Chapter 1

- 1. Various geometric formulas are given in Appendix E.
- (a) Expressing the radius of the Earth as

$$R = (6.37 \times 10^6 \text{ m})(10^{-3} \text{ km/m}) = 6.37 \times 10^3 \text{ km},$$

its circumference is $s = 2\pi R = 2\pi (6.37 \times 10^3 \text{ km}) = 4.00 \times 10^4 \text{ km}.$

- (b) The surface area of Earth is $A = 4\pi R^2 = 4\pi \left(6.37 \times 10^3 \text{ km}\right)^2 = 5.10 \times 10^8 \text{ km}^2$.
- (c) The volume of Earth is $V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} (6.37 \times 10^3 \text{ km})^3 = 1.08 \times 10^{12} \text{ km}^3$.
- 2. The conversion factors are: 1 gry = 1/10 line, 1 line = 1/12 inch and 1 point = 1/72 inch. The factors imply that

1 gry =
$$(1/10)(1/12)(72 \text{ points}) = 0.60 \text{ point}.$$

Thus, $1 \text{ gry}^2 = (0.60 \text{ point})^2 = 0.36 \text{ point}^2$, which means that $0.50 \text{ gry}^2 = 0.18 \text{ point}^2$.

- 3. The metric prefixes (micro, pico, nano, ...) are given for ready reference on the inside front cover of the textbook (see also Table 1–2).
- (a) Since 1 km = 1×10^3 m and 1 m = $1 \times 10^6 \mu$ m,

$$1 \text{km} = 10^3 \text{ m} = (10^3 \text{ m})(10^6 \ \mu \text{ m/m}) = 10^9 \ \mu \text{m}.$$

The given measurement is 1.0 km (two significant figures), which implies our result should be written as $1.0 \times 10^9 \ \mu m$.

(b) We calculate the number of microns in 1 centimeter. Since 1 cm = 10^{-2} m,

$$1 \text{cm} = 10^{-2} \text{ m} = (10^{-2} \text{m})(10^6 \,\mu\,\text{m/m}) = 10^4 \,\mu\text{m}.$$

We conclude that the fraction of one centimeter equal to 1.0 μ m is 1.0×10^{-4} .

(c) Since 1 yd = (3 ft)(0.3048 m/ft) = 0.9144 m,

$$1.0 \text{ yd} = (0.91 \text{ m})(10^6 \ \mu \text{ m/m}) = 9.1 \times 10^5 \ \mu \text{m}.$$

4. (a) Using the conversion factors 1 inch = 2.54 cm exactly and 6 picas = 1 inch, we obtain

$$0.80 \text{ cm} = (0.80 \text{ cm}) \left(\frac{1 \text{ inch}}{2.54 \text{ cm}}\right) \left(\frac{6 \text{ picas}}{1 \text{ inch}}\right) \approx 1.9 \text{ picas}.$$

(b) With 12 points = 1 pica, we have

$$0.80 \text{ cm} = (0.80 \text{ cm}) \left(\frac{1 \text{ inch}}{2.54 \text{ cm}}\right) \left(\frac{6 \text{ picas}}{1 \text{ inch}}\right) \left(\frac{12 \text{ points}}{1 \text{ pica}}\right) \approx 23 \text{ points}.$$

5. Given that 1 furlong = 201.168 m, 1 rod = 5.0292 m and 1 chain = 20.117 m, we find the relevant conversion factors to be

1.0 furlong = 201.168 m =
$$(201.168 \text{ m}) \frac{1 \text{ rod}}{5.0292 \text{ m}} = 40 \text{ rods},$$

and

1.0 furlong = 201.168 m =
$$(201.168 \text{ pr}) \frac{1 \text{ chain}}{20.117 \text{ pr}} = 10 \text{ chains}$$
.

Note the cancellation of m (meters), the unwanted unit. Using the given conversion factors, we find

(a) the distance *d* in rods to be

$$d = 4.0 \text{ furlongs} = (4.0 \text{ furlongs}) \frac{40 \text{ rods}}{1 \text{ furlong}} = 160 \text{ rods},$$

(b) and that distance in chains to be

$$d = 4.0 \text{ furlongs} = (4.0 \text{ furlongs}) \frac{10 \text{ chains}}{1 \text{ furlong}} = 40 \text{ chains}.$$

- 6. We make use of Table 1-6.
- (a) We look at the first ("cahiz") column: 1 fanega is equivalent to what amount of cahiz? We note from the already completed part of the table that 1 cahiz equals a dozen fanega. Thus, 1 fanega = $\frac{1}{12}$ cahiz, or 8.33×10^{-2} cahiz. Similarly, "1 cahiz = 48 cuartilla" (in the already completed part) implies that 1 cuartilla = $\frac{1}{48}$ cahiz, or 2.08×10^{-2} cahiz. Continuing in this way, the remaining entries in the first column are 6.94×10^{-3} and 3.47×10^{-3} .

- (b) In the second ("fanega") column, we find 0.250, 8.33×10^{-2} , and 4.17×10^{-2} for the last three entries.
- (c) In the third ("cuartilla") column, we obtain 0.333 and 0.167 for the last two entries.
- (d) Finally, in the fourth ("almude") column, we get $\frac{1}{2} = 0.500$ for the last entry.
- (e) Since the conversion table indicates that 1 almude is equivalent to 2 medios, our amount of 7.00 almudes must be equal to 14.0 medios.
- (f) Using the value (1 almude = 6.94×10^{-3} cahiz) found in part (a), we conclude that 7.00 almudes is equivalent to 4.86×10^{-2} cahiz.
- (g) Since each decimeter is 0.1 meter, then 55.501 cubic decimeters is equal to 0.055501 m^3 or 55501 cm^3 . Thus, 7.00 almudes $=\frac{7.00}{12}$ fanega $=\frac{7.00}{12}$ (55501 cm^3) $= 3.24 \times 10^4$ cm³.
- 7. We use the conversion factors found in Appendix D.

