Tutorial 3: Lists, Stacks and Queues

September 5, 2022

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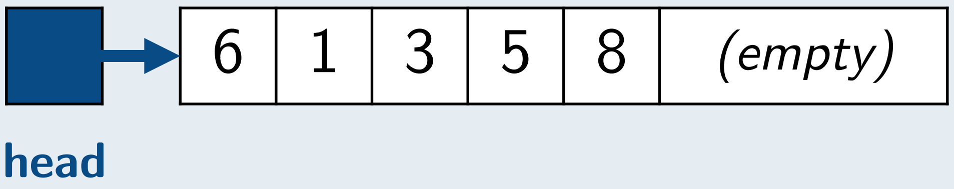
** Partly adopted from tutorial slides by [Wang Zhi Jian](#).*

Arrays & Linked Lists

What is the best way to implement an ADT?

Implementation of List/Queue/Stack ADT

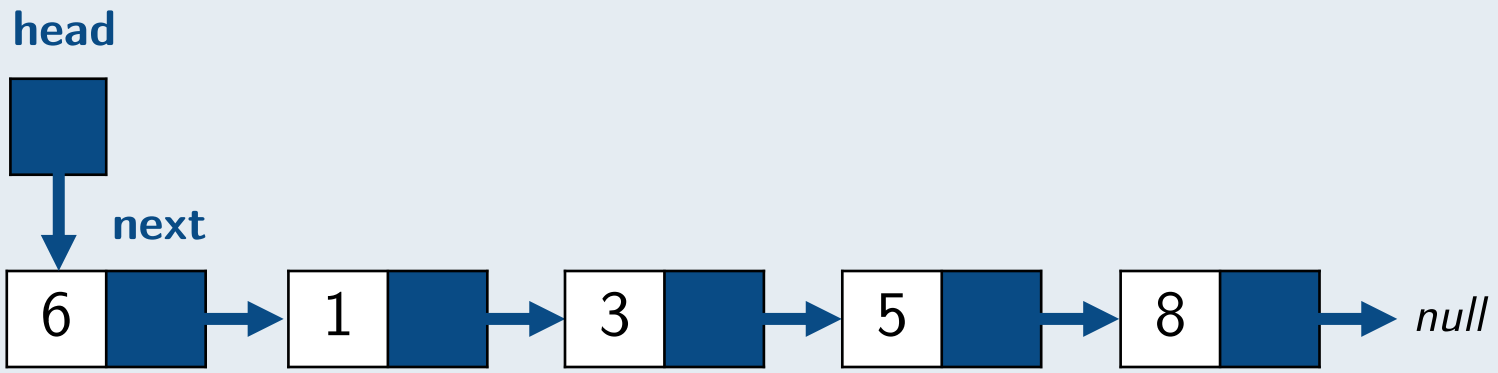
As array:



- **Pros**: Fast random access: $O(1)$ time to access any element; faster to scan the whole list due to *cache locality* (though all take $O(n)$).
- **Cons**: may waste some space; slow insertion/deletion/resizing.

Implementation of List/Queue/Stack ADT

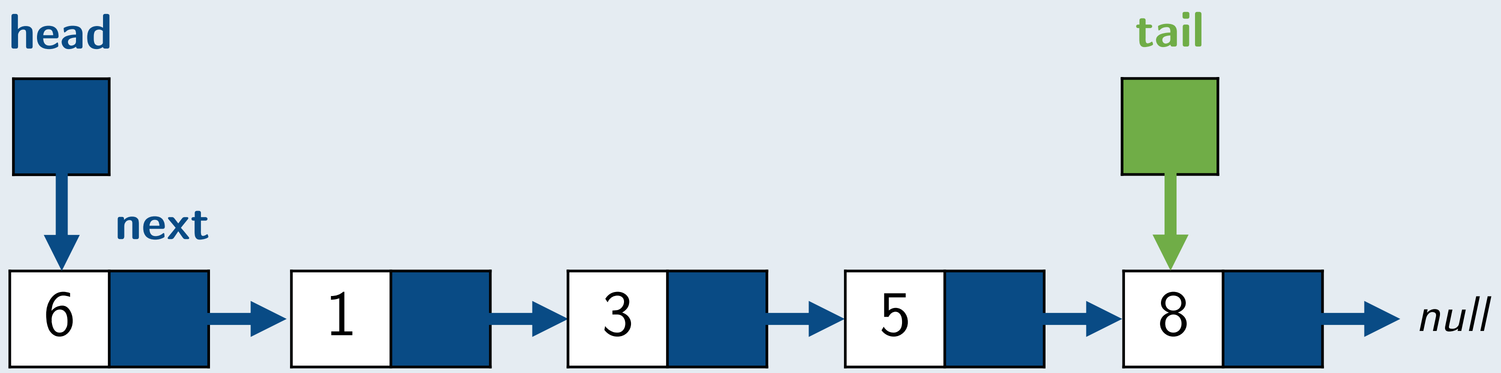
As linked list:



- **Pros**: Dynamic size and flexible memory usage; faster insertion/deletion (especially at head).
- **Cons**: Needs more space to store the pointers, no random access.

Variants of Linked Lists

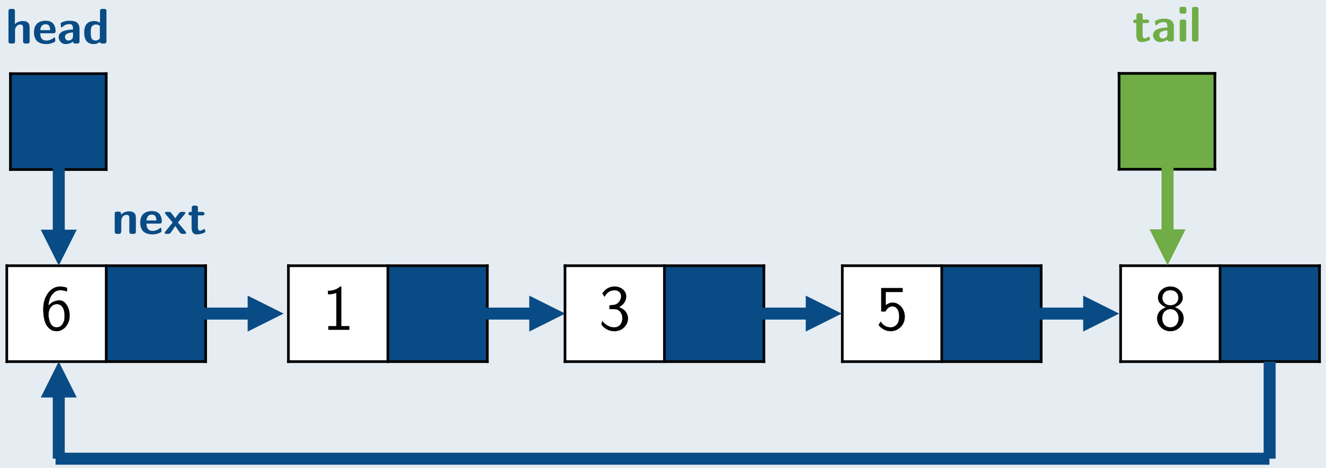
Tailed linked list:



- $O(1)$ access and insert at the tail.

