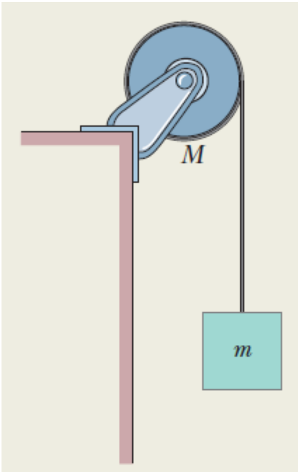
《大学物理 III》期末考试 A 卷答案

I. (30pts) Multiple Choices

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	D	B	C	D	B	A	D	A	A	C	D	D	C	B

II (10 pts). In the figure, a wheel of radius 0.20 m is mounted on a frictionless horizontal axis. The rotational inertia of the wheel about the axis is 0.40 $kg \cdot m^2$. A massless cord wrapped around the wheel's circumference is attached to a 6.0 kg box. The system is released from rest. When the box has a kinetic energy of 6.0 J, what are (a) the wheel's rotational kinetic energy and (b) the distance the box has fallen?



Solution: We employ energy methods in this solution; thus, considerations of positive versus negative sense (regarding the rotation of the wheel) are not relevant.

(a) The speed of the box is related to the angular speed of the wheel by $v = R\omega$, so that

$K_{box} = \frac{1}{2}m_{box}v^2$, and $v = \sqrt{\frac{2K_{box}}{m_{box}}} = 1.41\ m/s$,

which implies that the angular speed is ω = 1.41/0.20 = 7.1 rad/s.

Thus, the kinetic energy of rotation is $\frac{1}{2}I\omega^2 = 10.0\ J$

(b) Since it was released from rest at what we will consider to be the reference position for gravitational potential, then (with SI units understood) energy conservation requires

$$K_0 + U_0 = K + U$$

i.e.,

$$0 + 0 = (6.0 + 10.0) + m_{box}g(-h)$$

Therefore, h = 16.0/58.8 = 0.27 m.

III (10 pts). A 1.2 kg block sliding on a horizontal frictionless surface is attached to a horizontal spring with k = 480 N/m. Let x be the displacement of the block from the position at which the spring is unstretched. At t = 0 the block passes through x = 0 with a speed of 5.2 m/s in the positive x direction.What are the (a) frequency and (b) amplitude of the block's motion? (c) Write an expression for x as a function of time.

Solution: We note that for a horizontal spring, the relaxed position is the equilibrium position (in a regular simple harmonic motion setting); thus, we infer that the given v = 5.2 m/s at x = 0 is the maximum value v_m (which equals ωx_m where $\omega = k / m$ = 20 rad/s).

(a) Since $\omega = 2\pi f$, we find f = 3.2 Hz.

(b) We have v_m = 5.2 m/s = (20 rad/s) x_m, which leads to x_m = 0.26 m.

(c) With meters, seconds, and radians understood,

$$x = (0.26m)cos(20t + \phi)$$

$$v = -(5.2m/s)sin(20t + \phi)$$

The requirement that x = 0 at t = 0 implies (from the first equation above) that either $\phi = +\pi/2$ or $\phi = -\pi/2$. Only one of these choices meets the further requirement that $v > 0$ when t = 0; that choice is $\phi = -\pi/2$. Therefore,

$$x = (0.26m)cos(20t - \pi/2) = (0.26m)sin(20t)$$

IV (10 pts) A sinusoidal transverse wave traveling in the negative direction of an *x* axis has an amplitude of 1.00 cm, a frequency of 550 Hz, and a speed of 330 m/s. If the wave equation is of the form $y(x,t) = y_m sin(kx \pm \omega t)$, what are (a) y_m, (b) ω, (c) k, and (d) the correct choice of sign in front of ω?

Solution:

(a) The amplitude is y_m = 1.00 cm = 0.01 m, as given in the problem.

