Chapter 3

1. The x and the y components of a vector $\vec{a}$ lying on the xy plane are given by

$$a_x = a\cos\theta, \qquad a_y = a\sin\theta$$

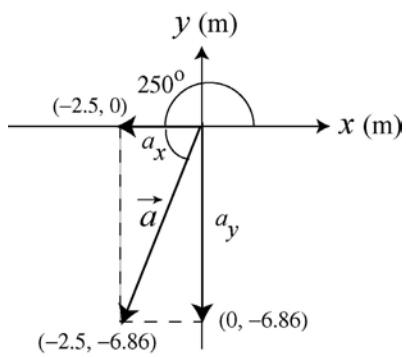
where $a = |\vec{a}|$ is the magnitude and θ is the angle between $\vec{a}$ and the positive x axis.

(a) The x component of $\vec{a}$ is given by $a_x = a\cos\theta = (7.3\text{ m})\cos 250° = -2.50\text{ m}$.

(b) Similarly, the y component is given by

$$a_y = a\sin\theta = (7.3\text{ m})\sin 250° = -6.86\text{ m} \approx -6.9\text{ m}.$$

The results are depicted in the figure below:



In considering the variety of ways to compute these, we note that the vector is 70° below the – x axis, so the components could also have been found from

$$a_x = -(7.3\text{ m})\cos 70° = -2.50\text{ m}, \quad a_y = -(7.3\text{ m})\sin 70° = -6.86\text{ m}.$$

Similarly, we note that the vector is 20° to the left from the – y axis, so one could also achieve the same results by using

$$a_x = -(7.3\text{ m})\sin 20° = -2.50\text{ m}, \quad a_y = -(7.3\text{ m})\cos 20° = -6.86\text{ m}.$$

As a consistency check, we note that

$$\sqrt{a_x^2 + a_y^2} = \sqrt{(-2.50\text{ m})^2 + (-6.86\text{ m})^2} = 7.3\text{ m}$$

and

$$\tan^{-1}\left(a_y / a_x\right) = \tan^{-1}[(-6.86\ \text{m})/(-2.50\ \text{m})] = 250°,$$

which are indeed the values given in the problem statement.

2. (a) With r = 15 m and θ = 30°, the x component of $\vec{r}$ is given by

$$r_x = r\cos\theta = (15\ \text{m})\cos 30° = 13\ \text{m}.$$

(b) Similarly, the y component is given by $r_y = r\sin\theta = (15\ \text{m})\sin 30° = 7.5\ \text{m}$.

3. A vector $\vec{a}$ can be represented in the *magnitude-angle* notation (a, θ), where

$$a = \sqrt{a_x^2 + a_y^2}$$

is the magnitude and

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

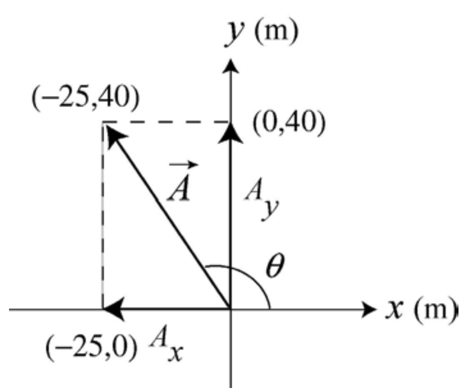
is the angle $\vec{a}$ makes with the positive x axis.

(a) Given A_x = –25.0 m and A_y = 40.0 m, $A = \sqrt{(-25.0\ \text{m})^2 + (40.0\ \text{m})^2} = 47.2\ \text{m}$.

(b) Recalling that tan θ = tan (θ + 180°),

$$\tan^{-1}[(40.0\ \text{m})/(-25.0\ \text{m})] = -58° \text{ or } 122°.$$

Noting that the vector is in the third quadrant (by the signs of its x and y components) we see that 122° is the correct answer. The graphical calculator "shortcuts" mentioned above are designed to correctly choose the right possibility. The results are depicted in the figure below:



We can check our answers by noting that the x- and the y- components of $\vec{A}$ can be written as

$$A_x = A\cos\theta, \qquad A_y = A\sin\theta$$

Substituting the results calculated above, we obtain

$$A_x = (47.2\ \text{m})\cos 122° = -25.0\ \text{m}, \quad A_y = (47.2\ \text{m})\sin 122° = +40.0\ \text{m}$$

which indeed are the values given in the problem statement.

4. The angle described by a full circle is 360° = 2π rad, which is the basis of our conversion factor.

(a) $20.0° = (20.0°)\dfrac{2\pi\ \text{rad}}{360°} = 0.349\ \text{rad}$.

(b) $50.0° = (50.0°)\dfrac{2\pi\ \text{rad}}{360°} = 0.873\ \text{rad}$.

(c) $100° = (100°)\dfrac{2\pi\ \text{rad}}{360°} = 1.75\ \text{rad}$.

(d) $0.330\ \text{rad} = (0.330\ \text{rad})\dfrac{360°}{2\pi\ \text{rad}} = 18.9°$.

(e) $2.10\ \text{rad} = (2.10\ \text{rad})\dfrac{360°}{2\pi\ \text{rad}} = 120°$.

(f) $7.70\ \text{rad} = (7.70\ \text{rad})\dfrac{360°}{2\pi\ \text{rad}} = 441°$.

5. The vector sum of the displacements $\vec{d}_{\text{storm}}$ and $\vec{d}_{\text{new}}$ must give the same result as its originally intended displacement $\vec{d}_o = (120\ \text{km})\hat{\text{j}}$ where east is $\hat{\text{i}}$, north is $\hat{\text{j}}$. Thus, we write

$$\vec{d}_{\text{storm}} = (100\ \text{km})\,\hat{\text{i}}, \quad \vec{d}_{\text{new}} = A\hat{\text{i}} + B\hat{\text{j}}.$$

(a) The equation $\vec{d}_{\text{storm}} + \vec{d}_{\text{new}} = \vec{d}_o$ readily yields A = –100 km and B = 120 km. The magnitude of $\vec{d}_{\text{new}}$ is therefore equal to $|\vec{d}_{\text{new}}| = \sqrt{A^2 + B^2} = 156\ \text{km}$.

(b) The direction is

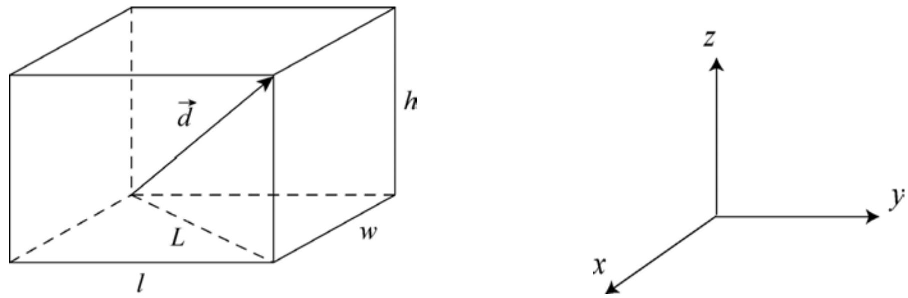
$$\tan^{-1}(B/A) = -50.2° \text{ or } 180° + (-50.2°) = 129.8°.$$

We choose the latter value since it indicates a vector pointing in the second quadrant, which is what we expect here. The answer can be phrased several equivalent ways: 129.8° counterclockwise from east, or 39.8° west from north, or 50.2° north from west.

6. (a) The height is $h = d\sin\theta$, where d = 12.5 m and θ = 20.0°. Therefore, h = 4.28 m.

(b) The horizontal distance is $d\cos\theta$ = 11.7 m.

7. The displacement of the fly is illustrated in the figure below:



A coordinate system such as the one shown (above right) allows us to express the displacement as a three-dimensional vector.

(a) The magnitude of the displacement from one corner to the diagonally opposite corner is

$$d = |\vec{d}| = \sqrt{w^2 + l^2 + h^2}$$

Substituting the values given, we obtain

$$d = |\vec{d}| = \sqrt{w^2 + l^2 + h^2} = \sqrt{(3.70\text{ m})^2 + (4.30\text{ m})^2 + (3.00\text{ m})^2} = 6.42\text{ m}\,.$$

(b) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than the magnitude of the displacement.

(c) It can be greater, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be

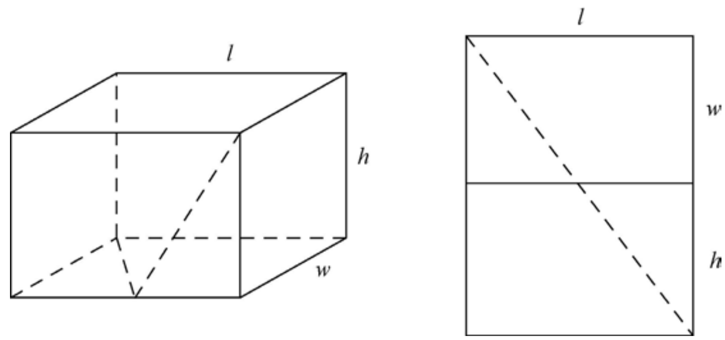
$$\ell + w + h = 11.0\text{ m.}$$

(d) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

(e) We take the x axis to be out of the page, the y axis to be to the right, and the z axis to be upward. Then the x component of the displacement is w = 3.70 m, the y component of the displacement is 4.30 m, and the z component is 3.00 m. Thus,

$$\vec{d} = (3.70\text{ m})\hat{\text{i}} + (4.30\text{ m})\hat{\text{j}} + (3.00\text{ m})\hat{\text{k}}\,.$$

An equally correct answer is gotten by interchanging the length, width, and height.



(f) Suppose the path of the fly is as shown by the dotted lines on the upper diagram. Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown on the lower diagram. The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is

$$L_{\min} = \sqrt{(w + h)^2 + \ell^2} = \sqrt{(3.70 \text{ m} + 3.00 \text{ m})^2 + (4.30 \text{ m})^2} = 7.96 \text{ m} .$$

To show that the shortest path is indeed given by $L_{\min}$, we write the length of the path as

$$L = \sqrt{y^2 + w^2} + \sqrt{(l - y)^2 + h^2} .$$

The condition for minimum is given by

$$\frac{dL}{dy} = \frac{y}{\sqrt{y^2 + w^2}} - \frac{l - y}{\sqrt{(l - y)^2 + h^2}} = 0 .$$

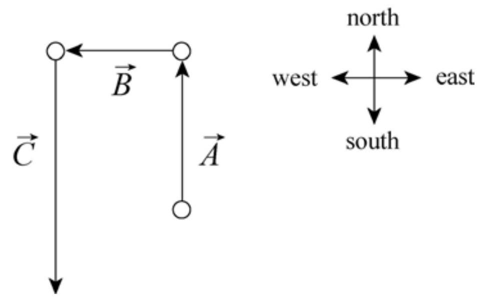
A little algebra shows that the condition is satisfied when $y = lw/(w + h)$, which gives

$$L_{\min} = \sqrt{w^2\left(1 + \frac{l^2}{(w+h)^2}\right)} + \sqrt{h^2\left(1 + \frac{l^2}{(w+h)^2}\right)} = \sqrt{(w+h)^2 + l^2} .$$

Any other path would be longer than 7.96 m.