Variants of Linked Lists

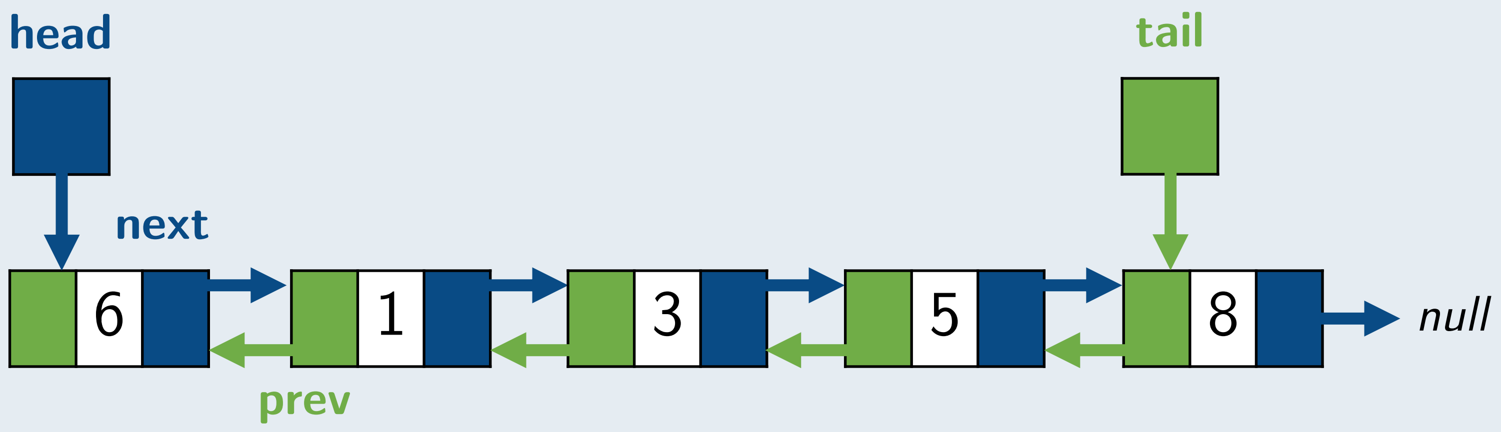
Circular linked list:



- Allow cycling through the list repeatedly.

Variants of Linked Lists

Doubly linked list:



- Allow fast accessing previous items.

Problem 1.a

True or false: *Deletion in any Linked List can always be done in $O(1)$ time.*

False. To delete an element in linked list:

1. Start from head pointer and find the item to be deleted. ($O(n)$ time)
2. Delete the item by modifying pointers. ($O(1)$ time)

Problem 1.b

True or false: *A search operation in a Doubly Linked List will only take $O(\log n)$ time.*

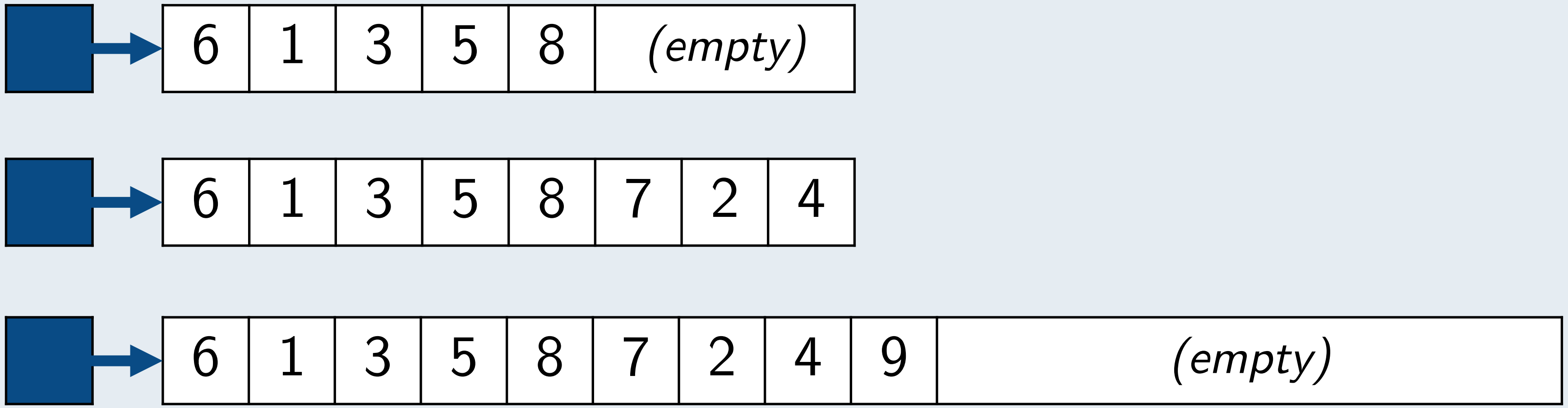
False. To use binary search on a list,

1. List should be sorted,
2. Should allow random access to "jump around" the list,

Doubly linked list has neither.

Problem 1.c

True or false: *All operations in a stack are $O(1)$ time when implemented using an array.*



Inserting needs $O(n)$ in worst case when resizing:

1. Allocate a new space of size $2n$.
2. Copy the array to the new space.

Problem 1.c

True or false: *All operations in a stack are $O(1)$ time when implemented using an array.*

False. The *push* operation needs in worst case $O(n)$.

You can also say true. You will need n $O(1)$ push operations so that the next push operation cost $O(n)$. So the total *amortized cost* is still $O(1)$.

** See more on amortized analysis in the appendix!*

Problem 1.d

True or false: *A stack can be implemented with a Singly Linked List with no tail reference with $O(1)$ time for all operations.*

True. *push* and *pop* only require insert and delete at the head of linked list, which cost $O(1)$.

Problem 1.e

True or false: *All operations in a queue are $O(1)$ time when implemented using a Doubly Linked List with no modification.*

True. Doubly Linked List has a tail reference so both enqueue at front and dequeue at the end costs $O(1)$.

