Characteristics of Gratings and Measurement of Wavelengths of Light Waves

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1 Abstract

Spectrum measurement and spectral analysis play pivotal roles in exploring the composition of substances and probing the structures of atoms and molecules. At the heart of these methodologies lies the indispensable dispersive element known as a grating, which effectively separates the emitted light from a source into distinct spectral lines, organizing them systematically according to wavelength. Diffractive gratings, a key component in this process, comprise numerous narrow slits that are uniformly wide, equally spaced, and aligned in parallel. Broadly categorized, diffractive gratings fall into two main types: transmission gratings, functioning with transmitted light, and reflection gratings, operating with reflected light.

2 Purpose of the experiment

A.Gain a deeper understanding of the construction, use, and adjustment methods of the spectrophotometer.
B.Acquire knowledge about the characteristics of gratings and utilize grating diffraction methods to measure the wavelength, angular dispersion, and resolving power of light waves.

3 Experimental apparatus

Spectrophotometer, Low-pressure mercury lamp, Grating plate, Plane mirror and some other experimental

Figure 1: JJY1 spectrophotometer

- **Spectrophotometer:** An optical instrument used to measure the intensity of light at different wavelengths in a spectrum. It typically consists of a light source, a sample holder, a monochromator, and a detector.
- **Low-pressure Mercury Lamp:** A type of gas-discharge lamp that emits ultraviolet light, primarily at 254 nm, due to the presence of mercury vapor. It is commonly used as a light source in spectroscopy experiments.
- **Grating Plate:** A thin plate containing a diffractive grating, which disperses incident light into its component wavelengths. Grating plates are essential in spectrometry for analyzing and resolving spectral lines.
- **Plane Mirror:** A flat mirror with a reflective surface, used to direct or reflect light beams. In spectroscopy, a plane mirror may be employed to redirect light within an optical setup or to facilitate specific measurements.

- **Other Experimental Equipment:** Additional tools and apparatus used in the experimental setup, depending on the specific requirements of the study. These could include lenses, filters, detectors, and other optical elements tailored to the experiment's objectives.

Figure 2: Low-pressure mercury lamp, Grating plate, Plane mirror and some other experimental

4 Experimental principles

4.1 Grating Constant and Grating Equation

A diffractive grating is an optical element composed of numerous narrow slits that are equally wide, equally spaced, and arranged in parallel. In gratings designed for the visible light range, the number of slits per millimeter can range from several hundred to over a thousand. Let a be the slit width, and b be the width of the non-transparent part between adjacent slits. The distance between slits, denoted as $d = a + b$, is referred to as the grating constant.

According to the theory of Fraunhofer diffraction, when parallel beams of wavelength λ are vertically projected onto the plane of a grating, diffraction occurs at each slit, and the diffracted light from each slit interferes at the overlapping points, with interference results determined by the optical path difference. Because the spacing between each slit of the grating is equal, the optical path difference for the diffracted light beams along the θ direction for adjacent slits is $d \sin\theta$. Here, θ is the angle between the diffracted light beam and the normal to the grating, known as the diffraction angle.

Placing a converging lens behind the grating with the lens axis parallel to the normal of the grating, the lens converges the diffracted light with an angle θ on the plane to the focal point P . According to the principle of multi-beam interference, interference main maxima will occur when θ satisfies the following equation, and the point P will be a bright spot

$$d \sin\theta = K\lambda \quad (k = 0, \pm 1, \pm 2, \dots) \tag{1}$$

The actual number of slits in a grating is usually large, and as the slit width decreases, when a light source with a narrow and elongated slit generates parallel light, the diffraction pattern of the grating will consist of finely sharp bright lines arranged in parallel. These bright lines are, in fact, diffraction interference fringes produced by the narrow slit of the light source.