(b) Since the frequency is f = 550 Hz, the angular frequency is $\omega = 2\pi f = 3.46 \times 10^3$ rad/s.

(c) The angular wave number is $k = \omega / v$ = $(3.46 \times 10^3$ rad/s) /(330 m/s) =10.5 rad/m.

(d) Since the wave is traveling in the –*x* direction, the sign in front of ω is plus and the argument of the trig function is $kx + \omega t$.

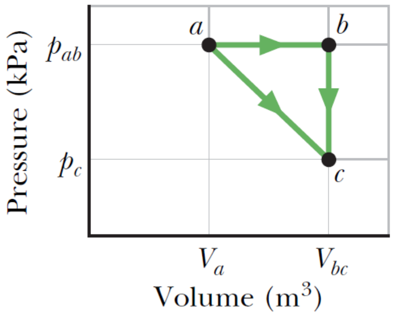
The results may be summarized as

$$y(x,t) = y_m sin(kx + \omega t) = y_m sin\left[2\pi f\left(\frac{x}{v} + t\right)\right]$$

$$= (0.01\ m) sin\left[2\pi(550\ Hz)\left(\frac{x}{330\ m/s} + t\right)\right]$$

$$= (0.01\ m) sin\left[\left(10.5\ \frac{rad}{m}\right)x + (3.46 \times 10^3\ rad/s)t\right]$$

V (10 pts). One mole of an ideal diatomic gas goes from a to c along the diagonal path in the figure. The scale of the vertical axis is set by $p_{ab} = 5.0\ kPa$ and $p_c = 2.0\ kPa$, and the scale of the horizontal axis is set by $V_{bc} = 4.0\ m^3$ and $V_a = 2.0\ m^3$. During the transition, (a) what is the change in internal energy of the gas, and (b) how much energy is added to the gas as heat? (c) How much heat is required if the gas goes from *a* to *c* along the indirect path *abc*?



Solution: Two formulas (other than the first law of thermodynamics) will be used. For any process that is depicted as a *straight line* on the *pV* diagram, the work is

$$W_{straight} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. On the other hand, we have

$$E_{int} = n\left(\frac{f}{2}\right)RT = \left(\frac{f}{2}\right)pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in joules, pressure should be in pascals (N/m^2) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is $f = 5$.

(a) The internal energy change is

$$E_{int,c} - E_{int,a} = \frac{5}{2}(p_c V_c - p_a V_a)$$

$$= \frac{5}{2}\left((2.0 \times 10^3\ Pa)(4.0\ m^3) - (5.0 \times 10^3\ Pa)(2.0\ m^3)\right) = -5.0 \times 10^3\ J$$

(b) The work done during the process represented by the diagonal path is

$$W_{diag} = \left(\frac{p_a + p_c}{2}\right)(V_c - V_a) = (3.5 \times 10^3\ Pa)(2.0\ m^3) = 7.0 \times 10^3\ J$$

Consequently, the first law of thermodynamics gives

$$Q_{diag} = \Delta E_{int} + W_{diag} = -5.0 \times 10^3\ J + 7.0 \times 10^3\ J = 2.0 \times 10^3\ J$$

(c) The fact that ΔE_{int} only depends on the initial and final states, and not on the details of the "path" between them, means we can write $\Delta E_{int} = E_{int,c} - E_{int,a} = -5.0 \times 10^3\ J$ for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{indirect} = \left(\frac{p_a + p_c}{2}\right)(V_c - V_a) = (5.0 \times 10^3\ Pa)(2.0\ m^3) + 0 = 10.0 \times 10^3\ J$$

Now, the first law of thermodynamics leads to

$$Q_{indirect} = \Delta E_{int} + W_{indirect} = -5.0 \times 10^3\ J + 10.0 \times 10^3\ J = 5.0 \times 10^3\ J$$

VI (10 pts). A heat pump is used to heat a building. The outside temperature is 25.0°C, and the temperature inside the building is to be maintained at 22°C.The pump's coefficient of performance is 3.8, and the heat pump delivers 7.54 MJ as heat to the building each hour. If the heat pump is a Carnot engine working in reverse, at what rate must work be done to run it?