1 acre
$$\cdot$$
 ft = $(43,560 \text{ ft}^2) \cdot$ ft = $43,560 \text{ ft}^3$

Since 2 in. = (1/6) ft, the volume of water that fell during the storm is

$$V = (26 \text{ km}^2)(1/6 \text{ ft}) = (26 \text{ km}^2)(3281 \text{ ft/km})^2(1/6 \text{ ft}) = 4.66 \times 10^7 \text{ ft}^3$$
.

Thus,

$$V = \frac{4.66 \times 10^7 \text{ ft}^3}{4.3560 \times 10^4 \text{ ft}^3/\text{acre} \cdot \text{ft}} = 1.1 \times 10^3 \text{ acre} \cdot \text{ft}.$$

- 8. From Fig. 1-4, we see that 212 S is equivalent to 258 W and 212 32 = 180 S is equivalent to 216 60 = 156 Z. The information allows us to convert S to W or Z.
- (a) In units of W, we have

$$50.0 \text{ S} = (50.0 \text{ S}) \left(\frac{258 \text{ W}}{212 \text{ S}} \right) = 60.8 \text{ W}$$

(b) In units of Z, we have

$$50.0 \text{ S} = (50.0 \text{ S}) \left(\frac{156 \text{ Z}}{180 \text{ S}} \right) = 43.3 \text{ Z}$$

9. The volume of ice is given by the product of the semicircular surface area and the thickness. The area of the semicircle is $A = \pi r^2/2$, where r is the radius. Therefore, the volume is

$$V = \frac{\pi}{2}r^2 z$$

where z is the ice thickness. Since there are 10^3 m in 1 km and 10^2 cm in 1 m, we have

$$r = (2000 \,\mathrm{km}) \left(\frac{10^3 \,\mathrm{m}}{1 \,\mathrm{km}} \right) \left(\frac{10^2 \,\mathrm{cm}}{1 \,\mathrm{m}} \right) = 2000 \times 10^5 \,\mathrm{cm}.$$

In these units, the thickness becomes

$$z = 3000 \,\mathrm{m} = (3000 \,\mathrm{m}) \left(\frac{10^2 \,\mathrm{cm}}{1 \,\mathrm{m}} \right) = 3000 \times 10^2 \,\mathrm{cm}$$

which yields
$$V = \frac{\pi}{2} (2000 \times 10^5 \text{ cm})^2 (3000 \times 10^2 \text{ cm}) = 1.9 \times 10^{22} \text{ cm}^3$$
.

- 10. Since a change of longitude equal to 360° corresponds to a 24 hour change, then one expects to change longitude by $360^{\circ}/24=15^{\circ}$ before resetting one's watch by 1.0 h.
- 11. (a) Presuming that a French decimal day is equivalent to a regular day, then the ratio of weeks is simply 10/7 or (to 3 significant figures) 1.43.
- (b) In a regular day, there are 86400 seconds, but in the French system described in the problem, there would be 10^5 seconds. The ratio is therefore 0.864.
- 12. A day is equivalent to 86400 seconds and a meter is equivalent to a million micrometers, so

$$\frac{(3.7 \text{ m})(10^6 \,\mu\,\text{m/m})}{(14 \,\text{day})(86400 \,\text{s/day})} = 3.1 \,\mu\,\text{m/s}.$$

13. The time on any of these clocks is a straight-line function of that on another, with slopes $\neq 1$ and y-intercepts $\neq 0$. From the data in the figure we deduce

$$t_C = \frac{2}{7}t_B + \frac{594}{7}, \quad t_B = \frac{33}{40}t_A - \frac{662}{5}.$$

These are used in obtaining the following results.

(a) We find

$$t_B' - t_B = \frac{33}{40} (t_A' - t_A) = 495 \text{ s}$$

when $t'_{A} - t_{A} = 600 \text{ s.}$

(b) We obtain
$$t_C' - t_C = \frac{2}{7} (t_B' - t_B) = \frac{2}{7} (495) = 141 \text{ s.}$$

- (c) Clock *B* reads $t_B = (33/40)(400) (662/5) \approx 198$ s when clock *A* reads $t_A = 400$ s.
- (d) From $t_C = 15 = (2/7)t_B + (594/7)$, we get $t_B \approx -245$ s.
- 14. The metric prefixes (micro (μ), pico, nano, ...) are given for ready reference on the inside front cover of the textbook (also Table 1–2).

(a) 1
$$\mu$$
century = $\left(10^{-6} \text{ century}\right) \left(\frac{100 \text{ y}}{1 \text{ century}}\right) \left(\frac{365 \text{ day}}{1 \text{ y}}\right) \left(\frac{24 \text{ h}}{1 \text{ day}}\right) \left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 52.6 \text{ min}.$

(b) The percent difference is therefore

$$\frac{52.6 \text{ min} - 50 \text{ min}}{52.6 \text{ min}} = 4.9\%.$$

- 15. A week is 7 days, each of which has 24 hours, and an hour is equivalent to 3600 seconds. Thus, two weeks (a fortnight) is 1209600 s. By definition of the micro prefix, this is roughly $1.21 \times 10^{12} \, \mu s$.
- 16. We denote the pulsar rotation rate f (for frequency).

$$f = \frac{1 \text{ rotation}}{1.55780644887275 \times 10^{-3} \text{ s}}$$

(a) Multiplying f by the time-interval t = 7.00 days (which is equivalent to 604800 s, if we ignore *significant figure* considerations for a moment), we obtain the number of rotations:

$$N = \left(\frac{1 \text{ rotation}}{1.55780644887275 \times 10^{-3} \text{ s}}\right) (604800 \text{ s}) = 388238218.4$$

which should now be rounded to 3.88×10^8 rotations since the time-interval was specified in the problem to three significant figures.

(b) We note that the problem specifies the *exact* number of pulsar revolutions (one million). In this case, our unknown is t, and an equation similar to the one we set up in part (a) takes the form N = ft, or

$$1 \times 10^6 = \left(\frac{1 \text{ rotation}}{1.55780644887275 \times 10^{-3} \text{ s}}\right) t$$

which yields the result t = 1557.80644887275 s (though students who do this calculation on their calculator might not obtain those last several digits).