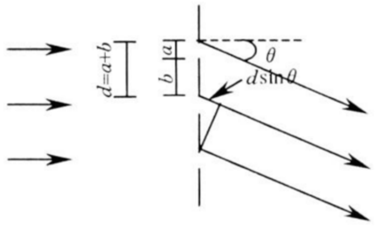

Figure 3: Diffractive grating

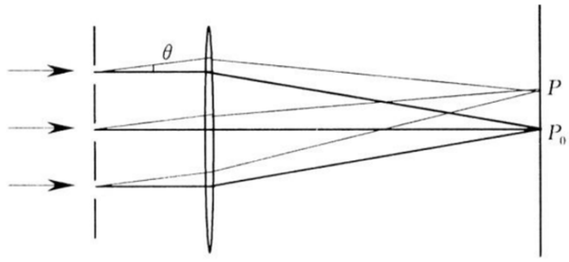

Figure 4: Diffraction grating diagram

4.2 Grating Spectrum

When the incident light is polychromatic, according to the grating equation, for a given constant d of the grating, only when $k = 0, i.e., \theta = 0$, will the central maximum of various wavelengths contained in the polychromatic light overlap. This overlap forms a bright central zero-order line on the focal plane of the lens. For other values of k, the central maxima of various wavelengths do not overlap, and fine, sharp lines of different wavelengths appear at different positions of the diffraction angle. The spectrum formed in this way is called a grating spectrum.

Lines of various wavelengths with the same series k are symmetrically arranged on both sides of the zero-order line in the order from short to long wavelengths, forming a spectrum. When $k = 1$, it is the first-order spectrum; when $k = 2$, it is the second-order spectrum, and so on. The fine, sharp lines of various wavelengths are called spectral lines. Figure is an illustrative diagram of the diffraction spectrum of a low-pressure mercury lamp. If the grating constant d and the series k are known, accurately measuring the diffraction angle of the spectral lines can determine the wavelength of the light waves. Conversely, the grating constant can be determined from known wavelengths.

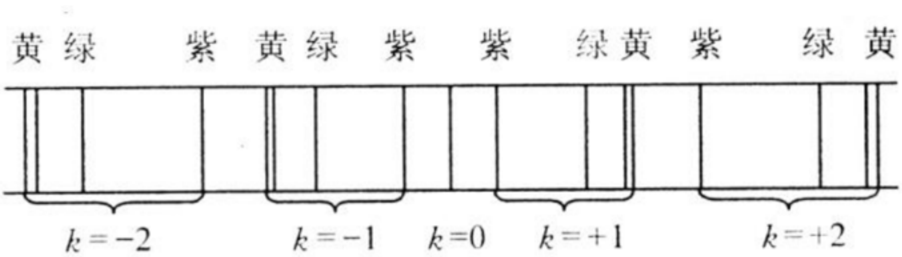

Figure 5: Schematic diagram of the diffraction spectrum

4.3 Grating Spectrum

Dispersion and resolving power are two important characteristics of a grating. A diffractive grating can separate polychromatic light into different wavelength lines on the focal plane of a lens, indicating its dispersive effect. Due to diffraction, the spectral lines broaden into wide bright fringes, limiting the resolving power of the grating. According to theoretical derivation, the dispersive ability of a grating can be represented by the angular dispersion

$$D = \frac{k}{d\cos\theta} \tag{2}$$

Resolving power R characterizes the ability of the grating to resolve details in the spectrum. If the grating can just separate two spectral lines λ and $\lambda + d\lambda$, then

$$R = \frac{\lambda}{d\lambda} = kN \tag{3}$$

5 Contents and Steps

5.1 Adjusting Spectrophotometer

- **Spectrophotometer Adjustment Requirements:**
  - The collimator produces parallel light.
  - The telescope focuses at infinity.
  - The optical axis of the collimator and the telescope are perpendicular to the instrument's rotation axis.
  - The optical axes of the collimator and the telescope are aligned on the same horizontal line.
- **Alignment Procedure:**
  1. Align the collimator directly with the light source.
  2. Adjust the collimator to produce parallel light.
  3. Adjust the slit width to 1mm 2mm.
  4. Rotate the telescope to align the crosshairs of the reticle with the center of the slit.
  5. Fix the telescope and place the grating on the stage.
  6. Visually align the grating plane so that it is perpendicular and bisects the line connecting ”1” and ”2” as much as possible, with ”3” lying within the grating plane.
  7. Rotate the vernier disk to roughly align the grating perpendicular to the optical axis of the telescope.
- **Auto-Collimation Adjustment:**
  1. Use the auto-collimation method.
  2. Precisely adjust leveling screws ”1” and ”2” under the stage (do not adjust the leveling screw of the telescope).
  3. Adjust until the green crosshair emitted by the telescope, reflected back from the grating plane, is in the corresponding position.
  4. At this point, the grating plane is perpendicular to the optical axis of the collimator and parallel to the instrument's rotation axis.
  5. Secure the vernier disk in place.
- **Observation and Adjustment of Spectral Lines:**
  1. Loosen the telescope's fixing screw.
  2. Rotate the telescope and observe the positive and negative first-order spectral lines.
  3. Check if the intersection of the crosshairs is at the center of each spectral line.
  4. If not centered, adjust screw ”3” under the stage to ensure the center of the positive and negative first-order spectral lines passes through the intersection of the crosshairs.
  5. Both sides of the spectral lines should be at the same height.

