Compute and plot the convolution y[n]=x[n]*h[n], where

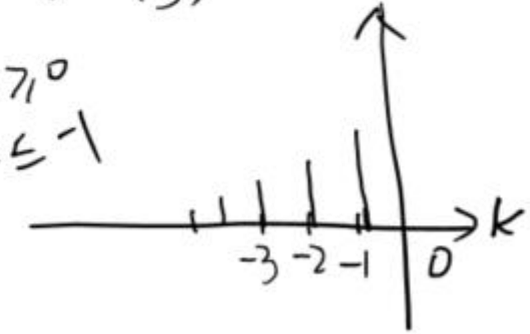
$$x[n]=\left(\frac{1}{3}\right)^{-n}u[-n-1] \text{ and } h[n]=u[n-1]$$

解法一：

① $x[k]=\left(\frac{1}{3}\right)^{-k}u[-k-1]$

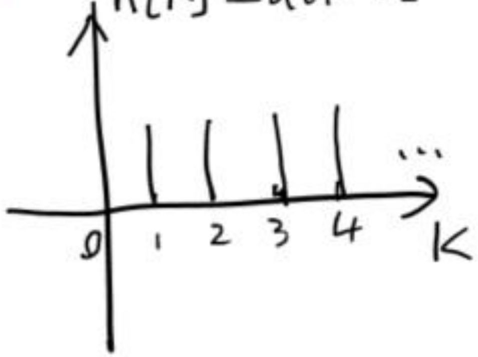
$-k-1\geq 0$

$\Rightarrow k\leq -1$



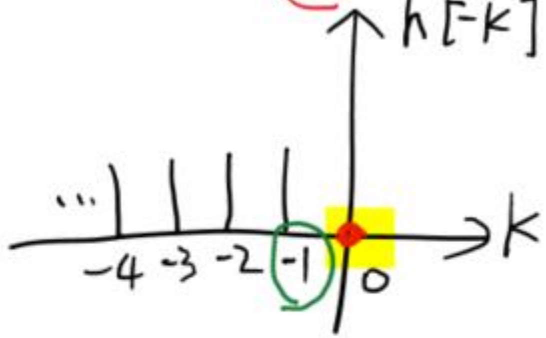
② $k\geq 1$

$h[k]=u[k-1]$

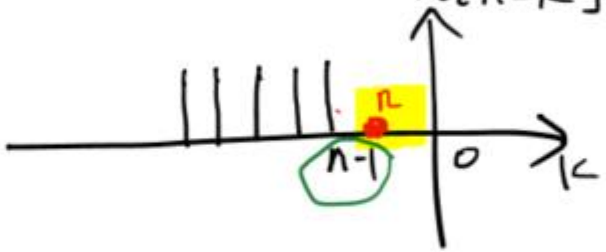


$\Rightarrow$

③ $h[-k]$



$\Rightarrow$ ④ $h[n-k]$



$$\Rightarrow\begin{cases} \text{当 } n\leq 0 \text{ 时 } (n-1\leq -1) & \displaystyle\sum_{k=-\infty}^{n-1} x[k]h[n-k]=\frac{1}{2}\cdot 3^{n} \\ \text{当 } n>0 \text{ 时 } (n-1>-1) & \displaystyle\sum_{k=-\infty}^{-1} x[k]h[n-1]=\frac{1}{2} \end{cases}$$

解法二： $y[n]=\sum_{k=-\infty}^{+\infty}x[k]h[n-k]=\sum_{k=-\infty}^{+\infty}\left(\frac{1}{3}\right)^{-k}u[-k-1]u[n-k-1]$

其中，$u[-k-1]$的非零区间为 $k\le -1$，

$u[n-k-1]$的非零区间为 $k\le n-1$.

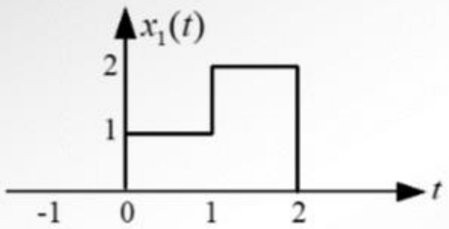
所以，当 $n\le 0$ 时，共享非零区间为 $k\le n-1$，

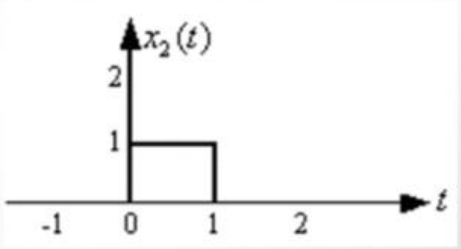
$$y[n]=\sum_{k=-\infty}^{n-1}\left(\frac{1}{3}\right)^{-k}=\sum_{k=1-n}^{+\infty}\left(\frac{1}{3}\right)^{k}=\frac{\left(\frac{1}{3}\right)^{1-n}}{1-\frac{1}{3}}=\frac{3^n}{2}$$

当 $n>0$ 时，共享非零区间为 $k\le -1$，

$$y[n]=\sum_{k=-\infty}^{-1}\left(\frac{1}{3}\right)^{-k}=\sum_{k=1}^{+\infty}\left(\frac{1}{3}\right)^{k}=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{2}$$

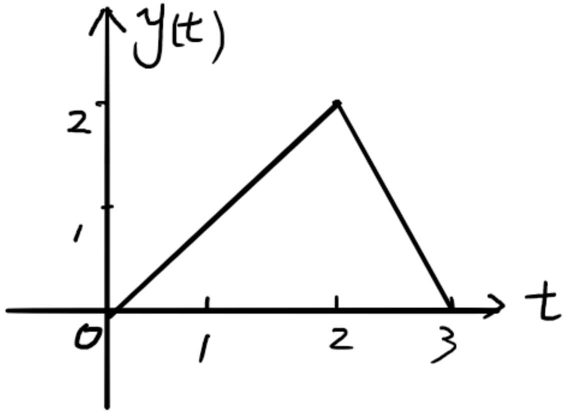
1. Two signals $x_1(t)$ and $x_2(t)$ are shown below, please determine $y(t)=x_1(t)*x_2(t)$

