 0) {
260             continue;
261         }
262         if (t == 2) {
263             if (pointOnSegment(v, l) && v != l.a && v != l.b) {
264                 if (cross(v - u, w - v) > 0) {
265                     return false;
266                 }
267             }
268         } else {
269             if (p1 != u && p1 != v) {
270                 if (pointOnLineLeft(l.a, Line(v, u))
271                     || pointOnLineLeft(l.b, Line(v, u))) {
272                     return false;
273                 }
274             } else if (p1 == v) {
275                 if (l.a == v) {
276                     if (pointOnLineLeft(u, l)) {
277                         if (pointOnLineLeft(w, l)
278                             && pointOnLineLeft(w, Line(u, v))) {
279                             return false;
280                         }
281                     }
282                 } else {
283                     }
284                 }
285             }
286         }
287     }
288 }

```

```

281         if (pointOnLineLeft(w, l)
282             || pointOnLineLeft(w, Line(u, v))) {
283             return false;
284         }
285     }
286     } else if (l.b == v) {
287         if (pointOnLineLeft(u, Line(l.b, l.a))) {
288             if (pointOnLineLeft(w, Line(l.b, l.a))
289                 && pointOnLineLeft(w, Line(u, v))) {
290                 return false;
291             }
292         } else {
293             if (pointOnLineLeft(w, Line(l.b, l.a))
294                 || pointOnLineLeft(w, Line(u, v))) {
295                 return false;
296             }
297         }
298     } else {
299         if (pointOnLineLeft(u, l)) {
300             if (pointOnLineLeft(w, Line(l.b, l.a))
301                 || pointOnLineLeft(w, Line(u, v))) {
302                 return false;
303             }
304         } else {
305             if (pointOnLineLeft(w, l)
306                 || pointOnLineLeft(w, Line(u, v))) {
307                 return false;
308             }
309         }
310     }
311 }
312 }
313 }
314 return true;
315 }
316
317 template<class T>
318 vector<Point<T>> hp(vector<Line<T>> lines) {
319     sort(lines.begin(), lines.end(), [&](auto l1, auto l2) {
320         auto d1 = l1.b - l1.a;
321         auto d2 = l2.b - l2.a;
322
323         if (sgn(d1) != sgn(d2)) {
324             return sgn(d1) == 1;
325         }
326
327         return cross(d1, d2) > 0;
328     });
329
330     deque<Line<T>> ls;
331     deque<Point<T>> ps;
332     for (auto l : lines) {
333         if (ls.empty()) {
334             ls.push_back(l);
335             continue;
336         }
337
338         while (!ps.empty() && !pointOnLineLeft(ps.back(), l)) {
339             ps.pop_back();
340             ls.pop_back();
341         }
342
343         while (!ps.empty() && !pointOnLineLeft(ps[0], l)) {
344             ps.pop_front();

```

```

345         ls.pop_front();
346     }
347
348     if (cross(l.b - l.a, ls.back().b - ls.back().a) == 0) {
349         if (dot(l.b - l.a, ls.back().b - ls.back().a) > 0) {
350
351             if (!pointOnLineLeft(ls.back().a, l)) {
352                 assert(ls.size() == 1);
353                 ls[0] = l;
354             }
355             continue;
356         }
357         return {};
358     }
359
360     ps.push_back(lineIntersection(ls.back(), l));
361     ls.push_back(l);
362 }
363
364 while (!ps.empty() && !pointOnLineLeft(ps.back(), ls[0])) {
365     ps.pop_back();
366     ls.pop_back();
367 }
368 if (ls.size() <= 2) {
369     return {};
370 }
371 ps.push_back(lineIntersection(ls[0], ls.back()));
372
373 return vector(ps.begin(), ps.end());
374 }
375
376 using real = long double;
377 using P = Point<real>;
378
379 constexpr real eps = 0;

```

3.9.2 平面几何 (with. complex)

```

1  using Point = complex<long double>;
2
3  #define x real
4  #define y imag
5
6  long double dot(const Point &a, const Point &b) {
7      return (conj(a) * b).x();
8  }
9
10 long double cross(const Point &a, const Point &b) {
11     return (conj(a) * b).y();
12 }
13
14 long double length(const Point &a) {
15     return sqrt(dot(a, a));
16 }
17
18 long double dist(const Point &a, const Point &b) {
19     return length(a - b);
20 }
21
22 long double get(const Point &a, const Point &b, const Point &c, const Point &d) {
23     auto e = a + (b - a) * cross(c - a, d - a) / cross(b - a, d - c);
24     return dist(d, e);

```

25 }

3.10 立体几何 (Point)

```

1  using i64 = long long;
2  using real = double;
3
4  struct Point {
5      real x = 0;
6      real y = 0;
7      real z = 0;
8  };
9
10 Point operator+(const Point &a, const Point &b) {
11     return {a.x + b.x, a.y + b.y, a.z + b.z};
12 }
13
14 Point operator-(const Point &a, const Point &b) {
15     return {a.x - b.x, a.y - b.y, a.z - b.z};
16 }
17
18 Point operator*(const Point &a, real b) {
19     return {a.x * b, a.y * b, a.z * b};
20 }
21
22 Point operator/(const Point &a, real b) {
23     return {a.x / b, a.y / b, a.z / b};
24 }
25
26 real length(const Point &a) {
27     return hypot(a.x, a.y, a.z);
28 }
29
30 Point normalize(const Point &a) {
31     real l = length(a);
32     return {a.x / l, a.y / l, a.z / l};
33 }
34
35 real getAng(real a, real b, real c) {
36     return acos((a * a + b * b - c * c) / 2 / a / b);
37 }
38
39 ostream &operator<<(ostream &os, const Point &a) {
40     return os << "(" << a.x << ", " << a.y << ", " << a.z << ")";
41 }
42
43 real dot(const Point &a, const Point &b) {
44     return a.x * b.x + a.y * b.y + a.z * b.z;
45 }
46
47 Point cross(const Point &a, const Point &b) {
48     return {
49         a.y * b.z - a.z * b.y,
50         a.z * b.x - a.x * b.z,
51         a.x * b.y - a.y * b.x
52     };
53 }

```

3.11 静态凸包

3.11.1 静态凸包 (with. Point, 新版)

```

1  struct Point {
2      i64 x;
3      i64 y;
4      Point() : x{0}, y{0} {}
5      Point(i64 x_, i64 y_) : x{x_}, y{y_} {}
6  };
7
8  i64 dot(Point a, Point b) {
9      return a.x * b.x + a.y * b.y;
10 }
11
12 i64 cross(Point a, Point b) {
13     return a.x * b.y - a.y * b.x;
14 }
15
16 Point operator+(Point a, Point b) {
17     return Point(a.x + b.x, a.y + b.y);
18 }
19
20 Point operator-(Point a, Point b) {
21     return Point(a.x - b.x, a.y - b.y);
22 }
23
24 auto getHull(vector<Point> p) {
25     sort(p.begin(), p.end(),
26         [&](auto a, auto b) {
27             return a.x < b.x || (a.x == b.x && a.y < b.y);
28         });
29
30     vector<Point> hi, lo;
31     for (auto p : p) {
32         while (hi.size() > 1 && cross(hi.back() - hi[hi.size() - 2], p - hi.back())
33             >= 0) {
34             hi.pop_back();
35         }
36         while (!hi.empty() && hi.back().x == p.x) {
37             hi.pop_back();
38         }
39         hi.push_back(p);
40         while (lo.size() > 1 && cross(lo.back() - lo[lo.size() - 2], p - lo.back())
41             <= 0) {
42             lo.pop_back();
43         }
44         if (lo.empty() || lo.back().x < p.x) {
45             lo.push_back(p);
46         }
47     }
48     return make_pair(hi, lo);
49 }
50
51 const double inf = INFINITY;

```

3.11.2 静态凸包 (with. complex)

```

1  using Point = complex<i64>;
2
3  #define x real
4  #define y imag
5
6  auto dot(const Point &a, const Point &b) {
7      return (conj(a) * b).x();
8  }
9
10 auto cross(const Point &a, const Point &b) {
11     return (conj(a) * b).y();
12 }
13
14 auto rot(const Point &p) {
15     return Point(-p.y(), p.x());
16 }
17
18 auto complexHull(vector<Point> a) {
19     sort(a.begin(), a.end(), [&](auto a, auto b) {
20         if (a.x() != b.x()) {
21             return a.x() < b.x();
22         } else {
23             return a.y() < b.y();
24         }
25     });
26
27     vector<Point> l, h;
28
29     for (auto p : a) {
30         while (l.size() > 1 && cross(l.back() - l[l.size() - 2], p - l.back()) <= 0)
31         {
32             l.pop_back();
33         }
34         while (h.size() > 1 && cross(h.back() - h[h.size() - 2], p - h.back()) >= 0)
35         {
36             h.pop_back();
37         }
38         l.push_back(p);
39         h.push_back(p);
40     }
41
42     reverse(h.begin(), h.end());
43
44     h.insert(h.end(), l.begin() + 1, l.end() - 1);
45
46     return h;
47 }
48
49 int sgn(Point p) {
50     if (p.y() > 0 || (p.y() == 0 && p.x() < 0)) {
51         return 0;
52     } else {
53         return 1;
54     }
55 }

```


3.12 多项式

3.12.1 多项式 (Poly, 旧版)

```

1  constexpr int C = 1024;
2  constexpr int P = 998244353;
3  vector<int> rev, roots{0, 1};
4  int power(int a, int b) {
5      int res = 1;
6      for (; b; b >>= 1, a = 1ll * a * a % P)
7          if (b & 1)
8              res = 1ll * res * a % P;
9      return res;
10 }
11 void dft(vector<int> &a) {
12     int n = a.size();
13     if (int(rev.size()) != n) {
14         int k = __builtin_ctz(n) - 1;
15         rev.resize(n);
16         for (int i = 0; i < n; ++i)
17             rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
18     }
19     for (int i = 0; i < n; ++i)
20         if (rev[i] < i)
21             swap(a[i], a[rev[i]]);
22     if (int(roots.size()) < n) {
23         int k = __builtin_ctz(roots.size());
24         roots.resize(n);
25         while ((1 << k) < n) {
26             int e = power(3, (P - 1) >> (k + 1));
27             for (int i = 1 << (k - 1); i < (1 << k); ++i) {
28                 roots[2 * i] = roots[i];
29                 roots[2 * i + 1] = 1ll * roots[i] * e % P;
30             }
31             ++k;
32         }
33     }
34     for (int k = 1; k < n; k *= 2) {
35         for (int i = 0; i < n; i += 2 * k) {
36             for (int j = 0; j < k; ++j) {
37                 int u = a[i + j];
38                 int v = 1ll * a[i + j + k] * roots[k + j] % P;
39                 int x = u + v;
40                 if (x >= P)
41                     x -= P;
42                 a[i + j] = x;
43                 x = u - v;
44                 if (x < 0)
45                     x += P;
46                 a[i + j + k] = x;
47             }
48         }
49     }
50 }
51 void idft(vector<int> &a) {
52     int n = a.size();
53     reverse(a.begin() + 1, a.end());
54     dft(a);
55     int inv = power(n, P - 2);
56     for (int i = 0; i < n; ++i)
57         a[i] = 1ll * a[i] * inv % P;
58 }
59 struct Poly {

```

```

60     vector<int> a;
61     Poly() {}
62     Poly(int a0) {
63         if (a0)
64             a = {a0};
65     }
66     Poly(const vector<int> &a1) : a(a1) {
67         while (!a.empty() && !a.back())
68             a.pop_back();
69     }
70     int size() const {
71         return a.size();
72     }
73     int operator[](int idx) const {
74         if (idx < 0 || idx >= size())
75             return 0;
76         return a[idx];
77     }
78     Poly mulxk(int k) const {
79         auto b = a;
80         b.insert(b.begin(), k, 0);
81         return Poly(b);
82     }
83     Poly modxk(int k) const {
84         k = min(k, size());
85         return Poly(vector<int>(a.begin(), a.begin() + k));
86     }
87     Poly divxk(int k) const {
88         if (size() <= k)
89             return Poly();
90         return Poly(vector<int>(a.begin() + k, a.end()));
91     }
92     friend Poly operator+(const Poly a, const Poly &b) {
93         vector<int> res(max(a.size(), b.size()));
94         for (int i = 0; i < int(res.size()); ++i) {
95             res[i] = a[i] + b[i];
96             if (res[i] >= P)
97                 res[i] -= P;
98         }
99         return Poly(res);
100     }
101     friend Poly operator-(const Poly a, const Poly &b) {
102         vector<int> res(max(a.size(), b.size()));
103         for (int i = 0; i < int(res.size()); ++i) {
104             res[i] = a[i] - b[i];
105             if (res[i] < 0)
106                 res[i] += P;
107         }
108         return Poly(res);
109     }
110     friend Poly operator*(Poly a, Poly b) {
111         int sz = 1, tot = a.size() + b.size() - 1;
112         while (sz < tot)
113             sz *= 2;
114         a.a.resize(sz);
115         b.a.resize(sz);
116         dft(a.a);
117         dft(b.a);
118         for (int i = 0; i < sz; ++i)
119             a.a[i] = 1ll * a[i] * b[i] % P;
120         idft(a.a);
121         return Poly(a.a);
122     }
123     Poly &operator+=(Poly b) {

```

```

124     return (*this) = (*this) + b;
125 }
126 Poly &operator--(Poly b) {
127     return (*this) = (*this) - b;
128 }
129 Poly &operator*=(Poly b) {
130     return (*this) = (*this) * b;
131 }
132 Poly deriv() const {
133     if (a.empty())
134         return Poly();
135     vector<int> res(size() - 1);
136     for (int i = 0; i < size() - 1; ++i)
137         res[i] = 11l * (i + 1) * a[i + 1] % P;
138     return Poly(res);
139 }
140 Poly integr() const {
141     if (a.empty())
142         return Poly();
143     vector<int> res(size() + 1);
144     for (int i = 0; i < size(); ++i)
145         res[i + 1] = 11l * a[i] * power(i + 1, P - 2) % P;
146     return Poly(res);
147 }
148 Poly inv(int m) const {
149     Poly x(power(a[0], P - 2));
150     int k = 1;
151     while (k < m) {
152         k *= 2;
153         x = (x * (2 - modxk(k) * x)).modxk(k);
154     }
155     return x.modxk(m);
156 }
157 Poly log(int m) const {
158     return (deriv() * inv(m)).integr().modxk(m);
159 }
160 Poly exp(int m) const {
161     Poly x(1);
162     int k = 1;
163     while (k < m) {
164         k *= 2;
165         x = (x * (1 - x.log(k) + modxk(k))).modxk(k);
166     }
167     return x.modxk(m);
168 }
169 Poly sqrt(int m) const {
170     Poly x(1);
171     int k = 1;
172     while (k < m) {
173         k *= 2;
174         x = (x + (modxk(k) * x.inv(k)).modxk(k)) * ((P + 1) / 2);
175     }
176     return x.modxk(m);
177 }
178 Poly mult(Poly b) const {
179     if (b.size() == 0)
180         return Poly();
181     int n = b.size();
182     reverse(b.a.begin(), b.a.end());
183     return ((*this) * b).divxk(n - 1);
184 }
185 vector<int> eval(vector<int> x) const {
186     if (size() == 0)
187         return vector<int>(x.size(), 0);

```

```

188     const int n = max(int(x.size()), size());
189     vector<Poly> q(4 * n);
190     vector<int> ans(x.size());
191     x.resize(n);
192     function<void(int, int, int)> build = [&](int p, int l, int r) {
193         if (r - l == 1) {
194             q[p] = vector<int>{1, (P - x[l]) % P};
195         } else {
196             int m = (l + r) / 2;
197             build(2 * p, l, m);
198             build(2 * p + 1, m, r);
199             q[p] = q[2 * p] * q[2 * p + 1];
200         }
201     };
202     build(1, 0, n);
203     function<void(int, int, int, const Poly &)> work = [&](int p, int l, int r,
const Poly &num) {
204         if (r - l == 1) {
205             if (l < int(ans.size()))
206                 ans[l] = num[0];
207         } else {
208             int m = (l + r) / 2;
209             work(2 * p, l, m, num.mulT(q[2 * p + 1]).modxk(m - 1));
210             work(2 * p + 1, m, r, num.mulT(q[2 * p]).modxk(r - m));
211         }
212     };
213     work(1, 0, n, mulT(q[1].inv(n)));
214     return ans;
215 }
216 };
217 using i64 = long long;
218 void dft(vector<vector<int>> &a) {
219     int n = a.size();
220     for (auto &v : a) {
221         dft(v);
222     }
223     for (int i = 0; i < int(a[0].size()); i++) {
224         vector<int> v(n);
225         for (int j = 0; j < n; j++) {
226             v[j] = a[j][i];
227         }
228         dft(v);
229         for (int j = 0; j < n; j++) {
230             a[j][i] = v[j];
231         }
232     }
233 }
234 void idft(vector<vector<int>> &a) {
235     int n = a.size();
236     for (auto &v : a) {
237         idft(v);
238     }
239     for (int i = 0; i < int(a[0].size()); i++) {
240         vector<int> v(n);
241         for (int j = 0; j < n; j++) {
242             v[j] = a[j][i];
243         }
244         idft(v);
245         for (int j = 0; j < n; j++) {
246             a[j][i] = v[j];
247         }
248     }
249 }
250 auto inv(const vector<vector<int>> &a) {

```