5.2 Measuring Wavelengths

- **Securing Adjusted Spectrophotometer:**
  - After adjusting the spectrophotometer as required, secure the stage and vernier disk.
  - Allow only the telescope to rotate around the main axis.
- **Observation of Diffraction Spectrum:**
  - Turn the telescope to the left and right to comprehensively observe the diffraction spectrum of the grating.
- **Measurement of Spectral Lines:**

- Measure the positions of various spectral lines.
- During measurements, start from the central bright fringe and measure towards both the left and right sides.
- Alternatively, measure from the left (or right) to the right (or left), rotating the telescope in a single direction.
- Record the data.

- **Post-Measurement Steps:**
  - After completing the measurements, remove the plane grating from the stage.
  - Ensure not to disturb the previously adjusted spectrophotometer settings.

6 Data processing

6.1 Measuring 1^{st} green light wavelength

Table 1: Measurement of Green Light with Given Grating Constant

Measurement	1st Right Spectral Line		1st Left Spectral Line	
	Left Window	Right Window	Left Window	Right Window
1	40°45'	220°40'	47°00'	226°56'
2	40°47'	220°47'	47°02'	227°04'
3	40°45'	220°40'	47°01'	227°00'

According to the formula:

$$\sin\theta = \sin(\frac{1}{4}\left|\theta_1 - \theta_2\right| + \frac{1}{4}\left|\theta_1' - \theta_2'\right|) \tag{4}$$

we can derive the wavelengths by

$$\lambda = \frac{d\sin\theta}{k} \qquad (k=1) \tag{5}$$

Since we are calculating the first spectral line, here we take $k = 1$. Then the wavelengths are as follows.

Table 2: Wavelength of green light

NO.	1st Right Spectral Line		1st Left Spectral Line		Wavelength(nm)
	Left Window	Right Window	Left Window	Right Window	
1	40°45'	220°40'	47°00'	226°56'	545.14
2	40°47'	220°47'	47°02'	227°04'	546.59
3	40°45'	220°40'	47°01'	227°00'	544.97

Then take the mean value and we have

$$\bar{\lambda} = 545.57nm \tag{6}$$

The relative error is

$$\sigma = \frac{|545.57 - 546.07|}{546.07} = 0.092\% \tag{7}$$

Table 3: Given the grating constant, measure the yellow light

Measurement	1st Right Spectral Line		1st Left Spectral Line	
	Left Window	Right Window	Left Window	Right Window
1	40°31’	220°31’	47°10’	227°11’
2	40°33’	220°32’	47°11’	227°11’
3	40°34’	220°35’	47°10’	227°10’

6.2 Measuring 1^{st} yellow light wavelength

According to the formula:

$$\sin\theta = \sin(\frac{1}{4}\left|\theta_1 - \theta_2\right| + \frac{1}{4}\left|\theta_1^{'} - \theta_2^{'}\right|) \tag{8}$$

we can derive the wavelengths by

$$\lambda = \frac{d\sin\theta}{k} \qquad (k=1) \tag{9}$$

Since we are calculating the first spectral line, here we take $k = 1$. Then the wavelengths are as follows.

Table 4: Wavelength of yellow light

NO.	1st Right Spectral Line		1st Left Spectral Line		Wavelength(nm)
	Left Window	Right Window	Left Window	Right Window	
1	40°31’	220°31’	47°10’	227°11’	579.13
2	40°33’	220°32’	47°11’	227°11’	579.27
3	40°34’	220°35’	47°10’	227°10’	574.91

Then take the mean value and we have

$$\bar{\lambda} = 577.77nm \tag{10}$$

The relative error is

$$\sigma = \frac{|577.77 - 576.96|}{576.96} = 0.14\% \tag{11}$$

6.3 Measuring 2^{nd} yellow light wavelength

Table 5: Given the grating constant, measure the yellow light

Measurement	2nd Right Spectral Line		2nd Left Spectral Line	
	Left Window	Right Window	Left Window	Right Window
1	37°16’	217°17’	50°30’	230°31’
2	37°14’	217°15’	50°30’	230°30’
3	37°15’	217°16’	50°31’	230°32’