Solution:

The coefficient of performance for a refrigerator is given by K = $|Q_L|/|W|$, where Q_L is the energy absorbed from the cold reservoir as heat and W is the work done during the refrigeration cycle, a negative value.

The first law of thermodynamics yields $Q_H + Q_L - W = 0$ for an integer number of cycles. Here Q_H is the energy ejected to the hot reservoir as heat. Thus, $Q_L = W - Q_H$. Q_H is negative and greater in magnitude than W, so $|Q_L| = |Q_H| - |W|$.

Thus,

$$K = \frac{|Q_H| - |W|}{|W|}$$

The solution for |W| is $|W| = |Q_H|/(K + 1)$. In one hour,

$$|W| = \frac{|Q_H|}{K+1} = \frac{7.54\ MJ}{3.8 + 1} = 1.57\ MJ$$

The rate at which work is done is $(1.57 \times 10^6$ J)/(3600 s) = 440 W.

VII (10 pts). White light is sent downward onto a horizontal thin film that is sandwiched between two materials. The indexes of refraction are 1.80 for the top material, 1.70 for the thin film, and 1.50 for the bottom material. The film thickness is $500\ nm$. Of the visible wavelengths (400 to 700 nm) that result in fully constructive interference at an observer above the film, which is the (a) longer and (b) shorter wavelength? The materials and film are then heated so that the film thickness increases. (c) Does the light resulting in fully constructive interference shift toward longer or shorter wavelengths?

Solution: (a) We are dealing with a thin film (material 2) in a situation where $n_1 > n_2 > n_3$ looking for strong *reflections*. Therefore, with lengths in nm and L = 500 nm and $n_2 = 1.70$, we have

$$\lambda = \frac{2n_2L}{m} = \begin{cases} 1700 & for\ m = 1 \\ 850 & for\ m = 2 \\ 567 & for\ m = 3 \\ 425 & for\ m = 4 \end{cases}$$

from which we see the latter *two* values are in the given range.

The longer wavelength (m=3) is $\lambda = 567$ nm.

(b) The shorter wavelength (m = 4) is $\lambda = 425$ nm.

(c) We assume the temperature dependence of the refraction index is negligible. From the proportionality evident in the part (a) equation, longer L means longer λ.

VIII (10 pts). A single slit is illuminated by light of wavelengths λ_a and λ_b, chosen so that the first diffraction minimum of the λ_a component coincides with the second minimum of the λ_b component. (a) If $\lambda_b = 350\,nm$, what is λ_a? For what order number m_b (if any) does a minimum of the λ_b component coincide with the minimum of the λ_a component in the order number (b) $m_a = 2$ and (c) $m_a = 3$?. Let m_a be the integer associated with a minimum in the pattern produced by light with wavelength λ_a, and m_b is defined in a similar way.

Solution:

(a) The condition for a minimum in a single-slit diffraction pattern is given by

$$a \sin \theta = m\lambda,$$

where a is the slit width, λ is the wavelength, and m is an integer.

For $\lambda = \lambda_a$ and $m = 1$, the angle θ is the same as for $\lambda = \lambda_b$ and $m = 2$. Thus,

$$\lambda_a = 2\lambda_b = 2(350\,nm) = 700\,nm$$

(b) A minimum in one pattern coincides with a minimum in the other if they occur at the same angle. This means

$$m_a\lambda_a = m_b\lambda_b$$

Since $\lambda_a = 2\lambda_b$, the minima coincide if $2m_a = m_b$.

Consequently, every other minimum of the λ_b pattern coincides with a minimum of the m_a pattern. With $m_a = 2$, we have $m_b = 4$.

(c) With $m_a = 3$, we have $m_b = 6$.