(c) Careful reading of the problem shows that the time-uncertainty *per revolution* is $\pm 3 \times 10^{-17}$ s. We therefore expect that as a result of one million revolutions, the uncertainty should be $(\pm 3 \times 10^{-17})(1 \times 10^6) = \pm 3 \times 10^{-11}$ s.

17. None of the clocks advance by exactly 24 h in a 24-h period but this is not the most important criterion for judging their quality for measuring time intervals. What is important is that the clock advance by the same amount in each 24-h period. The clock reading can then easily be adjusted to give the correct interval. If the clock reading jumps around from one 24-h period to another, it cannot be corrected since it would impossible to tell what the correction should be. The following gives the corrections (in seconds) that must be applied to the reading on each clock for each 24-h period. The entries were determined by subtracting the clock reading at the end of the interval from the clock reading at the beginning.

CLOCK	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.
	-Mon.	-Tues.	-Wed.	-Thurs.	-Fri.	-Sat.
A	-16	-16	-15	-17	-15	-15
В	-3	+5	-10	+5	+6	-7
С	-58	-58	-58	-58	-58	-58
D	+67	+67	+67	+67	+67	+67
Е	+70	+55	+2	+20	+10	+10

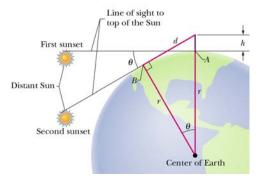
Clocks C and D are both good timekeepers in the sense that each is consistent in its daily drift (relative to WWF time); thus, C and D are easily made "perfect" with simple and predictable corrections. The correction for clock C is less than the correction for clock D, so we judge clock C to be the best and clock D to be the next best. The correction that must be applied to clock A is in the range from 15 s to 17s. For clock B it is the range from -5 s to +10 s, for clock E it is in the range from -70 s to -2 s. After C and D, A has the smallest range of correction, B has the next smallest range, and E has the greatest range. From best to worst, the ranking of the clocks is C, D, A, B, E.

18. The last day of the 20 centuries is longer than the first day by

The average day during the 20 centuries is (0 + 0.02)/2 = 0.01 s longer than the first day. Since the increase occurs uniformly, the cumulative effect T is

or roughly two hours.

19. When the Sun first disappears while lying down, your line of sight to the top of the Sun is tangent to the Earth's surface at point A shown in the figure. As you stand, elevating your eyes by a height h, the line of sight to the Sun is tangent to the Earth's surface at point B.



Let d be the distance from point B to your eyes. From the Pythagorean theorem, we have

$$d^{2} + r^{2} = (r+h)^{2} = r^{2} + 2rh + h^{2}$$

or $d^2 = 2rh + h^2$, where r is the radius of the Earth. Since $r \gg h$, the second term can be dropped, leading to $d^2 \approx 2rh$. Now the angle between the two radii to the two tangent points A and B is θ , which is also the angle through which the Sun moves about Earth during the time interval t = 11.1 s. The value of θ can be obtained by using

$$\frac{\theta}{360^{\circ}} = \frac{t}{24 \text{ h}}.$$

This yields

$$\theta = \frac{(360^{\circ})(11.1 \text{ s})}{(24 \text{ h})(60 \text{ min/h})(60 \text{ s/min})} = 0.04625^{\circ}.$$

Using $d = r \tan \theta$, we have $d^2 = r^2 \tan^2 \theta = 2rh$, or

$$r = \frac{2h}{\tan^2 \theta}$$

Using the above value for θ and h = 1.7 m, we have $r = 5.2 \times 10^6$ m.

20. (a) We find the volume in cubic centimeters

193 gal = (193 gal)
$$\left(\frac{231 \text{ in}^3}{1 \text{ gal}}\right) \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^3 = 7.31 \times 10^5 \text{ cm}^3$$

and subtract this from 1×10^6 cm³ to obtain 2.69×10^5 cm³. The conversion gal \rightarrow in³ is given in Appendix D (immediately below the table of Volume conversions).

(b) The volume found in part (a) is converted (by dividing by $(100 \text{ cm/m})^3$) to 0.731 m^3 , which corresponds to a mass of

$$(1000 \text{ kg/m}^3) (0.731 \text{ m}^2) = 731 \text{ kg}$$

using the density given in the problem statement. At a rate of 0.0018 kg/min, this can be filled in

$$\frac{731 \text{kg}}{0.0018 \text{ kg/min}} = 4.06 \times 10^5 \text{ min} = 0.77 \text{ y}$$

after dividing by the number of minutes in a year (365 days)(24 h/day) (60 min/h).

21. If M_E is the mass of Earth, m is the average mass of an atom in Earth, and N is the number of atoms, then $M_E = Nm$ or $N = M_E/m$. We convert mass m to kilograms using Appendix D (1 u = 1.661 × 10⁻²⁷ kg). Thus,

$$N = \frac{M_E}{m} = \frac{5.98 \times 10^{24} \text{ kg}}{(40 \text{ u}) (1.661 \times 10^{-27} \text{ kg/u})} = 9.0 \times 10^{49}.$$

22. The density of gold is

$$\rho = \frac{m}{V} = \frac{19.32 \text{ g}}{1 \text{ cm}^3} = 19.32 \text{ g/cm}^3.$$

(a) We take the volume of the leaf to be its area A multiplied by its thickness z. With density $\rho = 19.32 \text{ g/cm}^3$ and mass m = 27.63 g, the volume of the leaf is found to be

$$V = \frac{m}{\rho} = 1.430 \text{ cm}^3.$$

We convert the volume to SI units:

$$V = (1.430 \text{ cm}^3) \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 = 1.430 \times 10^{-6} \text{ m}^3.$$

Since V = Az with $z = 1 \times 10^{-6}$ m (metric prefixes can be found in Table 1–2), we obtain

$$A = \frac{1.430 \times 10^{-6} \text{ m}^3}{1 \times 10^{-6} \text{ m}} = 1.430 \text{ m}^2.$$

(b) The volume of a cylinder of length ℓ is $V = A\ell$ where the cross-section area is that of a circle: $A = \pi r^2$. Therefore, with $r = 2.500 \times 10^{-6}$ m and $V = 1.430 \times 10^{-6}$ m³, we obtain

$$\ell = \frac{V}{\pi r^2} = 7.284 \times 10^4 \text{ m} = 72.84 \text{ km}.$$

23. We introduce the notion of density:

$$\rho = \frac{m}{V}$$

and convert to SI units: $1 \text{ g} = 1 \times 10^{-3} \text{ kg}$.