```

251     int m = 1;
252     vector g(1, vector{Poly(a[0]).inv(C).a});
253     while (m < C) {
254         vector a0(4 * m, vector<int>(4 * C));
255         for (int i = 0; i < 2 * m; i++) {
256             for (int j = 0; j < C; j++) {
257                 a0[i][j] = a[i][j];
258             }
259         }
260         dft(a0);
261         g.resize(4 * m);
262         for (auto &v : g) {
263             v.resize(4 * C);
264         }
265         dft(g);
266         for (int i = 0; i < 4 * m; i++) {
267             for (int j = 0; j < 4 * C; j++) {
268                 g[i][j] = i64(g[i][j]) * (2 + i64(P - a0[i][j]) * g[i][j] % P) % P;
269             }
270         }
271         idft(g);
272         m *= 2;
273         g.resize(m);
274         for (auto &v : g) {
275             v.resize(C);
276         }
277     }
278     return g;
279 }

```

3.12.2 多项式 (Poly, with. MInt & MLong)

```

1  vector<int> rev;
2  template<int P>
3  vector<MInt<P>> roots{0, 1};
4
5  template<int P>
6  constexpr MInt<P> findPrimitiveRoot() {
7      MInt<P> i = 2;
8      int k = __builtin_ctz(P - 1);
9      while (true) {
10         if (power(i, (P - 1) / 2) != 1) {
11             break;
12         }
13         i += 1;
14     }
15     return power(i, (P - 1) >> k);
16 }
17
18 template<int P>
19 constexpr MInt<P> primitiveRoot = findPrimitiveRoot<P>();
20
21 template<>
22 constexpr MInt<998244353> primitiveRoot<998244353> {31};
23
24 template<int P>
25 constexpr void dft(vector<MInt<P>> &a) {
26     int n = a.size();
27
28     if (int(rev.size()) != n) {
29         int k = __builtin_ctz(n) - 1;
30         rev.resize(n);
31         for (int i = 0; i < n; i++) {

```

```

32         rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
33     }
34 }
35
36 for (int i = 0; i < n; i++) {
37     if (rev[i] < i) {
38         swap(a[i], a[rev[i]]);
39     }
40 }
41 if (roots<P>.size() < n) {
42     int k = __builtin_ctz(roots<P>.size());
43     roots<P>.resize(n);
44     while ((1 << k) < n) {
45         auto e = power(primitiveRoot<P>, 1 << (__builtin_ctz(P - 1) - k - 1));
46         for (int i = 1 << (k - 1); i < (1 << k); i++) {
47             roots<P>[2 * i] = roots<P>[i];
48             roots<P>[2 * i + 1] = roots<P>[i] * e;
49         }
50         k++;
51     }
52 }
53 for (int k = 1; k < n; k *= 2) {
54     for (int i = 0; i < n; i += 2 * k) {
55         for (int j = 0; j < k; j++) {
56             MInt<P> u = a[i + j];
57             MInt<P> v = a[i + j + k] * roots<P>[k + j];
58             a[i + j] = u + v;
59             a[i + j + k] = u - v;
60         }
61     }
62 }
63 }
64
65 template<int P>
66 constexpr void idft(vector<MInt<P>> &a) {
67     int n = a.size();
68     reverse(a.begin() + 1, a.end());
69     dft(a);
70     MInt<P> inv = (1 - P) / n;
71     for (int i = 0; i < n; i++) {
72         a[i] *= inv;
73     }
74 }
75
76 template<int P = 998244353>
77 struct Poly : public vector<MInt<P>> {
78     using Value = MInt<P>;
79
80     Poly() : vector<Value>() {}
81     explicit constexpr Poly(int n) : vector<Value>(n) {}
82
83     explicit constexpr Poly(const vector<Value> &a) : vector<Value>(a) {}
84     constexpr Poly(const initializer_list<Value> &a) : vector<Value>(a) {}
85
86     template<class InputIt, class = _RequireInputIter<InputIt>>
87     explicit constexpr Poly(InputIt first, InputIt last) : vector<Value>(first,
88 last) {}
89
90     template<class F>
91     explicit constexpr Poly(int n, F f) : vector<Value>(n) {
92         for (int i = 0; i < n; i++) {
93             (*this)[i] = f(i);
94         }
95     }

```

```

95
96 constexpr Poly shift(int k) const {
97     if (k >= 0) {
98         auto b = *this;
99         b.insert(b.begin(), k, 0);
100         return b;
101     } else if (this->size() <= -k) {
102         return Poly();
103     } else {
104         return Poly(this->begin() + (-k), this->end());
105     }
106 }
107 constexpr Poly trunc(int k) const {
108     Poly f = *this;
109     f.resize(k);
110     return f;
111 }
112 constexpr friend Poly operator+(const Poly &a, const Poly &b) {
113     Poly res(max(a.size(), b.size()));
114     for (int i = 0; i < a.size(); i++) {
115         res[i] += a[i];
116     }
117     for (int i = 0; i < b.size(); i++) {
118         res[i] += b[i];
119     }
120     return res;
121 }
122 constexpr friend Poly operator-(const Poly &a, const Poly &b) {
123     Poly res(max(a.size(), b.size()));
124     for (int i = 0; i < a.size(); i++) {
125         res[i] += a[i];
126     }
127     for (int i = 0; i < b.size(); i++) {
128         res[i] -= b[i];
129     }
130     return res;
131 }
132 constexpr friend Poly operator-(const Poly &a) {
133     vector<Value> res(a.size());
134     for (int i = 0; i < int(res.size()); i++) {
135         res[i] = -a[i];
136     }
137     return Poly(res);
138 }
139 constexpr friend Poly operator*(Poly a, Poly b) {
140     if (a.size() == 0 || b.size() == 0) {
141         return Poly();
142     }
143     if (a.size() < b.size()) {
144         swap(a, b);
145     }
146     int n = 1, tot = a.size() + b.size() - 1;
147     while (n < tot) {
148         n *= 2;
149     }
150     if (((P - 1) & (n - 1)) != 0 || b.size() < 128) {
151         Poly c(a.size() + b.size() - 1);
152         for (int i = 0; i < a.size(); i++) {
153             for (int j = 0; j < b.size(); j++) {
154                 c[i + j] += a[i] * b[j];
155             }
156         }
157         return c;
158     }

```

```

159     a.resize(n);
160     b.resize(n);
161     dft(a);
162     dft(b);
163     for (int i = 0; i < n; ++i) {
164         a[i] *= b[i];
165     }
166     idft(a);
167     a.resize(tot);
168     return a;
169 }
170 constexpr friend Poly operator*(Value a, Poly b) {
171     for (int i = 0; i < int(b.size()); i++) {
172         b[i] *= a;
173     }
174     return b;
175 }
176 constexpr friend Poly operator*(Poly a, Value b) {
177     for (int i = 0; i < int(a.size()); i++) {
178         a[i] *= b;
179     }
180     return a;
181 }
182 constexpr friend Poly operator/(Poly a, Value b) {
183     for (int i = 0; i < int(a.size()); i++) {
184         a[i] /= b;
185     }
186     return a;
187 }
188 constexpr Poly &operator+=(Poly b) {
189     return (*this) = (*this) + b;
190 }
191 constexpr Poly &operator-=(Poly b) {
192     return (*this) = (*this) - b;
193 }
194 constexpr Poly &operator*=(Poly b) {
195     return (*this) = (*this) * b;
196 }
197 constexpr Poly &operator*=(Value b) {
198     return (*this) = (*this) * b;
199 }
200 constexpr Poly &operator/=(Value b) {
201     return (*this) = (*this) / b;
202 }
203 constexpr Poly deriv() const {
204     if (this->empty()) {
205         return Poly();
206     }
207     Poly res(this->size() - 1);
208     for (int i = 0; i < this->size() - 1; ++i) {
209         res[i] = (i + 1) * (*this)[i + 1];
210     }
211     return res;
212 }
213 constexpr Poly integr() const {
214     Poly res(this->size() + 1);
215     for (int i = 0; i < this->size(); ++i) {
216         res[i + 1] = (*this)[i] / (i + 1);
217     }
218     return res;
219 }
220 constexpr Poly inv(int m) const {
221     Poly x((*this)[0].inv());
222     int k = 1;

```



```

223     while (k < m) {
224         k *= 2;
225         x = (x * (Poly{2} - trunc(k) * x)).trunc(k);
226     }
227     return x.trunc(m);
228 }
229 constexpr Poly log(int m) const {
230     return (deriv() * inv(m)).integr().trunc(m);
231 }
232 constexpr Poly exp(int m) const {
233     Poly x{1};
234     int k = 1;
235     while (k < m) {
236         k *= 2;
237         x = (x * (Poly{1} - x.log(k) + trunc(k))).trunc(k);
238     }
239     return x.trunc(m);
240 }
241 constexpr Poly pow(int k, int m) const {
242     int i = 0;
243     while (i < this->size() && (*this)[i] == 0) {
244         i++;
245     }
246     if (i == this->size() || 1LL * i * k >= m) {
247         return Poly(m);
248     }
249     Value v = (*this)[i];
250     auto f = shift(-i) * v.inv();
251     return (f.log(m - i * k) * k).exp(m - i * k).shift(i * k) * power(v, k);
252 }
253 constexpr Poly sqrt(int m) const {
254     Poly x{1};
255     int k = 1;
256     while (k < m) {
257         k *= 2;
258         x = (x + (trunc(k) * x.inv(k)).trunc(k)) * CInv<2, P>;
259     }
260     return x.trunc(m);
261 }
262 constexpr Poly mulT(Poly b) const {
263     if (b.size() == 0) {
264         return Poly();
265     }
266     int n = b.size();
267     reverse(b.begin(), b.end());
268     return ((*this) * b).shift(-(n - 1));
269 }
270 constexpr vector<Value> eval(vector<Value> x) const {
271     if (this->size() == 0) {
272         return vector<Value>(x.size(), 0);
273     }
274     const int n = max(x.size(), this->size());
275     vector<Poly> q(4 * n);
276     vector<Value> ans(x.size());
277     x.resize(n);
278     function<void(int, int, int)> build = [&](int p, int l, int r) {
279         if (r - l == 1) {
280             q[p] = Poly{1, -x[l]};
281         } else {
282             int m = (l + r) / 2;
283             build(2 * p, l, m);
284             build(2 * p + 1, m, r);
285             q[p] = q[2 * p] * q[2 * p + 1];
286         }

```

```

287     };
288     build(1, 0, n);
289     function<void(int, int, int, const Poly &)> work = [&](int p, int l, int r,
const Poly &num) {
290         if (r - l == 1) {
291             if (l < int(ans.size())) {
292                 ans[l] = num[0];
293             }
294         } else {
295             int m = (l + r) / 2;
296             work(2 * p, l, m, num.mulT(q[2 * p + 1]).trunc(m - 1));
297             work(2 * p + 1, m, r, num.mulT(q[2 * p]).trunc(r - m));
298         }
299     };
300     work(1, 0, n, mulT(q[1].inv(n)));
301     return ans;
302 }
303 };
304
305 template<int P = 998244353>
306 Poly<P> berlekampMassey(const Poly<P> &s) {
307     Poly<P> c;
308     Poly<P> oldC;
309     int f = -1;
310     for (int i = 0; i < s.size(); i++) {
311         auto delta = s[i];
312         for (int j = 1; j <= c.size(); j++) {
313             delta -= c[j - 1] * s[i - j];
314         }
315         if (delta == 0) {
316             continue;
317         }
318         if (f == -1) {
319             c.resize(i + 1);
320             f = i;
321         } else {
322             auto d = oldC;
323             d *= -1;
324             d.insert(d.begin(), 1);
325             MInt<P> df1 = 0;
326             for (int j = 1; j <= d.size(); j++) {
327                 df1 += d[j - 1] * s[f + 1 - j];
328             }
329             assert(df1 != 0);
330             auto coef = delta / df1;
331             d *= coef;
332             Poly<P> zeros(i - f - 1);
333             zeros.insert(zeros.end(), d.begin(), d.end());
334             d = zeros;
335             auto temp = c;
336             c += d;
337             if (i - temp.size() > f - oldC.size()) {
338                 oldC = temp;
339                 f = i;
340             }
341         }
342     }
343     c *= -1;
344     c.insert(c.begin(), 1);
345     return c;
346 }
347
348 template<int P = 998244353>
349 MInt<P> linearRecurrence(Poly<P> p, Poly<P> q, i64 n) {

```

```

350     int m = q.size() - 1;
351     while (n > 0) {
352         auto newq = q;
353         for (int i = 1; i <= m; i += 2) {
354             newq[i] *= -1;
355         }
356         auto newp = p * newq;
357         newq = q * newq;
358         for (int i = 0; i < m; i++) {
359             p[i] = newp[i * 2 + n % 2];
360         }
361         for (int i = 0; i <= m; i++) {
362             q[i] = newq[i * 2];
363         }
364         n /= 2;
365     }
366     return p[0] / q[0];
367 }
368
369 struct Comb {
370     int n;
371     vector<Z> _fac;
372     vector<Z> _invfac;
373     vector<Z> _inv;
374
375     Comb() : n{0}, _fac{1}, _invfac{1}, _inv{0} {}
376     Comb(int n) : Comb() {
377         init(n);
378     }
379
380     void init(int m) {
381         m = min(m, Z::getMod() - 1);
382         if (m <= n) return;
383         _fac.resize(m + 1);
384         _invfac.resize(m + 1);
385         _inv.resize(m + 1);
386
387         for (int i = n + 1; i <= m; i++) {
388             _fac[i] = _fac[i - 1] * i;
389         }
390         _invfac[m] = _fac[m].inv();
391         for (int i = m; i > n; i--) {
392             _invfac[i - 1] = _invfac[i] * i;
393             _inv[i] = _invfac[i] * _fac[i - 1];
394         }
395         n = m;
396     }
397
398     Z fac(int m) {
399         if (m > n) init(2 * m);
400         return _fac[m];
401     }
402     Z invfac(int m) {
403         if (m > n) init(2 * m);
404         return _invfac[m];
405     }
406     Z inv(int m) {
407         if (m > n) init(2 * m);
408         return _inv[m];
409     }
410     Z binom(int n, int m) {
411         if (n < m || m < 0) return 0;
412         return fac(n) * invfac(m) * invfac(n - m);
413     }

```

```

414 } comb;
415
416 Poly<P> get(int n, int m) {
417     if (m == 0) {
418         return Poly(n + 1);
419     }
420     if (m % 2 == 1) {
421         auto f = get(n, m - 1);
422         Z p = 1;
423         for (int i = 0; i <= n; i++) {
424             f[n - i] += comb.binom(n, i) * p;
425             p *= m;
426         }
427         return f;
428     }
429     auto f = get(n, m / 2);
430     auto fm = f;
431     for (int i = 0; i <= n; i++) {
432         fm[i] *= comb.fac(i);
433     }
434     Poly pw(n + 1);
435     pw[0] = 1;
436     for (int i = 1; i <= n; i++) {
437         pw[i] = pw[i - 1] * (m / 2);
438     }
439     for (int i = 0; i <= n; i++) {
440         pw[i] *= comb.invfac(i);
441     }
442     fm = fm.mulT(pw);
443     for (int i = 0; i <= n; i++) {
444         fm[i] *= comb.invfac(i);
445     }
446     return f + fm;
447 }

```

3.12.3 多项式乘法

```

1  constexpr int P = 998244353;
2
3  int power(int a, int b) {
4      int res = 1;
5      for (; b; b /= 2, a = 1LL * a * a % P) {
6          if (b % 2) {
7              res = 1LL * res * a % P;
8          }
9      }
10     return res;
11 }
12
13 vector<int> rev, roots {0, 1};
14
15 void dft(vector<int> &a) {
16     int n = a.size();
17     if (int(rev.size()) != n) {
18         int k = __builtin_ctz(n) - 1;
19         rev.resize(n);
20         for (int i = 0; i < n; i++) {
21             rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
22         }
23     }
24     for (int i = 0; i < n; i++) {
25         if (rev[i] < i) {
26             swap(a[i], a[rev[i]]);

```

```

27     }
28 }
29 if (roots.size() < n) {
30     int k = __builtin_ctz(roots.size());
31     roots.resize(n);
32     while ((1 << k) < n) {
33         int e = power(31, 1 << (__builtin_ctz(P - 1) - k - 1));
34         for (int i = 1 << (k - 1); i < (1 << k); i++) {
35             roots[2 * i] = roots[i];
36             roots[2 * i + 1] = 1LL * roots[i] * e % P;
37         }
38         k++;
39     }
40 }
41
42 for (int k = 1; k < n; k *= 2) {
43     for (int i = 0; i < n; i += 2 * k) {
44         for (int j = 0; j < k; j++) {
45             int u = a[i + j];
46             int v = 1LL * a[i + j + k] * roots[k + j] % P;
47             a[i + j] = (u + v) % P;
48             a[i + j + k] = (u - v) % P;
49         }
50     }
51 }
52 }
53
54 void idft(vector<int> &a) {
55     int n = a.size();
56     reverse(a.begin() + 1, a.end());
57     dft(a);
58     int inv = (1 - P) / n;
59     for (int i = 0; i < n; i++) {
60         a[i] = 1LL * a[i] * inv % P;
61     }
62 }
63
64 vector<int> mul(vector<int> a, vector<int> b) {
65     int n = 1, tot = a.size() + b.size() - 1;
66     while (n < tot) {
67         n *= 2;
68     }
69     if (tot < 128) {
70         vector<int> c(a.size() + b.size() - 1);
71         for (int i = 0; i < a.size(); i++) {
72             for (int j = 0; j < b.size(); j++) {
73                 c[i + j] = (c[i + j] + 1LL * a[i] * b[j]) % P;
74             }
75         }
76         return c;
77     }
78     a.resize(n);
79     b.resize(n);
80     dft(a);
81     dft(b);
82     for (int i = 0; i < n; i++) {
83         a[i] = 1LL * a[i] * b[i] % P;
84     }
85     idft(a);
86     a.resize(tot);
87     return a;
88 }

```

3.13 生成函数

3.13.1 生成函数 (q-int)