According to the formula:

$$\sin\theta = \sin(\frac{1}{4}\left|\theta_1 - \theta_2\right| + \frac{1}{4}\left|\theta_1^{'} - \theta_2^{'}\right|) \tag{12}$$

we can derive the wavelengths by

$$\lambda = \frac{d\sin\theta}{k} \qquad (k=2) \tag{13}$$

Table 6: Wavelength of yellow light

NO.	2nd Right Spectral Line		2nd Left Spectral Line		Wavelength(nm)
	Left Window	Right Window	Left Window	Right Window	
1	37°16’	217°17’	50°30’	230°31’	577.03
2	37°14’	217°15’	50°30’	230°30’	577.21
3	37°15’	217°16’	50°31’	230°32’	577.03

Since we are calculating the first spectral line, here we take $k = 1$. Then the wavelengths are as follows.

Then take the mean value and we have

$$\bar{\lambda} = 577.09nm \tag{14}$$

The relative error is

$$\sigma = \frac{|577.09 - 576.96|}{576.96} = 0.023\% \tag{15}$$

6.4 Measuring the grating constant

Table 7: Given the green light wavelength(546.07nm), measure the grating constant

Measurement	1st Right Spectral Line		1st Left Spectral Line	
	Left Window	Right Window	Left Window	Right Window
1	262°35’	82°32’	243°45’	63°47’
2	262°35’	82°31’	243°45’	63°39’
3	262°37’	82°31’	243°45’	63°39’

According to the formula

$$d = \frac{k\lambda}{\sin\theta} \tag{16}$$

We can calculate the grating constant as follows

Table 8: Wavelength of yellow light

NO.	1st Right Spectral Line		1st Left Spectral Line		Grating frequency
	Left Window	Right Window	Left Window	Right Window	
1	262°35’	82°32’	243°45’	63°47’	300
2	262°35’	82°31’	243°45’	63°39’	299
3	262°37’	82°31’	243°45’	63°39’	300

And the mean grating frequency is

$$\bar{n} = 300 \tag{17}$$

7 Conclusion and analysis

7.1 Conclusion

Through a series of operations and calculations, the wavelength of the first order green light is $\lambda_{green} = 545.57nm$. The wavelength of the first order yellow light is $\lambda_{yellow1} = 577.77nm$. The wavelength of the second order yellow light is $\lambda_{yellow2} = 577.09nm$. The grating constant is $n = 300$.

7.2 Error analysis

- **Experimenter's Leveling Adjustment:**
  - The leveling adjustment of the spectrophotometer may not be precise, leading to deviations.
- **Measurement Inaccuracies:**
  - Instruments used may have certain measurement inaccuracies.
  - Manufacturing deviations in the grating can contribute to inaccuracies.
  - The accuracy of the spectrophotometer may be limited.
- **Strategies for Improvement:**
  - Utilize a computer for leveling adjustments or data readings to enhance precision.
  - Increase the number of experiments to reduce both systematic and random errors.

A Experimental data recording graph


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已知光栅常数，测定绿光

次数	右边一级谱线		左边一级谱线		波长
	左窗	右窗	左窗	右窗	
1	40°45′	220°40′	47°0′	226°56′	545.14
2	40°47′	220°47′	47°2′	227°4′	
3	40°45′	220°40′	47°1′	227°0′	

已知光栅常数，测定 1 级黄光

次数	右边一级谱线		左边一级谱线		波长
	左窗	右窗	左窗	右窗	
1	40°31′	220°31′	47°10′	227°11′	579.13
2	40°33′	220°32′	47°11′	227°11′	
3	40°34′	220°35′	47°10′	227°10′	

已知光栅常数，测定 2 级黄光

次数	右边 2 级谱线		左边 2 级谱线		波长
	左窗	右窗	左窗	右窗	
1	37°16′	217°17′	50°30′	230°31′	577.03
2	37°14′	217°15′	50°30′	230°30′	
3	37°15′	217°16′	50°31′	230°32′	

测定未知光栅常数（已知绿光波长λ=546.07nm）

次数	右边一级谱线		左边一级谱线		光栅常数
	左窗	右窗	左窗	右窗	
1	34°26′	214°25′	53°19′	233°18′	300
2	34°27′	214°26′	53°20′	233°19′	
3	34°25′	214°25′	53°19′	233°19′	

Figure 6: Experimental data recording graph