You can also say false by assuming* the doubly linked list may not have a tail reference.

** Reasonable assumptions are accepted as correct answer in CS2040S exams.*

Problem 1.f

True or false: *Three items A, B, C are inserted (in this order) into an unknown data structure X. If the first element removed from X is B, X can be a queue.*

False. If X is a queue (FIFO), then the first element removed is A.

Summary

Operations	Array	Linked List
`getItemAtIndex`	$O(1)$	$O(n)$
`getFirst/getLast`	$O(1)$	$O(1)^*$
`addAtIndex/removeAtIndex`	$O(n)$	$O(n)$
`addFront/removeFront`	$O(n)$	$O(1)$
`addBack/removeBack`	$O(n)$ ($O(1)$ amortized)	$O(1)^*$

Lists: depends on application, usually needs `getItemAtIndex`.

Stack (LIFO): needs `addFront/removeFront`.

Queue (FIFO): needs `addFront/removeBack`.

** Needs tail reference for $O(1)$ operation at back.*

New Operations

How to enable other operations in those ADT?

Problem Solving Strategies

1. **Think of a trivial answer.**

   Brute-force algorithms? Some algorithms we've learned?

2. **Try with some simple examples.**

   Some small or trivial input?

3. **See where we can improve.**

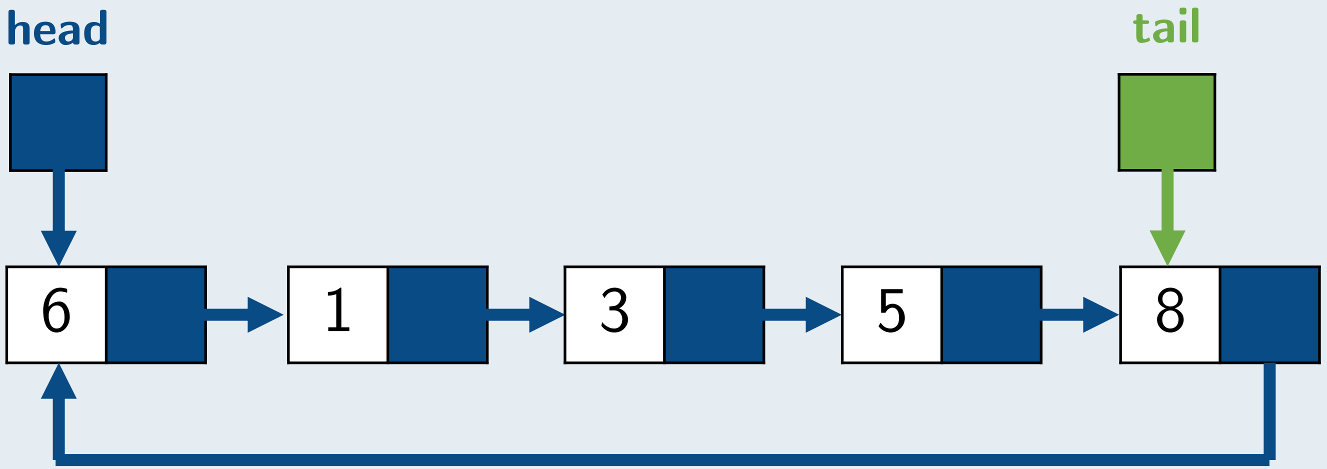
   Use better data structure? Delete some redundant work?

4. **Consider special and boundary cases.**

   What if input size is small, say 1 or 2? What if input is special?

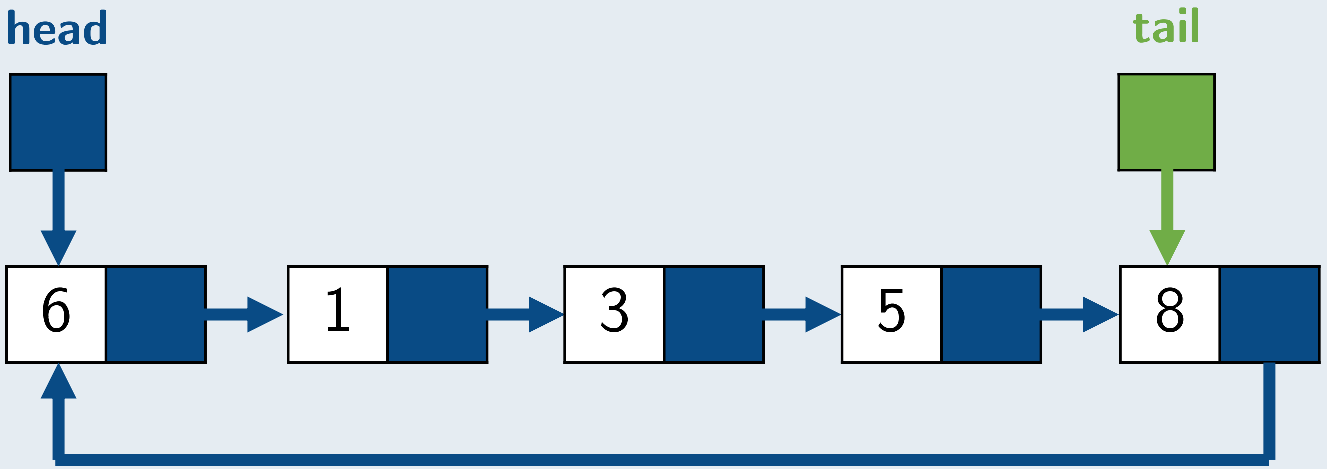
Problem 2

Goal: enable *swap* operation (swapping adjacent elements) in a circular linked list.



Problem 2

Simple Example: Try to swap element 3 and 5.