8. We label the displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\mathrm{i}}$ direction (+*x* direction) and *north* as the $\hat{\mathrm{j}}$ direction (+*y* direction). All distances are understood to be in kilometers.

(a) The vector diagram representing the motion is shown next:



$\vec{A} = (3.1\text{ km})\,\hat{\text{j}}$
$\vec{B} = (-2.4\text{ km})\,\hat{\text{i}}$
$\vec{C} = (-5.2\text{ km})\,\hat{\text{j}}$

(b) The final point is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = (-2.4\text{ km})\hat{\text{i}} + (-2.1\text{ km})\hat{\text{j}}$$

whose magnitude is

$$|\vec{r}| = \sqrt{(-2.4\text{ km})^2 + (-2.1\text{ km})^2} \approx 3.2\text{ km}\ .$$

(c) There are two possibilities for the angle:

$$\theta = \tan^{-1}\left(\frac{-2.1\text{ km}}{-2.4\text{ km}}\right) = 41°, \text{or } 221°\ .$$

We choose the latter possibility since $\vec{r}$ is in the third quadrant. It should be noted that many graphical calculators have polar $\leftrightarrow$ rectangular “shortcuts” that automatically produce the correct answer for angle (measured counterclockwise from the +*x* axis). We may phrase the angle, then, as 221° counterclockwise from East (a phrasing that sounds peculiar, at best) or as 41° south from west or 49° west from south. The resultant $\vec{r}$ is not shown in our sketch; it would be an arrow directed from the “tail” of $\vec{A}$ to the “head” of $\vec{C}$.

9. All distances in this solution are understood to be in meters.

(a) $\vec{a} + \vec{b} = [4.0 + (-1.0)]\,\hat{\text{i}} + [(-3.0) + 1.0]\,\hat{\text{j}} + (1.0 + 4.0)\hat{\text{k}} = (3.0\,\hat{\text{i}} - 2.0\,\hat{\text{j}} + 5.0\,\hat{\text{k}})$ m.

(b) $\vec{a} - \vec{b} = [4.0 - (-1.0)]\hat{\text{i}} + [(-3.0) - 1.0]\hat{\text{j}} + (1.0 - 4.0)\hat{\text{k}} = (5.0\,\hat{\text{i}} - 4.0\,\hat{\text{j}} - 3.0\,\hat{\text{k}})$ m.

(c) The requirement $\vec{a} - \vec{b} + \vec{c} = 0$ leads to $\vec{c} = \vec{b} - \vec{a}$, which we note is the opposite of what we found in part (b). Thus, $\vec{c} = (-5.0\,\hat{\text{i}} + 4.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}})$ m.

10. The *x*, *y*, and *z* components of $\vec{r} = \vec{c} + \vec{d}$ are, respectively,

(a) $r_x = c_x + d_x = 7.4\text{ m} + 4.4\text{ m} = 12\text{ m}$,

(b) $r_y = c_y + d_y = -3.8\text{ m} - 2.0\text{ m} = -5.8\text{ m}$, and

(c) $r_z = c_z + d_z = -6.1\text{ m} + 3.3\text{ m} = -2.8\text{ m}$.

11. We write $\vec{r} = \vec{a} + \vec{b}$. When not explicitly displayed, the units here are assumed to be meters.

(a) The x and the y components of $\vec{r}$ are $r_x = a_x + b_x$ = (4.0 m) – (13 m) = –9.0 m and $r_y = a_y + b_y$ = (3.0 m) + (7.0 m) = 10 m, respectively. Thus $\vec{r} = (-9.0\text{ m})\,\hat{\mathrm{i}} + (10\text{ m})\,\hat{\mathrm{j}}$.

(b) The magnitude of $\vec{r}$ is

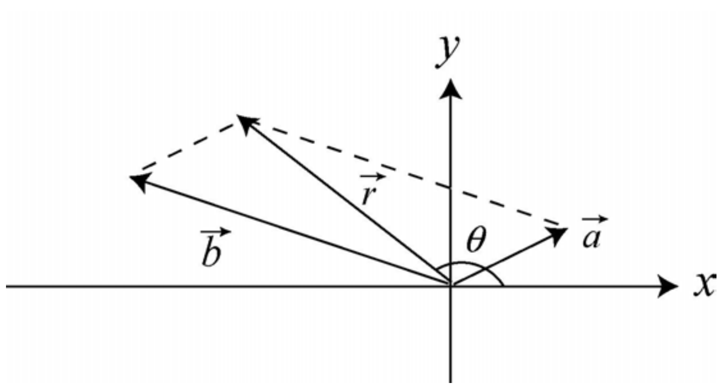
$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0\text{ m})^2 + (10\text{ m})^2} = 13\text{ m}.$$

(c) The angle between the resultant and the $+x$ axis is given by

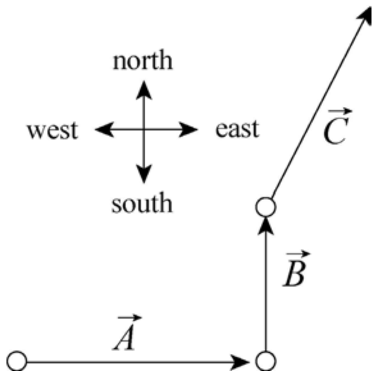
$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{10.0\text{ m}}{-9.0\text{ m}}\right) = -48^\circ \text{ or } 132^\circ.$$

Since the x component of the resultant is negative and the y component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from $+x$ axis).

The addition of the two vectors is depicted in the figure below (not to scale). Indeed, we expect $\vec{r}$ to be in the second quadrant.



12. We label the displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\mathrm{i}}$ direction ($+x$ direction) and *north* as the $\hat{\mathrm{j}}$ direction ($+y$ direction). We note that the angle between $\vec{C}$ and the x axis is 60°. Thus,



$$\vec{A} = (50\text{ km})\,\hat{\text{i}}$$
$$\vec{B} = (30\text{ km})\,\hat{\text{j}}$$
$$\vec{C} = (25\text{ km})\cos(60°)\,\hat{\text{i}} + (25\text{ km})\sin(60°)\,\hat{\text{j}}$$

(a) The total displacement of the car from its initial position is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = (62.5\text{ km})\,\hat{\text{i}} + (51.7\text{ km})\,\hat{\text{j}}$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{(62.5\,\text{km})^2 + (51.7\,\text{km})^2} = 81\text{ km}.$$

(b) The angle (counterclockwise from $+x$ axis) is $\tan^{-1}$ (51.7 km/62.5 km) = 40°, which is to say that it points 40° *north of east*. Although the resultant $\vec{r}$ is shown in our sketch, it would be a direct line from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

13. We find the components and then add them (as scalars, not vectors). With d = 3.40 km and θ = 35.0° we find $d \cos\theta + d \sin\theta$ = 4.74 km.

14. (a) Summing the x components, we have

$$20\text{ m} + b_x - 20\text{ m} - 60\text{ m} = -140\text{ m},$$

which gives $b_x = -80\text{ m}$.

(b) Summing the y components, we have

$$60\text{ m} - 70\text{ m} + c_y - 70\text{ m} = 30\text{ m},$$

which implies c_y =110 m.

(c) Using the Pythagorean theorem, the magnitude of the overall displacement is given by $\sqrt{(-140\text{ m})^2 + (30\text{ m})^2} \approx 143\text{ m}.$

(d) The angle is given by $\tan^{-1}(30/(-140)) = -12°$, (which would be 12° measured clockwise from the $-x$ axis, or 168° measured counterclockwise from the $+x$ axis).

15. It should be mentioned that an efficient way to work this vector addition problem is with the cosine law for general triangles (and since $\vec{a}, \vec{b}$, and $\vec{r}$ form an isosceles triangle, the angles are easy to figure). However, in the interest of reinforcing the usual systematic approach to vector addition, we note that the angle $\vec{b}$ makes with the +*x* axis is 30° +105° = 135° and apply Eq. 3-5 and Eq. 3-6 where appropriate.

(a) The *x* component of $\vec{r}$ is r_x = (10.0 m) cos 30° + (10.0 m) cos 135° = 1.59 m.

(b) The *y* component of $\vec{r}$ is r_y = (10.0 m) sin 30° + (10.0 m) sin 135° = 12.1 m.

(c) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{(1.59 \text{ m})^2 + (12.1 \text{ m})^2} = 12.2$ m.

(d) The angle between $\vec{r}$ and the +*x* direction is $\tan^{-1}$[(12.1 m)/(1.59 m)] = 82.5°.

16. (a) $\vec{a} + \vec{b} = (3.0\,\hat{\text{i}} + 4.0\,\hat{\text{j}})\text{ m} + (5.0\,\hat{\text{i}} - 2.0\,\hat{\text{j}})\text{ m} = (8.0\text{ m})\,\hat{\text{i}} + (2.0\text{ m})\,\hat{\text{j}}.$

(b) The magnitude of $\vec{a} + \vec{b}$ is

$$|\vec{a} + \vec{b}| = \sqrt{(8.0 \text{ m})^2 + (2.0 \text{ m})^2} = 8.2 \text{ m}.$$

(c) The angle between this vector and the +*x* axis is

$$\tan^{-1}[(2.0 \text{ m})/(8.0 \text{ m})] = 14°.$$

(d) $\vec{b} - \vec{a} = (5.0\,\hat{\text{i}} - 2.0\,\hat{\text{j}})\text{ m} - (3.0\,\hat{\text{i}} + 4.0\,\hat{\text{j}})\text{ m} = (2.0\text{ m})\,\hat{\text{i}} - (6.0\text{ m})\,\hat{\text{j}}.$

(e) The magnitude of the difference vector $\vec{b} - \vec{a}$ is

$$|\vec{b} - \vec{a}| = \sqrt{(2.0 \text{ m})^2 + (-6.0 \text{ m})^2} = 6.3 \text{ m}.$$

(f) The angle between this vector and the +*x* axis is $\tan^{-1}$[(–6.0 m)/(2.0 m)] = –72°. The vector is 72° *clockwise* from the axis defined by $\hat{\text{i}}$.

17. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6). Where the length unit is not displayed, the unit meter should be understood.

(a) Using unit-vector notation,

$$\vec{a} = (50\text{ m})\cos(30°)\hat{\text{i}} + (50\text{ m})\sin(30°)\hat{\text{j}}$$
$$\vec{b} = (50\text{ m})\cos(195°)\hat{\text{i}} + (50\text{ m})\sin(195°)\hat{\text{j}}$$
$$\vec{c} = (50\text{ m})\cos(315°)\hat{\text{i}} + (50\text{ m})\sin(315°)\hat{\text{j}}$$
$$\vec{a} + \vec{b} + \vec{c} = (30.4\text{ m})\hat{\text{i}} - (23.3\text{ m})\hat{\text{j}}.$$

The magnitude of this result is $\sqrt{(30.4\text{ m})^2 + (-23.3\text{ m})^2} = 38\text{ m}$.

(b) The two possibilities presented by a simple calculation for the angle between the vector described in part (a) and the +*x* direction are $\tan^{-1}$[(–23.2 m)/(30.4 m)] = –37.5°, and 180° + (–37.5°) = 142.5°. The former possibility is the correct answer since the vector is in the fourth quadrant (indicated by the signs of its components). Thus, the angle is –37.5°, which is to say that it is 37.5° *clockwise* from the +*x* axis. This is equivalent to 322.5° counterclockwise from +*x*.