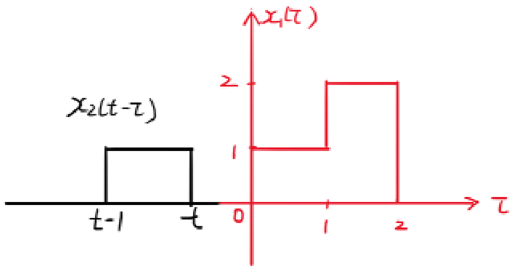




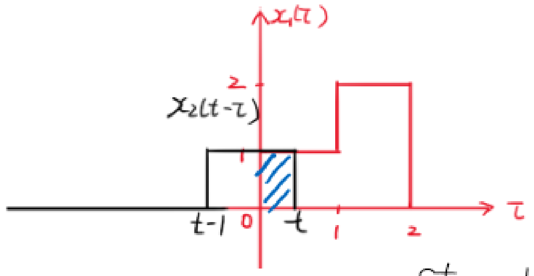
Solution:

$$y(t)=\begin{cases}0, & t<0\\ t, & 0\le t<2\\ 6-2t, & 2\le t<3\\ 0, & t\ge 3\end{cases}$$

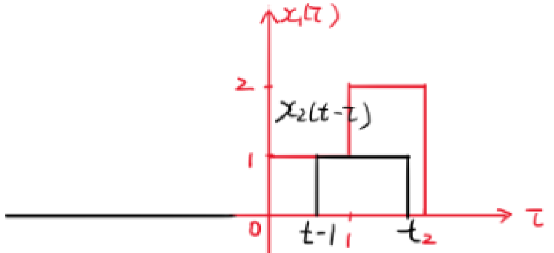




① $t<0$, $y(t)=0$



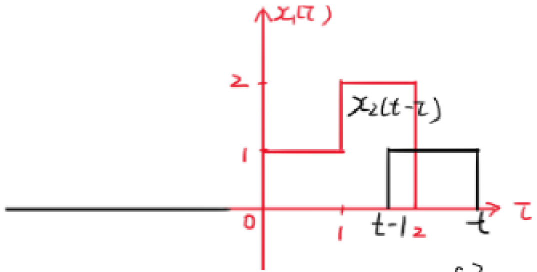
② $0\le t<1$, $y(t)=\int_0^t 1\cdot 1\,d\tau=t$



③ $1\le t<2$, $y(t)=\int_{t-1}^{1}1\cdot 1\,d\tau+\int_{1}^{t}1\cdot 2\,d\tau$

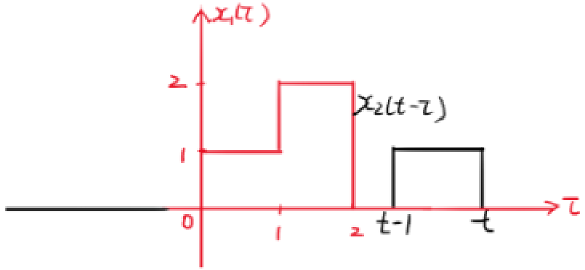
$= 2-t+2(t-1)$

$= t$



④ $(1\le t-1<2)$ $\Rightarrow 2\le t<3$, $y(t)=\int_{t-1}^{2}1\cdot 2\,d\tau=2(2-t+1)$

$=6-2t$

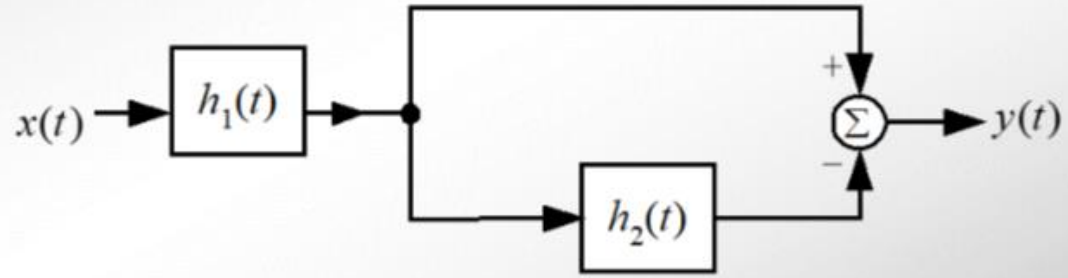


⑤ $(t-1\ge 2)$ $\Rightarrow t\ge 3$, $y(t)=0$

2. An interconnection of LTI systems is depicted in following figure.

If $h_1(t)=e^{-2t}u(t)$, $h_2(t)=\delta(t-2)$,

please determine the unit impulse response $h(t)$ of the overall system.



Solution:

$$h(t)=h_1(t)*[\delta(t)-h_2(t)]=h_1(t)-h_1(t)*h_2(t)$$

$$=e^{-2t}u(t)-e^{-2t}u(t)*\delta(t-2)$$

$$=e^{-2t}u(t)-e^{-2(t-2)}u(t-2)$$

$$x(t)*\delta(t)=x(t)$$

$$x(t)*\delta(t-t_0)=x(t-t_0)$$

$$h(t)=\begin{cases} 0, & t<0 \\ e^{-2}, & 0<t<2 \\ e^{-2t}(1-e^{4}), & t>2 \end{cases}$$

The impulse response of an identical system is $\delta(t)$.