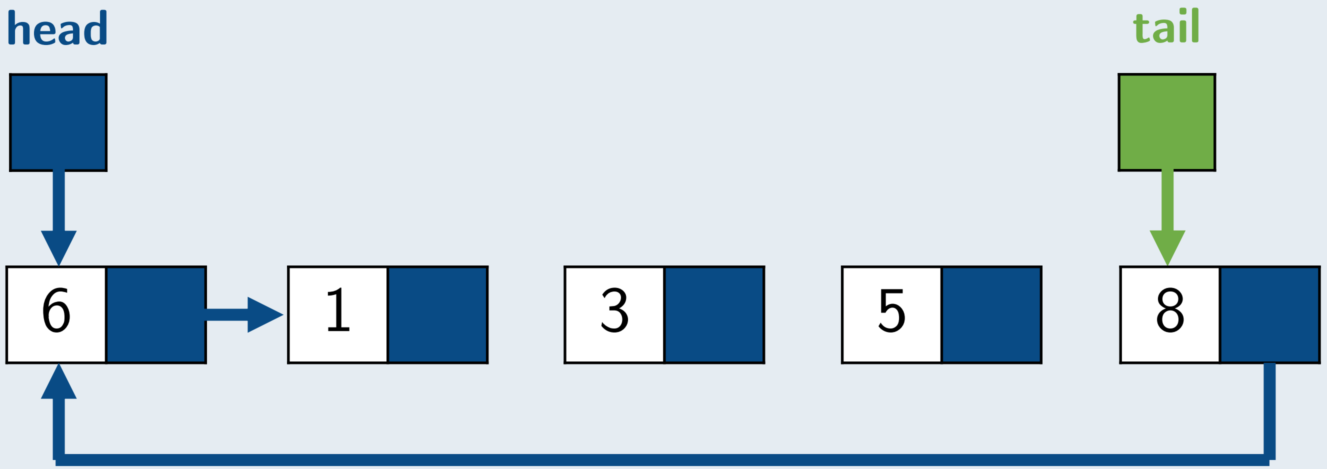


Which pointers do we need to modify?

We basically want $1 \to 5 \to 3 \to 8$.

Problem 2

Simple Example: Try to swap element 3 and 5.



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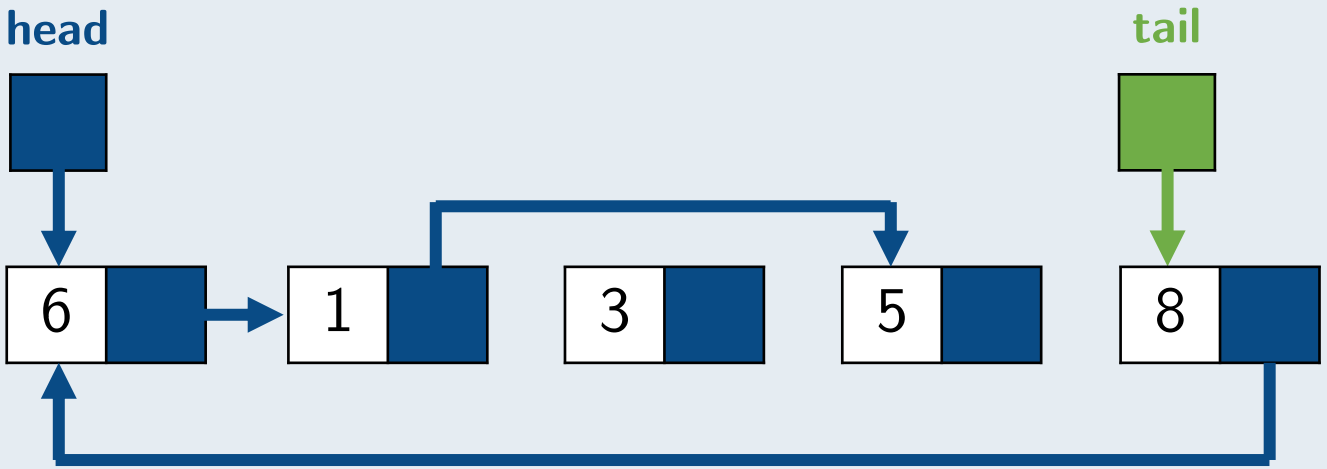

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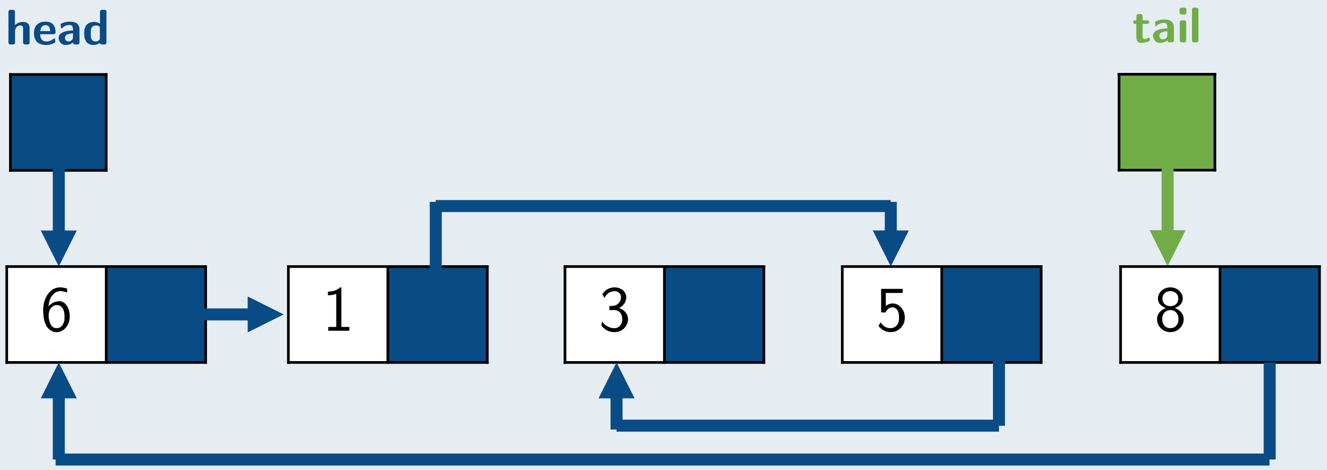

