PHIL 220: Introduction to Logic

Week 13 Discussion (11/21/2025)

Today's Goals: Review rules for quantifiers and practice proofs involving quantifiers.

Rules for quantifiers:

Easy rules:

l	$\forall v\varphi$	
m	$\varphi(\tau/v)$	$\forall$E, l

l	$\varphi(\tau/v)$	
m	$\exists v\varphi$	$\exists$I, l

Tricky rules:

l	$\varphi(\tau/v)$	
m	$\forall v\varphi$	$\forall$I, l

provided τ does not occur in φ or in any undischarged assumption.

k	$\exists v\varphi$	
l	$\varphi(a/x) \to \psi$	
n	ψ	E$\exists$ k, l

where a occurs neither in ψ nor in $\exists v\varphi$ nor in an undischarged assumption.

Exercises 1 *Which lines of the following proofs are wrong, and why?*

(1).

1.$AxAyRxy$: *Assumption*

2.Rab : $EA1$

(2).

1.Rac : *Assumption*

2.$ExEyRxy$: $IE1$

(3).

1.$AxRxa$: *Assumption*

2.Rba : $EA1$

3.$AyRby$: $IA2$

(4).

1.$Pa \to ExRxa$: *Assumption*

2.$ExPx$: *assumption*

3.$ExRxa$: $EE1, 2$

(5).

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1. Ex Rxb :assumption
2. ┌ Rbb :assumption
3. │ ExRxx :IE2
4. Rbb→ExRxx :I→2-3
5. ExRxx :EE1,4
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Exercises 2 *Prove the following arguments:*

(1). $\forall x(Px \to Rxa), \forall x \neg Rbx \vdash \neg Pb$

(2). $(\forall xPx \wedge \forall xQx) \vdash \forall x(Px \wedge Qx)$

(3). $\forall x(Px \to Rxb), \exists yPy \vdash \exists zRzb$

(4). $\forall x(\exists y(Qy \wedge Rxy) \to Px), \exists x(Sx \wedge \exists y(Qy \wedge Rxy)) \vdash \exists x(Sx \wedge Px)$