(a) For volume conversion, we find 1 cm³ = $(1 \times 10^{-2} \text{m})^3 = 1 \times 10^{-6} \text{m}^3$. Thus, the density in kg/m³ is

$$1 \text{ g/cm}^3 = \left(\frac{1 \text{ g}}{\text{cm}^3}\right) \left(\frac{10^{-3} \text{ kg}}{\text{g}}\right) \left(\frac{\text{cm}^3}{10^{-6} \text{ m}^3}\right) = 1 \times 10^3 \text{ kg/m}^3.$$

Thus, the mass of a cubic meter of water is 1000 kg.

(b) We divide the mass of the water by the time taken to drain it. The mass is found from $M = \rho V$ (the product of the volume of water and its density):

$$M = (5700 \text{ m}^3) (1 \times 10^3 \text{ kg/m}^3) = 5.70 \times 10^6 \text{ kg.}$$

The time is $t = (10h)(3600 \text{ s/h}) = 3.6 \times 10^4 \text{ s}$, so the mass flow rate R is

$$R = \frac{M}{t} = \frac{5.70 \times 10^6 \text{ kg}}{3.6 \times 10^4 \text{ s}} = 158 \text{ kg/s}.$$

24. The metric prefixes (micro (μ), pico, nano, ...) are given for ready reference on the inside front cover of the textbook (see also Table 1–2). The surface area A of each grain of sand of radius $r = 50 \ \mu\text{m} = 50 \times 10^{-6} \ \text{m}$ is given by $A = 4\pi (50 \times 10^{-6})^2 = 3.14 \times 10^{-8} \ \text{m}^2$ (Appendix E contains a variety of geometry formulas). We introduce the notion of

density, $\rho = m/V$, so that the mass can be found from $m = \rho V$, where $\rho = 2600 \text{ kg/m}^3$. Thus, using $V = 4\pi r^3/3$, the mass of each grain is

$$m = \rho V = \rho \left(\frac{4\pi r^3}{3}\right) = \left(2600 \frac{\text{kg}}{\text{m}^3}\right) \frac{4\pi \left(50 \times 10^{-6} \text{ m}\right)^3}{3} = 1.36 \times 10^{-9} \text{ kg}.$$

We observe that (because a cube has six equal faces) the indicated surface area is 6 m^2 . The number of spheres (the grains of sand) N that have a total surface area of 6 m^2 is given by

$$N = \frac{6 \text{ m}^2}{3.14 \times 10^{-8} \text{ m}^2} = 1.91 \times 10^8.$$

Therefore, the total mass *M* is $M = Nm = (1.91 \times 10^8) (1.36 \times 10^{-9} \text{ kg}) = 0.260 \text{ kg}.$

- 25. The volume of the section is $(2500 \text{ m})(800 \text{ m})(2.0 \text{ m}) = 4.0 \times 10^6 \text{ m}^3$. Letting "d" stand for the thickness of the mud after it has (uniformly) distributed in the valley, then its volume there would be (400 m)(400 m)d. Requiring these two volumes to be equal, we can solve for d. Thus, d = 25 m. The volume of a small part of the mud over a patch of area of 4.0 m^2 is $(4.0)d = 100 \text{ m}^3$. Since each cubic meter corresponds to a mass of 1900 kg (stated in the problem), then the mass of that small part of the mud is $1.9 \times 10^5 \text{ kg}$.
- 26. (a) The volume of the cloud is $(3000 \text{ m})\pi(1000 \text{ m})^2 = 9.4 \times 10^9 \text{ m}^3$. Since each cubic meter of the cloud contains from 50×10^6 to 500×10^6 water drops, then we conclude that the entire cloud contains from 4.7×10^{18} to 4.7×10^{19} drops. Since the volume of each drop is $\frac{4}{3}\pi(10 \times 10^{-6} \text{ m})^3 = 4.2 \times 10^{-15} \text{ m}^3$, then the total volume of water in a cloud is from 2×10^3 to 2×10^4 m³.
- (b) Using the fact that $1 L = 1 \times 10^3 \text{ cm}^3 = 1 \times 10^{-3} \text{ m}^3$, the amount of water estimated in part (a) would fill from 2×10^6 to 2×10^7 bottles.
- (c) At 1000 kg for every cubic meter, the mass of water is from 2×10^6 to 2×10^7 kg. The coincidence in numbers between the results of parts (b) and (c) of this problem is due to the fact that each liter has a mass of one kilogram when water is at its normal density (under standard conditions).
- 27. We introduce the notion of density, $\rho = m/V$, and convert to SI units: 1000 g = 1 kg, and 100 cm = 1 m.
- (a) The density ρ of a sample of iron is

$$\rho = (7.87 \text{ g/cm}^3) \left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = 7870 \text{ kg/m}^3.$$

If we ignore the empty spaces between the close-packed spheres, then the density of an individual iron atom will be the same as the density of any iron sample. That is, if M is the mass and V is the volume of an atom, then

$$V = \frac{M}{\rho} = \frac{9.27 \times 10^{-26} \text{ kg}}{7.87 \times 10^3 \text{ kg/m}^3} = 1.18 \times 10^{-29} \text{ m}^3.$$

(b) We set $V = 4\pi R^3/3$, where R is the radius of an atom (Appendix E contains several geometry formulas). Solving for R, we find

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3(1.18 \times 10^{-29} \text{ m}^3)}{4\pi}\right)^{1/3} = 1.41 \times 10^{-10} \text{ m}.$$

The center-to-center distance between atoms is twice the radius, or 2.82×10^{-10} m.