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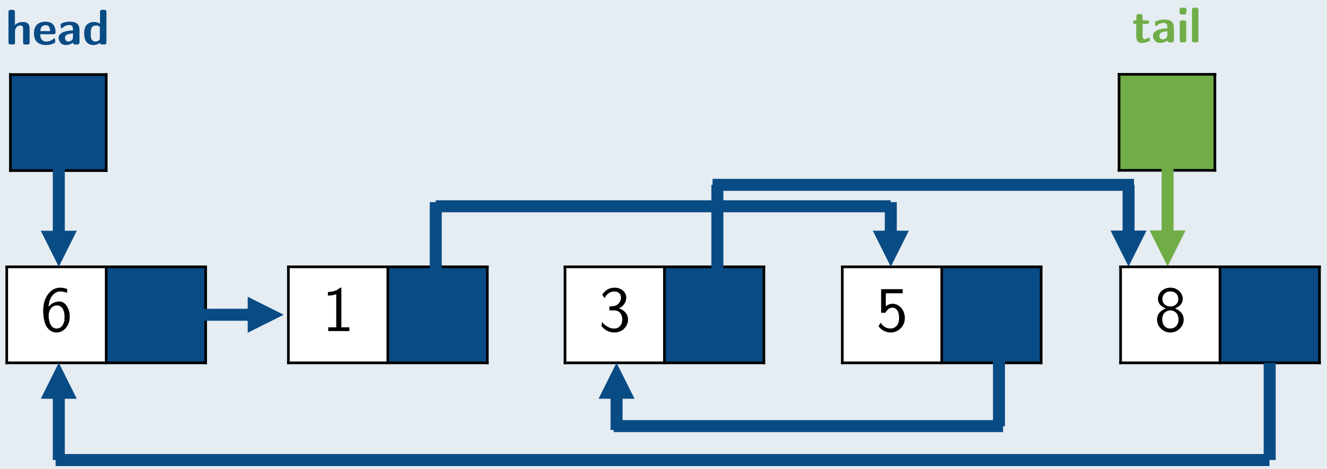
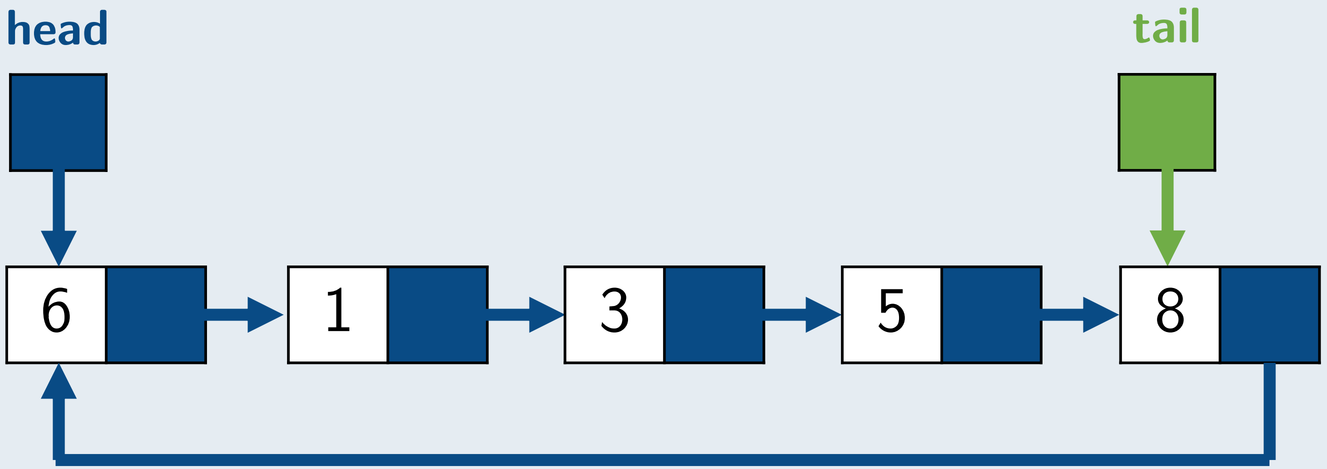


Need to modify 3 pointers. the previous one and the two we are swapping.

Problem 2

Boundary Case: *What if the items are at head and/or tail?*

Try to swap element 5 and 8.



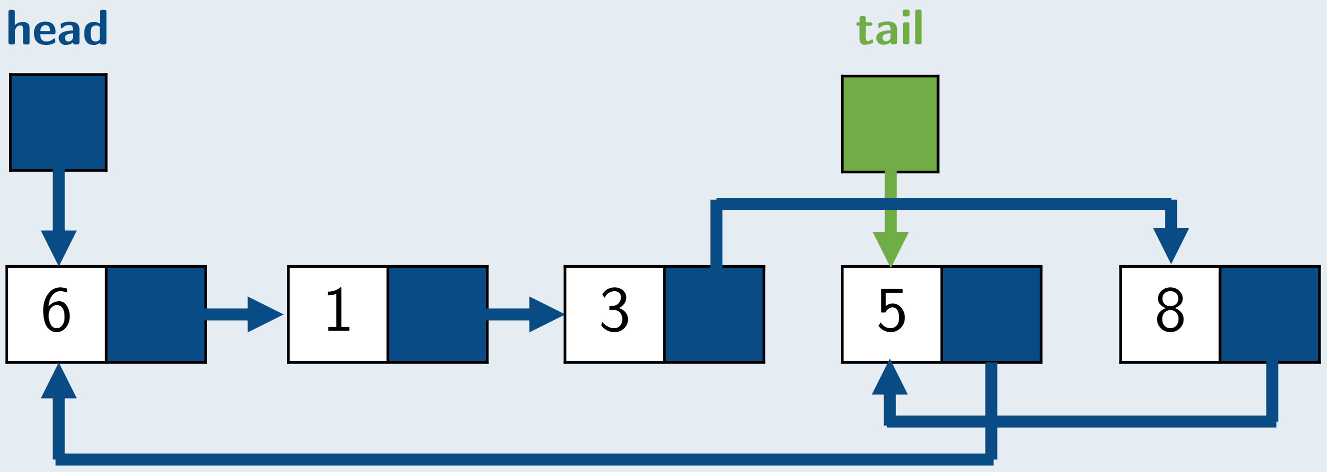
Which pointers do we need to modify?

We basically want $3 \to 8 \to 5 \to 6$.

Problem 2

Boundary Case: *What if the items are at head and/or tail?*

Try to swap element 5 and 8.

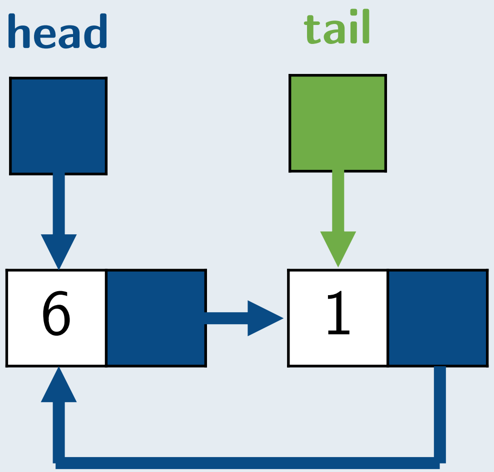


Need to modify the 3 pointers plus head and/or tail pointer.

Problem 2

Special Case: *What if there are less than 3 pointers to modify?*

Say we have only two elements.