(c) We find

$$\vec{a} - \vec{b} + \vec{c} = [43.3 - (-48.3) + 35.4]\hat{\text{i}} - [25 - (-12.9) + (-35.4)]\hat{\text{j}} = (127\hat{\text{i}} + 2.60\hat{\text{j}})\text{ m}$$

in unit-vector notation. The magnitude of this result is

$$|\vec{a} - \vec{b} + \vec{c}| = \sqrt{(127\text{ m})^2 + (2.6\text{ m})^2} \approx 1.30 \times 10^2\text{ m}.$$

(d) The angle between the vector described in part (c) and the +*x* axis is $\tan^{-1}(2.6\text{ m}/127\text{ m}) \approx 1.2°$.

(e) Using unit-vector notation, $\vec{d}$ is given by $\vec{d} = \vec{a} + \vec{b} - \vec{c} = (-40.4\,\hat{\text{i}} + 47.4\,\hat{\text{j}})\text{ m}$, which has a magnitude of $\sqrt{(-40.4\text{ m})^2 + (47.4\text{ m})^2} = 62\text{ m}$.

(f) The two possibilities presented by a simple calculation for the angle between the vector described in part (e) and the +*x* axis are $\tan^{-1}(47.4/(-40.4)) = -50.0°$, and $180° + (-50.0°) = 130°$. We choose the latter possibility as the correct one since it indicates that $\vec{d}$ is in the second quadrant (indicated by the signs of its components).

18. If we wish to use Eq. 3-5 in an unmodified fashion, we should note that the angle between $\vec{C}$ and the +*x* axis is 180° + 20.0° = 200°.

(a) The *x* and *y* components of $\vec{B}$ are given by

$$B_x = C_x - A_x = (15.0\text{ m})\cos 200° - (12.0\text{ m})\cos 40° = -23.3\text{ m},$$
$$B_y = C_y - A_y = (15.0\text{ m})\sin 200° - (12.0\text{ m})\sin 40° = -12.8\text{ m}.$$

Consequently, its magnitude is $|\vec{B}| = \sqrt{(-23.3\text{ m})^2 + (-12.8\text{ m})^2} = 26.6\text{ m}$.

(b) The two possibilities presented by a simple calculation for the angle between $\vec{B}$ and the +x axis are $\tan^{-1}$[(–12.8 m)/(–23.3 m)] = 28.9°, and 180° + 28.9° = 209°. We choose the latter possibility as the correct one since it indicates that $\vec{B}$ is in the third quadrant (indicated by the signs of its components). We note, too, that the answer can be equivalently stated as $-151°$.

19. (a) With $\hat{\text{i}}$ directed forward and $\hat{\text{j}}$ directed leftward, the resultant is (5.00 $\hat{\text{i}}$ + 2.00 $\hat{\text{j}}$) m . The magnitude is given by the Pythagorean theorem: $\sqrt{(5.00\text{ m})^2 + (2.00\text{ m})^2}$ = 5.385 m ≈ 5.39 m.

(b) The angle is $\tan^{-1}$(2.00/5.00) ≈ 21.8º (left of forward).

20. The desired result is the displacement vector, in units of km, $\vec{A}$ = (5.6 km), 90º (measured counterclockwise from the +x axis), or $\vec{A} = (5.6\text{ km})\hat{\text{j}}$, where $\hat{\text{j}}$ is the unit vector along the positive y axis (north). This consists of the sum of two displacements: during the whiteout, $\vec{B} = (7.8\text{ km}),\ 50°$, or

$$\vec{B} = (7.8\text{ km})(\cos 50°\hat{\text{i}} + \sin 50°\ \hat{\text{j}}) = (5.01\text{ km})\hat{\text{i}} + (5.98\text{ km})\hat{\text{j}}$$

and the unknown $\vec{C}$. Thus, $\vec{A} = \vec{B} + \vec{C}$.

(a) The desired displacement is given by $\vec{C} = \vec{A} - \vec{B} = (-5.01\text{ km})\ \hat{\text{i}} - (0.38\text{ km})\ \hat{\text{j}}$. The magnitude is $\sqrt{(-5.01\text{ km})^2 + (-0.38\text{ km})^2} = 5.0\text{ km}$.

(b) The angle is $\tan^{-1}[(-0.38\text{ km})/(-5.01\text{ km})] = 4.3°$, south of due west.

21. Reading carefully, we see that the (x, y) specifications for each "dart" are to be interpreted as (Δx, Δy) descriptions of the corresponding displacement vectors. We combine the different parts of this problem into a single exposition.

(a) Along the x axis, we have (with the centimeter unit understood)

$$30.0 + b_x - 20.0 - 80.0 = -140,$$

which gives b_x = –70.0 cm.

(b) Along the y axis we have

$$40.0 - 70.0 + c_y - 70.0 = -20.0$$

which yields c_y = 80.0 cm.

(c) The magnitude of the final location (–140 , –20.0) is $\sqrt{(-140)^2 + (-20.0)^2} = 141$ cm.

(d) Since the displacement is in the third quadrant, the angle of the overall displacement is given by $\pi + \tan^{-1}[(-20.0)/(-140)]$ or 188° counterclockwise from the +x axis (or $-172°$ counterclockwise from the +x axis).

22. Angles are given in ‘standard’ fashion, so Eq. 3-5 applies directly. We use this to write the vectors in unit-vector notation before adding them. However, a very different-looking approach using the special capabilities of most graphical calculators can be imagined. Wherever the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Allowing for the different angle units used in the problem statement, we arrive at

$$\begin{aligned}
\vec{E} &= 3.73\ \hat{\mathrm{i}} + 4.70\ \hat{\mathrm{j}} \\
\vec{F} &= 1.29\ \hat{\mathrm{i}} - 4.83\ \hat{\mathrm{j}} \\
\vec{G} &= 1.45\ \hat{\mathrm{i}} + 3.73\ \hat{\mathrm{j}} \\
\vec{H} &= -5.20\ \hat{\mathrm{i}} + 3.00\ \hat{\mathrm{j}} \\
\vec{E} + \vec{F} + \vec{G} + \vec{H} &= 1.28\ \hat{\mathrm{i}}\ + 6.60\ \hat{\mathrm{j}}.
\end{aligned}$$

(b) The magnitude of the vector sum found in part (a) is $\sqrt{(1.28\text{ m})^2 + (6.60\text{ m})^2} = 6.72\text{ m}$.

(c) Its angle measured counterclockwise from the +x axis is $\tan^{-1}(6.60/1.28)$ = 79.0°.

(d) Using the conversion factor $\pi\ \text{rad} = 180°$, 79.0° = 1.38 rad.

23. The resultant (along the y axis, with the same magnitude as $\vec{C}$) forms (along with $\vec{C}$) a side of an isosceles triangle (with $\vec{B}$ forming the base). If the angle between $\vec{C}$ and the y axis is $\theta = \tan^{-1}(3/4) = 36.87°$, then it should be clear that (referring to the magnitudes of the vectors) $B = 2C\sin(\theta/2)$. Thus (since C = 5.0) we find B = 3.2.

24. As a vector addition problem, we express the situation (described in the problem statement) as $\vec{A} + \vec{B} = (3A)\hat{\mathrm{j}}$, where $\vec{A} = A\,\hat{\mathrm{i}}$ and B = 7.0 m. Since $\hat{\mathrm{i}} \perp \hat{\mathrm{j}}$ we may use the Pythagorean theorem to express B in terms of the magnitudes of the other two vectors:

$$B = \sqrt{(3A)^2 + A^2} \qquad \Rightarrow \qquad A = \frac{1}{\sqrt{10}} B \;=\; 2.2 \text{ m} .$$

25. The strategy is to find where the camel is ($\vec{C}$) by adding the two consecutive displacements described in the problem, and then finding the difference between that location and the oasis ($\vec{B}$). Using the magnitude-angle notation

$$\vec{C} = (24 \angle\; -15°) + (8.0 \angle\; 90°) = (23.25 \angle\; 4.41°)$$

so

$$\vec{B} - \vec{C} \;=\; (25 \angle\; 0°) - (23.25 \angle\; 4.41°) = (2.5 \angle -45°)$$

which is efficiently implemented using a vector-capable calculator in polar mode. The distance is therefore 2.6 km.

26. The vector equation is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. Expressing $\vec{B}$ and $\vec{D}$ in unit-vector notation, we have $(1.69\hat{\text{i}} + 3.63\hat{\text{j}})$ m and $(-2.87\hat{\text{i}} + 4.10\hat{\text{j}})$ m , respectively. Where the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Adding corresponding components, we obtain $\vec{R} = (-3.18 \text{ m})\hat{\text{i}} + (4.72 \text{ m})\hat{\text{j}}$.

(b) Using Eq. 3-6, the magnitude is

$$|\vec{R}| = \sqrt{(-3.18 \text{ m})^2 + (4.72 \text{ m})^2} = 5.69 \text{ m}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{4.72 \text{ m}}{-3.18 \text{ m}}\right) = -56.0° \;\; (\text{with } -x \text{ axis}).$$

If measured counterclockwise from +*x*-axis, the angle is then $180° - 56.0° = 124°$. Thus, converting the result to polar coordinates, we obtain

$$(-3.18, 4.72) \;\rightarrow\; (5.69 \angle\; 124°)$$

27. Solving the simultaneous equations yields the answers:

(a) $\vec{d}_1 \;=\; 4\,\vec{d}_3 \;= 8\,\hat{\text{i}} + 16\,\hat{\text{j}}$, and

(b) $\vec{d}_2 \;=\; \vec{d}_3 \;= 2\,\hat{\text{i}} + 4\,\hat{\text{j}}$.

28. Let $\vec{A}$ represent the first part of Beetle 1's trip (0.50 m east or $0.5\,\hat{\text{i}}$) and $\vec{C}$ represent the first part of Beetle 2's trip intended voyage (1.6 m at 50º north of east). For

their respective second parts: $\vec{B}$ is 0.80 m at 30º north of east and $\vec{D}$ is the unknown. The final position of Beetle 1 is

$$\vec{A}+\vec{B}=(0.5\text{ m})\hat{\text{i}}+(0.8\text{ m})(\cos 30^\circ\ \hat{\text{i}}+\sin 30^\circ\ \hat{\text{j}})=(1.19\text{ m})\ \hat{\text{i}}+(0.40\text{ m})\ \hat{\text{j}}.$$

The equation relating these is $\vec{A}+\vec{B}=\vec{C}+\vec{D}$, where

$$\vec{C}=(1.60\text{ m})(\cos 50.0^\circ\hat{\text{i}}+\sin 50.0^\circ\hat{\text{j}})=(1.03\text{ m})\hat{\text{i}}+(1.23\text{ m})\hat{\text{j}}$$

(a) We find $\vec{D}=\vec{A}+\vec{B}-\vec{C}=(0.16\text{ m})\hat{\text{i}}+(-0.83\text{ m})\hat{\text{j}}$, and the magnitude is D = 0.84 m.

(b) The angle is $\tan^{-1}(-0.83/0.16)=-79^\circ$, which is interpreted to mean 79º south of east (or 11º east of south).