```

1  i64 power(i64 a, i64 b, i64 p) {
2      i64 res = 1;
3      for (; b; b /= 2, a = i128(a) * a % p) {
4          if (b % 2) {
5              res = i128(res) * a % p;
6          }
7      }
8      return res;
9  }
10
11 pair<int, int> qint(int q, int n, int p) {
12     q %= p;
13     for (int x = 2; x * x <= n; x++) {
14         if (n % x == 0) {
15             auto [v1, e1] = qint(q, x, p);
16             auto [v2, e2] = qint(power(q, x, p), n / x, p);
17             return {1LL * v1 * v2 % p, e1 + e2};
18         }
19     }
20     if (q == 1) {
21         if (n == p) {
22             return {0, 1};
23         }
24         return {n, 0};
25     }
26     // cerr << q << " " << n << " " << p << "\n";
27     i64 v = 1 - power(q, n, 1LL * p * p);
28     if (v < 0) {
29         v += 1LL * p * p;
30     }
31     assert(v != 0);
32     int inv = power(1 - q + p, p - 2, p);
33     if (v % p == 0) {
34         return {(v / p) * inv % p, 1};
35     } else {
36         return {v % p * inv % p, 0};
37     }
38 }

```

3.13.2 生成函数 (q-Binomial)

```

1  int power(int a, int b, int p) {
2      int res = 1;
3      for (; b; b /= 2, a = 1LL * a * a % p) {
4          if (b % 2) {
5              res = 1LL * res * a % p;
6          }
7      }
8      return res;
9  }
10
11 int qint(int n, int q, int p) {
12     return 1LL * (power(q, n, p) - 1) * power(q - 1, p - 2, p) % p;
13 }
14
15 int qBinomial(int n, int k, int q, int p) {
16     if (q == 0) {
17         return 1;

```

```

18     }
19     int r = 0;
20     int x = 1;
21     do {
22         x = 1LL * x * q % p;
23         r++;
24     } while (x != 1);
25
26     if (n / r > k / r + (n - k) / r) {
27         return 0;
28     }
29     int num = 1, den = 1;
30     for (int i = 1; i <= k % r; i++) {
31         num = 1LL * num * qint(n % r - i + 1, q, p) % p;
32         den = 1LL * den * qint(i, q, p) % p;
33     }
34     n /= r, k /= r;
35     while (n > 0 || k > 0) {
36         if (n % p < k % p) {
37             return 0;
38         }
39         for (int i = 1; i <= k % p; i++) {
40             num = 1LL * num * (n % p - i + 1) % p;
41             den = 1LL * den * i % p;
42         }
43         n /= p, k /= p;
44     }
45     int ans = 1LL * num * power(den, p - 2, p) % p;
46     return ans;
47 }

```

3.13.3 生成函数 (Binomial 任意模数二项式)

```

1  vector<pair<int, int>> factorize(int n) {
2      vector<pair<int, int>> factors;
3      for (int i = 2; static_cast<long long>(i) * i <= n; i++) {
4          if (n % i == 0) {
5              int t = 0;
6              for (; n % i == 0; n /= i)
7                  ++t;
8              factors.emplace_back(i, t);
9          }
10     }
11     if (n > 1)
12         factors.emplace_back(n, 1);
13     return factors;
14 }
15 constexpr int power(int base, i64 exp) {
16     int res = 1;
17     for (; exp > 0; base *= base, exp /= 2) {
18         if (exp % 2 == 1) {
19             res *= base;
20         }
21     }
22     return res;
23 }
24 constexpr int power(int base, i64 exp, int mod) {
25     int res = 1 % mod;
26     for (; exp > 0; base = 1LL * base * base % mod, exp /= 2) {
27         if (exp % 2 == 1) {
28             res = 1LL * res * base % mod;
29         }
30     }

```

```

31     return res;
32 }
33 int inverse(int a, int m) {
34     int g = m, r = a, x = 0, y = 1;
35     while (r != 0) {
36         int q = g / r;
37         g %= r;
38         swap(g, r);
39         x -= q * y;
40         swap(x, y);
41     }
42     return x < 0 ? x + m : x;
43 }
44 int solveModuloEquations(const vector<pair<int, int>> &e) {
45     int m = 1;
46     for (size_t i = 0; i < e.size(); i++) {
47         m *= e[i].first;
48     }
49     int res = 0;
50     for (size_t i = 0; i < e.size(); i++) {
51         int p = e[i].first;
52         res = (res + 1LL * e[i].second * (m / p) * inverse(m / p, p)) % m;
53     }
54     return res;
55 }
56 constexpr int N = 1E5;
57 class Binomial {
58     const int mod;
59 private:
60     const vector<pair<int, int>> factors;
61     vector<int> pk;
62     vector<vector<int>> prod;
63     static constexpr i64 exponent(i64 n, int p) {
64         i64 res = 0;
65         for (n /= p; n > 0; n /= p) {
66             res += n;
67         }
68         return res;
69     }
70     int product(i64 n, size_t i) {
71         int res = 1;
72         int p = factors[i].first;
73         for (; n > 0; n /= p) {
74             res = 1LL * res * power(prod[i].back(), n / pk[i], pk[i]) % pk[i] *
prod[i][n % pk[i]] % pk[i];
75         }
76         return res;
77     }
78 public:
79     Binomial(int mod) : mod(mod), factors(factorize(mod)) {
80         pk.resize(factors.size());
81         prod.resize(factors.size());
82         for (size_t i = 0; i < factors.size(); i++) {
83             int p = factors[i].first;
84             int k = factors[i].second;
85             pk[i] = power(p, k);
86             prod[i].resize(min(N + 1, pk[i]));
87             prod[i][0] = 1;
88             for (int j = 1; j < prod[i].size(); j++) {
89                 if (j % p == 0) {
90                     prod[i][j] = prod[i][j - 1];
91                 } else {
92                     prod[i][j] = 1LL * prod[i][j - 1] * j % pk[i];
93                 }

```



```

94     }
95     }
96 }
97 int operator()(i64 n, i64 m) {
98     if (n < m || m < 0) {
99         return 0;
100     }
101     vector<pair<int, int>> ans(factors.size());
102     for (int i = 0; i < factors.size(); i++) {
103         int p = factors[i].first;
104         int k = factors[i].second;
105         int e = exponent(n, p) - exponent(m, p) - exponent(n - m, p);
106         if (e >= k) {
107             ans[i] = make_pair(pk[i], 0);
108         } else {
109             int pn = product(n, i);
110             int pm = product(m, i);
111             int pd = product(n - m, i);
112             int res = 1LL * pn * inverse(pm, pk[i]) % pk[i] * inverse(pd,
pk[i]) % pk[i] * power(p, e) % pk[i];
113             ans[i] = make_pair(pk[i], res);
114         }
115     }
116     return solveModuloEquations(ans);
117 }
118 };

```

3.14 自适应辛普森法 (Simpson)

```

1  const double Pi = acos(-1.0);
2  constexpr double EPS = 1e-9;
3  double v, r, d;
4  double f(double x) {
5      double s = sin(x);
6      return 1 / v / (sqrt(s * s + 3) - s);
7  }
8  double simpson(double l, double r) {
9      return (f(l) + 4 * f((l + r) / 2) + f(r)) * (r - l) / 6;
10 }
11 double integral(double l, double r, double eps, double st) {
12     double mid = (l + r) / 2;
13     double sl = simpson(l, mid);
14     double sr = simpson(mid, r);
15     if (abs(sl + sr - st) <= 15 * eps)
16         return sl + sr + (sl + sr - st) / 15;
17     return integral(l, mid, eps / 2, sl) + integral(mid, r, eps / 2, sr);
18 }
19 double integral(double l, double r) {
20     return integral(l, r, EPS, simpson(l, r));
21 }

```

3.15 矩阵 (Matrix)

```

1  using u64 = unsigned long long;
2  using Matrix = array<u64, 65>;
3
4  Matrix operator*(const Matrix &a, const Matrix &b) {
5      Matrix c{};
6      for (int i = 0; i <= 64; i++) {
7          for (int j = 0; j <= 64; j++) {
8              if (j == 64 ? i == 64 : (a[i] >> j & 1)) {

```

```

9         c[i] ^= b[j];
10     }
11 }
12 }
13 return c;
14 }
15
16 u64 operator*(u64 a, const Matrix &b) {
17     u64 c = 0;
18     for (int i = 0; i <= 64; i++) {
19         if (i == 64 || (a >> i & 1)) {
20             c ^= b[i];
21         }
22     }
23     return c;
24 }
25
26 Matrix readMatrix() {
27     int m;
28     cin >> m;
29
30     Matrix f{};
31     for (int i = 0; i < m; i++) {
32         int s, o;
33         u64 A;
34         cin >> s >> o >> A;
35
36         if (o == 0) {
37             for (int j = 0; j < 64; j++) {
38                 if (A >> ((j + s) % 64) & 1) {
39                     f[64] ^= 1ULL << ((j + s) % 64);
40                 } else {
41                     f[j] ^= 1ULL << ((j + s) % 64);
42                 }
43             }
44         } else {
45             for (int j = 0; j < 64; j++) {
46                 if (A >> ((j + s) % 64) & 1) {
47                     f[j] ^= 1ULL << ((j + s) % 64);
48                 }
49             }
50         }
51     }
52
53     u64 B;
54     cin >> B;
55     f[64] ^= B;
56
57     return f;
58 }

```

3.16 高斯消元法 (gaussian elimination) 【久远】

```

1  /** 高斯消元法 (gaussian elimination) 【久远】 */
2  vector<int> operator*(const vector<int> &lhs, const vector<int> &rhs) {
3      vector<int> res(lhs.size() + rhs.size() - 1);
4      for (int i = 0; i < int(lhs.size()); ++i)
5          for (int j = 0; j < int(rhs.size()); ++j)
6              res[i + j] = (res[i + j] + 1ll * lhs[i] * rhs[j]) % P;
7      return res;
8  }
9  vector<int> operator%(const vector<int> &lhs, const vector<int> &rhs) {

```

```

10     auto res = lhs;
11     int m = rhs.size() - 1;
12     int inv = power(rhs.back(), P - 2);
13     for (int i = res.size() - 1; i >= m; --i) {
14         int x = 1ll * inv * res[i] % P;
15         for (int j = 0; j < m; ++j)
16             res[i - m + j] = (res[i - m + j] + 1ll * (P - x) * rhs[j]) % P;
17     }
18     if (int(res.size()) > m)
19         res.resize(m);
20     return res;
21 }
22 vector<int> gauss(vector<vector<int>> a, vector<int> b) {
23     int n = a.size();
24     for (int i = 0; i < n; ++i) {
25         int r = i;
26         while (a[r][i] == 0)
27             ++r;
28         swap(a[i], a[r]);
29         swap(b[i], b[r]);
30         int inv = power(a[i][i], P - 2);
31         for (int j = i; j < n; ++j)
32             a[i][j] = 1ll * a[i][j] * inv % P;
33         b[i] = 1ll * b[i] * inv % P;
34         for (int j = 0; j < n; ++j) {
35             if (i == j)
36                 continue;
37             int x = a[j][i];
38             for (int k = i; k < n; ++k)
39                 a[j][k] = (a[j][k] + 1ll * (P - x) * a[i][k]) % P;
40             b[j] = (b[j] + 1ll * (P - x) * b[i]) % P;
41         }
42     }
43     return b;
44 }
45 /** 高斯消元法 (gaussian elimination) 【久远】 */
46 vector<double> gauss(vector<vector<double>> a, vector<double> b) {
47     int n = a.size();
48     for (int i = 0; i < n; ++i) {
49         double x = a[i][i];
50         for (int j = i; j < n; ++j) a[i][j] /= x;
51         b[i] /= x;
52         for (int j = 0; j < n; ++j) {
53             if (i == j) continue;
54             x = a[j][i];
55             for (int k = i; k < n; ++k) a[j][k] -= a[i][k] * x;
56             b[j] -= b[i] * x;
57         }
58     }
59     return b;
60 }

```

/END/

4 数据结构

4.1 树状数组 (Fenwick)

```

1  template <typename T>
2  struct Fenwick {
3      int n;
4      vector<T> a;
5
6      Fenwick(int n_ = 0) {
7          init(n_);
8      }
9
10     void init(int n_) {
11         n = n_;
12         a.assign(n, T{});
13     }
14
15     void add(int x, const T &v) {
16         for (int i = x + 1; i <= n; i += i & -i) {
17             a[i - 1] = a[i - 1] + v;
18         }
19     }
20
21     T sum(int x) {
22         T ans{};
23         for (int i = x; i > 0; i -= i & -i) {
24             ans = ans + a[i - 1];
25         }
26         return ans;
27     }
28
29     T rangeSum(int l, int r) {
30         return sum(r) - sum(l);
31     }
32
33     int select(const T &k) {
34         int x = 0;
35         T cur{};
36         for (int i = 1 << __lg(n); i; i /= 2) {
37             if (x + i <= n && cur + a[x + i - 1] <= k) {
38                 x += i;
39                 cur = cur + a[x - 1];
40             }
41         }
42         return x;
43     }
44 };

```

4.2 并查集

4.2.1 并查集 (DSU)

```

1  struct DSU {
2      vector<int> f, siz;
3
4      DSU() {}
5      DSU(int n) {
6          init(n);
7      }
8

```

```

9   void init(int n) {
10       f.resize(n);
11       iota(f.begin(), f.end(), 0);
12       siz.assign(n, 1);
13   }
14
15   int find(int x) {
16       while (x != f[x]) {
17           x = f[x] = f[f[x]];
18       }
19       return x;
20   }
21
22   bool same(int x, int y) {
23       return find(x) == find(y);
24   }
25
26   bool merge(int x, int y) {
27       x = find(x);
28       y = find(y);
29       if (x == y) {
30           return false;
31       }
32       siz[x] += siz[y];
33       f[y] = x;
34       return true;
35   }
36
37   int size(int x) {
38       return siz[find(x)];
39   }
40 };

```

4.2.2 可撤销并查集 (DSU With Rollback)

```

1   struct DSU {
2       vector<int> siz;
3       vector<int> f;
4       vector<array<int, 2>> his;
5
6       DSU(int n) : siz(n + 1, 1), f(n + 1) {
7           iota(f.begin(), f.end(), 0);
8       }
9
10      int find(int x) {
11          while (f[x] != x) {
12              x = f[x];
13          }
14          return x;
15      }
16
17      bool merge(int x, int y) {
18          x = find(x);
19          y = find(y);
20          if (x == y) {
21              return false;
22          }
23          if (siz[x] < siz[y]) {
24              swap(x, y);
25          }
26          his.push_back({x, y});
27          siz[x] += siz[y];
28          f[y] = x;

```

```

29     return true;
30 }
31
32 int time() {
33     return his.size();
34 }
35
36 void revert(int tm) {
37     while (his.size() > tm) {
38         auto [x, y] = his.back();
39         his.pop_back();
40         f[y] = y;
41         siz[x] -= siz[y];
42     }
43 }
44 };

```

4.3 线段树

4.3.1 线段树 (SegmentTree+Info 区间加+单点修改)

```

1  struct SegmentTree {
2      int n;
3      vector<int> tag;
4      vector<Info> info;
5      SegmentTree(int n_) : n(n_), tag(4 * n), info(4 * n) {}
6
7      void pull(int p) {
8          info[p] = info[2 * p] + info[2 * p + 1];
9      }
10
11     void add(int p, int v) {
12         tag[p] += v;
13         info[p].max += v;
14     }
15
16     void push(int p) {
17         add(2 * p, tag[p]);
18         add(2 * p + 1, tag[p]);
19         tag[p] = 0;
20     }
21
22     Info query(int p, int l, int r, int x, int y) {
23         if (l >= y || r <= x) {
24             return {};
25         }
26         if (l >= x && r <= y) {
27             return info[p];
28         }
29         int m = (l + r) / 2;
30         push(p);
31         return query(2 * p, l, m, x, y) + query(2 * p + 1, m, r, x, y);
32     }
33
34     Info query(int x, int y) {
35         return query(1, 0, n, x, y);
36     }
37
38     void rangeAdd(int p, int l, int r, int x, int y, int v) {
39         if (l >= y || r <= x) {
40             return;
41         }

```

```

42     if (l >= x && r <= y) {
43         return add(p, v);
44     }
45     int m = (l + r) / 2;
46     push(p);
47     rangeAdd(2 * p, l, m, x, y, v);
48     rangeAdd(2 * p + 1, m, r, x, y, v);
49     pull(p);
50 }
51
52 void rangeAdd(int x, int y, int v) {
53     rangeAdd(1, 0, n, x, y, v);
54 }
55
56 void modify(int p, int l, int r, int x, const Info &v) {
57     if (r - l == 1) {
58         info[p] = v;
59         return;
60     }
61     int m = (l + r) / 2;
62     push(p);
63     if (x < m) {
64         modify(2 * p, l, m, x, v);
65     } else {
66         modify(2 * p + 1, m, r, x, v);
67     }
68     pull(p);
69 }
70
71 void modify(int x, const Info &v) {
72     modify(1, 0, n, x, v);
73 }
74 };

```

4.3.2 线段树 (SegmentTree 区间乘+单点加)

```

1  struct SegmentTree {
2      int n;
3      vector<int> tag, sum;
4      SegmentTree(int n_) : n(n_), tag(4 * n, 1), sum(4 * n) {}
5
6      void pull(int p) {
7          sum[p] = (sum[2 * p] + sum[2 * p + 1]) % P;
8      }
9
10     void mul(int p, int v) {
11         tag[p] = 1LL * tag[p] * v % P;
12         sum[p] = 1LL * sum[p] * v % P;
13     }
14
15     void push(int p) {
16         mul(2 * p, tag[p]);
17         mul(2 * p + 1, tag[p]);
18         tag[p] = 1;
19     }
20
21     int query(int p, int l, int r, int x, int y) {
22         if (l >= y || r <= x) {
23             return 0;
24         }
25         if (l >= x && r <= y) {
26             return sum[p];
27         }

```