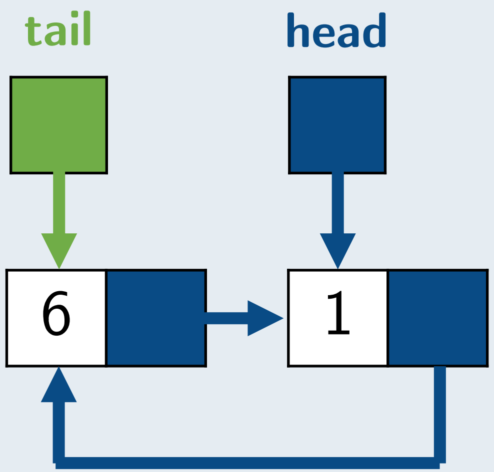


Which pointers do we need to modify?

Problem 2

Special Case: *What if there are less than 3 pointers?*

Say we have only two elements.

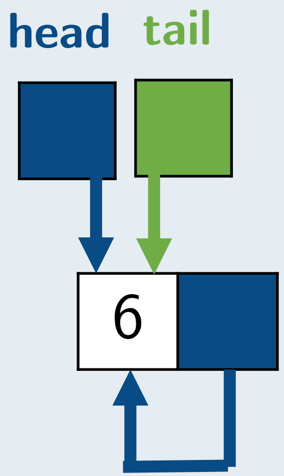


Only the head and tail pointers!

Problem 2

Special Case: *What if there are less than 3 pointers?*

Say we have only one element.

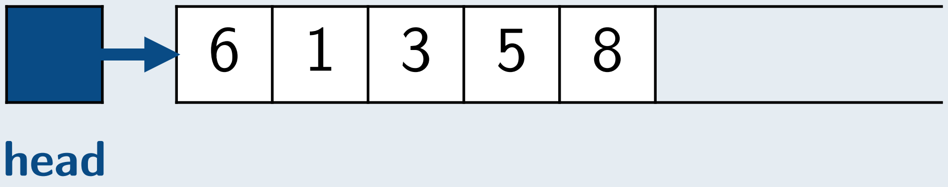


No need to do anything :-D

* *See `CircularLinkedList.java` for a detailed solution.*

Problem 3

- **Goal**: enable *leave* operation that allows a certain item to leave the queue (as an array).



Simple Example: suppose we want to delete item 3.

Problem 3

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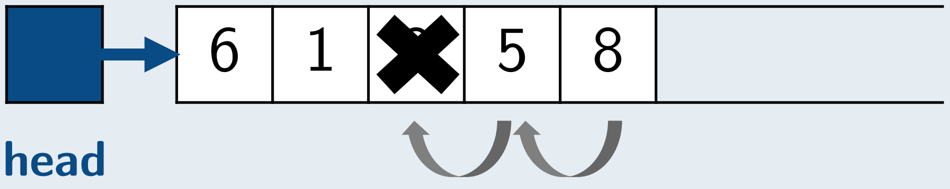

Trivial answer: Simply delete the item from the array.

Time complexity of *leave*: $O(n)$.

Problem 3

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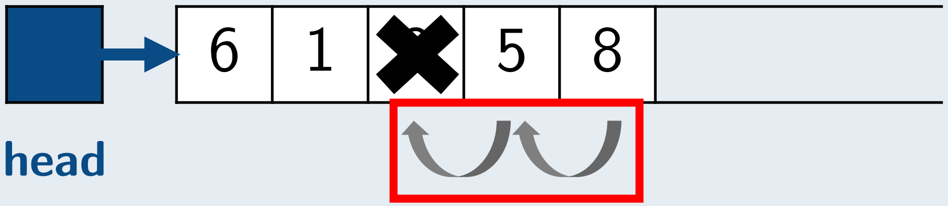

Not necessary if we remember who has left!

Note. Actually we don't need the queue to be correct. We just need to make sure other operations *enqueue* and *dequeue* return the correct result.

Problem 3

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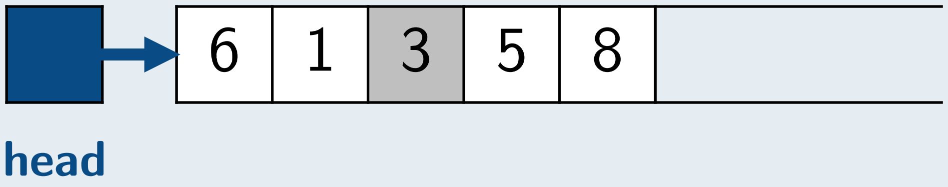

Idea 1 (Lazy deletion):

1. Simply mark the item as *left*.
2. *dequeue* skips the *left* items and continue on to next item.

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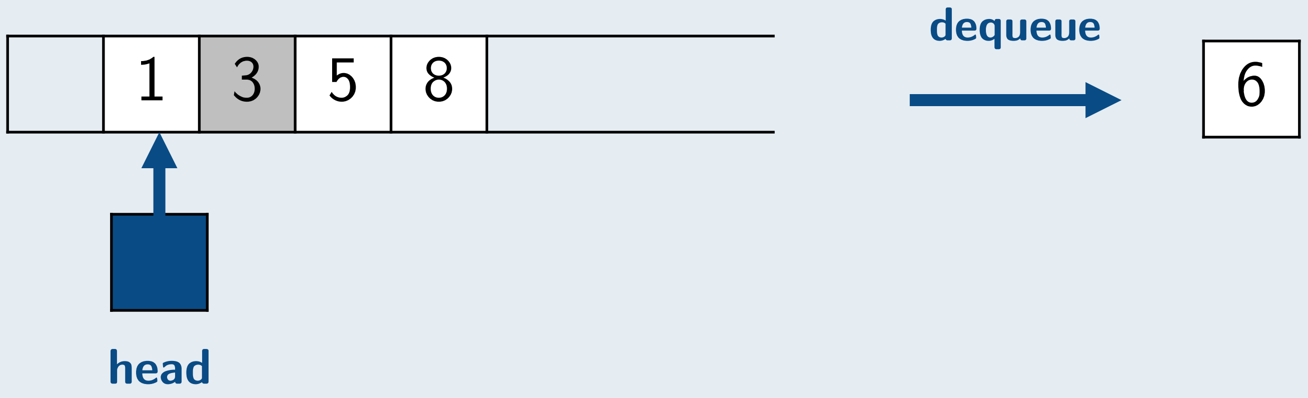