29. Let $l_0=2.0\text{ cm}$ be the length of each segment. The nest is located at the endpoint of segment *w*.

(a) Using unit-vector notation, the displacement vector for point *A* is

$$\begin{aligned}\vec{d}_A&=\vec{w}+\vec{v}+\vec{i}+\vec{h}=l_0(\cos 60^\circ\hat{\text{i}}+\sin 60^\circ\ \hat{\text{j}})+\left(l_0\ \hat{\text{j}}\right)+l_0(\cos 120^\circ\hat{\text{i}}+\sin 120^\circ\ \hat{\text{j}})+\left(l_0\ \hat{\text{j}}\right)\\&=(2+\sqrt{3})l_0\ \hat{\text{j}}.\end{aligned}$$

Therefore, the magnitude of $\vec{d}_A$ is $|\vec{d}_A|=(2+\sqrt{3})(2.0\text{ cm})=7.5\text{ cm}$.

(b) The angle of $\vec{d}_A$ is $\theta=\tan^{-1}(d_{A,y}/d_{A,x})=\tan^{-1}(\infty)=90^\circ$.

(c) Similarly, the displacement for point *B* is

$$\begin{aligned}\vec{d}_B&=\vec{w}+\vec{v}+\vec{j}+\vec{p}+\vec{o}\\&=l_0(\cos 60^\circ\hat{\text{i}}+\sin 60^\circ\ \hat{\text{j}})+\left(l_0\ \hat{\text{j}}\right)+l_0(\cos 60^\circ\hat{\text{i}}+\sin 60^\circ\ \hat{\text{j}})+l_0(\cos 30^\circ\hat{\text{i}}+\sin 30^\circ\ \hat{\text{j}})+\left(l_0\ \hat{\text{i}}\right)\\&=(2+\sqrt{3}/2)l_0\ \hat{\text{i}}+(3/2+\sqrt{3})l_0\ \hat{\text{j}}.\end{aligned}$$

Therefore, the magnitude of $\vec{d}_B$ is

$$|\vec{d}_B|=l_0\sqrt{(2+\sqrt{3}/2)^2+(3/2+\sqrt{3})^2}=(2.0\text{ cm})(4.3)=8.6\text{ cm}.$$

(d) The direction of $\vec{d}_B$ is

$$\theta_B = \tan^{-1}\left(\frac{d_{B,y}}{d_{B,x}}\right) = \tan^{-1}\left(\frac{3/2+\sqrt{3}}{2+\sqrt{3}/2}\right) = \tan^{-1}(1.13) = 48°.$$

30. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6).

(a) The magnitude of $\vec{a}$ is $a = \sqrt{(4.0\text{ m})^2 + (-3.0\text{ m})^2} = 5.0\text{ m}$.

(b) The angle between $\vec{a}$ and the +*x* axis is $\tan^{-1}$ [(–3.0 m)/(4.0 m)] = –37°. The vector is 37° *clockwise* from the axis defined by $\hat{\text{i}}$.

(c) The magnitude of $\vec{b}$ is $b = \sqrt{(6.0\text{ m})^2 + (8.0\text{ m})^2} = 10\text{ m}$.

(d) The angle between $\vec{b}$ and the +*x* axis is $\tan^{-1}$[(8.0 m)/(6.0 m)] = 53°.

(e) $\vec{a}+\vec{b} = (4.0\text{ m}+6.0\text{ m})\,\hat{\text{i}} + [(-3.0\text{ m})+8.0\text{ m}]\hat{\text{j}} = (10\text{ m})\hat{\text{i}} + (5.0\text{ m})\hat{\text{j}}$. The magnitude of this vector is $|\vec{a}+\vec{b}| = \sqrt{(10\text{ m})^2 + (5.0\text{ m})^2} = 11\text{ m}$; we round to two significant figures in our results.

(f) The angle between the vector described in part (e) and the +*x* axis is $\tan^{-1}$[(5.0 m)/(10 m)] = 27°.

(g) $\vec{b}-\vec{a} = (6.0\text{ m} - 4.0\text{ m})\,\hat{\text{i}} + [8.0\text{ m} - (-3.0\text{ m})]\,\hat{\text{j}} = (2.0\text{ m})\,\hat{\text{i}} + (11\text{ m})\,\hat{\text{j}}$. The magnitude of this vector is $|\vec{b}-\vec{a}| = \sqrt{(2.0\text{ m})^2 + (11\text{ m})^2} = 11\text{ m}$, which is, interestingly, the same result as in part (e) (exactly, not just to 2 significant figures) (this curious coincidence is made possible by the fact that $\vec{a} \perp \vec{b}$).

(h) The angle between the vector described in part (g) and the +*x* axis is $\tan^{-1}$[(11 m)/(2.0 m)] = 80°.

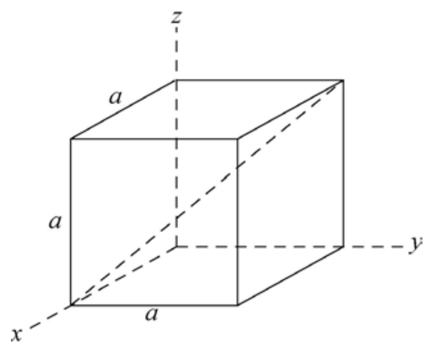
(i) $\vec{a}-\vec{b} = (4.0\text{ m} - 6.0\text{ m})\,\hat{\text{i}} + [(-3.0\text{ m}) - 8.0\text{ m}]\,\hat{\text{j}} = (-2.0\text{ m})\,\hat{\text{i}} + (-11\text{ m})\,\hat{\text{j}}$. The magnitude of this vector is $|\vec{a}-\vec{b}| = \sqrt{(-2.0\text{ m})^2 + (-11\text{ m})^2} = 11\text{ m}$.

(j) The two possibilities presented by a simple calculation for the angle between the vector described in part (i) and the +*x* direction are $\tan^{-1}$ [(–11 m)/(–2.0 m)] = 80°, and 180° + 80° = 260°. The latter possibility is the correct answer (see part (k) for a further observation related to this result).

(k) Since $\vec{a} - \vec{b} = (-1)(\vec{b} - \vec{a})$, they point in opposite (anti-parallel) directions; the angle between them is 180°.

31. (a) As can be seen from Figure 3-30, the point diametrically opposite the origin (0,0,0) has position vector $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$ and this is the vector along the "body diagonal."

(b) From the point (a, 0, 0), which corresponds to the position vector $a\,\hat{\mathrm{i}}$, the diametrically opposite point is (0, a, a) with the position vector $a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$. Thus, the vector along the line is the difference $-a\hat{\mathrm{i}} + a\hat{\mathrm{j}} + a\hat{\mathrm{k}}$.



(c) If the starting point is (0, a, 0) with the corresponding position vector $a\,\hat{\mathrm{j}}$, the diametrically opposite point is (a, 0, a) with the position vector $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{k}}$. Thus, the vector along the line is the difference $a\,\hat{\mathrm{i}} - a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$.

(d) If the starting point is (a, a, 0) with the corresponding position vector $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}}$, the diametrically opposite point is (0, 0, a) with the position vector $a\,\hat{\mathrm{k}}$. Thus, the vector along the line is the difference $-a\,\hat{\mathrm{i}} - a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$.

(e) Consider the vector from the back lower left corner to the front upper right corner. It is $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$. We may think of it as the sum of the vector $a\,\hat{\mathrm{i}}$ parallel to the x axis and the vector $a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$ perpendicular to the x axis. The tangent of the angle between the vector and the x axis is the perpendicular component divided by the parallel component. Since the magnitude of the perpendicular component is $\sqrt{a^2 + a^2} = a\sqrt{2}$ and the magnitude of the parallel component is a, $\tan\theta = \left(a\sqrt{2}\right)/a = \sqrt{2}$. Thus θ = 54.7°. The angle between the vector and each of the other two adjacent sides (the y and z axes) is the same as is the angle between any of the other diagonal vectors and any of the cube sides adjacent to them.

(f) The length of any of the diagonals is given by $\sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$.

32. (a) With a = 17.0 m and θ = 56.0° we find $a_x = a \cos \theta$ = 9.51 m.

(b) Similarly, $a_y = a \sin \theta$ = 14.1 m.

(c) The angle relative to the new coordinate system is θ' = (56.0° – 18.0°) = 38.0°. Thus, $a'_x = a\cos\theta' = 13.4$ m.

(d) Similarly, $a'_y = a \sin \theta'$ = 10.5 m.

33. Examining the figure, we see that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $|\vec{a} \times \vec{b}| = (3.0)(4.0) = 12$ since the angle between them is 90º.

(b) Using the Right-Hand Rule, the vector $\vec{a}\times\vec{b}$ points in the $\hat{\mathrm{i}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}}$, or the +z direction.

(c) $|\vec{a} \times \vec{c}| = |\vec{a} \times (-\vec{a} - \vec{b})| = |-(\vec{a} \times \vec{b})| = 12.$

(d) The vector $-\vec{a}\times\vec{b}$ points in the $-\hat{\mathrm{i}}\times\hat{\mathrm{j}}=-\hat{\mathrm{k}}$, or the $-z$ direction.

(e) $|\vec{b} \times \vec{c}| = |\vec{b} \times (-\vec{a} - \vec{b})| = |-(\vec{b} \times \vec{a})| = |(\vec{a} \times \vec{b})| = 12.$

(f) The vector points in the +z direction, as in part (a).

34. We apply Eq. 3-30 and Eq. 3-23.

(a) $\vec{a}\times\vec{b} = (a_x b_y - a_y b_x)\,\hat{\mathrm{k}}$ since all other terms vanish, due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any z components. Consequently, we obtain $[(3.0)(4.0)-(5.0)(2.0)]\hat{\mathrm{k}} = 2.0\,\hat{\mathrm{k}}$.

(b) $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y$ yields (3.0)(2.0) + (5.0)(4.0) = 26.

(c) $\vec{a}+\vec{b} = (3.0 + 2.0)\,\hat{\mathrm{i}} + (5.0 + 4.0)\,\hat{\mathrm{j}} \Rightarrow (\vec{a} + \vec{b})\cdot\vec{b} = (5.0)\,(2.0) + (9.0)\,(4.0) = 46$.

(d) Several approaches are available. In this solution, we will construct a $\hat{b}$ unit-vector and “dot” it (take the scalar product of it) with $\vec{a}$. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}}}{\sqrt{(2.0)^2 + (4.0)^2}}\,.$$

We therefore obtain

$$a_b = \vec{a}\cdot\hat{b} = \frac{(3.0)(2.0)+(5.0)(4.0)}{\sqrt{(2.0)^2+(4.0)^2}} = 5.8.$$

35. (a) The scalar (dot) product is (4.50)(7.30)cos(320º – 85.0º) = – 18.8 .

(b) The vector (cross) product is in the $\hat{k}$ direction (by the right-hand rule) with magnitude |(4.50)(7.30) sin(320º – 85.0º)| = 26.9 .