- 28. If we estimate the "typical" large domestic cat mass as 10 kg, and the "typical" atom (in the cat) as $10 \text{ u} \approx 2 \times 10^{-26} \text{ kg}$, then there are roughly $(10 \text{ kg})/(2 \times 10^{-26} \text{ kg}) \approx 5 \times 10^{26}$ atoms. This is close to being a factor of a thousand greater than Avogadro's number. Thus this is roughly a kilomole of atoms.
- 29. The mass in kilograms is

$$(28.9 \text{ piculs}) \left(\frac{100 \text{ gin}}{1 \text{ picul}}\right) \left(\frac{16 \text{ tahil}}{1 \text{ gin}}\right) \left(\frac{10 \text{ chee}}{1 \text{ tahil}}\right) \left(\frac{10 \text{ hoon}}{1 \text{ chee}}\right) \left(\frac{0.3779 \text{ g}}{1 \text{ hoon}}\right)$$

which yields 1.747×10^6 g or roughly 1.75×10^3 kg.

- 30. To solve the problem, we note that the first derivative of the function with respect to time gives the rate. Setting the rate to zero gives the time at which an extreme value of the variable mass occurs; here that extreme value is a maximum.
- (a) Differentiating $m(t) = 5.00t^{0.8} 3.00t + 20.00$ with respect to t gives

$$\frac{dm}{dt} = 4.00t^{-0.2} - 3.00.$$

The water mass is the greatest when dm/dt = 0, or at $t = (4.00/3.00)^{1/0.2} = 4.21$ s.

(b) At t = 4.21 s, the water mass is

$$m(t = 4.21 \text{ s}) = 5.00(4.21)^{0.8} - 3.00(4.21) + 20.00 = 23.2 \text{ g}.$$

(c) The rate of mass change at $t = 2.00 \,\mathrm{s}$ is

$$\frac{dm}{dt}\bigg|_{t=2.00 \text{ s}} = \left[4.00(2.00)^{-0.2} - 3.00\right] \text{g/s} = 0.48 \text{ g/s} = 0.48 \frac{\text{g}}{\text{s}} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} \cdot \frac{60 \text{ s}}{1 \text{ min}}$$
$$= 2.89 \times 10^{-2} \text{ kg/min}.$$

(d) Similarly, the rate of mass change at t = 5.00 s is

$$\frac{dm}{dt}\bigg|_{t=2.00s} = \left[4.00(5.00)^{-0.2} - 3.00\right] g/s = -0.101 g/s = -0.101 \frac{g}{s} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} \cdot \frac{60 \text{ s}}{1 \text{ min}}$$
$$= -6.05 \times 10^{-3} \text{ kg/min}.$$

31. The mass density of the candy is

$$\rho = \frac{m}{V} = \frac{0.0200 \text{ g}}{50.0 \text{ mm}^3} = 4.00 \times 10^{-4} \text{ g/mm}^3 = 4.00 \times 10^{-4} \text{ kg/cm}^3.$$

If we neglect the volume of the empty spaces between the candies, then the total mass of the candies in the container when filled to height h is $M = \rho Ah$, where $A = (14.0 \text{ cm})(17.0 \text{ cm}) = 238 \text{ cm}^2$ is the base area of the container that remains unchanged. Thus, the rate of mass change is given by

$$\frac{dM}{dt} = \frac{d(\rho Ah)}{dt} = \rho A \frac{dh}{dt} = (4.00 \times 10^{-4} \text{ kg/cm}^3)(238 \text{ cm}^2)(0.250 \text{ cm/s})$$
$$= 0.0238 \text{ kg/s} = 1.43 \text{ kg/min}.$$

32. The total volume V of the real house is that of a triangular prism (of height h = 3.0 m and base area $A = 20 \times 12 = 240$ m²) in addition to a rectangular box (height h' = 6.0 m and same base). Therefore,

$$V = \frac{1}{2} hA + h'A = \left(\frac{h}{2} + h'\right) A = 1800 \text{ m}^3.$$

(a) Each dimension is reduced by a factor of 1/12, and we find

$$V_{\text{doll}} = (1800 \text{ m}^3) \left(\frac{1}{12}\right)^3 \approx 1.0 \text{ m}^3.$$

(b) In this case, each dimension (relative to the real house) is reduced by a factor of 1/144. Therefore,

$$V_{\text{miniature}} = (1800 \text{ m}^3) \left(\frac{1}{144}\right)^3 \approx 6.0 \times 10^{-4} \text{ m}^3.$$

33. In this problem we are asked to differentiate between three types of tons: displacement ton, freight ton and register ton, all of which are units of volume. The three different tons are given in terms of barrel bulk, with

1 barrel bulk =
$$0.1415 \text{ m}^3 = 4.0155 \text{ U.S.}$$
 bushels

using $1 \text{ m}^3 = 28.378 \text{ U.S.}$ bushels. Thus, in terms of U.S. bushels, we have

1 displacement ton = (7 barrels bulk) ×
$$\left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right)$$
 = 28.108 U.S. bushels

1 freight ton = (8 barrels bulk) × $\left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right)$ = 32.124 U.S. bushels

1 register ton = (20 barrels bulk) × $\left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right)$ = 80.31 U.S. bushels

(a) The difference between 73 "freight" tons and 73 "displacement" tons is

$$\Delta V = 73$$
(freight tons – displacement tons) = 73(32.124 U.S. bushels – 28.108 U.S. bushels) = 293.168 U.S. bushels ≈ 293 U.S. bushels

(b) Similarly, the difference between 73 "register" tons and 73 "displacement" tons is

$$\Delta V = 73 \text{(register tons - displacement tons)} = 73 (80.31 \text{ U.S. bushels} - 28.108 \text{ U.S. bushels})$$

= 3810.746 U.S. bushels $\approx 3.81 \times 10^3 \text{ U.S. bushels}$

34. The customer expects a volume $V_1 = 20 \times 7056$ in³ and receives $V_2 = 20 \times 5826$ in.³, the difference being $\Delta V = V_1 - V_2 = 24600$ in.³, or

$$\Delta V = (24600 \text{ in.}^3) \left(\frac{2.54 \text{ cm}}{1 \text{ inch}}\right)^3 \left(\frac{1 \text{L}}{1000 \text{ cm}^3}\right) = 403 \text{L}$$

where Appendix D has been used.