```

28     int m = (1 + r) / 2;
29     push(p);
30     return (query(2 * p, 1, m, x, y) + query(2 * p + 1, m, r, x, y)) % P;
31 }
32
33 int query(int x, int y) {
34     return query(1, 0, n, x, y);
35 }
36
37 void rangeMul(int p, int l, int r, int x, int y, int v) {
38     if (l >= y || r <= x) {
39         return;
40     }
41     if (l >= x && r <= y) {
42         return mul(p, v);
43     }
44     int m = (1 + r) / 2;
45     push(p);
46     rangeMul(2 * p, 1, m, x, y, v);
47     rangeMul(2 * p + 1, m, r, x, y, v);
48     pull(p);
49 }
50
51 void rangeMul(int x, int y, int v) {
52     rangeMul(1, 0, n, x, y, v);
53 }
54
55 void add(int p, int l, int r, int x, int v) {
56     if (r - l == 1) {
57         sum[p] = (sum[p] + v) % P;
58         return;
59     }
60     int m = (1 + r) / 2;
61     push(p);
62     if (x < m) {
63         add(2 * p, 1, m, x, v);
64     } else {
65         add(2 * p + 1, m, r, x, v);
66     }
67     pull(p);
68 }
69
70 void add(int x, int v) {
71     add(1, 0, n, x, v);
72 }
73 };

```

4.3.3 线段树 (SegmentTree+Info 初始赋值+单点修改+查找前驱后继)

```

1  template<class Info> struct SegmentTree {
2      int n;
3      vector<Info> info;
4      SegmentTree() : n(0) {}
5      SegmentTree(int n_, Info v_ = Info()) {
6          init(n_, v_);
7      }
8      template<class T>
9      SegmentTree(vector<T> init_) {
10         init(init_);
11     }
12     void init(int n_, Info v_ = Info()) {
13         init(vector(n_, v_));
14     }

```



```

15     template<class T>
16     void init(vector<T> init_) {
17         n = init_.size();
18         info.assign(4 << __lg(n), Info());
19         function<void(int, int, int)> build = [&](int p, int l, int r) {
20             if (r - l == 1) {
21                 info[p] = init_[l];
22                 return;
23             }
24             int m = (l + r) / 2;
25             build(2 * p, l, m);
26             build(2 * p + 1, m, r);
27             pull(p);
28         };
29         build(1, 0, n);
30     }
31     void pull(int p) {
32         info[p] = info[2 * p] + info[2 * p + 1];
33     }
34     void modify(int p, int l, int r, int x, const Info &v) {
35         if (r - l == 1) {
36             info[p] = v;
37             return;
38         }
39         int m = (l + r) / 2;
40         if (x < m) {
41             modify(2 * p, l, m, x, v);
42         } else {
43             modify(2 * p + 1, m, r, x, v);
44         }
45         pull(p);
46     }
47     void modify(int p, const Info &v) {
48         modify(1, 0, n, p, v);
49     }
50     Info rangeQuery(int p, int l, int r, int x, int y) {
51         if (l >= y || r <= x) {
52             return Info();
53         }
54         if (l >= x && r <= y) {
55             return info[p];
56         }
57         int m = (l + r) / 2;
58         return rangeQuery(2 * p, l, m, x, y) + rangeQuery(2 * p + 1, m, r, x, y);
59     }
60     Info rangeQuery(int l, int r) {
61         return rangeQuery(1, 0, n, l, r);
62     }
63     template<class F>
64     int findFirst(int p, int l, int r, int x, int y, F &&pred) {
65         if (l >= y || r <= x) {
66             return -1;
67         }
68         if (l >= x && r <= y && !pred(info[p])) {
69             return -1;
70         }
71         if (r - l == 1) {
72             return l;
73         }
74         int m = (l + r) / 2;
75         int res = findFirst(2 * p, l, m, x, y, pred);
76         if (res == -1) {
77             res = findFirst(2 * p + 1, m, r, x, y, pred);
78         }

```

```

79     return res;
80 }
81 template<class F>
82 int findFirst(int l, int r, F &&pred) {
83     return findFirst(1, 0, n, l, r, pred);
84 }
85 template<class F>
86 int findLast(int p, int l, int r, int x, int y, F &&pred) {
87     if (l >= y || r <= x) {
88         return -1;
89     }
90     if (l >= x && r <= y && !pred(info[p])) {
91         return -1;
92     }
93     if (r - l == 1) {
94         return l;
95     }
96     int m = (l + r) / 2;
97     int res = findLast(2 * p + 1, m, r, x, y, pred);
98     if (res == -1) {
99         res = findLast(2 * p, l, m, x, y, pred);
100    }
101    return res;
102 }
103 template<class F>
104 int findLast(int l, int r, F &&pred) {
105     return findLast(1, 0, n, l, r, pred);
106 }
107 };

```

4.3.4 线段树 (SegmentTree+Info+Merge 初始赋值+单点修改+区间合并)

```

1  template<class Info, class Merge = plus<Info>> struct SegmentTree {
2      const int n;
3      const Merge merge;
4      vector<Info> info;
5      SegmentTree(int n) : n(n), merge(Merge()), info(4 << __lg(n)) {}
6      SegmentTree(vector<Info> init) : SegmentTree(init.size()) {
7          function<void(int, int, int)> build = [&](int p, int l, int r) {
8              if (r - l == 1) {
9                  info[p] = init[l];
10                 return;
11             }
12             int m = (l + r) / 2;
13             build(2 * p, l, m);
14             build(2 * p + 1, m, r);
15             pull(p);
16         };
17         build(1, 0, n);
18     }
19     void pull(int p) {
20         info[p] = merge(info[2 * p], info[2 * p + 1]);
21     }
22     void modify(int p, int l, int r, int x, const Info &v) {
23         if (r - l == 1) {
24             info[p] = v;
25             return;
26         }
27         int m = (l + r) / 2;
28         if (x < m) {
29             modify(2 * p, l, m, x, v);

```

```

30     } else {
31         modify(2 * p + 1, m, r, x, v);
32     }
33     pull(p);
34 }
35 void modify(int p, const Info &v) {
36     modify(1, 0, n, p, v);
37 }
38 Info rangeQuery(int p, int l, int r, int x, int y) {
39     if (l >= y || r <= x) {
40         return Info();
41     }
42     if (l >= x && r <= y) {
43         return info[p];
44     }
45     int m = (l + r) / 2;
46     return merge(rangeQuery(2 * p, l, m, x, y), rangeQuery(2 * p + 1, m, r, x,
47 y));
48 }
49 Info rangeQuery(int l, int r) {
50     return rangeQuery(1, 0, n, l, r);
51 };

```

4.4 懒标记线段树 (LazySegmentTree)

```

1  template<class Info, class Tag> struct LazySegmentTree {
2      int n;
3      vector<Info> info;
4      vector<Tag> tag;
5      LazySegmentTree() : n(0) {}
6      LazySegmentTree(int n_, Info v_ = Info()) {
7          init(n_, v_);
8      }
9      template<class T>
10     LazySegmentTree(vector<T> init_) {
11         init(init_);
12     }
13     void init(int n_, Info v_ = Info()) {
14         init(vector(n_, v_));
15     }
16     template<class T>
17     void init(vector<T> init_) {
18         n = init_.size();
19         info.assign(4 << __lg(n), Info());
20         tag.assign(4 << __lg(n), Tag());
21         function<void(int, int, int)> build = [&](int p, int l, int r) {
22             if (r - l == 1) {
23                 info[p] = init_[l];
24                 return;
25             }
26             int m = (l + r) / 2;
27             build(2 * p, l, m);
28             build(2 * p + 1, m, r);
29             pull(p);
30         };
31         build(1, 0, n);
32     }
33     void pull(int p) {
34         info[p] = info[2 * p] + info[2 * p + 1];
35     }
36     void apply(int p, const Tag &v) {

```

```

37     info[p].apply(v);
38     tag[p].apply(v);
39 }
40 void push(int p) {
41     apply(2 * p, tag[p]);
42     apply(2 * p + 1, tag[p]);
43     tag[p] = Tag();
44 }
45 void modify(int p, int l, int r, int x, const Info &v) {
46     if (r - l == 1) {
47         info[p] = v;
48         return;
49     }
50     int m = (l + r) / 2;
51     push(p);
52     if (x < m) {
53         modify(2 * p, l, m, x, v);
54     } else {
55         modify(2 * p + 1, m, r, x, v);
56     }
57     pull(p);
58 }
59 void modify(int p, const Info &v) {
60     modify(1, 0, n, p, v);
61 }
62 Info rangeQuery(int p, int l, int r, int x, int y) {
63     if (l >= y || r <= x) {
64         return Info();
65     }
66     if (l >= x && r <= y) {
67         return info[p];
68     }
69     int m = (l + r) / 2;
70     push(p);
71     return rangeQuery(2 * p, l, m, x, y) + rangeQuery(2 * p + 1, m, r, x, y);
72 }
73 Info rangeQuery(int l, int r) {
74     return rangeQuery(1, 0, n, l, r);
75 }
76 void rangeApply(int p, int l, int r, int x, int y, const Tag &v) {
77     if (l >= y || r <= x) {
78         return;
79     }
80     if (l >= x && r <= y) {
81         apply(p, v);
82         return;
83     }
84     int m = (l + r) / 2;
85     push(p);
86     rangeApply(2 * p, l, m, x, y, v);
87     rangeApply(2 * p + 1, m, r, x, y, v);
88     pull(p);
89 }
90 void rangeApply(int l, int r, const Tag &v) {
91     return rangeApply(1, 0, n, l, r, v);
92 }
93 void half(int p, int l, int r) {
94     if (info[p].act == 0) {
95         return;
96     }
97     if ((info[p].min + 1) / 2 == (info[p].max + 1) / 2) {
98         apply(p, {-(info[p].min + 1) / 2});
99         return;
100    }

```

```

101     int m = (l + r) / 2;
102     push(p);
103     half(2 * p, l, m);
104     half(2 * p + 1, m, r);
105     pull(p);
106 }
107 void half() {
108     half(1, 0, n);
109 }
110
111 template<class F>
112 int findFirst(int p, int l, int r, int x, int y, F &&pred) {
113     if (l >= y || r <= x) {
114         return -1;
115     }
116     if (l >= x && r <= y && !pred(info[p])) {
117         return -1;
118     }
119     if (r - l == 1) {
120         return l;
121     }
122     int m = (l + r) / 2;
123     push(p);
124     int res = findFirst(2 * p, l, m, x, y, pred);
125     if (res == -1) {
126         res = findFirst(2 * p + 1, m, r, x, y, pred);
127     }
128     return res;
129 }
130 template<class F>
131 int findFirst(int l, int r, F &&pred) {
132     return findFirst(1, 0, n, l, r, pred);
133 }
134 template<class F>
135 int findLast(int p, int l, int r, int x, int y, F &&pred) {
136     if (l >= y || r <= x) {
137         return -1;
138     }
139     if (l >= x && r <= y && !pred(info[p])) {
140         return -1;
141     }
142     if (r - l == 1) {
143         return l;
144     }
145     int m = (l + r) / 2;
146     push(p);
147     int res = findLast(2 * p + 1, m, r, x, y, pred);
148     if (res == -1) {
149         res = findLast(2 * p, l, m, x, y, pred);
150     }
151     return res;
152 }
153 template<class F>
154 int findLast(int l, int r, F &&pred) {
155     return findLast(1, 0, n, l, r, pred);
156 }
157
158 void maintainL(int p, int l, int r, int pre) {
159     if (info[p].difl > 0 && info[p].maxlowl < pre) {
160         return;
161     }
162     if (r - l == 1) {
163         info[p].max = info[p].maxlowl;
164         info[p].maxl = info[p].maxr = l;

```

```

165         info[p].maxlowl = info[p].maxlowr = -inf;
166         return;
167     }
168     int m = (1 + r) / 2;
169     push(p);
170     maintainL(2 * p, l, m, pre);
171     pre = max(pre, info[2 * p].max);
172     maintainL(2 * p + 1, m, r, pre);
173     pull(p);
174 }
175 void maintainL() {
176     maintainL(1, 0, n, -1);
177 }
178 void maintainR(int p, int l, int r, int suf) {
179     if (info[p].difr > 0 && info[p].maxlowr < suf) {
180         return;
181     }
182     if (r - l == 1) {
183         info[p].max = info[p].maxlowl;
184         info[p].maxl = info[p].maxr = l;
185         info[p].maxlowl = info[p].maxlowr = -inf;
186         return;
187     }
188     int m = (1 + r) / 2;
189     push(p);
190     maintainR(2 * p + 1, m, r, suf);
191     suf = max(suf, info[2 * p + 1].max);
192     maintainR(2 * p, l, m, suf);
193     pull(p);
194 }
195 void maintainR() {
196     maintainR(1, 0, n, -1);
197 }
198 };
199
200 struct Tag {
201     int x = 0;
202     void apply(const Tag &t) & {
203         x = max(x, t.x);
204     }
205 };
206
207 struct Info {
208     int x = 0;
209     void apply(const Tag &t) & {
210         x = max(x, t.x);
211     }
212 };
213
214 Info operator+(const Info &a, const Info &b) {
215     return {max(a.x, b.x)};
216 }

```

4.5 取模类

4.5.1 取模类 (Z 旧版)

```

1  constexpr int P = 998244353;
2  // assume -P <= x < 2P
3  int norm(int x) {
4      if (x < 0) {
5          x += P;

```

```

6     }
7     if (x >= P) {
8         x -= P;
9     }
10    return x;
11 }
12 template<class T>
13 T power(T a, i64 b) {
14     T res = 1;
15     for (; b; b /= 2, a *= a) {
16         if (b % 2) {
17             res *= a;
18         }
19     }
20     return res;
21 }
22 struct Z {
23     int x;
24     Z(int x = 0) : x(norm(x)) {}
25     Z(i64 x) : x(norm(x % P)) {}
26     int val() const {
27         return x;
28     }
29     Z operator-() const {
30         return Z(norm(P - x));
31     }
32     Z inv() const {
33         assert(x != 0);
34         return power(*this, P - 2);
35     }
36     Z &operator*=(const Z &rhs) {
37         x = i64(x) * rhs.x % P;
38         return *this;
39     }
40     Z &operator+=(const Z &rhs) {
41         x = norm(x + rhs.x);
42         return *this;
43     }
44     Z &operator-=(const Z &rhs) {
45         x = norm(x - rhs.x);
46         return *this;
47     }
48     Z &operator/=(const Z &rhs) {
49         return *this *= rhs.inv();
50     }
51     friend Z operator*(const Z &lhs, const Z &rhs) {
52         Z res = lhs;
53         res *= rhs;
54         return res;
55     }
56     friend Z operator+(const Z &lhs, const Z &rhs) {
57         Z res = lhs;
58         res += rhs;
59         return res;
60     }
61     friend Z operator-(const Z &lhs, const Z &rhs) {
62         Z res = lhs;
63         res -= rhs;
64         return res;
65     }
66     friend Z operator/(const Z &lhs, const Z &rhs) {
67         Z res = lhs;
68         res /= rhs;
69         return res;

```

```

70     }
71     friend istream &operator>>(istream &is, Z &a) {
72         i64 v;
73         is >> v;
74         a = Z(v);
75         return is;
76     }
77     friend ostream &operator<<(ostream &os, const Z &a) {
78         return os << a.val();
79     }
80 };

```

4.5.2 取模类 (MLong & MInt 新版)

```

1  /** 取模类 (MLong & MInt 新版)
2   * 根据输入内容动态修改 MOD 的方法 : Z::setMod(p) 。
3   **/
4  template<class T>
5  constexpr T power(T a, i64 b) {
6      T res = 1;
7      for (; b; b /= 2, a *= a) {
8          if (b % 2) {
9              res *= a;
10         }
11     }
12     return res;
13 }
14
15 constexpr i64 mul(i64 a, i64 b, i64 p) {
16     i64 res = a * b - i64(1.L * a * b / p) * p;
17     res %= p;
18     if (res < 0) {
19         res += p;
20     }
21     return res;
22 }
23 template<i64 P>
24 struct MLong {
25     i64 x;
26     constexpr MLong() : x{} {}
27     constexpr MLong(i64 x) : x{norm(x % getMod())} {}
28
29     static i64 Mod;
30     constexpr static i64 getMod() {
31         if (P > 0) {
32             return P;
33         } else {
34             return Mod;
35         }
36     }
37     constexpr static void setMod(i64 Mod_) {
38         Mod = Mod_;
39     }
40     constexpr i64 norm(i64 x) const {
41         if (x < 0) {
42             x += getMod();
43         }
44         if (x >= getMod()) {
45             x -= getMod();
46         }
47         return x;
48     }
49     constexpr i64 val() const {

```



```

50     return x;
51 }
52 explicit constexpr operator i64() const {
53     return x;
54 }
55 constexpr MLong operator-() const {
56     MLong res;
57     res.x = norm(getMod() - x);
58     return res;
59 }
60 constexpr MLong inv() const {
61     assert(x != 0);
62     return power(*this, getMod() - 2);
63 }
64 constexpr MLong &operator*=(MLong rhs) & {
65     x = mul(x, rhs.x, getMod());
66     return *this;
67 }
68 constexpr MLong &operator+=(MLong rhs) & {
69     x = norm(x + rhs.x);
70     return *this;
71 }
72 constexpr MLong &operator--(MLong rhs) & {
73     x = norm(x - rhs.x);
74     return *this;
75 }
76 constexpr MLong &operator/=(MLong rhs) & {
77     return *this *= rhs.inv();
78 }
79 friend constexpr MLong operator*(MLong lhs, MLong rhs) {
80     MLong res = lhs;
81     res *= rhs;
82     return res;
83 }
84 friend constexpr MLong operator+(MLong lhs, MLong rhs) {
85     MLong res = lhs;
86     res += rhs;
87     return res;
88 }
89 friend constexpr MLong operator-(MLong lhs, MLong rhs) {
90     MLong res = lhs;
91     res -= rhs;
92     return res;
93 }
94 friend constexpr MLong operator/(MLong lhs, MLong rhs) {
95     MLong res = lhs;
96     res /= rhs;
97     return res;
98 }
99 friend constexpr istream &operator>>(istream &is, MLong &a) {
100     i64 v;
101     is >> v;
102     a = MLong(v);
103     return is;
104 }
105 friend constexpr ostream &operator<<(ostream &os, const MLong &a) {
106     return os << a.val();
107 }
108 friend constexpr bool operator==(MLong lhs, MLong rhs) {
109     return lhs.val() == rhs.val();
110 }
111 friend constexpr bool operator!=(MLong lhs, MLong rhs) {
112     return lhs.val() != rhs.val();
113 }

```