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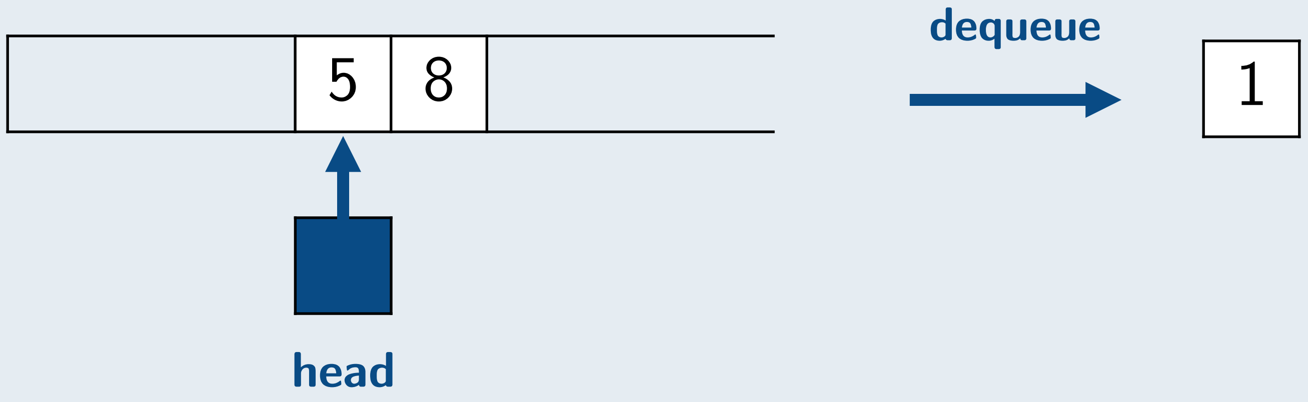
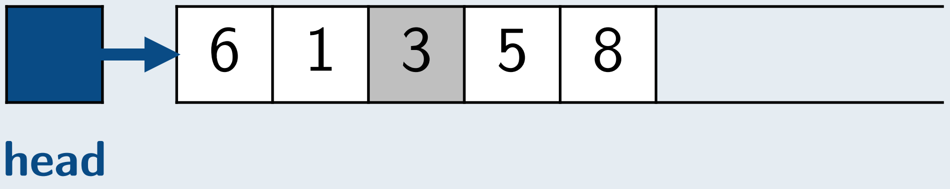


Note. You may also need a tail pointer for fast enqueue and indicate which is the last item.

Problem 3

- **Goal**: enable *leave* operation that allows a certain item to leave the queue (as an array).

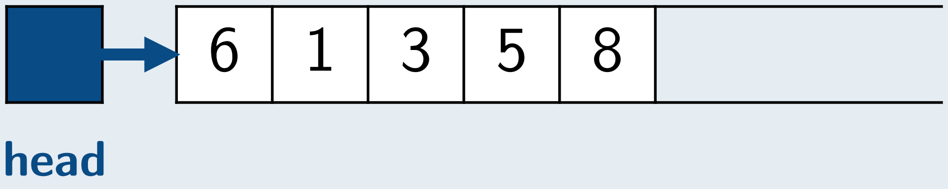


Idea 1 (Lazy deletion):

- *leave* costs only $O(1)$.
- *dequeue* may cost $O(n)$ if $O(n)$ items have left (still $O(1)$ amortized).

Problem 3

- **Goal**: enable *leave* operation that allows a certain item to leave the queue (as an array).



Left items
3

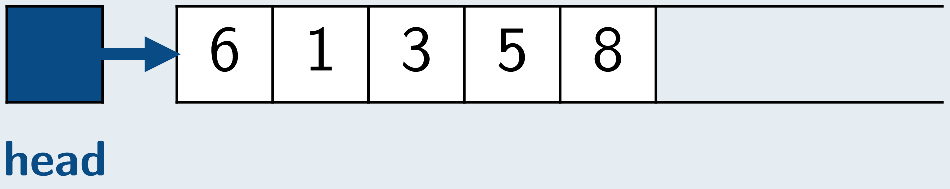
Idea 2:

1. Keep a dictionary (which data structure?) on who has left.
2. *dequeue* skips items in the dictionary and continue onto next ones

** See `WaitingQueue.java` for a detailed solution.*

Problem 3

- **Goal**: enable *leave* operation that allows a certain item to leave the queue (as an array).



Left items
3

Idea 2:

1. If we use hash table, leave costs $O(1)$.
2. dequeue also costs $O(1)$ amortized.
3. Needs $O(n)$ additional space for the hash table.

Applications of ADT

How to efficiently use lists, stacks & queues?

Problem 4

- **Goal**: evaluate Lisp expression.

```
( + ( - 6 ) ( * 2 3 4 ) )
```

Problem 4

- **Goal**: evaluate Lisp expression.

`( + ( - 6 ) ( * 2 3 4 ) )`

We need to ...

1. Match each pair of parenthesis, ← Last-in, first-out
2. Evaluate expression inside each parenthesis **in correct order**.
   e.g. `( / 4 2 1 )` should be `4/2/1=2` instead of `1/2/4=0.125`.

First-in, first-out

Problem 4

- **Goal**: evaluate Lisp expression.

```
( + ( - 6 ) ( * 2 3 4 ) )
↑
```

To match parenthesis, let's start with a stack.

(	

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(	+	(	-	6	

A right parenthesis!

Problem 4

- **Goal**: evaluate Lisp expression.

(+ (- 6) (* 2 3 4))

To match parenthesis, let's start with a stack.

(	+	(	-	

$\xrightarrow{\text{pop}}$ 6

pop until the last left parenthesis we've pushed.

Problem 4

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(+ (- 6) (* 2 3 4))

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(	+	(		$\xrightarrow{\text{pop}}$	6	-

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Problem 4

- **Goal**: evaluate Lisp expression.