35. The first two conversions are easy enough that a *formal* conversion is not especially called for, but in the interest of *practice makes perfect* we go ahead and proceed formally:

(a) 11 tuffets =
$$\left(11 \text{ tuffets}\right) \left(\frac{2 \text{ peck}}{1 \text{ tuffet}}\right) = 22 \text{ pecks}$$
.

(b) 11 tuffets =
$$(11 \text{ tuffets}) \left(\frac{0.50 \text{ Imperial bushel}}{1 \text{ tuffet}} \right) = 5.5 \text{ Imperial bushels}$$
.

(c) 11 tuffets =
$$(5.5 \text{ Imperial bushel}) \left(\frac{36.3687 \text{ L}}{1 \text{ Imperial bushel}} \right) \approx 200 \text{ L}.$$

- 36. Table 7 can be completed as follows:
- (a) It should be clear that the first column (under "wey") is the reciprocal of the first row so that $\frac{9}{10} = 0.900, \frac{3}{40} = 7.50 \times 10^{-2}$, and so forth. Thus, 1 pottle = 1.56×10^{-3} wey and 1 gill = 8.32×10^{-6} wey are the last two entries in the first column.
- (b) In the second column (under "chaldron"), clearly we have 1 chaldron = 1 chaldron (that is, the entries along the "diagonal" in the table must be 1's). To find out how many chaldron are equal to one bag, we note that 1 wey = 10/9 chaldron = 40/3 bag so that $\frac{1}{12}$ chaldron = 1 bag. Thus, the next entry in that second column is $\frac{1}{12} = 8.33 \times 10^{-2}$. Similarly, 1 pottle = 1.74×10^{-3} chaldron and 1 gill = 9.24×10^{-6} chaldron.
- (c) In the third column (under "bag"), we have 1 chaldron = 12.0 bag, 1 bag = 1 bag, 1 pottle = 2.08×10^{-2} bag, and 1 gill = 1.11×10^{-4} bag.
- (d) In the fourth column (under "pottle"), we find 1 chaldron = 576 pottle, 1 bag = 48 pottle, 1 pottle = 1 pottle, and 1 gill = 5.32×10^{-3} pottle.
- (e) In the last column (under "gill"), we obtain 1 chaldron = 1.08×10^5 gill, 1 bag = 9.02×10^3 gill, 1 pottle = 188 gill, and, of course, 1 gill = 1 gill.
- (f) Using the information from part (c), 1.5 chaldron = (1.5)(12.0) = 18.0 bag. And since each bag is 0.1091 m³ we conclude 1.5 chaldron = (18.0)(0.1091) = 1.96 m³.
- 37. The volume of one unit is $1~\text{cm}^3=1\times10^{-6}~\text{m}^3$, so the volume of a mole of them is $6.02\times10^{23}~\text{cm}^3=6.02\times10^{17}~\text{m}^3$. The cube root of this number gives the edge length: $8.4\times10^5~\text{m}^3$. This is equivalent to roughly $8\times10^2~\text{km}$.
- 38. (a) Using the fact that the area A of a rectangle is (width) \times (length), we find

$$A_{\text{total}} = (3.00 \,\text{acre}) + (25.0 \,\text{perch})(4.00 \,\text{perch})$$

= $(3.00 \,\text{acre}) \left(\frac{(40 \,\text{perch})(4 \,\text{perch})}{1 \,\text{acre}} \right) + 100 \,\text{perch}^2$
= $580 \,\text{perch}^2$.

We multiply this by the perch² \rightarrow rood conversion factor (1 rood/40 perch²) to obtain the answer: $A_{\text{total}} = 14.5$ roods.

(b) We convert our intermediate result in part (a):

$$A_{\text{total}} = (580 \text{ perch}^2) \left(\frac{16.5 \text{ ft}}{1 \text{ perch}}\right)^2 = 1.58 \times 10^5 \text{ ft}^2.$$

Now, we use the feet \rightarrow meters conversion given in Appendix D to obtain

$$A_{\text{total}} = (1.58 \times 10^5 \text{ ft}^2) \left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)^2 = 1.47 \times 10^4 \text{ m}^2.$$

39. This problem compares the U.K gallon with U.S. gallon, two non-SI units for volume. The interpretation of the type of gallons, whether U.K. or U.S., affects the amount of gasoline one calculates for traveling a given distance.

If the fuel consumption rate is R (in miles/gallon), then the amount of gasoline (in gallons) needed for a trip of distance d (in miles) would be

$$V(\text{gallon}) = \frac{d \text{ (miles)}}{R \text{ (miles/gallon)}}$$

Since the car was manufactured in the U.K., the fuel consumption rate is calibrated based on U.K. gallon, and the correct interpretation should be "40 miles per U.K. gallon." In U.K., one would think of gallon as U.K. gallon; however, in the U.S., the word "gallon" would naturally be interpreted as U.S. gallon. Note also that since 1 U.K. gallon = 4.5460900 L and 1 U.S. gallon = 3.7854118 L, the relationship between the two is

1 U.K. gallon =
$$(4.5460900 \text{ L}) \left(\frac{1 \text{ U.S. gallon}}{3.7854118 \text{ L}} \right) = 1.20095 \text{ U.S. gallons}$$

(a) The amount of gasoline actually required is

$$V' = \frac{750 \text{ miles}}{40 \text{ miles/U. K. gallon}} = 18.75 \text{ U. K. gallons} \approx 18.8 \text{ U. K. gallons}$$

This means that the driver mistakenly believes that the car should need 18.8 U.S. gallons.