36. First, we rewrite the given expression as 4($\vec{d}_{\text{plane}} \cdot \vec{d}_{\text{cross}}$) where $\vec{d}_{\text{plane}} = \vec{d}_1 + \vec{d}_2$ and in the plane of $\vec{d}_1$ and $\vec{d}_2$, and $\vec{d}_{\text{cross}} = \vec{d}_1 \times \vec{d}_2$. Noting that $\vec{d}_{\text{cross}}$ is perpendicular to the plane of $\vec{d}_1$ and $\vec{d}_2$, we see that the answer must be 0 (the scalar [dot] product of perpendicular vectors is zero).

37. We apply Eq. 3-30 and Eq.3-23. If a vector-capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method.

(a) We note that $\vec{b}\times\vec{c} = -8.0\hat{\text{i}} + 5.0\hat{\text{j}} + 6.0\hat{\text{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (3.0)(-8.0) + (3.0)(5.0) + (-2.0)(6.0) = -21.$$

(b) We note that $\vec{b}+\vec{c} = 1.0\hat{\text{i}} - 2.0\hat{\text{j}} + 3.0\hat{\text{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}+\vec{c}) = (3.0)(1.0) + (3.0)(-2.0) + (-2.0)(3.0) = -9.0.$$

(c) Finally,

$$\begin{aligned}\vec{a}\times(\vec{b}+\vec{c}) &= [(3.0)(3.0)-(-2.0)(-2.0)]\,\hat{\text{i}} + [(-2.0)(1.0)-(3.0)(3.0)]\,\hat{\text{j}} \\ &\quad + [(3.0)(-2.0)-(3.0)(1.0)]\,\hat{\text{k}} \\ &= 5\hat{\text{i}} - 11\hat{\text{j}} - 9\hat{\text{k}}\end{aligned}$$

38. Using the fact that

$$\hat{\text{i}}\times\hat{\text{j}} = \hat{\text{k}},\quad \hat{\text{j}}\times\hat{\text{k}} = \hat{\text{i}},\quad \hat{\text{k}}\times\hat{\text{i}} = \hat{\text{j}}$$

we obtain

$$2\vec{A}\times\vec{B} = 2\left(2.00\hat{\text{i}} + 3.00\hat{\text{j}} - 4.00\hat{\text{k}}\right)\times\left(-3.00\hat{\text{i}} + 4.00\hat{\text{j}} + 2.00\hat{\text{k}}\right) = 44.0\hat{\text{i}} + 16.0\hat{\text{j}} + 34.0\hat{\text{k}}.$$

Next, making use of

$$\hat{\text{i}}\cdot\hat{\text{i}} = \hat{\text{j}}\cdot\hat{\text{j}} = \hat{\text{k}}\cdot\hat{\text{k}} = 1$$
$$\hat{\text{i}}\cdot\hat{\text{j}} = \hat{\text{j}}\cdot\hat{\text{k}} = \hat{\text{k}}\cdot\hat{\text{i}} = 0$$

we have

$$3\vec{C}\cdot\left(2\vec{A}\times\vec{B}\right)=3\left(7.00\,\hat{\mathrm{i}}-8.00\hat{\mathrm{j}}\right)\cdot\left(44.0\,\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\,\hat{\mathrm{k}}\right)$$
$$=3[(7.00)(44.0)+(-8.00)(16.0)+(0)(34.0)]=540.$$

39. From the definition of the dot product between $\vec{A}$ and $\vec{B}$, $\vec{A}\cdot\vec{B}=AB\cos\theta$, we have

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{AB}$$

With $A=6.00$, $B=7.00$ and $\vec{A}\cdot\vec{B}=14.0$, $\cos\theta=0.333$, or $\theta=70.5°$.

40. The displacement vectors can be written as (in meters)

$$\vec{d}_1=(4.50\text{ m})(\cos 63°\,\hat{\mathrm{j}}+\sin 63°\,\hat{\mathrm{k}})=(2.04\text{ m})\,\hat{\mathrm{j}}+(4.01\text{ m})\,\hat{\mathrm{k}}$$
$$\vec{d}_2=(1.40\text{ m})(\cos 30°\,\hat{\mathrm{i}}+\sin 30°\,\hat{\mathrm{k}})=(1.21\text{ m})\,\hat{\mathrm{i}}+(0.70\text{ m})\,\hat{\mathrm{k}}\,.$$

(a) The dot product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\vec{d}_1\cdot\vec{d}_2=(2.04\,\hat{\mathrm{j}}+4.01\hat{\mathrm{k}})\cdot(1.21\hat{\mathrm{i}}+0.70\,\hat{\mathrm{k}})=(4.01\hat{\mathrm{k}})\cdot(0.70\,\hat{\mathrm{k}})=2.81\,\mathrm{m}^2.$$

(b) The cross product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\begin{aligned}\vec{d}_1\times\vec{d}_2&=(2.04\,\hat{\mathrm{j}}+4.01\hat{\mathrm{k}})\times(1.21\hat{\mathrm{i}}+0.70\,\hat{\mathrm{k}})\\&=(2.04)(1.21)(-\hat{\mathrm{k}})+(2.04)(0.70)\hat{\mathrm{i}}+(4.01)(1.21)\hat{\mathrm{j}}\\&=(1.43\,\hat{\mathrm{i}}+4.86\,\hat{\mathrm{j}}-2.48\,\hat{\mathrm{k}})\,\mathrm{m}^2.\end{aligned}$$

(c) The magnitudes of $\vec{d}_1$ and $\vec{d}_2$ are

$$d_1=\sqrt{(2.04\text{ m})^2+(4.01\text{ m})^2}=4.50\text{ m}$$
$$d_2=\sqrt{(1.21\text{ m})^2+(0.70\text{ m})^2}=1.40\text{ m}.$$

Thus, the angle between the two vectors is

$$\theta=\cos^{-1}\left(\frac{\vec{d}_1\cdot\vec{d}_2}{d_1d_2}\right)=\cos^{-1}\left(\frac{2.81\text{ m}^2}{(4.50\text{ m})(1.40\text{ m})}\right)=63.5°.$$

41. Since $ab\cos\phi=a_xb_x+a_yb_y+a_zb_z$,

$$\cos\phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab}.$$

The magnitudes of the vectors given in the problem are

$$a = |\vec{a}| = \sqrt{(3.00)^2 + (3.00)^2 + (3.00)^2} = 5.20$$

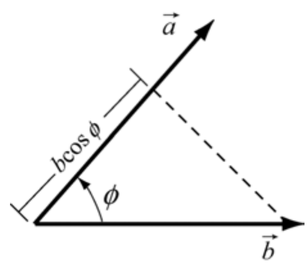
$$b = |\vec{b}| = \sqrt{(2.00)^2 + (1.00)^2 + (3.00)^2} = 3.74.$$

The angle between them is found from

$$\cos\phi = \frac{(3.00)(2.00) + (3.00)(1.00) + (3.00)(3.00)}{(5.20)(3.74)} = 0.926.$$

The angle is $\phi = 22°$.

As the name implies, the scalar product (or dot product) between two vectors is a scalar quantity. It can be regarded as the product between the magnitude of one of the vectors and the scalar component of the second vector along the direction of the first one, as illustrated below (see also in Fig. 3-18 of the text):



$$\vec{a}\cdot\vec{b} = ab\cos\phi = (a)(b\cos\phi)$$

42. The two vectors are written as, in unit of meters,

$$\vec{d}_1 = 4.0\,\hat{\mathrm{i}} + 5.0\,\hat{\mathrm{j}} = d_{1x}\hat{\mathrm{i}} + d_{1y}\hat{\mathrm{j}}, \qquad \vec{d}_2 = -3.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}} = d_{2x}\hat{\mathrm{i}} + d_{2y}\hat{\mathrm{j}}$$

(a) The vector (cross) product gives

$$\vec{d}_1 \times \vec{d}_2 = (d_{1x}d_{2y} - d_{1y}d_{2x})\hat{\mathrm{k}} = [(4.0)(4.0) - (5.0)(-3.0)]\hat{\mathrm{k}} = 31\ \hat{\mathrm{k}}$$

(b) The scalar (dot) product gives

$$\vec{d}_1 \cdot \vec{d}_2 = d_{1x}d_{2x} + d_{1y}d_{2y} = (4.0)(-3.0) + (5.0)(4.0) = 8.0.$$

(c)

$$(\vec{d}_1 + \vec{d}_2)\cdot\vec{d}_2 = \vec{d}_1\cdot\vec{d}_2 + d_2^2 = 8.0 + (-3.0)^2 + (4.0)^2 = 33.$$

(d) Note that the magnitude of the d_1 vector is $\sqrt{16+25} = 6.4$. Now, the dot product is $(6.4)(5.0)\cos\theta = 8$. Dividing both sides by 32 and taking the inverse cosine yields $\theta = 75.5°$. Therefore the component of the d_1 vector along the direction of the d_2 vector is $6.4\cos\theta \approx 1.6$.

43. From the figure, we note that $\vec{c} \perp \vec{b}$, which implies that the angle between $\vec{c}$ and the $+x$ axis is $\theta + 90°$. In unit-vector notation, the three vectors can be written as

$$\vec{a} = a_x\hat{\mathrm{i}}$$
$$\vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} = (b\cos\theta)\hat{\mathrm{i}} + (b\sin\theta)\hat{\mathrm{j}}$$
$$\vec{c} = c_x\hat{\mathrm{i}} + c_y\hat{\mathrm{j}} = [c\cos(\theta + 90°)]\hat{\mathrm{i}} + [c\sin(\theta + 90°)]\hat{\mathrm{j}}$$

The above expressions allow us to evaluate the components of the vectors.

(a) The x-component of $\vec{a}$ is $a_x = a \cos 0° = a = 3.00$ m.

(b) Similarly, the y-componnet of $\vec{a}$ is $a_y = a \sin 0° = 0$.

(c) The x-component of $\vec{b}$ is $b_x = b \cos 30° = (4.00\text{ m}) \cos 30° = 3.46$ m,

(d) and the y-component is $b_y = b \sin 30° = (4.00\text{ m}) \sin 30° = 2.00$ m.

(e) The x-component of $\vec{c}$ is $c_x = c \cos 120° = (10.0\text{ m}) \cos 120° = –5.00$ m,

(f) and the y-component is $c_y = c \sin 30° = (10.0\text{ m}) \sin 120° = 8.66$ m.

(g) The fact that $\vec{c} = p\vec{a} + q\vec{b}$ implies

$$\vec{c} = c_x\hat{\mathrm{i}} + c_y\hat{\mathrm{j}} = p(a_x\hat{\mathrm{i}}) + q(b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}}) = (pa_x + qb_x)\hat{\mathrm{i}} + qb_y\hat{\mathrm{j}}$$

or

$$c_x = pa_x + qb_x, \qquad c_y = qb_y$$

Substituting the values found above, we have

$$-5.00\text{ m} = p\,(3.00\text{ m}) + q\,(3.46\text{ m})$$
$$8.66\text{ m} = q\,(2.00\text{ m}).$$

Solving these equations, we find $p = –6.67$.

(h) Similarly, $q = 4.33$ (note that it’s easiest to solve for q first). The numbers p and q have no units.