```

114 };
115
116 template<>
117 i64 MLong<0LL>::Mod = i64(1E18) + 9;
118
119 template<int P>
120 struct MInt {
121     int x;
122     constexpr MInt() : x{} {}
123     constexpr MInt(i64 x) : x{norm(x % getMod())} {}
124
125     static int Mod;
126     constexpr static int getMod() {
127         if (P > 0) {
128             return P;
129         } else {
130             return Mod;
131         }
132     }
133     constexpr static void setMod(int Mod_) {
134         Mod = Mod_;
135     }
136     constexpr int norm(int x) const {
137         if (x < 0) {
138             x += getMod();
139         }
140         if (x >= getMod()) {
141             x -= getMod();
142         }
143         return x;
144     }
145     constexpr int val() const {
146         return x;
147     }
148     explicit constexpr operator int() const {
149         return x;
150     }
151     constexpr MInt operator-() const {
152         MInt res;
153         res.x = norm(getMod() - x);
154         return res;
155     }
156     constexpr MInt inv() const {
157         assert(x != 0);
158         return power(*this, getMod() - 2);
159     }
160     constexpr MInt &operator*=(MInt rhs) & {
161         x = 1LL * x * rhs.x % getMod();
162         return *this;
163     }
164     constexpr MInt &operator+=(MInt rhs) & {
165         x = norm(x + rhs.x);
166         return *this;
167     }
168     constexpr MInt &operator-=(MInt rhs) & {
169         x = norm(x - rhs.x);
170         return *this;
171     }
172     constexpr MInt &operator/=(MInt rhs) & {
173         return *this *= rhs.inv();
174     }
175     friend constexpr MInt operator*(MInt lhs, MInt rhs) {
176         MInt res = lhs;
177         res *= rhs;

```

```

178     return res;
179 }
180 friend constexpr MInt operator+(MInt lhs, MInt rhs) {
181     MInt res = lhs;
182     res += rhs;
183     return res;
184 }
185 friend constexpr MInt operator-(MInt lhs, MInt rhs) {
186     MInt res = lhs;
187     res -= rhs;
188     return res;
189 }
190 friend constexpr MInt operator/(MInt lhs, MInt rhs) {
191     MInt res = lhs;
192     res /= rhs;
193     return res;
194 }
195 friend constexpr istream &operator>>(istream &is, MInt &a) {
196     i64 v;
197     is >> v;
198     a = MInt(v);
199     return is;
200 }
201 friend constexpr ostream &operator<<(ostream &os, const MInt &a) {
202     return os << a.val();
203 }
204 friend constexpr bool operator==(MInt lhs, MInt rhs) {
205     return lhs.val() == rhs.val();
206 }
207 friend constexpr bool operator!=(MInt lhs, MInt rhs) {
208     return lhs.val() != rhs.val();
209 }
210 };
211
212 template<>
213 int MInt<0>::Mod = 998244353;
214
215 template<int V, int P>
216 constexpr MInt<P> CInv = MInt<P>(V).inv();
217
218 constexpr int P = 1000000007;
219 using Z = MInt<P>;

```

4.5.3 动态取模类 (ModIntBase)

```

1  template<typename T>
2  constexpr T power(T a, u64 b) {
3      T res {1};
4      for (; b != 0; b /= 2, a *= a) {
5          if (b % 2 == 1) {
6              res *= a;
7          }
8      }
9      return res;
10 }
11
12 template<u32 P>
13 constexpr u32 mulMod(u32 a, u32 b) {
14     return 1ULL * a * b % P;
15 }
16
17 template<u64 P>
18 constexpr u64 mulMod(u64 a, u64 b) {

```

```

19     u64 res = a * b - u64(1.L * a * b / P - 0.5L) * P;
20     res %= P;
21     return res;
22 }
23
24 template<typename U, U P>
25 requires unsigned_integral<U>
26 struct ModIntBase {
27 public:
28     constexpr ModIntBase() : x {0} {}
29
30     template<typename T>
31     requires integral<T>
32     constexpr ModIntBase(T x_) : x {norm(x_ % T {P})} {}
33
34     constexpr static U norm(U x) {
35         if ((x >> (8 * sizeof(U) - 1) & 1) == 1) {
36             x += P;
37         }
38         if (x >= P) {
39             x -= P;
40         }
41         return x;
42     }
43
44     constexpr U val() const {
45         return x;
46     }
47
48     constexpr ModIntBase operator-() const {
49         ModIntBase res;
50         res.x = norm(P - x);
51         return res;
52     }
53
54     constexpr ModIntBase inv() const {
55         return power(*this, P - 2);
56     }
57
58     constexpr ModIntBase &operator*=(const ModIntBase &rhs) & {
59         x = mulMod<P>(x, rhs.val());
60         return *this;
61     }
62
63     constexpr ModIntBase &operator+=(const ModIntBase &rhs) & {
64         x = norm(x + rhs.x);
65         return *this;
66     }
67
68     constexpr ModIntBase &operator-=(const ModIntBase &rhs) & {
69         x = norm(x - rhs.x);
70         return *this;
71     }
72
73     constexpr ModIntBase &operator/=(const ModIntBase &rhs) & {
74         return *this *= rhs.inv();
75     }
76
77     friend constexpr ModIntBase operator*(ModIntBase lhs, const ModIntBase &rhs) {
78         lhs *= rhs;
79         return lhs;
80     }
81
82     friend constexpr ModIntBase operator+(ModIntBase lhs, const ModIntBase &rhs) {

```

```

83     lhs += rhs;
84     return lhs;
85 }
86
87 friend constexpr ModIntBase operator-(ModIntBase lhs, const ModIntBase &rhs) {
88     lhs -= rhs;
89     return lhs;
90 }
91
92 friend constexpr ModIntBase operator/(ModIntBase lhs, const ModIntBase &rhs) {
93     lhs /= rhs;
94     return lhs;
95 }
96
97 friend constexpr ostream &operator<<(ostream &os, const ModIntBase &a) {
98     return os << a.val();
99 }
100
101 friend constexpr bool operator==(ModIntBase lhs, ModIntBase rhs) {
102     return lhs.val() == rhs.val();
103 }
104
105 friend constexpr bool operator!=(ModIntBase lhs, ModIntBase rhs) {
106     return lhs.val() != rhs.val();
107 }
108
109 friend constexpr bool operator<(ModIntBase lhs, ModIntBase rhs) {
110     return lhs.val() < rhs.val();
111 }
112
113 private:
114     U x;
115 };
116
117 template<u32 P>
118 using ModInt = ModIntBase<u32, P>;
119
120 template<u64 P>
121 using ModInt64 = ModIntBase<u64, P>;
122
123 constexpr u32 P = 998244353;
124 using Z = ModInt<P>;

```

4.6 状压RMQ (RMQ)

```

1  template<class T, class Cmp = less<T>> struct RMQ {
2      const Cmp cmp = Cmp();
3      static constexpr unsigned B = 64;
4      using u64 = unsigned long long;
5      int n;
6      vector<vector<T>> a;
7      vector<T> pre, suf, ini;
8      vector<u64> stk;
9      RMQ() {}
10     RMQ(const vector<T> &v) {
11         init(v);
12     }
13     void init(const vector<T> &v) {
14         n = v.size();
15         pre = suf = ini = v;
16         stk.resize(n);
17         if (!n) {

```

```

18         return;
19     }
20     const int M = (n - 1) / B + 1;
21     const int lg = __lg(M);
22     a.assign(lg + 1, vector<T>(M));
23     for (int i = 0; i < M; i++) {
24         a[0][i] = v[i * B];
25         for (int j = 1; j < B && i * B + j < n; j++) {
26             a[0][i] = min(a[0][i], v[i * B + j], cmp);
27         }
28     }
29     for (int i = 1; i < n; i++) {
30         if (i % B) {
31             pre[i] = min(pre[i], pre[i - 1], cmp);
32         }
33     }
34     for (int i = n - 2; i >= 0; i--) {
35         if (i % B != B - 1) {
36             suf[i] = min(suf[i], suf[i + 1], cmp);
37         }
38     }
39     for (int j = 0; j < lg; j++) {
40         for (int i = 0; i + (2 << j) <= M; i++) {
41             a[j + 1][i] = min(a[j][i], a[j][i + (1 << j)], cmp);
42         }
43     }
44     for (int i = 0; i < M; i++) {
45         const int l = i * B;
46         const int r = min(1U * n, l + B);
47         u64 s = 0;
48         for (int j = 1; j < r; j++) {
49             while (s && cmp(v[j], v[__lg(s) + 1])) {
50                 s ^= 1ULL << __lg(s);
51             }
52             s |= 1ULL << (j - 1);
53             stk[j] = s;
54         }
55     }
56 }
57 T operator()(int l, int r) {
58     if (l / B != (r - 1) / B) {
59         T ans = min(suf[l], pre[r - 1], cmp);
60         l = l / B + 1;
61         r = r / B;
62         if (l < r) {
63             int k = __lg(r - l);
64             ans = min({ans, a[k][l], a[k][r - (1 << k)]}, cmp);
65         }
66         return ans;
67     } else {
68         int x = B * (l / B);
69         return ini[__builtin_ctzll(stk[r - 1] >> (1 - x)) + 1];
70     }
71 }
72 };

```

4.7 Splay

```

1 struct Node {
2     Node *l = nullptr;
3     Node *r = nullptr;
4     int cnt = 0;

```

```

5     i64 sum = 0;
6 };
7
8 Node *add(Node *t, int l, int r, int p, int v) {
9     Node *x = new Node;
10    if (t) {
11        *x = *t;
12    }
13    x->cnt += 1;
14    x->sum += v;
15    if (r - l == 1) {
16        return x;
17    }
18    int m = (l + r) / 2;
19    if (p < m) {
20        x->l = add(x->l, l, m, p, v);
21    } else {
22        x->r = add(x->r, m, r, p, v);
23    }
24    return x;
25 }
26
27 int find(Node *tl, Node *tr, int l, int r, int x) {
28     if (r <= x) {
29         return -1;
30     }
31     if (l >= x) {
32         int cnt = (tr ? tr->cnt : 0) - (tl ? tl->cnt : 0);
33         if (cnt == 0) {
34             return -1;
35         }
36         if (r - l == 1) {
37             return l;
38         }
39     }
40     int m = (l + r) / 2;
41     int res = find(tl ? tl->l : tl, tr ? tr->l : tr, l, m, x);
42     if (res == -1) {
43         res = find(tl ? tl->r : tl, tr ? tr->r : tr, m, r, x);
44     }
45     return res;
46 }
47
48 pair<int, i64> get(Node *t, int l, int r, int x, int y) {
49     if (l >= y || r <= x || !t) {
50         return {0, 0LL};
51     }
52     if (l >= x && r <= y) {
53         return {t->cnt, t->sum};
54     }
55     int m = (l + r) / 2;
56     auto [cl, sl] = get(t->l, l, m, x, y);
57     auto [cr, sr] = get(t->r, m, r, x, y);
58     return {cl + cr, sl + sr};
59 }
60
61 struct Tree {
62     int add = 0;
63     int val = 0;
64     int id = 0;
65     Tree *ch[2] = {};
66     Tree *p = nullptr;
67 };
68

```

```

69 int pos(Tree *t) {
70     return t->p->ch[1] == t;
71 }
72
73 void add(Tree *t, int v) {
74     t->val += v;
75     t->add += v;
76 }
77
78 void push(Tree *t) {
79     if (t->ch[0]) {
80         add(t->ch[0], t->add);
81     }
82     if (t->ch[1]) {
83         add(t->ch[1], t->add);
84     }
85     t->add = 0;
86 }
87
88 void rotate(Tree *t) {
89     Tree *q = t->p;
90     int x = !pos(t);
91     q->ch[!x] = t->ch[x];
92     if (t->ch[x]) t->ch[x]->p = q;
93     t->p = q->p;
94     if (q->p) q->p->ch[pos(q)] = t;
95     t->ch[x] = q;
96     q->p = t;
97 }
98
99 void splay(Tree *t) {
100     vector<Tree *> s;
101     for (Tree *i = t; i->p; i = i->p) s.push_back(i->p);
102     while (!s.empty()) {
103         push(s.back());
104         s.pop_back();
105     }
106     push(t);
107     while (t->p) {
108         if (t->p->p) {
109             if (pos(t) == pos(t->p)) rotate(t->p);
110             else rotate(t);
111         }
112         rotate(t);
113     }
114 }
115
116 void insert(Tree *&t, Tree *x, Tree *p = nullptr) {
117     if (!t) {
118         t = x;
119         x->p = p;
120         return;
121     }
122
123     push(t);
124     if (x->val < t->val) {
125         insert(t->ch[0], x, t);
126     } else {
127         insert(t->ch[1], x, t);
128     }
129 }
130
131 void dfs(Tree *t) {
132     if (!t) {

```



```

133         return;
134     }
135     push(t);
136     dfs(t->ch[0]);
137     cerr << t->val << " ";
138     dfs(t->ch[1]);
139 }
140
141 pair<Tree *, Tree *> split(Tree *t, int x) {
142     if (!t) {
143         return {t, t};
144     }
145     Tree *v = nullptr;
146     Tree *j = t;
147     for (Tree *i = t; i; ) {
148         push(i);
149         j = i;
150         if (i->val >= x) {
151             v = i;
152             i = i->ch[0];
153         } else {
154             i = i->ch[1];
155         }
156     }
157
158     splay(j);
159     if (!v) {
160         return {j, nullptr};
161     }
162
163     splay(v);
164
165     Tree *u = v->ch[0];
166     if (u) {
167         v->ch[0] = u->p = nullptr;
168     }
169     // cerr << "split " << x << "\n";
170     // dfs(u);
171     // cerr << "\n";
172     // dfs(v);
173     // cerr << "\n";
174     return {u, v};
175 }
176
177 Tree *merge(Tree *l, Tree *r) {
178     if (!l) {
179         return r;
180     }
181     if (!r) {
182         return l;
183     }
184     Tree *i = l;
185     while (i->ch[1]) {
186         i = i->ch[1];
187     }
188     splay(i);
189     i->ch[1] = r;
190     r->p = i;
191     return i;
192 }

```

```

1 struct Matrix : array<array<i64, 4>, 4> {
2     Matrix(i64 v = 0) {

```

```

3         for (int i = 0; i < 4; i++) {
4             for (int j = 0; j < 4; j++) {
5                 (*this)[i][j] = (i == j ? v : inf);
6             }
7         }
8     }
9 };
10
11 Matrix operator*(const Matrix &a, const Matrix &b) {
12     Matrix c(inf);
13     for (int i = 0; i < 3; i++) {
14         for (int j = 0; j < 3; j++) {
15             for (int k = 0; k < 4; k++) {
16                 c[i][k] = min(c[i][k], a[i][j] + b[j][k]);
17             }
18         }
19         c[i][3] = min(c[i][3], a[i][3]);
20     }
21     c[3][3] = 0;
22     return c;
23 }
24
25 struct Node {
26     Node *ch[2], *p;
27     i64 sumg = 0;
28     i64 sumh = 0;
29     i64 sumb = 0;
30     i64 g = 0;
31     i64 h = 0;
32     i64 b = 0;
33     Matrix mat;
34     Matrix prd;
35     array<i64, 4> ans{};
36     Node() : ch{nullptr, nullptr}, p(nullptr) {}
37
38     void update() {
39         mat = Matrix(inf);
40         mat[0][0] = b + h - g + sumg;
41         mat[1][1] = mat[1][2] = mat[1][3] = h + sumh;
42         mat[2][0] = mat[2][1] = mat[2][2] = mat[2][3] = b + h + sumb;
43         mat[3][3] = 0;
44     }
45 };
46 void push(Node *t) {
47
48 }
49 void pull(Node *t) {
50     t->prd = (t->ch[0] ? t->ch[0]->prd : Matrix()) * t->mat * (t->ch[1] ? t->ch[1]-
51 >prd : Matrix());
52 }
53 bool isroot(Node *t) {
54     return t->p == nullptr || (t->p->ch[0] != t && t->p->ch[1] != t);
55 }
56 int pos(Node *t) {
57     return t->p->ch[1] == t;
58 }
59 void pushAll(Node *t) {
60     if (!isroot(t)) {
61         pushAll(t->p);
62     }
63     push(t);
64 }
65 void rotate(Node *t) {
66     Node *q = t->p;

```

```

66     int x = !pos(t);
67     q->ch[!x] = t->ch[x];
68     if (t->ch[x]) {
69         t->ch[x]->p = q;
70     }
71     t->p = q->p;
72     if (!isroot(q)) {
73         q->p->ch[pos(q)] = t;
74     }
75     t->ch[x] = q;
76     q->p = t;
77     pull(q);
78 }
79 void splay(Node *t) {
80     pushAll(t);
81     while (!isroot(t)) {
82         if (!isroot(t->p)) {
83             if (pos(t) == pos(t->p)) {
84                 rotate(t->p);
85             } else {
86                 rotate(t);
87             }
88         }
89         rotate(t);
90     }
91     pull(t);
92 }
93
94 array<i64, 4> get(Node *t) {
95     array<i64, 4> ans;
96     ans.fill(inf);
97     ans[3] = 0;
98     for (int i = 0; i < 3; i++) {
99         for (int j = 0; j < 4; j++) {
100             ans[i] = min(ans[i], t->prd[i][j]);
101         }
102     }
103     return ans;
104 }
105
106 void access(Node *t) {
107     array<i64, 4> old{};
108     for (Node *i = t, *q = nullptr; i; q = i, i = i->p) {
109         splay(i);
110         if (i->ch[1]) {
111             auto res = get(i->ch[1]);
112             i->sumg += res[0];
113             i->sumh += min({res[1], res[2], res[3]});
114             i->sumb += min({res[0], res[1], res[2], res[3]});
115         }
116         i->ch[1] = q;
117         i->sumg -= old[0];
118         i->sumh -= min({old[1], old[2], old[3]});
119         i->sumb -= min({old[0], old[1], old[2], old[3]});
120         old = get(i);
121         i->update();
122         pull(i);
123     }
124     splay(t);
125 }

```