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( + ( - 6 ) ( * 2 3 4 ) )
```

To match parenthesis, let's start with a stack.

Idea: Use another stack to reverse the order!

Problem 4

- **Goal**: evaluate Lisp expression.

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( + ( - 6 ) ( * 2 3 4 ) )
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To match parenthesis, let's start with a stack. Another stack for reversing order.

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(	+	-6	(	*	2	3	4

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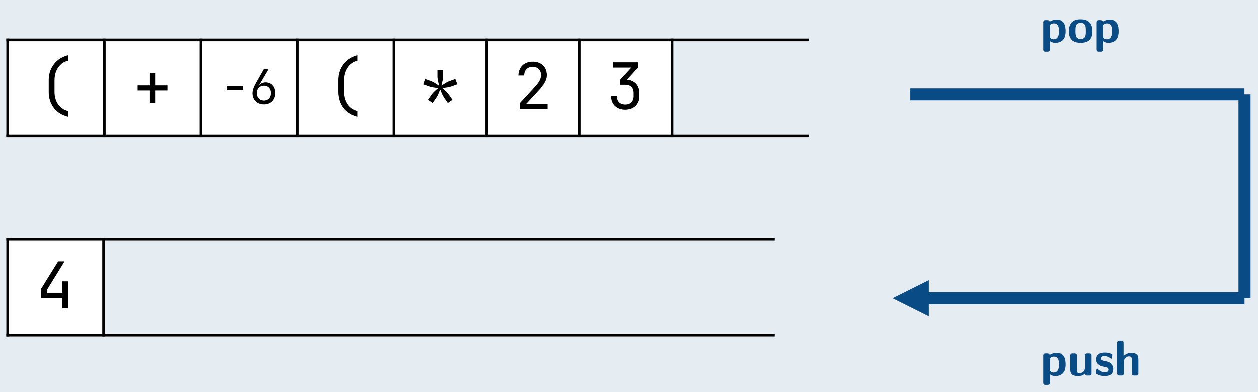

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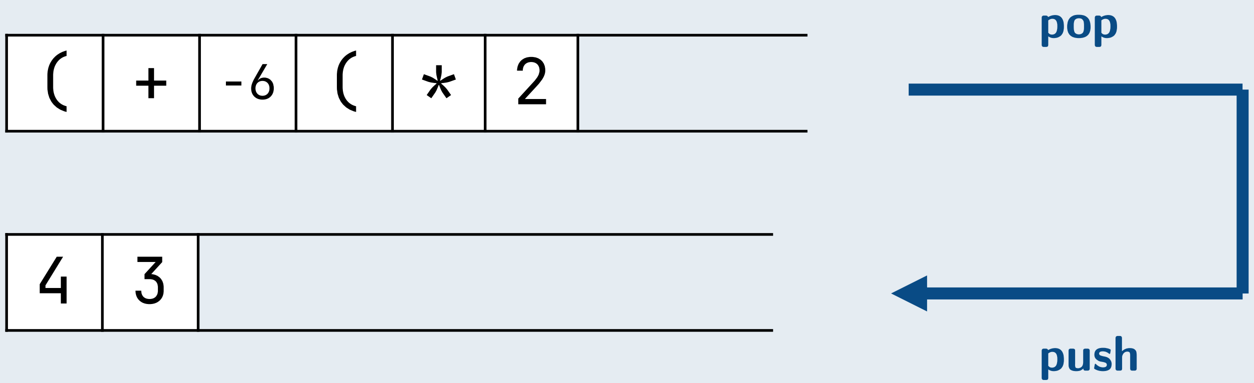

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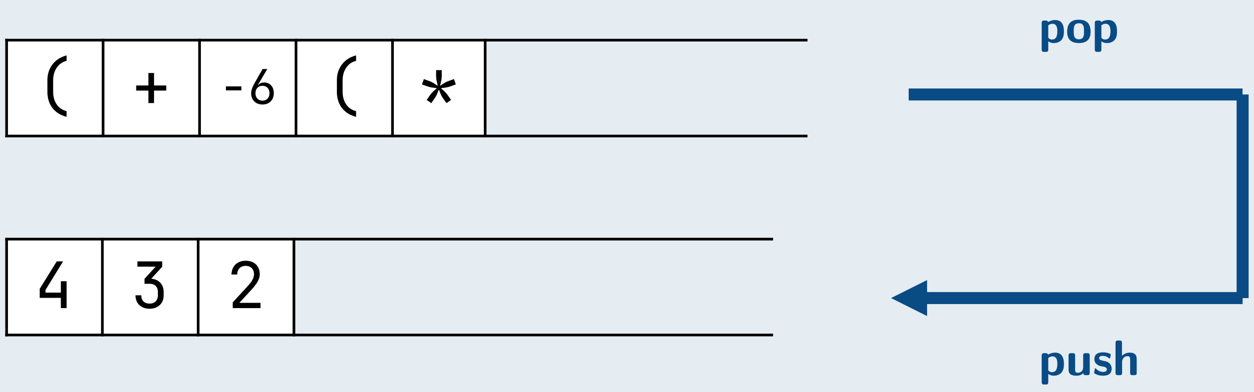

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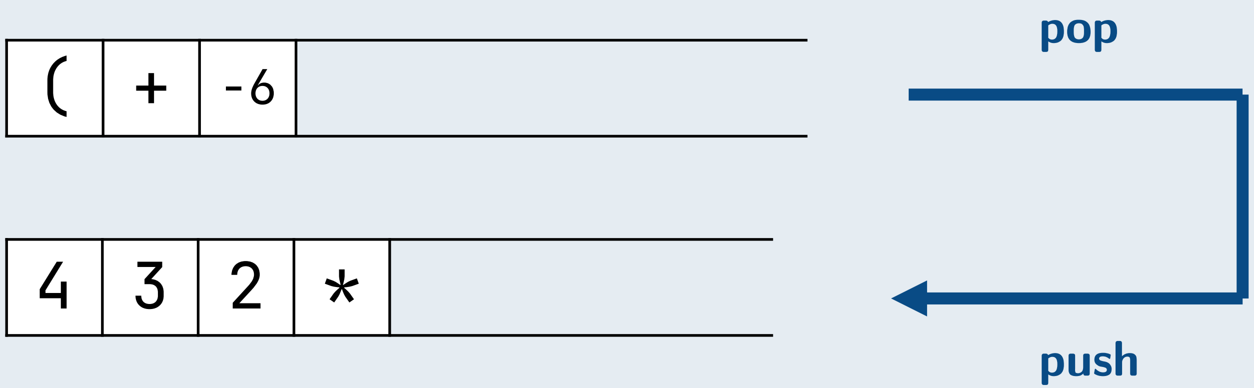

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`( + ( - 6 ) ( * 2 3 4 ) )`

Each element pushed and popped twice: $O(n)$ time.

** See* `ExpEval.java` *for a detailed solution.*

Appendix

Amortized Analysis *

- **Purpose**: take the frequency of an operation into account.
- **Example**: I eat at the Deck and spend \$5 for lunch every day. Once every month I decide to reward myself and have a \$100 lunch.
    - **Worst case analysis**: $\leq$ \$100 for lunch every day.
    - **Amortized analysis**: $\leq$ \$8 for lunch every day. (reserve \$3 for each lunch at the Deck, and spend them all on the rewarding lunch)

A more reasonable way to describe expensive operations that rarely happen!

Amortized Analysis *

- **Example**: Inserting at the back of an array usually costs $O(1)$. After $O(n)$ insertions, the next insert will cost $O(n)$ for resizing.
  - **Worst case analysis**: $O(n)$ per insert.
  - **Amortized analysis**: $O(1)$ per insert. (reserve $O(1)$ time for each regular insert, and spend them all on the resizing insert)

End of File

Thank you very much for your attention :-)