(b) Using the conversion factor found above, the actual amount required is equivalent to

$$V' = (18.75 \text{ U. K. gallons}) \times \left(\frac{1.20095 \text{ U.S. gallons}}{1 \text{ U.K. gallon}}\right) \approx 22.5 \text{ U.S. gallons}.$$

- 40. Equation 1-9 gives (to very high precision!) the conversion from atomic mass units to kilograms. Since this problem deals with the ratio of total mass (1.0 kg) divided by the mass of one atom (1.0 u, but converted to kilograms), then the computation reduces to simply taking the reciprocal of the number given in Eq. 1-9 and rounding off appropriately. Thus, the answer is 6.0×10^{26} .
- 41. Using the (exact) conversion 1 in = 2.54 cm = 0.0254 m, we find that

1 ft = 12 in. = (12 in.) ×
$$\left(\frac{0.0254 \text{ m}}{1 \text{ in.}}\right)$$
 = 0.3048 m

and $1 \text{ ft}^3 = (0.3048 \text{ m})^3 = 0.0283 \text{ m}^3$ for volume (these results also can be found in Appendix D). Thus, the volume of a cord of wood is $V = (8 \text{ ft}) \times (4 \text{ ft}) \times (4 \text{ ft}) = 128 \text{ ft}^3$. Using the conversion factor found above, we obtain

$$V = 1 \text{ cord} = 128 \text{ ft}^3 = (128 \text{ ft}^3) \times \left(\frac{0.0283 \text{ m}^3}{1 \text{ ft}^3}\right) = 3.625 \text{ m}^3$$

which implies that 1 m³ = $\left(\frac{1}{3.625}\right)$ cord = 0.276 cord \approx 0.3 cord .

42. (a) In atomic mass units, the mass of one molecule is (16 + 1 + 1)u = 18 u. Using Eq. 1-9, we find

$$18u = (18u) \left(\frac{1.6605402 \times 10^{-27} \text{ kg}}{1u} \right) = 3.0 \times 10^{-26} \text{ kg}.$$

(b) We divide the total mass by the mass of each molecule and obtain the (approximate) number of water molecules:

$$N \approx \frac{1.4 \times 10^{21}}{3.0 \times 10^{-26}} \approx 5 \times 10^{46}.4.7$$
 \forall \forall

- 43. A million milligrams comprise a kilogram, so 2.3 kg/week is 2.3×10^6 mg/week. Figuring 7 days a week, 24 hours per day, 3600 second per hour, we find 604800 seconds are equivalent to one week. Thus, $(2.3 \times 10^6 \text{ mg/week})/(604800 \text{ s/week}) = 3.8 \text{ mg/s}$.
- 44. The volume of the water that fell is

$$V = (26 \text{ km}^2) (2.0 \text{ in.}) = (26 \text{ km}^2) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right)^2 (2.0 \text{ in.}) \left(\frac{0.0254 \text{ m}}{1 \text{ in.}}\right)$$
$$= (26 \times 10^6 \text{ m}^2) (0.0508 \text{ m})$$
$$= 1.3 \times 10^6 \text{ m}^3.$$

We write the mass-per-unit-volume (density) of the water as:

$$\rho = \frac{m}{V} = 1 \times 10^3 \text{ kg/m}^3.$$

The mass of the water that fell is therefore given by $m = \rho V$:

$$m = (1 \times 10^3 \text{ kg/m}^3) (1.3 \times 10^6 \text{ m}^3) = 1.3 \times 10^9 \text{ kg}.$$

45. The number of seconds in a year is 3.156×10^7 . This is listed in Appendix D and results from the product

- (a) The number of shakes in a second is 10^8 ; therefore, there are indeed more shakes per second than there are seconds per year.
- (b) Denoting the age of the universe as 1 u-day (or 86400 u-sec), then the time during which humans have existed is given by

$$\frac{10^6}{10^{10}} = 10^{-4} \text{ u-day},$$

which may also be expressed as $(10^{-4} \text{ u-day}) \left(\frac{86400 \text{ u-sec}}{1 \text{ u-day}} \right) = 8.6 \text{ u-sec}$.

46. The volume removed in one year is

$$V = (75 \times 10^4 \text{ m}^2) (26 \text{ m}) \approx 2 \times 10^7 \text{ m}^3$$

which we convert to cubic kilometers: $V = (2 \times 10^7 \text{ m}^3) \left(\frac{1 \text{ km}}{1000 \text{ m}}\right)^3 = 0.020 \text{ km}^3$.

47. We convert meters to astronomical units, and seconds to minutes, using

1000 m = 1 km

$$1 \text{ AU} = 1.50 \times 10^8 \text{ km}$$

 $60 \text{ s} = 1 \text{ min}$.

Thus, 3.0×10^8 m/s becomes

$$\left(\frac{3.0\times10^8~\text{m}}{\text{s}}\right)\!\left(\frac{1~\text{km}}{1000~\text{m}}\right)\!\left(\frac{\text{AU}}{1.50\times10^8~\text{km}}\right)\!\left(\frac{60~\text{s}}{\text{min}}\right) = 0.12~\text{AU/min}\,.$$

48. Since one atomic mass unit is $1 \text{ u} = 1.66 \times 10^{-24} \text{ g}$ (see Appendix D), the mass of one mole of atoms is about $m = (1.66 \times 10^{-24} \text{ g})(6.02 \times 10^{23}) = 1 \text{ g}$. On the other hand, the mass of one mole of atoms in the common Eastern mole is

$$m' = \frac{75 \text{ g}}{7.5} = 10 \text{ g}$$

Therefore, in atomic mass units, the average mass of one atom in the common Eastern mole is

$$\frac{m'}{N_A} = \frac{10 \text{ g}}{6.02 \times 10^{23}} = 1.66 \times 10^{-23} \text{ g} = 10 \text{ u}.$$

49. (a) Squaring the relation 1 ken = 1.97 m, and setting up the ratio, we obtain

$$\frac{1 \text{ ken}^2}{1 \text{ m}^2} = \frac{1.97^2 \text{ m}^2}{1 \text{ m}^2} = 3.88.$$

(b) Similarly, we find

$$\frac{1 \text{ ken}^3}{1 \text{ m}^3} = \frac{1.97^3 \text{ m}^3}{1 \text{ m}^3} = 7.65.$$