```

1  constexpr int D = 27;
2  struct Info {

```

```

3     int up[D][2] {};
4     int down[D][2] {};
5     int t = 0;
6     i64 ans = 0;
7 };
8
9 Info operator+(const Info &a, const Info &b) {
10     Info c;
11     c.t = a.t ^ b.t;
12     c.ans = a.ans + b.ans;
13     for (int i = 0; i < D; i++) {
14         for (int j = 0; j < 2; j++) {
15             c.ans += (1LL << i) * a.down[i][j] * b.up[i][j ^ 1];
16             c.up[i][j] += a.up[i][j] + b.up[i][j ^ (a.t >> i & 1)];
17             c.down[i][j] += b.down[i][j] + a.down[i][j ^ (b.t >> i & 1)];
18         }
19     }
20     return c;
21 }
22 struct Node {
23     Node *ch[2], *p;
24     Info val;
25     Info tot;
26     int cnt[D][2];
27     i64 pair[D][2];
28     i64 sum;
29     Node() : ch{nullptr, nullptr}, p(nullptr), cnt {}, pair {}, sum {} {}
30 };
31 void pull(Node *t) {
32     t->tot = (t->ch[0] ? t->ch[0]->tot : Info {}) + t->val + (t->ch[1] ? t->ch[1]-
33 >tot : Info {});
34 }
35 bool isroot(Node *t) {
36     return t->p == nullptr || (t->p->ch[0] != t && t->p->ch[1] != t);
37 }
38 int pos(Node *t) {
39     return t->p->ch[1] == t;
40 }
41 void rotate(Node *t) {
42     Node *q = t->p;
43     int x = !pos(t);
44     q->ch[!x] = t->ch[x];
45     if (t->ch[x]) {
46         t->ch[x]->p = q;
47     }
48     t->p = q->p;
49     if (!isroot(q)) {
50         q->p->ch[pos(q)] = t;
51     }
52     t->ch[x] = q;
53     q->p = t;
54     pull(q);
55 }
56 void update(Node *t) {
57     t->val.ans = t->val.t + t->sum;
58     for (int i = 0; i < D; i++) {
59         t->val.ans += (1LL << i) * t->pair[i][t->val.t >> i & 1];
60         for (int j = 0; j < 2; j++) {
61             t->val.up[i][j] = t->cnt[i][j ^ (t->val.t >> i & 1)];
62             t->val.down[i][j] = t->cnt[i][j ^ (t->val.t >> i & 1)];
63         }
64         t->val.up[i][t->val.t >> i & 1]++;
65         t->val.down[i][t->val.t >> i & 1]++;
66     }
67 }

```

```

66     pull(t);
67 }
68 void splay(Node *t) {
69     while (!isroot(t)) {
70         if (!isroot(t->p)) {
71             if (pos(t) == pos(t->p)) {
72                 rotate(t->p);
73             } else {
74                 rotate(t);
75             }
76         }
77         rotate(t);
78     }
79     pull(t);
80 }
81 void add(Node *t, Info s) {
82     for (int i = 0; i < D; i++) {
83         for (int x = 0; x < 2; x++) {
84             t->pair[i][x] += s.up[i][1 ^ x];
85             for (int j = 0; j < 2; j++) {
86                 t->pair[i][x] += t->cnt[i][j] * s.up[i][j ^ 1 ^ x];
87             }
88         }
89         for (int j = 0; j < 2; j++) {
90             t->cnt[i][j] += s.up[i][j];
91         }
92     }
93     t->sum += s.ans;
94 }
95 void del(Node *t, Info s) {
96     t->sum -= s.ans;
97     for (int i = 0; i < D; i++) {
98         for (int j = 0; j < 2; j++) {
99             t->cnt[i][j] -= s.up[i][j];
100         }
101         for (int x = 0; x < 2; x++) {
102             for (int j = 0; j < 2; j++) {
103                 t->pair[i][x] -= t->cnt[i][j] * s.up[i][j ^ 1 ^ x];
104             }
105             t->pair[i][x] -= s.up[i][1 ^ x];
106         }
107     }
108 }
109 void access(Node *t, int v) {
110     Info lst;
111     for (Node *i = t, *q = nullptr; i; q = i, i = i->p) {
112         splay(i);
113         if (i->ch[1]) {
114             add(i, i->ch[1]->tot);
115         }
116         i->ch[1] = q;
117         if (q) {
118             del(i, lst);
119         } else {
120             i->val.t = v;
121         }
122         lst = i->tot;
123         update(i);
124     }
125     splay(t);
126 }

```

4.8 其他平衡树

```

1 struct Node {
2     Node *l = nullptr;
3     Node *r = nullptr;
4     int sum = 0;
5     int sumodd = 0;
6
7     Node(Node *t) {
8         if (t) {
9             *this = *t;
10        }
11    }
12 };
13
14 Node *add(Node *t, int l, int r, int x, int v) {
15     t = new Node(t);
16     t->sum += v;
17     t->sumodd += (x % 2) * v;
18     if (r - l == 1) {
19         return t;
20     }
21     int m = (l + r) / 2;
22     if (x < m) {
23         t->l = add(t->l, l, m, x, v);
24     } else {
25         t->r = add(t->r, m, r, x, v);
26     }
27     return t;
28 }
29
30 int query1(Node *t1, Node *t2, int l, int r, int k) {
31     if (r - l == 1) {
32         return 1;
33     }
34     int m = (l + r) / 2;
35     int odd = (t1 && t1->r ? t1->r->sumodd : 0) - (t2 && t2->r ? t2->r->sumodd : 0);
36     int cnt = (t1 && t1->r ? t1->r->sum : 0) - (t2 && t2->r ? t2->r->sum : 0);
37     if (odd > 0 || cnt > k) {
38         return query1(t1 ? t1->r : t1, t2 ? t2->r : t2, m, r, k);
39     } else {
40         return query1(t1 ? t1->l : t1, t2 ? t2->l : t2, l, m, k - cnt);
41     }
42 }
43
44 array<int, 3> query2(Node *t1, Node *t2, int l, int r, int k) {
45     if (r - l == 1) {
46         int cnt = (t1 ? t1->sumodd : 0) - (t2 ? t2->sumodd : 0);
47         return {1, cnt, k};
48     }
49     int m = (l + r) / 2;
50     int cnt = (t1 && t1->r ? t1->r->sumodd : 0) - (t2 && t2->r ? t2->r->sumodd : 0);
51     if (cnt > k) {
52         return query2(t1 ? t1->r : t1, t2 ? t2->r : t2, m, r, k);
53     } else {
54         return query2(t1 ? t1->l : t1, t2 ? t2->l : t2, l, m, k - cnt);
55     }
56 }

```

```

1 struct Node {
2     Node *l = nullptr;
3     Node *r = nullptr;

```

```

4     int cnt = 0;
5 };
6
7 Node *add(Node *t, int l, int r, int x) {
8     if (t) {
9         t = new Node(*t);
10    } else {
11        t = new Node;
12    }
13    t->cnt += 1;
14    if (r - l == 1) {
15        return t;
16    }
17    int m = (l + r) / 2;
18    if (x < m) {
19        t->l = add(t->l, l, m, x);
20    } else {
21        t->r = add(t->r, m, r, x);
22    }
23    return t;
24 }
25
26 int query(Node *t1, Node *t2, int l, int r, int x) {
27     int cnt = (t2 ? t2->cnt : 0) - (t1 ? t1->cnt : 0);
28     if (cnt == 0 || l >= x) {
29         return -1;
30     }
31     if (r - l == 1) {
32         return l;
33     }
34     int m = (l + r) / 2;
35     int res = query(t1 ? t1->r : t1, t2 ? t2->r : t2, m, r, x);
36     if (res == -1) {
37         res = query(t1 ? t1->l : t1, t2 ? t2->l : t2, l, m, x);
38     }
39     return res;
40 }

```

```

1 struct Info {
2     int imp = 0;
3     int id = 0;
4 };
5
6 Info operator+(Info a, Info b) {
7     return {max(a.imp, b.imp), 0};
8 }
9
10 struct Node {
11     int w = rng();
12     Info info;
13     Info sum;
14     int siz = 1;
15     Node *l = nullptr;
16     Node *r = nullptr;
17 };
18
19 void pull(Node *t) {
20     t->sum = t->info;
21     t->siz = 1;
22     if (t->l) {
23         t->sum = t->l->sum + t->sum;
24         t->siz += t->l->siz;
25     }

```

```

26     if (t->r) {
27         t->sum = t->sum + t->r->sum;
28         t->siz += t->r->siz;
29     }
30 }
31
32 pair<Node *, Node *> splitAt(Node *t, int p) {
33     if (!t) {
34         return {t, t};
35     }
36     if (p <= (t->l ? t->l->siz : 0)) {
37         auto [l, r] = splitAt(t->l, p);
38         t->l = r;
39         pull(t);
40         return {l, t};
41     } else {
42         auto [l, r] = splitAt(t->r, p - 1 - (t->l ? t->l->siz : 0));
43         t->r = l;
44         pull(t);
45         return {t, r};
46     }
47 }
48
49 void insertAt(Node *&t, int p, Node *x) {
50     if (!t) {
51         t = x;
52         return;
53     }
54     if (x->w < t->w) {
55         auto [l, r] = splitAt(t, p);
56         t = x;
57         t->l = l;
58         t->r = r;
59         pull(t);
60         return;
61     }
62     if (p <= (t->l ? t->l->siz : 0)) {
63         insertAt(t->l, p, x);
64     } else {
65         insertAt(t->r, p - 1 - (t->l ? t->l->siz : 0), x);
66     }
67     pull(t);
68 }
69
70 Node *merge(Node *a, Node *b) {
71     if (!a) {
72         return b;
73     }
74     if (!b) {
75         return a;
76     }
77
78     if (a->w < b->w) {
79         a->r = merge(a->r, b);
80         pull(a);
81         return a;
82     } else {
83         b->l = merge(a, b->l);
84         pull(b);
85         return b;
86     }
87 }
88
89 int query(Node *t, int v) {

```



```

90     if (!t) {
91         return 0;
92     }
93     if (t->sum.imp < v) {
94         return t->siz;
95     }
96     int res = query(t->r, v);
97     if (res != (t->r ? t->r->siz : 0)) {
98         return res;
99     }
100    if (t->info.imp > v) {
101        return res;
102    }
103    return res + 1 + query(t->l, v);
104 }
105
106 void dfs(Node *t) {
107     if (!t) {
108         return;
109     }
110     dfs(t->l);
111     cout << t->info.id << " ";
112     dfs(t->r);
113 }

```

```

1  struct Node {
2      Node *l = nullptr;
3      Node *r = nullptr;
4      int cnt = 0;
5      int cntnew = 0;
6  };
7
8  Node *add(int l, int r, int x, int isnew) {
9      Node *t = new Node;
10     t->cnt = 1;
11     t->cntnew = isnew;
12     if (r - l == 1) {
13         return t;
14     }
15     int m = (l + r) / 2;
16     if (x < m) {
17         t->l = add(l, m, x, isnew);
18     } else {
19         t->r = add(m, r, x, isnew);
20     }
21     return t;
22 }
23
24 struct Info {
25     Node *t = nullptr;
26     int psum = 0;
27     bool rev = false;
28 };
29
30 void pull(Node *t) {
31     t->cnt = (t->l ? t->l->cnt : 0) + (t->r ? t->r->cnt : 0);
32     t->cntnew = (t->l ? t->l->cntnew : 0) + (t->r ? t->r->cntnew : 0);
33 }
34
35 pair<Node *, Node *> split(Node *t, int l, int r, int x, bool rev) {
36     if (!t) {
37         return {t, t};
38     }

```

```

39     if (x == 0) {
40         return {nullptr, t};
41     }
42     if (x == t->cnt) {
43         return {t, nullptr};
44     }
45     if (r - l == 1) {
46         Node *t2 = new Node;
47         t2->cnt = t->cnt - x;
48         t->cnt = x;
49         return {t, t2};
50     }
51     Node *t2 = new Node;
52     int m = (l + r) / 2;
53     if (!rev) {
54         if (t->l && x <= t->l->cnt) {
55             tie(t->l, t2->l) = split(t->l, l, m, x, rev);
56             t2->r = t->r;
57             t->r = nullptr;
58         } else {
59             tie(t->r, t2->r) = split(t->r, m, r, x - (t->l ? t->l->cnt : 0), rev);
60         }
61     } else {
62         if (t->r && x <= t->r->cnt) {
63             tie(t->r, t2->r) = split(t->r, m, r, x, rev);
64             t2->l = t->l;
65             t->l = nullptr;
66         } else {
67             tie(t->l, t2->l) = split(t->l, l, m, x - (t->r ? t->r->cnt : 0), rev);
68         }
69     }
70     pull(t);
71     pull(t2);
72     return {t, t2};
73 }
74
75 Node *merge(Node *t1, Node *t2, int l, int r) {
76     if (!t1) {
77         return t2;
78     }
79     if (!t2) {
80         return t1;
81     }
82     if (r - l == 1) {
83         t1->cnt += t2->cnt;
84         t1->cntnew += t2->cntnew;
85         delete t2;
86         return t1;
87     }
88     int m = (l + r) / 2;
89     t1->l = merge(t1->l, t2->l, l, m);
90     t1->r = merge(t1->r, t2->r, m, r);
91     delete t2;
92     pull(t1);
93     return t1;
94 }

```

4.9 分数四则运算 (Frac)

```

1  template<class T>
2  struct Frac {
3      T num;
4      T den;
5      Frac(T num_, T den_) : num(num_), den(den_) {
6          if (den < 0) {
7              den = -den;
8              num = -num;
9          }
10     }
11     Frac() : Frac(0, 1) {}
12     Frac(T num_) : Frac(num_, 1) {}
13     explicit operator double() const {
14         return 1. * num / den;
15     }
16     Frac &operator+=(const Frac &rhs) {
17         num = num * rhs.den + rhs.num * den;
18         den *= rhs.den;
19         return *this;
20     }
21     Frac &operator-=(const Frac &rhs) {
22         num = num * rhs.den - rhs.num * den;
23         den *= rhs.den;
24         return *this;
25     }
26     Frac &operator*=(const Frac &rhs) {
27         num *= rhs.num;
28         den *= rhs.den;
29         return *this;
30     }
31     Frac &operator/=(const Frac &rhs) {
32         num *= rhs.den;
33         den *= rhs.num;
34         if (den < 0) {
35             num = -num;
36             den = -den;
37         }
38         return *this;
39     }
40     friend Frac operator+(Frac lhs, const Frac &rhs) {
41         return lhs += rhs;
42     }
43     friend Frac operator-(Frac lhs, const Frac &rhs) {
44         return lhs -= rhs;
45     }
46     friend Frac operator*(Frac lhs, const Frac &rhs) {
47         return lhs *= rhs;
48     }
49     friend Frac operator/(Frac lhs, const Frac &rhs) {
50         return lhs /= rhs;
51     }
52     friend Frac operator-(const Frac &a) {
53         return Frac(-a.num, a.den);
54     }
55     friend bool operator==(const Frac &lhs, const Frac &rhs) {
56         return lhs.num * rhs.den == rhs.num * lhs.den;
57     }
58     friend bool operator!=(const Frac &lhs, const Frac &rhs) {
59         return lhs.num * rhs.den != rhs.num * lhs.den;
60     }
61     friend bool operator<(const Frac &lhs, const Frac &rhs) {

```

```

62     return lhs.num * rhs.den < rhs.num * lhs.den;
63 }
64 friend bool operator>(const Frac &lhs, const Frac &rhs) {
65     return lhs.num * rhs.den > rhs.num * lhs.den;
66 }
67 friend bool operator<=(const Frac &lhs, const Frac &rhs) {
68     return lhs.num * rhs.den <= rhs.num * lhs.den;
69 }
70 friend bool operator>=(const Frac &lhs, const Frac &rhs) {
71     return lhs.num * rhs.den >= rhs.num * lhs.den;
72 }
73 friend ostream &operator<<(ostream &os, Frac x) {
74     T g = gcd(x.num, x.den);
75     if (x.den == g) {
76         return os << x.num / g;
77     } else {
78         return os << x.num / g << "/" << x.den / g;
79     }
80 }
81 };

```

4.10 线性基 (Basis)

```

1  struct Basis {
2      int a[20] {};
3      int t[20] {};
4
5      Basis() {
6          fill(t, t + 20, -1);
7      }
8
9      void add(int x, int y = 1E9) {
10         for (int i = 0; i < 20; i++) {
11             if (x >> i & 1) {
12                 if (y > t[i]) {
13                     swap(a[i], x);
14                     swap(t[i], y);
15                 }
16                 x ^= a[i];
17             }
18         }
19     }
20
21     bool query(int x, int y = 0) {
22         for (int i = 0; i < 20; i++) {
23             if ((x >> i & 1) && t[i] >= y) {
24                 x ^= a[i];
25             }
26         }
27         return x == 0;
28     }
29 };

```

4.11 高精度 (BigInt)

```

1  constexpr int N = 1000;
2
3  struct BigInt {
4      int a[N];
5      BigInt(int x = 0) : a{} {
6          for (int i = 0; x; i++) {

```

