Seat NO. ______ Major ______ Student ID □□□□□□□□□□□□ Chinese Name ______

（DONOT WRITE YOUR ANSWER IN THIS AREA）

Seal Line

WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

SCUT Final Exam

2022-2023-2《Calculus II》 Exam Paper A

Notice:

1. Make sure that you have filled the form on the left side of seal line.
2. Write your answers on the exam paper.
3. This is a close-book exam.
4. The exam with full score of 100 points lasts 120 minutes.

Question No.	1	2	3	4	5	6	7	8	Sum
Score									

1. Answer the following questions (30 points):

Score

(1) Classify the following series as absolutely convergent, conditionally convergent or divergent: $\sum_{n=1}^{\infty}(-1)^n \frac{\sin(n)}{n\sqrt{n}}$.

(2) Find the distance between the parallel planes $2x-3y+\sqrt{3}z=4$ and $2x-3y+\sqrt{3}z=9$.

(3) Let G be the spherical surface $x^2+y^2+z^2=a^2$. Evaluate the following surface integral

$$\iint_G \frac{x+y^3+\sin z}{1+z^4}\mathrm{d}S.$$

(4) Change the order of integration of $\int_{\frac{1}{2}}^{1}[\int_{x^3}^{x} f(x,y)\mathrm{d}y]\mathrm{d}x.$

(5) Find $\dfrac{\partial z}{\partial x}$, if equation $3x^2z+y^3-xyz^3=0$ defines an implicit function $z=f(x,y)$.

2. Evaluate the following problems (30 points):

Score

(1) Evaluate $\int_{1}^{2}\int_{0}^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\mathrm{d}y\mathrm{d}x.$

(2) Find the convergence set for the power series $\sum_{n=1}^{\infty}\frac{(3x+1)^n}{n2^n}$.

(3) Find the symmetric equation of the tangent line to the curve with equation $\vec{r}=2\cos t\vec{i}+6\sin t\vec{j}+t\vec{k}$, at $t=\frac{\pi}{3}$.

(4) Find the maximum and minimum values of $f(x,y,z)=-x+2y+2z$ on the ellipse $\begin{cases} x^2+y^2=2 \\ y+2z=1 \end{cases}$.

(5) Evaluate the line integrals $\int_{(0,0,0)}^{(1,1,1)} \left(6xy^3+2z^2\right)\mathrm{d}x+9x^2y^2\mathrm{d}y+\left(4xz+1\right)\mathrm{d}z$.

3.(6 points) Show the limit $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^3+y^3+z^3}$ does not exist.

Score

4.(8 points) Let S be the solid cylinder bounded by $x^2+y^2=4, \quad z=0, \quad z=3,$ and let $\vec{n}$ be the outer unit normal to the boundary ∂S. If $\vec{F}=\left(x^3+\tan yz\right)\vec{i}+\left(y^3-e^{xz}\right)\vec{j}+\left(3z+x^3\right)\vec{k},$ find the surface integral

$$\iint_{\partial S}\vec{F}\cdot\vec{n}\,\mathrm{d}S=\iint_{\partial S}\left(x^3+\tan yz\right)\mathrm{d}y\mathrm{d}z+\left(y^3-e^{xz}\right)\mathrm{d}z\mathrm{d}x+\left(3z+x^3\right)\mathrm{d}x\mathrm{d}y.$$

Score

5.(6 points) Calculate $\int_0^3\int_0^{\sqrt{9-x^2}}\int_0^2(x^2+y^2)^{1/2}\,\mathrm{d}z\mathrm{d}y\mathrm{d}x.$

Score

6. (6 points) Find the sum of constant series $\sum_{n=0}^{\infty}\frac{(n+1)^2}{n!}$.

Score

7. (8 points) Solve differential equation $y''-3y'+2y=e^x+1$.

Score

8. (6 points) Let C be the positive closed curve formed by $(x-a)^2+(y-a)^2=1$, and $f(x)$ is a positive continuous function. Prove that the line integral $\oint_C \frac{x}{f(y)}\mathrm{d}y - yf(x)\mathrm{d}x \geq 2\pi$.

Score