44. Applying Eq. 3-23, $\vec{F} = q\vec{v} \times \vec{B}$ (where q is a scalar) becomes

$$F_x\,\hat{\mathrm{i}} + F_y\,\hat{\mathrm{j}} + F_z\,\hat{\mathrm{k}} = q\left(v_yB_z - v_zB_y\right)\hat{\mathrm{i}} + q\left(v_zB_x - v_xB_z\right)\hat{\mathrm{j}} + q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}}$$

which — plugging in values — leads to three equalities:

$$\begin{aligned} 4.0 &= 2\,(4.0B_z - 6.0B_y) \\ -20 &= 2\,(6.0B_x - 2.0B_z) \\ 12 &= 2\,(2.0B_y - 4.0B_x) \end{aligned}$$

Since we are told that $B_x = B_y$, the third equation leads to $B_y = -3.0$. Inserting this value into the first equation, we find $B_z = -4.0$. Thus, our answer is

$$\vec{B} = -3.0\,\hat{\mathrm{i}} - 3.0\,\hat{\mathrm{j}} - 4.0\,\hat{\mathrm{k}}.$$

45. The two vectors are given by

$$\begin{aligned} \vec{A} &= 8.00(\cos 130°\,\hat{\mathrm{i}} + \sin 130°\,\hat{\mathrm{j}}) = -5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}} \\ \vec{B} &= B_x\hat{\mathrm{i}} + B_y\hat{\mathrm{j}} = -7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}. \end{aligned}$$

(a) The dot product of $5\vec{A}\cdot\vec{B}$ is

$$\begin{aligned} 5\vec{A}\cdot\vec{B} &= 5(-5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}})\cdot(-7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}) = 5[(-5.14)(-7.72) + (6.13)(-9.20)] \\ &= -83.4. \end{aligned}$$

(b) In unit vector notation

$$4\vec{A}\times 3\vec{B} = 12\vec{A}\times\vec{B} = 12(-5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}})\times(-7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}) = 12(94.6\,\hat{\mathrm{k}}) = 1.14\times 10^3\,\hat{\mathrm{k}}$$

(c) We note that the azimuthal angle is undefined for a vector along the z axis. Thus, our result is "1.14×10^3, θ not defined, and $\phi = 0°$."

(d) Since $\vec{A}$ is in the xy plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90°.

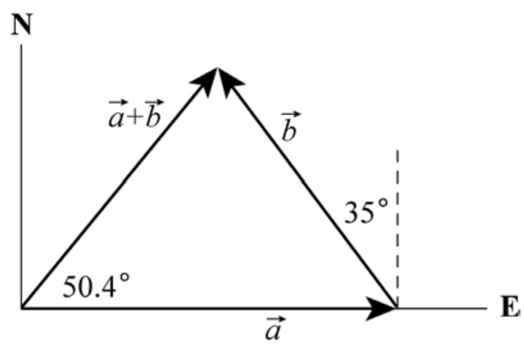
(e) Clearly, $\vec{A} + 3.00\,\hat{\mathrm{k}} = -5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}} + 3.00\,\hat{\mathrm{k}}$.

(f) The Pythagorean theorem yields magnitude $A = \sqrt{(5.14)^2 + (6.13)^2 + (3.00)^2} = 8.54$. The azimuthal angle is θ = 130°, just as it was in the problem statement ($\vec{A}$ <u>is</u> the projection onto the *xy* plane of the new vector created in part (e)). The angle measured from the +*z* axis is

$$\phi = \cos^{-1}(3.00/8.54) = 69.4°.$$

46. The vectors are shown on the diagram. The *x* axis runs from west to east and the *y* axis runs from south to north. Then a_x = 5.0 m, a_y = 0,

$$b_x = -(4.0\text{ m})\sin 35° = -2.29\text{ m},\quad b_y = (4.0\text{ m})\cos 35° = 3.28\text{ m}.$$



(a) Let $\vec{c} = \vec{a} + \vec{b}$. Then $c_x = a_x + b_x = 5.00\text{ m} - 2.29\text{ m} = 2.71\text{ m}$ and $c_y = a_y + b_y = 0 + 3.28\text{ m} = 3.28\text{ m}$. The magnitude of *c* is

$$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.71\text{m})^2 + (3.28\text{m})^2} = 4.2\text{ m}.$$

(b) The angle θ that $\vec{c} = \vec{a} + \vec{b}$ makes with the +*x* axis is

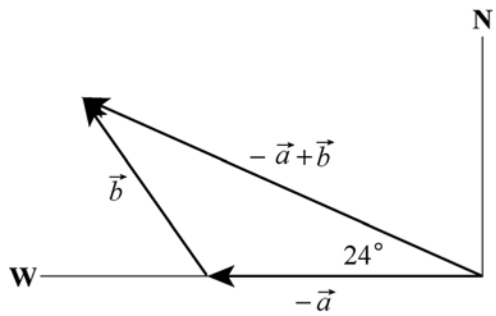
$$\theta = \tan^{-1}\left(\frac{c_y}{c_x}\right) = \tan^{-1}\left(\frac{3.28}{2.71}\right) = 50.5° \approx 50°.$$

The second possibility (θ = 50.4° + 180° = 230.4°) is rejected because it would point in a direction opposite to $\vec{c}$.

(c) The vector $\vec{b} - \vec{a}$ is found by adding $-\vec{a}$ to $\vec{b}$. The result is shown on the diagram to the right. Let $\vec{c} = \vec{b} - \vec{a}$. The components are

$$c_x = b_x - a_x = -2.29\text{ m} - 5.00\text{ m} = -7.29\text{ m}$$
$$c_y = b_y - a_y = 3.28\text{ m}.$$

The magnitude of $\vec{c}$ is $c = \sqrt{c_x^2 + c_y^2} = 8.0\,\text{m}$.



(d) The tangent of the angle θ that $\vec{c}$ makes with the $+x$ axis (east) is

$$\tan\theta = \frac{c_y}{c_x} = \frac{3.28\text{ m}}{-7.29\text{ m}} = -4.50.$$

There are two solutions: –24.2° and 155.8°. As the diagram shows, the second solution is correct. The vector $\vec{c} = -\vec{a} + \vec{b}$ is 24° north of west.

47. Noting that the given 130° is measured counterclockwise from the $+x$ axis, the two vectors can be written as

$$\vec{A} = 8.00(\cos 130°\,\hat{\text{i}} + \sin 130°\,\hat{\text{j}}) = -5.14\,\hat{\text{i}} + 6.13\,\hat{\text{j}}$$
$$\vec{B} = B_x\hat{\text{i}} + B_y\hat{\text{j}} = -7.72\,\hat{\text{i}} - 9.20\,\hat{\text{j}}.$$

(a) The angle between the negative direction of the y axis ($-\hat{\text{j}}$) and the direction of $\vec{A}$ is

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot(-\hat{\text{j}})}{A}\right) = \cos^{-1}\left(\frac{-6.13}{\sqrt{(-5.14)^2 + (6.13)^2}}\right) = \cos^{-1}\left(\frac{-6.13}{8.00}\right) = 140°.$$

Alternatively, one may say that the $-y$ direction corresponds to an angle of 270°, and the answer is simply given by 270°–130° = 140°.

(b) Since the y axis is in the xy plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90.0°.

(c) The vector can be simplified as

$$\vec{A}\times(\vec{B} + 3.00\,\hat{\text{k}}) = (-5.14\,\hat{\text{i}} + 6.13\,\hat{\text{j}})\times(-7.72\,\hat{\text{i}} - 9.20\,\hat{\text{j}} + 3.00\,\hat{\text{k}})$$
$$= 18.39\,\hat{\text{i}} + 15.42\,\hat{\text{j}} + 94.61\,\hat{\text{k}}$$

Its magnitude is $|\vec{A}\times(\vec{B}+3.00\hat{k})| = 97.6.$ The angle between the negative direction of the y axis ($-\hat{j}$) and the direction of the above vector is

$$\theta = \cos^{-1}\left(\frac{-15.42}{97.6}\right) = 99.1°.$$

48. Where the length unit is not displayed, the unit meter is understood.

(a) We first note that the magnitudes of the vectors are $a = |\vec{a}| = \sqrt{(3.2)^2 + (1.6)^2} = 3.58$ and $b = |\vec{b}| = \sqrt{(0.50)^2 + (4.5)^2} = 4.53$. Now,

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y = ab\cos\phi$$
$$(3.2)(0.50) + (1.6)(4.5) = (3.58)(4.53)\cos\phi$$

which leads to ϕ = 57° (the inverse cosine is double-valued as is the inverse tangent, but we know this is the right solution since both vectors are in the same quadrant).

(b) Since the angle (measured from +x) for $\vec{a}$ is $\tan^{-1}$(1.6/3.2) = 26.6°, we know the angle for $\vec{c}$ is 26.6° –90° = –63.4° (the other possibility, 26.6° + 90° would lead to a c_x < 0). Therefore,

$$c_x = c\cos(-63.4°) = (5.0)(0.45) = 2.2 \text{ m}.$$

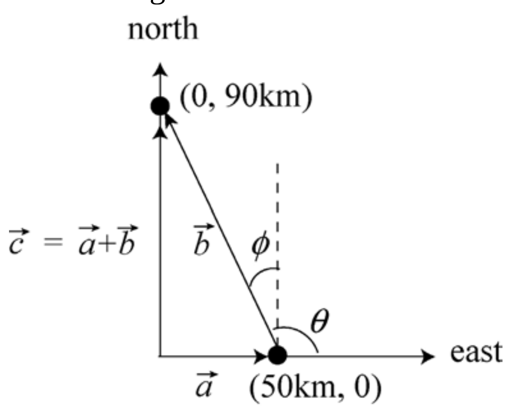
(c) Also, $c_y = c\sin(-63.4°) = (5.0)(-0.89) = -4.5$ m.

(d) And we know the angle for $\vec{d}$ to be 26.6° + 90° = 116.6°, which leads to

$$d_x = d\cos(116.6°) = (5.0)(-0.45) = -2.2 \text{ m}.$$

(e) Finally, $d_y = d\sin 116.6° = (5.0)(0.89) = 4.5$ m.

49. The situation is depicted in the figure below.



Let $\vec{a}$ represent the first part of his actual voyage (50.0 km east) and $\vec{c}$ represent the intended voyage (90.0 km north). We are looking for a vector $\vec{b}$ such that $\vec{c} = \vec{a} + \vec{b}$.

(a) Using the Pythagorean theorem, the distance traveled by the sailboat is
$b = \sqrt{(50.0\text{ km})^2 + (90.0\text{ km})^2} = 103\text{ km}.$

(b) The direction is

$$\phi = \tan^{-1}\left(\frac{50.0\text{ km}}{90.0\text{ km}}\right) = 29.1°$$

west of north (which is equivalent to 60.9° north of due west).

Note that this problem could also be solved by first expressing the vectors in unit-vector notation: $\vec{a} = (50.0\,\text{km})\hat{\text{i}}, \vec{c} = (90.0\,\text{km})\hat{\text{j}}$. This gives

$$\vec{b} = \vec{c} - \vec{a} = -(50.0\,\text{km})\hat{\text{i}} + (90.0\,\text{km})\hat{\text{j}}$$

The angle between $\vec{b}$ and the +*x*-axis is

$$\theta = \tan^{-1}\left(\frac{90.0\text{ km}}{-50.0\text{ km}}\right) = 119.1°$$

The angle θ is related to ϕ by $\theta = 90° + \phi$.