```

7         a[i] = x % 10;
8         x /= 10;
9     }
10 }
11 BigInt &operator*=(int x) {
12     for (int i = 0; i < N; i++) {
13         a[i] *= x;
14     }
15     for (int i = 0; i < N - 1; i++) {
16         a[i + 1] += a[i] / 10;
17         a[i] %= 10;
18     }
19     return *this;
20 }
21 BigInt &operator/=(int x) {
22     for (int i = N - 1; i >= 0; i--) {
23         if (i) {
24             a[i - 1] += a[i] % x * 10;
25         }
26         a[i] /= x;
27     }
28     return *this;
29 }
30 BigInt &operator+=(const BigInt &x) {
31     for (int i = 0; i < N; i++) {
32         a[i] += x.a[i];
33         if (a[i] >= 10) {
34             a[i + 1] += 1;
35             a[i] -= 10;
36         }
37     }
38     return *this;
39 }
40 };
41
42 ostream &operator<<(ostream &o, const BigInt &a) {
43     int t = N - 1;
44     while (a.a[t] == 0) {
45         t--;
46     }
47     for (int i = t; i >= 0; i--) {
48         o << a.a[i];
49     }
50     return o;
51 }

```

4.12 Link-Cut Tree

```

1 namespace SegT {
2     int tag[8 * N];
3     int64_t wsum[8 * N], sum[8 * N];
4     void add(int p, int l, int r, int v) {
5         sum[p] += v * (r - l);
6         wsum[p] += 1ll * v * (r - l) * (1 + r + l) / 2;
7         tag[p] += v;
8     }
9     void push(int p, int l, int r) {
10        int m = (l + r) / 2;
11        add(2 * p, l, m, tag[p]);
12        add(2 * p + 1, m, r, tag[p]);
13        tag[p] = 0;
14    }

```

```

15 void pull(int p) {
16     sum[p] = sum[2 * p] + sum[2 * p + 1];
17     wsum[p] = wsum[2 * p] + wsum[2 * p + 1];
18 }
19 void rangeAdd(int p, int l, int r, int x, int y, int v) {
20     if (l >= y || r <= x)
21         return;
22     if (l >= x && r <= y)
23         return add(p, l, r, v);
24     push(p, l, r);
25     int m = (l + r) / 2;
26     rangeAdd(2 * p, l, m, x, y, v);
27     rangeAdd(2 * p + 1, m, r, x, y, v);
28     pull(p);
29 }
30 int64_t query(int p, int l, int r, int x) {
31     if (l >= x)
32         return sum[p] * x;
33     if (r <= x)
34         return wsum[p];
35     int m = (l + r) / 2;
36     push(p, l, r);
37     return query(2 * p, l, m, x) + query(2 * p + 1, m, r, x);
38 }
39 int get(int p, int l, int r, int x) {
40     if (r - l == 1)
41         return sum[p];
42     int m = (l + r) / 2;
43     push(p, l, r);
44     if (x < m) {
45         return get(2 * p, l, m, x);
46     } else {
47         return get(2 * p + 1, m, r, x);
48     }
49 }
50 }
51 namespace LCT {
52     int ch[2 * N][2], p[2 * N], endp[2 * N], mn[2 * N], mx[2 * N];
53     bool isroot(int t) {
54         return ch[p[t]][0] != t && ch[p[t]][1] != t;
55     }
56     bool pos(int t) {
57         return ch[p[t]][1] == t;
58     }
59     void pull(int t) {
60         mn[t] = max(0, ch[t][0] ? mn[ch[t][0]] : SAM::len[SAM::link[t]]);
61         mx[t] = ch[t][1] ? mx[ch[t][1]] : SAM::len[t];
62     }
63     void rotate(int t) {
64         int k = !pos(t);
65         int q = p[t];
66         ch[q][!k] = ch[t][k];
67         if (ch[t][k])
68             p[ch[t][k]] = q;
69         p[t] = p[q];
70         if (isroot(q)) {
71             endp[t] = endp[q];
72         } else {
73             ch[p[q]][pos(q)] = t;
74         }
75         ch[t][k] = q;
76         p[q] = t;
77         pull(q);
78     }

```

```

79 void splay(int t) {
80     while (!isroot(t)) {
81         int q = p[t];
82         if (!isroot(q))
83             rotate(pos(t) == pos(q) ? q : t);
84         rotate(t);
85     }
86     pull(t);
87 }
88 void access(int t, int len) {
89     for (int i = t, u = 0; i; u = i, i = p[i]) {
90         splay(i);
91         if (ch[i][1])
92             endp[ch[i][1]] = endp[i];
93         ch[i][1] = 0;
94         pull(i);
95         if (u)
96             SegT::rangeAdd(1, 0, n, endp[i] - mx[i], endp[i] - mn[i], -1);
97         ch[i][1] = u;
98         pull(i);
99     }
100    splay(t);
101    endp[t] = len;
102    SegT::rangeAdd(1, 0, n, len - mx[t], len - mn[t], 1);
103 }
104 void cut(int t) {
105     splay(t);
106     if (ch[t][0]) {
107         endp[ch[t][0]] = endp[t];
108         p[ch[t][0]] = p[t];
109         p[t] = 0;
110         ch[t][0] = 0;
111         pull(t);
112     } else {
113         p[t] = 0;
114     }
115 }
116 void link(int t, int x) {
117     p[t] = x;
118 }
119 }

```

```

1 struct Node {
2     Node *ch[2], *p;
3     bool rev;
4     int siz = 1;
5     Node() : ch{nullptr, nullptr}, p(nullptr), rev(false) {}
6 };
7 void reverse(Node *t) {
8     if (t) {
9         swap(t->ch[0], t->ch[1]);
10        t->rev ^= 1;
11    }
12 }
13 void push(Node *t) {
14     if (t->rev) {
15         reverse(t->ch[0]);
16         reverse(t->ch[1]);
17         t->rev = false;
18    }
19 }
20 void pull(Node *t) {
21     t->siz = (t->ch[0] ? t->ch[0]->siz : 0) + 1 + (t->ch[1] ? t->ch[1]->siz : 0);

```

```

22 }
23 bool isroot(Node *t) {
24     return t->p == nullptr || (t->p->ch[0] != t && t->p->ch[1] != t);
25 }
26 int pos(Node *t) {
27     return t->p->ch[1] == t;
28 }
29 void pushAll(Node *t) {
30     if (!isroot(t)) {
31         pushAll(t->p);
32     }
33     push(t);
34 }
35 void rotate(Node *t) {
36     Node *q = t->p;
37     int x = !pos(t);
38     q->ch[!x] = t->ch[x];
39     if (t->ch[x]) {
40         t->ch[x]->p = q;
41     }
42     t->p = q->p;
43     if (!isroot(q)) {
44         q->p->ch[pos(q)] = t;
45     }
46     t->ch[x] = q;
47     q->p = t;
48     pull(q);
49 }
50 void splay(Node *t) {
51     pushAll(t);
52     while (!isroot(t)) {
53         if (!isroot(t->p)) {
54             if (pos(t) == pos(t->p)) {
55                 rotate(t->p);
56             } else {
57                 rotate(t);
58             }
59         }
60         rotate(t);
61     }
62     pull(t);
63 }
64 void access(Node *t) {
65     for (Node *i = t, *q = nullptr; i; q = i, i = i->p) {
66         splay(i);
67         i->ch[1] = q;
68         pull(i);
69     }
70     splay(t);
71 }
72 void makeroot(Node *t) {
73     access(t);
74     reverse(t);
75 }
76 void link(Node *x, Node *y) {
77     makeroot(x);
78     x->p = y;
79 }
80 void split(Node *x, Node *y) {
81     makeroot(x);
82     access(y);
83 }
84 void cut(Node *x, Node *y) {
85     split(x, y);

```



```
86     x->p = y->ch[0] = nullptr;  
87     pull(y);  
88 }  
89 int dist(Node *x, Node *y) {  
90     split(x, y);  
91     return y->siz - 1;  
92 }
```

/END/

5 字符串

5.1 马拉车 (Manacher)

```

1 vector<int> manacher(string s) {
2     string t = "#";
3     for (auto c : s) {
4         t += c;
5         t += '#';
6     }
7     int n = t.size();
8     vector<int> r(n);
9     for (int i = 0, j = 0; i < n; i++) {
10        if (2 * j - i >= 0 && j + r[j] > i) {
11            r[i] = min(r[2 * j - i], j + r[j] - i);
12        }
13        while (i - r[i] >= 0 && i + r[i] < n && t[i - r[i]] == t[i + r[i]]) {
14            r[i] += 1;
15        }
16        if (i + r[i] > j + r[j]) {
17            j = i;
18        }
19    }
20    return r;
21 }
22

```

5.2 z函数

```

1 vector<int> Z(string s) {
2     int n = s.size();
3     vector<int> z(n + 1);
4     z[0] = n;
5     for (int i = 1, j = 1; i < n; i++) {
6         z[i] = max(0, min(j + z[j] - i, z[i - j]));
7         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
8             z[i]++;
9         }
10        if (i + z[i] > j + z[j]) {
11            j = i;
12        }
13    }
14    return z;
15 }

```

5.3 后缀数组

5.3.1 后缀数组 (SuffixArray 旧版)

```

1 struct SuffixArray {
2     int n;
3     vector<int> sa, rk, lc;
4     SuffixArray(const string &s) {
5         n = s.length();
6         sa.resize(n);
7         lc.resize(n - 1);
8         rk.resize(n);
9         iota(sa.begin(), sa.end(), 0);
10        sort(sa.begin(), sa.end(), [&](int a, int b) {return s[a] < s[b];});

```

```

11     rk[sa[0]] = 0;
12     for (int i = 1; i < n; ++i)
13         rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
14     int k = 1;
15     vector<int> tmp, cnt(n);
16     tmp.reserve(n);
17     while (rk[sa[n - 1]] < n - 1) {
18         tmp.clear();
19         for (int i = 0; i < k; ++i)
20             tmp.push_back(n - k + i);
21         for (auto i : sa)
22             if (i >= k)
23                 tmp.push_back(i - k);
24         fill(cnt.begin(), cnt.end(), 0);
25         for (int i = 0; i < n; ++i)
26             ++cnt[rk[i]];
27         for (int i = 1; i < n; ++i)
28             cnt[i] += cnt[i - 1];
29         for (int i = n - 1; i >= 0; --i)
30             sa[--cnt[rk[tmp[i]]]] = tmp[i];
31         swap(rk, tmp);
32         rk[sa[0]] = 0;
33         for (int i = 1; i < n; ++i)
34             rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa[i - 1]] < tmp[sa[i]] || sa[i -
35 1] + k == n || tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
36         k *= 2;
37     }
38     for (int i = 0, j = 0; i < n; ++i) {
39         if (rk[i] == 0) {
40             j = 0;
41         } else {
42             for (j -= j > 0; i + j < n && sa[rk[i] - 1] + j < n && s[i + j] ==
43 s[sa[rk[i] - 1] + j]; )
44                 ++j;
45             lc[rk[i] - 1] = j;
46         }
47     }
48 };

```

5.3.2 后缀数组 (SA及其应用 新版)

```

1 struct SA {
2     int n;
3     vector<int> sa, rk, lc;
4
5     SA(string s) {
6         n = s.size();
7         sa.resize(n);
8         lc.resize(n - 1);
9         rk.resize(n);
10        iota(sa.begin(), sa.end(), 0);
11        sort(sa.begin(), sa.end(),
12            [&](int a, int b) {
13                return s[a] < s[b];
14            });
15        rk[sa[0]] = 0;
16        for (int i = 1; i < n; ++i) {
17            rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
18        }
19        int k = 1;
20        vector<int> tmp, cnt(n);
21        tmp.reserve(n);

```

```

22     while (rk[sa[n - 1]] < n - 1) {
23         tmp.clear();
24         for (int i = 0; i < k; i++) {
25             tmp.push_back(n - k + i);
26         }
27         for (auto i : sa) {
28             if (i >= k) {
29                 tmp.push_back(i - k);
30             }
31         }
32         fill(cnt.begin(), cnt.end(), 0);
33         for (int i = 0; i < n; i++) {
34             cnt[rk[i]]++;
35         }
36         for (int i = 1; i < n; i++) {
37             cnt[i] += cnt[i - 1];
38         }
39         for (int i = n - 1; i >= 0; i--) {
40             sa[--cnt[rk[tmp[i]]]] = tmp[i];
41         }
42         swap(rk, tmp);
43         rk[sa[0]] = 0;
44         for (int i = 1; i < n; i++) {
45             rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa[i - 1]] < tmp[sa[i]] || sa[i -
1] + k == n || tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
46         }
47         k *= 2;
48     }
49     for (int i = 0, j = 0; i < n; i++) {
50         if (rk[i] == 0) {
51             j = 0;
52         } else {
53             for (j -= j > 0; i + j < n && sa[rk[i] - 1] + j < n && s[i + j] ==
s[sa[rk[i] - 1] + j]; ) {
54                 j++;
55             }
56             lc[rk[i] - 1] = j;
57         }
58     }
59 }
60 };
61
62 void solve() {
63     constexpr int K = 21;
64     vector st(K, vector<int>(1 - 1));
65     st[0] = lc;
66     for (int j = 0; j < K - 1; j++) {
67         for (int i = 0; i + (2 << j) <= 1 - 1; i++) {
68             st[j + 1][i] = min(st[j][i], st[j][i + (1 << j)]);
69         }
70     }
71
72     auto rmq = [&](int l, int r) {
73         int k = __lg(r - l);
74         return min(st[k][l], st[k][r - (1 << k)]);
75     };
76
77     auto lcp = [&](int i, int j) {
78         if (i == j || i == n || j == n) {
79             return min(n - i, n - j);
80         }
81         int a = rk[i];
82         int b = rk[j];
83         if (a > b) {

```

```

84         swap(a, b);
85     }
86     return min({n - i, n - j, rmq(a, b)});
87 };
88
89 auto lcs = [&](int i, int j) {
90     if (i == j || i == 0 || j == 0) {
91         return min(i, j);
92     }
93     int a = rk[n + n - i];
94     int b = rk[n + n - j];
95     if (a > b) {
96         swap(a, b);
97     }
98     return min({i, j, rmq(a, b)});
99 };
100 }

```

5.4 后缀自动机

5.4.1 后缀自动机 (SuffixAutomaton 旧版)

```

1  struct SuffixAutomaton {
2      static constexpr int ALPHABET_SIZE = 26, N = 5e5;
3      struct Node {
4          int len;
5          int link;
6          int next[ALPHABET_SIZE];
7          Node() : len(0), link(0), next{} {}
8      } t[2 * N];
9      int cntNodes;
10     SuffixAutomaton() {
11         cntNodes = 1;
12         fill(t[0].next, t[0].next + ALPHABET_SIZE, 1);
13         t[0].len = -1;
14     }
15     int extend(int p, int c) {
16         if (t[p].next[c]) {
17             int q = t[p].next[c];
18             if (t[q].len == t[p].len + 1)
19                 return q;
20             int r = ++cntNodes;
21             t[r].len = t[p].len + 1;
22             t[r].link = t[q].link;
23             copy(t[q].next, t[q].next + ALPHABET_SIZE, t[r].next);
24             t[q].link = r;
25             while (t[p].next[c] == q) {
26                 t[p].next[c] = r;
27                 p = t[p].link;
28             }
29             return r;
30         }
31         int cur = ++cntNodes;
32         t[cur].len = t[p].len + 1;
33         while (!t[p].next[c]) {
34             t[p].next[c] = cur;
35             p = t[p].link;
36         }
37         t[cur].link = extend(p, c);
38         return cur;
39     }
40 };

```

5.4.2 后缀自动机 (SAM 新版)

```

1 struct SAM {
2     static constexpr int ALPHABET_SIZE = 26;
3     struct Node {
4         int len;
5         int link;
6         array<int, ALPHABET_SIZE> next;
7         Node() : len{}, link{}, next{} {}
8     };
9     vector<Node> t;
10    SAM() {
11        init();
12    }
13    void init() {
14        t.assign(2, Node());
15        t[0].next.fill(1);
16        t[0].len = -1;
17    }
18    int newNode() {
19        t.emplace_back();
20        return t.size() - 1;
21    }
22    int extend(int p, int c) {
23        if (t[p].next[c]) {
24            int q = t[p].next[c];
25            if (t[q].len == t[p].len + 1) {
26                return q;
27            }
28            int r = newNode();
29            t[r].len = t[p].len + 1;
30            t[r].link = t[q].link;
31            t[r].next = t[q].next;
32            t[q].link = r;
33            while (t[p].next[c] == q) {
34                t[p].next[c] = r;
35                p = t[p].link;
36            }
37            return r;
38        }
39        int cur = newNode();
40        t[cur].len = t[p].len + 1;
41        while (!t[p].next[c]) {
42            t[p].next[c] = cur;
43            p = t[p].link;
44        }
45        t[cur].link = extend(p, c);
46        return cur;
47    }
48    int extend(int p, char c, char offset = 'a') {
49        return extend(p, c - offset);
50    }
51
52    int next(int p, int x) {
53        return t[p].next[x];
54    }
55
56    int next(int p, char c, char offset = 'a') {
57        return next(p, c - 'a');
58    }
59
60    int link(int p) {
61        return t[p].link;

```

```

62     }
63
64     int len(int p) {
65         return t[p].len;
66     }
67
68     int size() {
69         return t.size();
70     }
71 };

```

5.5 回文自动机 (PAM)

