PHIL 220: Introduction to Logic

Week 3 Discussion (09/12/2025)

Our Goals for Today

We'll revisit and do more exercises on

1. The syntax (especially drawing the construction trees) of propositional logic.
2. The semantics (especially truth tables) of propositional logic.

1 Syntax

The syntax of the language of propositional logic has two parts:

- Vocabulary: three kinds of symbols for building sentences: propositional variables, truth-functional connectives, and parentheses.
- Grammar: How symbols can be combined to form legitimate sentences.

Identify if the following are sentences of the language of propositional logic. If not, explain why. If they are, draw the construction tree for each.

Exercise

1. $(\neg(\phi \wedge \neg\psi) \rightarrow (\neg\phi \vee \psi))$
2. $(p \wedge (\neg q \vee (r)))$
3. $(\neg\neg\neg p)$
4. $((p \wedge (q \rightarrow p)) \rightarrow (q \rightarrow (p \vee q)))$
5. $(((p \wedge q) \rightarrow \neg(\neg q \vee \neg p)) \rightarrow p)$

2 Semantics

Symbols themselves are meaningless; they are just marks on paper. To give symbols meanings, we use truth values to interpret the propositional variables and truth functions to interpret the

connectives of the language.

What are the truth connectives of the following sentences with their truth tables?

Exercise

p	q	$(p$	**?**	$q)$
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

p	q	**(?**	p	**?**	$q)$
T	T	F	T	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	F	T	F

Draw the truth tables for the following sentences.

Exercise

1. (1). $(p \wedge q) \vee (\neg p \wedge \neg q)$; (2). $(p \vee q) \rightarrow (p \wedge q)$

2. $((p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$

3. $(p \rightarrow q) \wedge (s \wedge \neg(\neg q \rightarrow \neg p))$

4. $\{p, \neg r, (p \wedge q) \rightarrow \neg r, q \rightarrow (p \vee r)\}$

3 Checklist for Week 3: The language of propositional logic

What you need to know	**No idea**	**Meh**	**Got it!**
Quotation: use and mention			
Syntax: vocabulary (what symbols do we have?), and grammar (what count as sentences?)			
The notational convention (for parentheses)			
Semantics: truth tables, tautology, contradiction, equivalence, and consistency			