(c) The volume of a cylinder is the circular area of its base multiplied by its height. Thus,

$$\pi r^2 h = \pi (3.00)^2 (5.50) = 156 \text{ ken}^3.$$

- (d) If we multiply this by the result of part (b), we determine the volume in cubic meters: $(155.5)(7.65) = 1.19 \times 10^3 \text{ m}^3$.
- 50. According to Appendix D, a nautical mile is 1.852 km, so 24.5 nautical miles would be 45.374 km. Also, according to Appendix D, a mile is 1.609 km, so 24.5 miles is 39.4205 km. The difference is 5.95 km.
- 51. (a) For the minimum (43 cm) case, 9 cubits converts as follows:

9 cubits =
$$(9 \text{ cubits}) \left(\frac{0.43 \text{ m}}{1 \text{ cubit}} \right) = 3.9 \text{ m}.$$

And for the maximum (53 cm) case we obtain

9 cubits =
$$(9 \text{ cubits}) \left(\frac{0.53 \text{ m}}{1 \text{ cubit}} \right) = 4.8 \text{ m}.$$

- (b) Similarly, with 0.43 m \rightarrow 430 mm and 0.53 m \rightarrow 530 mm, we find 3.9×10^3 mm and 4.8×10^3 mm, respectively.
- (c) We can convert length and diameter first and then compute the volume, or first compute the volume and then convert. We proceed using the latter approach (where d is diameter and ℓ is length).

$$V_{\text{cylinder, min}} = \frac{\pi}{4} \ell d^2 = 28 \text{ cubit}^3 = \left(28 \text{ cubit}^3\right) \left(\frac{0.43 \text{ m}}{1 \text{ cubit}}\right)^3 = 2.2 \text{ m}^3.$$

Similarly, with 0.43 m replaced by 0.53 m, we obtain $V_{\text{cylinder, max}} = 4.2 \text{ m}^3$.

52. Abbreviating wapentake as "wp" and assuming a hide to be 110 acres, we set up the ratio 25 wp/11 barn along with appropriate conversion factors:

$$\frac{\left(25~\text{wp}\right)\left(\frac{100~\text{hide}}{1~\text{wp}}\right)\left(\frac{110~\text{acre}}{1~\text{hide}}\right)\left(\frac{4047~\text{m}^2}{1~\text{acre}}\right)}{\left(11~\text{barn}\right)\left(\frac{1\times10^{-28}~\text{m}^2}{1~\text{barn}}\right)}\approx 1\times10^{36}.$$

53. The objective of this problem is to convert the Earth-Sun distance to parsecs and light-years. To relate parsec (pc) to AU, we note that when θ is measured in radians, it is equal to the arc length s divided by the radius R. For a very large radius circle and small value of θ , the arc may be approximated as the straight line-segment of length 1 AU. Thus,

$$\theta = 1 \text{ arcsec} = (1 \text{ arcsec}) \left(\frac{1 \text{ arcmin}}{60 \text{ arcsec}} \right) \left(\frac{1^{\circ}}{60 \text{ arcmin}} \right) \left(\frac{2\pi \text{ radian}}{360^{\circ}} \right) = 4.85 \times 10^{-6} \text{ rad}$$

Therefore, one parsec is

1 pc =
$$R_0 = \frac{s}{\theta} = \frac{1 \text{ AU}}{4.85 \times 10^{-6}} = 2.06 \times 10^5 \text{ AU}$$

Next, we relate AU to light-year (ly). Since a year is about 3.16×10^7 s, we have $1 \text{ly} = \left(186,000 \text{ mi/s}\right) \left(3.16 \times 10^7 \text{ s}\right) = 5.9 \times 10^{12} \text{ mi}.$

(a) Since $1 \text{ pc} = 2.06 \times 10^5 \text{ AU}$, inverting the relationship gives

$$R = 1 \text{ AU} = (1 \text{ AU}) \left(\frac{1 \text{ pc}}{2.06 \times 10^5 \text{ AU}} \right) = 4.9 \times 10^{-6} \text{ pc.}$$

(b) Given that $1AU = 92.9 \times 10^6$ mi and $11y = 5.9 \times 10^{12}$ mi, the two expressions together lead to

$$1 \text{ AU} = 92.9 \times 10^6 \text{ mi} = (92.9 \times 10^6 \text{ mi}) \left(\frac{1 \text{ ly}}{5.9 \times 10^{12} \text{ mi}} \right) = 1.57 \times 10^{-5} \text{ ly} \approx 1.6 \times 10^{-5} \text{ ly}.$$

Our results can be further combined to give 1 pc = 3.2 ly.

54. (a) Using Appendix D, we have 1 ft = 0.3048 m, 1 gal = 231 in.³, and 1 in.³ = 1.639×10^{-2} L. From the latter two items, we find that 1 gal = 3.79 L. Thus, the quantity 460 ft²/gal becomes

$$460 \text{ ft}^2/\text{gal} = \left(\frac{460 \text{ ft}^2}{\text{gal}}\right) \left(\frac{1 \text{ m}}{3.28 \text{ ft}}\right)^2 \left(\frac{1 \text{ gal}}{3.79 \text{ L}}\right) = 11.3 \text{ m}^2/\text{L}.$$

(b) Also, since 1 m³ is equivalent to 1000 L, our result from part (a) becomes

$$11.3 \text{ m}^2/\text{L} = \left(\frac{11.3 \text{ m}^2}{\text{L}}\right) \left(\frac{1000 \text{L}}{1 \text{ m}^3}\right) = 1.13 \times 10^4 \text{ m}^{-1}.$$

- (c) The inverse of the original quantity is $(460 \text{ ft}^2/\text{gal})^{-1} = 2.17 \times 10^{-3} \text{ gal/ft}^2$.
- (d) The answer in (c) represents the volume of the paint (in gallons) needed to cover a square foot of area. From this, we could also figure the paint thickness [it turns out to be about a tenth of a millimeter, as one sees by taking the reciprocal of the answer in part (b)].