```

1  struct PAM {
2      static constexpr int ALPHABET_SIZE = 26;
3      struct Node {
4          int len;
5          int link;
6          int cnt;
7          array<int, ALPHABET_SIZE> next;
8          Node() : len{}, link{}, cnt{}, next{} {}
9      };
10     vector<Node> t;
11     int suff;
12     string s;
13     PAM() {
14         init();
15     }
16     void init() {
17         t.assign(2, Node());
18         t[0].len = -1;
19         suff = 1;
20         s.clear();
21     }
22     int newNode() {
23         t.emplace_back();
24         return t.size() - 1;
25     }
26     bool add(char c) {
27         int pos = s.size();
28         s += c;
29         int let = c - 'a';
30         int cur = suff, curlen = 0;
31         while (true) {
32             curlen = t[cur].len;
33             if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) {
34                 break;
35             }
36             cur = t[cur].link;
37         }
38         if (t[cur].next[let]) {
39             suff = t[cur].next[let];
40             return false;
41         }
42         int num = newNode();
43         suff = num;
44         t[num].len = t[cur].len + 2;
45         t[cur].next[let] = num;
46         if (t[num].len == 1) {
47             t[num].link = 1;
48             t[num].cnt = 1;
49             return true;

```

```

50     }
51     while (true) {
52         cur = t[cur].link;
53         curlen = t[cur].len;
54         if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) {
55             t[num].link = t[cur].next[let];
56             break;
57         }
58     }
59     t[num].cnt = 1 + t[t[num].link].cnt;
60     return true;
61 }
62 int next(int p, int x) {
63     return t[p].next[x];
64 }
65 int link(int p) {
66     return t[p].link;
67 }
68 int len(int p) {
69     return t[p].len;
70 }
71 int size() {
72     return t.size();
73 }
74 };

```

5.6 AC自动机

5.6.1 AC自动机 (AC 旧版)

```

1  constexpr int N = 3e5 + 30, A = 26;
2
3  struct Node {
4      int fail;
5      int sum;
6      int next[A];
7      Node() : fail(-1), sum(0) {
8          memset(next, -1, sizeof(next));
9      }
10 } node[N];
11
12 int cnt = 0;
13 int bin[N];
14 int nBin = 0;
15
16 int newNode() {
17     int p = nBin > 0 ? bin[--nBin] : cnt++;
18     node[p] = Node();
19     return p;
20 }
21
22 struct AC {
23     vector<int> x;
24     AC(AC &&a) : x(move(a.x)) {}
25     AC(vector<string> s, vector<int> w) {
26         x = {newNode(), newNode()};
27         fill(node[x[0]].next, node[x[0]].next + A, x[1]);
28         node[x[1]].fail = x[0];
29
30         for (int i = 0; i < int(s.size()); i++) {
31             int p = x[1];
32             for (int j = 0; j < int(s[i].length()); j++) {

```



```

33         int c = s[i][j] - 'a';
34         if (node[p].next[c] == -1) {
35             int u = newNode();
36             x.push_back(u);
37             node[p].next[c] = u;
38         }
39         p = node[p].next[c];
40     }
41     node[p].sum += w[i];
42 }
43
44 queue<int> que;
45 que.push(x[1]);
46 while (!que.empty()) {
47     int u = que.front();
48     que.pop();
49     node[u].sum += node[node[u].fail].sum;
50     for (int c = 0; c < A; c++) {
51         if (node[u].next[c] == -1) {
52             node[u].next[c] = node[node[u].fail].next[c];
53         } else {
54             node[node[u].next[c]].fail = node[node[u].fail].next[c];
55             que.push(node[u].next[c]);
56         }
57     }
58 }
59 }
60 ~AC() {
61     for (auto p : x) {
62         bin[nBin++] = p;
63     }
64 }
65 i64 query(const string &s) const {
66     i64 ans = 0;
67     int p = x[1];
68     for (int i = 0; i < int(s.length()); i++) {
69         int c = s[i] - 'a';
70         p = node[p].next[c];
71         ans += node[p].sum;
72     }
73     return ans;
74 }
75 };

```

5.6.2 AC自动机 (AhoCorasick, with vector 新版)

```

1 struct AhoCorasick {
2     static constexpr int ALPHABET = 26;
3     struct Node {
4         int len;
5         int link;
6         array<int, ALPHABET> next;
7         Node() : link{}, next{} {}
8     };
9
10    vector<Node> t;
11
12    AhoCorasick() {
13        init();
14    }
15
16    void init() {
17        t.assign(2, Node());

```

```

18     t[0].next.fill(1);
19     t[0].len = -1;
20 }
21
22 int newNode() {
23     t.emplace_back();
24     return t.size() - 1;
25 }
26
27 int add(const vector<int> &a) {
28     int p = 1;
29     for (auto x : a) {
30         if (t[p].next[x] == 0) {
31             t[p].next[x] = newNode();
32             t[t[p].next[x]].len = t[p].len + 1;
33         }
34         p = t[p].next[x];
35     }
36     return p;
37 }
38
39 int add(const string &a, char offset = 'a') {
40     vector<int> b(a.size());
41     for (int i = 0; i < a.size(); i++) {
42         b[i] = a[i] - offset;
43     }
44     return add(b);
45 }
46
47 void work() {
48     queue<int> q;
49     q.push(1);
50
51     while (!q.empty()) {
52         int x = q.front();
53         q.pop();
54
55         for (int i = 0; i < ALPHABET; i++) {
56             if (t[x].next[i] == 0) {
57                 t[x].next[i] = t[t[x].link].next[i];
58             } else {
59                 t[t[x].next[i]].link = t[t[x].link].next[i];
60                 q.push(t[x].next[i]);
61             }
62         }
63     }
64 }
65
66 int next(int p, int x) {
67     return t[p].next[x];
68 }
69
70 int next(int p, char c, char offset = 'a') {
71     return next(p, c - 'a');
72 }
73
74 int link(int p) {
75     return t[p].link;
76 }
77
78 int len(int p) {
79     return t[p].len;
80 }
81

```

```

82     int size() {
83         return t.size();
84     }
85 };

```

5.6.3 AC自动机 (AhoCorasick, with string 新版)

```

1  struct AhoCorasick {
2      static constexpr int ALPHABET = 26;
3      struct Node {
4          int len;
5          int link;
6          array<int, ALPHABET> next;
7          Node() : len{0}, link{0}, next{} {}
8      };
9
10     vector<Node> t;
11
12     AhoCorasick() {
13         init();
14     }
15
16     void init() {
17         t.assign(2, Node());
18         t[0].next.fill(1);
19         t[0].len = -1;
20     }
21
22     int newNode() {
23         t.emplace_back();
24         return t.size() - 1;
25     }
26
27     int add(const string &a) {
28         int p = 1;
29         for (auto c : a) {
30             int x = c - 'a';
31             if (t[p].next[x] == 0) {
32                 t[p].next[x] = newNode();
33                 t[t[p].next[x]].len = t[p].len + 1;
34             }
35             p = t[p].next[x];
36         }
37         return p;
38     }
39
40     void work() {
41         queue<int> q;
42         q.push(1);
43
44         while (!q.empty()) {
45             int x = q.front();
46             q.pop();
47
48             for (int i = 0; i < ALPHABET; i++) {
49                 if (t[x].next[i] == 0) {
50                     t[x].next[i] = t[t[x].link].next[i];
51                 } else {
52                     t[t[x].next[i]].link = t[t[x].link].next[i];
53                     q.push(t[x].next[i]);
54                 }
55             }
56         }

```

```

57     }
58
59     int next(int p, int x) {
60         return t[p].next[x];
61     }
62
63     int link(int p) {
64         return t[p].link;
65     }
66
67     int len(int p) {
68         return t[p].len;
69     }
70
71     int size() {
72         return t.size();
73     }
74 };

```

5.7 字符串哈希（随机底模例题）

```

1  #include <bits/stdc++.h>
2
3  using i64 = long long;
4
5  bool isprime(int n) {
6      if (n <= 1) {
7          return false;
8      }
9      for (int i = 2; i * i <= n; i++) {
10         if (n % i == 0) {
11             return false;
12         }
13     }
14     return true;
15 }
16
17 int findPrime(int n) {
18     while (!isprime(n)) {
19         n++;
20     }
21     return n;
22 }
23
24 using Hash = array<int, 2>;
25
26 int main() {
27     ios::sync_with_stdio(false);
28     cin.tie(nullptr);
29
30     mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
31
32     const int P = findPrime(rng() % 900000000 + 100000000);
33
34     string s, x;
35     cin >> s >> x;
36
37     int n = s.length();
38     int m = x.length();
39
40     vector<int> h(n + 1), p(n + 1);
41     for (int i = 0; i < n; i++) {

```

```

42     h[i + 1] = (10LL * h[i] + s[i] - '0') % P;
43 }
44 p[0] = 1;
45 for (int i = 0; i < n; i++) {
46     p[i + 1] = 10LL * p[i] % P;
47 }
48
49 auto get = [&](int l, int r) {
50     return (h[r] + 1LL * (P - h[l]) * p[r - l]) % P;
51 };
52
53 int px = 0;
54 for (auto c : x) {
55     px = (10LL * px + c - '0') % P;
56 }
57
58 for (int i = 0; i <= n - 2 * (m - 1); i++) {
59     if ((get(i, i + m - 1) + get(i + m - 1, i + 2 * m - 2)) % P == px) {
60         cout << i + 1 << " " << i + m - 1 << "\n";
61         cout << i + m << " " << i + 2 * m - 2 << "\n";
62         return 0;
63     }
64 }
65
66 vector<int> z(m + 1), f(n + 1);
67 z[0] = m;
68
69 for (int i = 1, j = -1; i < m; i++) {
70     if (j != -1) {
71         z[i] = max(0, min(j + z[j] - i, z[i - j]));
72     }
73     while (z[i] + i < m && x[z[i]] == x[z[i] + i]) {
74         z[i]++;
75     }
76     if (j == -1 || i + z[i] > j + z[j]) {
77         j = i;
78     }
79 }
80 for (int i = 0, j = -1; i < n; i++) {
81     if (j != -1) {
82         f[i] = max(0, min(j + f[j] - i, z[i - j]));
83     }
84     while (f[i] + i < n && f[i] < m && x[f[i]] == s[f[i] + i]) {
85         f[i]++;
86     }
87     if (j == -1 || i + f[i] > j + f[j]) {
88         j = i;
89     }
90 }
91
92 for (int i = 0; i + m <= n; i++) {
93     int l = min(m, f[i]);
94
95     for (auto j : { m - 1, m - 1 - 1 }) {
96         if (j <= 0) {
97             continue;
98         }
99         if (j <= i && (get(i - j, i) + get(i, i + m)) % P == px) {
100             cout << i - j + 1 << " " << i << "\n";
101             cout << i + 1 << " " << i + m << "\n";
102             return 0;
103         }
104         if (i + m + j <= n && (get(i, i + m) + get(i + m, i + m + j)) % P ==
px) {

```

```

105         cout << i + 1 << " " << i + m << "\n";
106         cout << i + m + 1 << " " << i + m + j << "\n";
107         return 0;
108     }
109 }
110 }
111
112     return 0;
113 }

```

5.8 最长公共前缀 LCP (例题)

```

1  constexpr int L = 2E6 + 10;
2
3  int len[L];
4  int lnk[L];
5  int nxt[L][26];
6
7  int f[L];
8  int tot = 1;
9
10 vector<int> adj[L];
11
12 int extend(int p, int c) {
13     if (nxt[p][c]) {
14         int q = nxt[p][c];
15         if (len[q] == len[p] + 1) {
16             return q;
17         }
18         int r = ++tot;
19         len[r] = len[p] + 1;
20         lnk[r] = lnk[q];
21         copy(nxt[q], nxt[q] + 26, nxt[r]);
22         lnk[q] = r;
23         while (nxt[p][c] == q) {
24             nxt[p][c] = r;
25             p = lnk[p];
26         }
27         return r;
28     }
29     int cur = ++tot;
30     len[cur] = len[p] + 1;
31     while (!nxt[p][c]) {
32         nxt[p][c] = cur;
33         p = lnk[p];
34     }
35     lnk[cur] = extend(p, c);
36     return cur;
37 }
38
39 int main() {
40     ios::sync_with_stdio(false);
41     cin.tie(nullptr);
42
43     fill(nxt[0], nxt[0] + 26, 1);
44     len[0] = -1;
45
46     int N;
47     cin >> N;
48
49     vector<string> S(N);
50     for (int i = 0; i < N; i++) {

```

```

51     cin >> S[i];
52     int p = 1;
53     for (auto c : S[i]) {
54         p = extend(p, c - 'a');
55         if (f[p] != -1) {
56             if (f[p] == 0) {
57                 f[p] = i + 1;
58             } else if (f[p] != i + 1) {
59                 f[p] = -1;
60             }
61         }
62     }
63 }
64
65 for (int i = 1; i <= tot; i++) {
66     adj[lnk[i]].push_back(i);
67 }
68 }

```

5.9 字典树 Trie

```

1  constexpr i64 inf = 1E18;
2
3  constexpr int N = 1E6 + 10;
4
5  int trie[N][26];
6  int tot;
7
8  int newNode() {
9      tot++;
10     fill(trie[tot], trie[tot] + 26, 0);
11     val[tot] = inf;
12     return tot;
13 }
14
15 void solve() {
16     /* init
17     tot = 0;
18     newNode();
19
20     /* insert
21     for (int i = 0; i < N; i++) {
22         int p = 1;
23         int l = S[i].size();
24         for (int j = 0; j < l; j++) {
25             int x = S[i][j] - 'a';
26             if (!trie[p][x]) {
27                 trie[p][x] = newNode();
28             }
29             p = trie[p][x];
30             /* 处理
31             /* val[p] = min(val[p], 1 + K + f[(K - (1 - j - 1) % K) % K]);
32         }
33     }
34
35     /* query
36     for (int i = 0; i < L; i++) {
37         int p = 1;
38         for (int j = i; j < L; j++) {
39             int x = T[j] - 'a';
40             p = trie[p][x];
41             if (!p) {

```

```

42         continue;
43     }
44     /* 处理
45     /* dp[j + 1] = min(dp[j + 1], dp[i] + val[p]);
46     }
47 }
48 }

```

```

1  int tot;
2  int trie[N][2];
3  int f[N];
4
5  int newNode() {
6      int x = ++tot;
7      trie[x][0] = trie[x][1] = 0;
8      f[x] = inf;
9      return x;
10 }
11 void add(int x, int i) {
12     int p = 1;
13     for (int j = 29; j >= 0; j--) {
14         int &q = trie[p][x >> j & 1];
15         if (q == 0) {
16             q = newNode();
17         }
18         p = q;
19         f[p] = min(f[p], i);
20     }
21 }
22
23 int query(int a, int b) {
24     int ans1 = inf, ans2 = inf;
25     int p = 1;
26     for (int i = 29; i >= 0; i--) {
27         int d = a >> i & 1;
28         int e = b >> i & 1;
29         if (e) {
30             ans1 = min(ans1, f[trie[p][d]]);
31         } else {
32             ans2 = min(ans2, f[trie[p][d ^ 1]]);
33         }
34         p = trie[p][e ^ d];
35     }
36     ans1 = min(ans1, f[p]);
37     ans2 = min(ans2, f[p]);
38     if (ans1 == inf || ans2 == inf) {
39         return -1;
40     }
41     return max({1, ans1, ans2});
42 }

```

```

1  int trie[N][2];
2  int cnt[N][2];
3
4  int tot = 0;
5  int newNode() {
6      int x = ++tot;
7      trie[x][0] = trie[x][1] = 0;
8      cnt[x][0] = cnt[x][1] = 0;
9      return x;
10 }
11

```



```

12 void add(int x, int d, int t = 1) {
13     int p = 1;
14     cnt[p][d] += t;
15     for (int i = 29; i >= 0; i--) {
16         int u = x >> i & 1;
17         if (!trie[p][u]) {
18             trie[p][u] = newNode();
19         }
20         p = trie[p][u];
21         cnt[p][d] += t;
22     }
23 }
24
25 int query(int x, int d) {
26     int p = 1;
27     if (!cnt[p][d]) {
28         return 0;
29     }
30     int ans = 0;
31     for (int i = 29; i >= 0; i--) {
32         int u = x >> i & 1;
33         if (cnt[trie[p][u ^ 1]][d]) {
34             ans |= 1 << i;
35             p = trie[p][u ^ 1];
36         } else {
37             p = trie[p][u];
38         }
39     }
40     return ans;
41 }

```

```

1  constexpr int N = 1E7;
2  constexpr int inf = 1E9;
3  int tot;
4  int trie[N][2];
5  int f[N];
6
7  int newNode() {
8      int x = ++tot;
9      trie[x][0] = trie[x][1] = 0;
10     f[x] = inf;
11     return x;
12 }
13 void add(int x, int i) {
14     int p = 1;
15     for (int j = 29; j >= 0; j--) {
16         int &q = trie[p][x >> j & 1];
17         if (q == 0) {
18             q = newNode();
19         }
20         p = q;
21         f[p] = min(f[p], i);
22     }
23 }
24
25 int query(int a, int b) {
26     int ans1 = inf, ans2 = inf;
27     int p = 1;
28     for (int i = 29; i >= 0; i--) {
29         int d = a >> i & 1;
30         int e = b >> i & 1;
31         if (e) {
32             ans1 = min(ans1, f[trie[p][d]]);

```

```

33     } else {
34         ans2 = min(ans2, f[trie[p][d ^ 1]]);
35     }
36     p = trie[p][e ^ d];
37 }
38 ans1 = min(ans1, f[p]);
39 ans2 = min(ans2, f[p]);
40 if (ans1 == inf || ans2 == inf) {
41     return -1;
42 }
43 return max({1, ans1, ans2});
44 }

```

5.10 前缀函数 (KMP)

```

1  vector<int> kmp(string s) {
2      int n = s.size();
3      vector<int> f(n + 1);
4      for (int i = 1, j = 0; i < n; i++) {
5          while (j && s[i] != s[j]) {
6              j = f[j];
7          }
8          j += (s[i] == s[j]);
9          f[i + 1] = j;
10     }
11     return f;
12 }

```

/END/



Author by *Jiangly*
Tyrp by *Wida*

Version: 1.8.7 (2024.10.07)

github.com/hh2048
cnblogs.com/WIDA