50. The two vectors $\vec{d}_1$ and $\vec{d}_2$ are given by $\vec{d}_1 = -d_1\,\hat{\text{j}}$ and $\vec{d}_2 = d_2\,\hat{\text{i}}$.

(a) The vector $\vec{d}_2/4 = (d_2/4)\,\hat{\text{i}}$ points in the +*x* direction. The ¼ factor does not affect the result.

(b) The vector $\vec{d}_1/(-4) = (d_1/4)\hat{\text{j}}$ points in the +*y* direction. The minus sign (with the "–4") does affect the direction: –(–*y*) = + *y*.

(c) $\vec{d}_1 \cdot \vec{d}_2 = 0$ since $\hat{\text{i}} \cdot \hat{\text{j}} = 0$. The two vectors are perpendicular to each other.

(d) $\vec{d}_1 \cdot (\vec{d}_2/4) = (\vec{d}_1 \cdot \vec{d}_2)/4 = 0$, as in part (c).

(e) $\vec{d}_1 \times \vec{d}_2 = -d_1 d_2 (\hat{\text{j}} \times \hat{\text{i}}) = d_1 d_2\,\hat{\text{k}}$, in the +*z*-direction.

(f) $\vec{d}_2 \times \vec{d}_1 = -d_2 d_1 (\hat{\text{i}} \times \hat{\text{j}}) = -d_1 d_2\,\hat{\text{k}}$, in the –*z*-direction.

(g) The magnitude of the vector in (e) is $d_1 d_2$.

(h) The magnitude of the vector in (f) is $d_1 d_2$.

(i) Since $d_1 \times (\vec{d}_2 / 4) = (d_1 d_2 / 4)\hat{k}$, the magnitude is $d_1 d_2 / 4$.

(j) The direction of $\vec{d}_1 \times (\vec{d}_2 / 4) = (d_1 d_2 / 4)\hat{k}$ is in the +z-direction.

51. Although we think of this as a three-dimensional movement, it is rendered effectively two-dimensional by referring measurements to its well-defined plane of the fault.

(a) The magnitude of the net displacement is

$$|\overrightarrow{AB}| = \sqrt{|AD|^2 + |AC|^2} = \sqrt{(17.0 \text{ m})^2 + (22.0 \text{ m})^2} = 27.8 \text{m}.$$

(b) The magnitude of the vertical component of $\overrightarrow{AB}$ is $|AD| \sin 52.0° = 13.4$ m.

52. The three vectors are

$$\begin{aligned}
\vec{d}_1 &= 4.0\hat{i} + 5.0\hat{j} - 6.0\hat{k} \\
\vec{d}_2 &= -1.0\hat{i} + 2.0\hat{j} + 3.0\hat{k} \\
\vec{d}_3 &= 4.0\hat{i} + 3.0\hat{j} + 2.0\hat{k}
\end{aligned}$$

(a) $\vec{r} = \vec{d}_1 - \vec{d}_2 + \vec{d}_3 = (9.0 \text{ m})\hat{i} + (6.0 \text{ m})\hat{j} + (-7.0 \text{ m})\hat{k}$.

(b) The magnitude of $\vec{r}$ is $|\vec{r}| = \sqrt{(9.0 \text{ m})^2 + (6.0 \text{ m})^2 + (-7.0 \text{ m})^2} = 12.9 \text{ m}$. The angle between $\vec{r}$ and the z-axis is given by

$$\cos\theta = \frac{\vec{r} \cdot \hat{k}}{|\vec{r}|} = \frac{-7.0 \text{ m}}{12.9 \text{ m}} = -0.543$$

which implies $\theta = 123°$.

(c) The component of $\vec{d}_1$ along the direction of $\vec{d}_2$ is given by $d_{\parallel} = \vec{d}_1 \cdot \hat{u} = d_1 \cos\varphi$ where φ is the angle between $\vec{d}_1$ and $\vec{d}_2$, and $\hat{u}$ is the unit vector in the direction of $\vec{d}_2$. Using the properties of the scalar (dot) product, we have

$$d_{\parallel} = d_1 \left(\frac{\vec{d}_1 \cdot \vec{d}_2}{d_1 d_2} \right) = \frac{\vec{d}_1 \cdot \vec{d}_2}{d_2} = \frac{(4.0)(-1.0) + (5.0)(2.0) + (-6.0)(3.0)}{\sqrt{(-1.0)^2 + (2.0)^2 + (3.0)^2}} = \frac{-12}{\sqrt{14}} = -3.2 \text{ m}.$$

(d) Now we are looking for $d_\perp$ such that $d_1^2 = (4.0)^2 + (5.0)^2 + (-6.0)^2 = 77 = d_\parallel^2 + d_\perp^2$. From (c), we have

$$d_\perp = \sqrt{77 \text{ m}^2 - (-3.2 \text{ m})^2} = 8.2 \text{ m}.$$

This gives the magnitude of the perpendicular component (and is consistent with what one would get using Eq. 3-27), but if more information (such as the direction, or a full specification in terms of unit vectors) is sought then more computation is needed.

53. We apply Eq. 3-20 and Eq. 3-27 to calculate the scalar and vector products between two vectors:

$$\vec{a} \cdot \vec{b} = ab \cos \phi$$
$$|\vec{a} \times \vec{b}| = ab \sin \phi$$

(a) Given that $a = |\vec{a}| = 10$, $b = |\vec{b}| = 6.0$ and $\phi = 60°$, the scalar (dot) product of $\vec{a}$ and $\vec{b}$ is

$$\vec{a} \cdot \vec{b} = ab \cos \phi = (10)\,(6.0) \cos 60° = 30.$$

(b) Similarly, the magnitude of the vector (cross) product of the two vectors is

$$|\vec{a} \times \vec{b}| = ab \sin \phi = (10)\,(6.0) \sin 60° = 52.$$

When two vectors are parallel ($\phi = 0$), $\vec{a} \cdot \vec{b} = ab \cos \phi = ab$, and $|\vec{a} \times \vec{b}| = ab \sin \phi = 0$. On the other hand, when the vectors are perpendicular ($\phi = 90°$), $\vec{a} \cdot \vec{b} = ab \cos \phi = 0$ and $|\vec{a} \times \vec{b}| = ab \sin \phi = ab$.

54. From the figure, it is clear that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $\vec{a} \cdot \vec{b} = 0$ since the angle between them is 90º.

(b) $\vec{a} \cdot \vec{c} = \vec{a} \cdot (-\vec{a} - \vec{b}) = -|\vec{a}|^2 = -16$.

(c) Similarly, $\vec{b} \cdot \vec{c} = -9.0$.

55. We choose +x east and +y north and measure all angles in the "standard" way (positive ones are counterclockwise from +x). Thus, vector $\vec{d}_1$ has magnitude d_1 = 4.00 m (with the unit meter) and direction θ_1 = 225°. Also, $\vec{d}_2$ has magnitude d_2 = 5.00 m and direction θ_2 = 0°, and vector $\vec{d}_3$ has magnitude d_3 = 6.00 m and direction θ_3 = 60°.

(a) The x-component of $\vec{d}_1$ is $d_{1x} = d_1 \cos \theta_1 = -2.83$ m.

(b) The y-component of $\vec{d}_1$ is $d_{1y} = d_1 \sin \theta_1 = -2.83$ m.

(c) The x-component of $\vec{d}_2$ is $d_{2x} = d_2 \cos \theta_2 = 5.00$ m.

(d) The y-component of $\vec{d}_2$ is $d_{2y} = d_2 \sin \theta_2 = 0$.

(e) The x-component of $\vec{d}_3$ is $d_{3x} = d_3 \cos \theta_3 = 3.00$ m.

(f) The y-component of $\vec{d}_3$ is $d_{3y} = d_3 \sin \theta_3 = 5.20$ m.

(g) The sum of x-components is

$$d_x = d_{1x} + d_{2x} + d_{3x} = -2.83 \text{ m} + 5.00 \text{ m} + 3.00 \text{ m} = 5.17 \text{ m}.$$

(h) The sum of y-components is

$$d_y = d_{1y} + d_{2y} + d_{3y} = -2.83 \text{ m} + 0 + 5.20 \text{ m} = 2.37 \text{ m}.$$

(i) The magnitude of the resultant displacement is

$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{(5.17 \text{ m})^2 + (2.37 \text{ m})^2} = 5.69 \text{ m}.$$

(j) And its angle is

$$\theta = \tan^{-1}(2.37/5.17) = 24.6°,$$

which (recalling our coordinate choices) means it points at about 25° north of east.

(k) and (l) This new displacement (the direct line home) when vectorially added to the previous (net) displacement must give zero. Thus, the new displacement is the negative, or opposite, of the previous (net) displacement. That is, it has the same magnitude (5.69 m) but points in the opposite direction (25° south of west).

56. If we wish to use Eq. 3-5 directly, we should note that the angles for $\vec{Q}, \vec{R}$, and $\vec{S}$ are 100°, 250°, and 310°, respectively, if they are measured counterclockwise from the $+x$ axis.

(a) Using unit-vector notation, with the unit meter understood, we have

$$\vec{P} = 10.0\cos(25.0°)\,\hat{\mathrm{i}} + 10.0\sin(25.0°)\,\hat{\mathrm{j}}$$
$$\vec{Q} = 12.0\cos(100°)\,\hat{\mathrm{i}} + 12.0\sin(100°)\,\hat{\mathrm{j}}$$
$$\vec{R} = 8.00\cos(250°)\,\hat{\mathrm{i}} + 8.00\sin(250°)\,\hat{\mathrm{j}}$$
$$\vec{S} = 9.00\cos(310°)\,\hat{\mathrm{i}} + 9.00\sin(310°)\,\hat{\mathrm{j}}$$
$$\vec{P} + \vec{Q} + \vec{R} + \vec{S} = (10.0\text{ m})\hat{\mathrm{i}} + (1.63\text{ m})\hat{\mathrm{j}}$$

(b) The magnitude of the vector sum is $\sqrt{(10.0\text{ m})^2 + (1.63\text{ m})^2} = 10.2\text{ m}$.

(c) The angle is $\tan^{-1}$ (1.63 m/10.0 m) ≈ 9.24° measured counterclockwise from the $+x$ axis.

57. From the problem statement, we have

$$\vec{A} + \vec{B} = (6.0)\hat{\mathrm{i}} + (1.0)\hat{\mathrm{j}}$$
$$\vec{A} - \vec{B} = -(4.0)\hat{\mathrm{i}} + (7.0)\hat{\mathrm{j}}$$

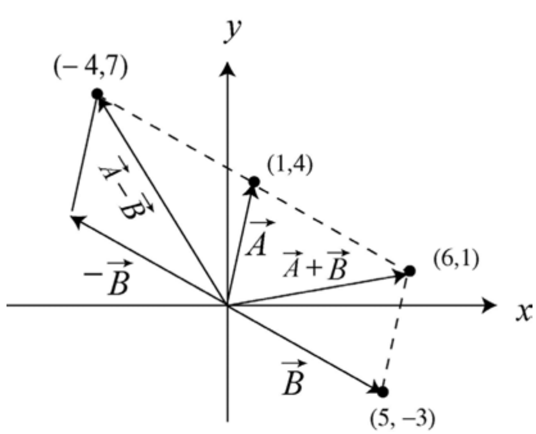
Adding the above equations and dividing by 2 leads to $\vec{A} = (1.0)\hat{\mathrm{i}} + (4.0)\hat{\mathrm{j}}$. Thus, the magnitude of $\vec{A}$ is

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(1.0)^2 + (4.0)^2} = 4.1$$

Similarly, the vector $\vec{B}$ is $\vec{B} = (5.0)\hat{\mathrm{i}} + (-3.0)\hat{\mathrm{j}}$, and its magnitude is

$$B = |\vec{B}| = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.0)^2 + (-3.0)^2} = 5.8.$$

The results are summarized in the figure below:



58. The vector can be written as $\vec{d} = (2.5\text{ m})\hat{\text{j}}$, where we have taken $\hat{\text{j}}$ to be the unit vector pointing north.

(a) The magnitude of the vector $\vec{a} = 4.0\,\vec{d}$ is (4.0)(2.5 m) = 10 m.

(b) The direction of the vector $\vec{a} = 4.0\vec{d}$ is the same as the direction of $\vec{d}$ (north).

(c) The magnitude of the vector $\vec{c} = -3.0\vec{d}$ is (3.0)(2.5 m) = 7.5 m.

(d) The direction of the vector $\vec{c} = -3.0\vec{d}$ is the opposite of the direction of $\vec{d}$. Thus, the direction of $\vec{c}$ is south.

59. Reference to Figure 3-18 (and the accompanying material in that section) is helpful. If we convert $\vec{B}$ to the magnitude-angle notation (as $\vec{A}$ already is) we have $\vec{B} = (14.4 \angle 33.7°)$ (appropriate notation especially if we are using a vector capable calculator in polar mode). Where the length unit is not displayed in the solution, the unit meter should be understood. In the magnitude-angle notation, rotating the axis by +20° amounts to subtracting that angle from the angles previously specified. Thus, $\vec{A} = (12.0 \angle 40.0°)'$ and $\vec{B} = (14.4 \angle 13.7°)'$, where the 'prime' notation indicates that the description is in terms of the new coordinates. Converting these results to (*x*, *y*) representations, we obtain

(a) $\vec{A} = (9.19\text{ m})\,\hat{\text{i}}' + (7.71\text{ m})\,\hat{\text{j}}'$.

(b) Similarly, $\vec{B} = (14.0\text{ m})\,\hat{\text{i}}' + (3.41\text{ m})\,\hat{\text{j}}'$.

60. The two vectors can be found be solving the simultaneous equations.

(a) If we add the equations, we obtain $2\vec{a} = 6\vec{c}$, which leads to $\vec{a} = 3\vec{c} = 9\,\hat{\text{i}} + 12\,\hat{\text{j}}$.

(b) Plugging this result back in, we find $\vec{b} = \vec{c} = 3\hat{\text{i}} + 4\hat{\text{j}}$.

61. The three vectors given are

$$\begin{aligned}\vec{a} &= 5.0\,\hat{\text{i}} + 4.0\,\hat{\text{j}} - 6.0\,\hat{\text{k}}\\ \vec{b} &= -2.0\,\hat{\text{i}} + 2.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}}\\ \vec{c} &= 4.0\,\hat{\text{i}} + 3.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}}\end{aligned}$$

(a) The vector equation $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ is

$$\begin{aligned}\vec{r} &= [5.0-(-2.0)+4.0]\hat{\mathrm{i}}+(4.0-2.0+3.0)\hat{\mathrm{j}}+(-6.0-3.0+2.0)\hat{\mathrm{k}}\\ &= 11\hat{\mathrm{i}}+5.0\hat{\mathrm{j}}-7.0\hat{\mathrm{k}}.\end{aligned}$$

(b) We find the angle from +z by "dotting" (taking the scalar product) $\vec{r}$ with $\hat{\mathrm{k}}$. Noting that $r = |\vec{r}\,| = \sqrt{(11.0)^2 + (5.0)^2 + (-7.0)^2} = 14,$ Eq. 3-20 with Eq. 3-23 leads to

$$\vec{r}\cdot\vec{\mathrm{k}} = -7.0 = (14)(1)\cos\phi \quad\Rightarrow\quad \phi = 120°.$$

(c) To find the component of a vector in a certain direction, it is efficient to "dot" it (take the scalar product of it) with a unit-vector in that direction. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}\,|} = \frac{-2.0\hat{\mathrm{i}}+2.0\hat{\mathrm{j}}+3.0\hat{\mathrm{k}}}{\sqrt{(-2.0)^2 + (2.0)^2 + (3.0)^2}}.$$

We therefore obtain

$$a_b = \vec{a}\cdot\hat{b} = \frac{(5.0)(-2.0) + (4.0)(2.0) + (-6.0)(3.0)}{\sqrt{(-2.0)^2 + (2.0)^2 + (3.0)^2}} = -4.9\,.$$

(d) One approach (if all we require is the magnitude) is to use the vector cross product, as the problem suggests; another (which supplies more information) is to subtract the result in part (c) (multiplied by $\hat{b}$) from $\vec{a}$. We briefly illustrate both methods. We note that if a cos θ (where θ is the angle between $\vec{a}$ and $\vec{b}$) gives a_b (the component along $\hat{b}$) then we expect a sin θ to yield the orthogonal component:

$$a\sin\theta = \frac{\left|\vec{a}\times\vec{b}\right|}{b} = 7.3$$

(alternatively, one might compute θ form part (c) and proceed more directly). The second method proceeds as follows:

$$\begin{aligned}\vec{a} - a_b\hat{b} &= (5.0 - 2.35)\hat{\mathrm{i}} + (4.0 - (-2.35))\hat{\mathrm{j}} + ((-6.0) - (-3.53))\hat{\mathrm{k}}\\ &= 2.65\hat{\mathrm{i}} + 6.35\hat{\mathrm{j}} - 2.47\hat{\mathrm{k}}\end{aligned}$$

This describes the perpendicular part of $\vec{a}$ completely. To find the magnitude of this part, we compute

$$\sqrt{(2.65)^2 + (6.35)^2 + (-2.47)^2} = 7.3$$

which agrees with the first method.

62. We choose $+x$ east and $+y$ north and measure all angles in the "standard" way (positive ones counterclockwise from $+x$, negative ones clockwise). Thus, vector $\vec{d}_1$ has magnitude d_1 = 3.66 (with the unit meter and three significant figures assumed) and direction θ_1 = 90°. Also, $\vec{d}_2$ has magnitude d_2 = 1.83 and direction θ_2 = –45°, and vector $\vec{d}_3$ has magnitude d_3 = 0.91 and direction θ_3 = –135°. We add the x and y components, respectively:

$$x:\ d_1 \cos\theta_1 + d_2 \cos\theta_2 + d_3 \cos\theta_3 = 0.65\ \text{m}$$
$$y:\ d_1 \sin\theta_1 + d_2 \sin\theta_2 + d_3 \sin\theta_3 = 1.7\ \text{m}.$$

(a) The magnitude of the direct displacement (the vector sum $\vec{d}_1 + \vec{d}_2 + \vec{d}_3$) is $\sqrt{(0.65\ \text{m})^2 + (1.7\ \text{m})^2} = 1.8$ m.

(b) The angle (understood in the sense described above) is $\tan^{-1}$ (1.7/0.65) = 69°. That is, the first putt must aim in the direction 69° north of east.

63. The three vectors are

$$\vec{d}_1 = -3.0\,\hat{\text{i}} + 3.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}}$$
$$\vec{d}_2 = -2.0\,\hat{\text{i}} - 4.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}}$$
$$\vec{d}_3 = 2.0\,\hat{\text{i}} + 3.0\,\hat{\text{j}} + 1.0\,\hat{\text{k}}.$$

(a) Since $\vec{d}_2 + \vec{d}_3 = 0\,\hat{\text{i}} - 1.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}}$, we have

$$\begin{aligned}\vec{d}_1 \cdot (\vec{d}_2 + \vec{d}_3) &= (-3.0\,\hat{\text{i}} + 3.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}}) \cdot (0\,\hat{\text{i}} - 1.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}}) \\ &= 0 - 3.0 + 6.0 = 3.0\ \text{m}^2.\end{aligned}$$

(b) Using Eq. 3-30, we obtain $\vec{d}_2 \times \vec{d}_3 = -10\,\hat{\text{i}} + 6.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}}.$ Thus,

$$\begin{aligned}\vec{d}_1 \cdot (\vec{d}_2 \times \vec{d}_3) &= (-3.0\,\hat{\text{i}} + 3.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}}) \cdot (-10\,\hat{\text{i}} + 6.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}}) \\ &= 30 + 18 + 4.0 = 52\ \text{m}^3.\end{aligned}$$

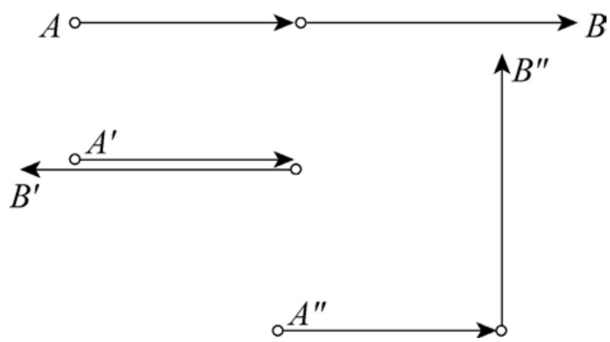
(c) We found $\vec{d}_2 + \vec{d}_3$ in part (a). Use of Eq. 3-30 then leads to

$$\begin{aligned}\vec{d}_1 \times (\vec{d}_2 + \vec{d}_3) &= (-3.0\,\hat{\text{i}} + 3.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}}) \times (0\,\hat{\text{i}} - 1.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}}) \\ &= (11\,\hat{\text{i}} + 9.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}}\,)\ \text{m}^2\end{aligned}$$

64. (a) The vectors should be parallel to achieve a resultant 7 m long (the unprimed case shown below),

(b) anti-parallel (in opposite directions) to achieve a resultant 1 m long (primed case shown),
(c) and perpendicular to achieve a resultant $\sqrt{3^2+4^2}=5\,\text{m}$ long (the double-primed case shown).

In each sketch, the vectors are shown in a "head-to-tail" sketch but the resultant is not shown. The resultant would be a straight line drawn from beginning to end; the beginning is indicated by *A* (with or without primes, as the case may be) and the end is indicated by *B*.



65. (a) This is one example of an answer: $(-40\,\hat{\text{i}} - 20\,\hat{\text{j}} + 25\,\hat{\text{k}})$ m, with $\hat{\text{i}}$ directed anti-parallel to the first path, $\hat{\text{j}}$ directed anti-parallel to the second path, and $\hat{\text{k}}$ directed upward (in order to have a right-handed coordinate system). Other examples include $(40\,\hat{\text{i}} + 20\,\hat{\text{j}} + 25\,\hat{\text{k}})$ m and $(40\hat{\text{i}} - 20\,\hat{\text{j}} - 25\,\hat{\text{k}})$ m (with slightly different interpretations for the unit vectors). Note that the product of the components is positive in each example.

(b) Using the Pythagorean theorem, we have $\sqrt{(40\text{ m})^2+(20\text{ m})^2}$ = 44.7 m ≈ 45 m.