PHIL 220: Introduction to Logic

Week 11 Discussion (11/07/2025)

Today: Do more exercises on translations and models of predicate logic.

Exercise 1: Translation

Domain: people

$P\_$ **: _ is a philosopher.**

$Q\_$ **: _ is a logician.**

$R\_\ \_$ **: _ admires _.**

$S\_\ \_$ **: _ is greater than _.**

a **: Plato**

b**: Aristotle**

c**: Hume**

1. Plato admires only philosophers.

Solution: $\forall x(Rax \to Px)$

2. No philosopher admires a logician.

Solution: $\neg\exists x(Px \land \exists y(Qy \land Rxy))$ or $\forall x(Px \to \forall y(Qy \to \neg Rxy))$

3. Not every logician admires Plato.

Solution: $\neg\forall x(Qx \to Rxa)$ or $\exists x(Qx \land \neg Rxa)$

4. Some logicians admire every philosopher.

Solution: $\exists x(Qx \land \forall y(Py \to Rxy))$

5. No logician admires every philosopher.

Solution: $\neg\exists x(Qx \land \forall y(Py \to Rxy))$ or $\forall x(Qx \to \exists y(Py \land \neg Rxy))$

6. Anyone who admires every philosopher is a logician.

Solution: $\forall x(\forall y(Py \to Rxy) \to Qx)$

7. Hume neither admires Plato nor admires anyone greater than Aristotle.

Solution: $\neg Rca \wedge \neg\exists x(Sxb \wedge Rcx)$ or $\neg Rca \wedge \forall x(Sxb \rightarrow \neg Rcx)$

8. No philosopher greater than Hume admires Aristotle.

Solution: $\neg\exists x((Px \wedge Sxc) \wedge Rxb)$ or $\forall x((Px \wedge Sxc) \rightarrow \neg Rxb)$

Exercise 2: Models

1. Consistency

(a). $Raa, (Raa \rightarrow Qa), (\neg Pa \wedge Qb), Rbc, (Rcb \rightarrow (Pc \wedge Qb)), \neg Rac$

Solution: Consistent.

Domain: $\{0, 1, 2\}$

$P(\_) : \{2\}$

$Q(\_) : \{0, 1\}$

$R(\_, \_) : \{[0, 0], [1, 2]\}$

$a : 0, b : 1, c : 2$

(b). $Qa \rightarrow (Raa \rightarrow Pa), \neg Pa, \forall x Rxa$

Solution: Consistent.

Domain: $\{0, 1\}$

$P(\_) : \{1\}$

$Q(\_) : \emptyset$

$R(\_, \_) : \{[0, 0], [1, 0]\}$

$a : 0$

(c). $\forall x Px \rightarrow \forall x Qx, \neg\forall x(Px \rightarrow Qx)$

Solution: Consistent.

Domain: $\{0, 1\}$

$P(\_) : \{0\}$

$Q(\_) : \emptyset$

2. Non-equivalence

(a). $\exists x \neg Qx, \forall x Qx$

Solution: Not equivalent (clearly contradictory).

Domain: $\{0\}$

For $\exists x \neg Qx$ true and $\forall x Qx$ false: $Q(\_) : \emptyset$

For $\forall x Qx$ true and $\exists x \neg Qx$ false: $Q(\_) : \{0\}$

(b). $(\forall x Px \vee \forall x Qx), \forall x(Px \vee Qx)$

Solution: Not equivalent.

Domain: $\{0, 1\}$

$P(\_) : \{0\}$

$Q(\_) : \{1\}$

This makes $\forall x(Px \vee Qx)$ true, but $(\forall x Px \vee \forall x Qx)$ false.

(c). $\forall x(Px \rightarrow Gx), \exists x Px \rightarrow \exists x Gx$

Solution: Not equivalent.

Domain: $\{0, 1, 2\}$

$P(\_) : \{0, 1, 2\}$

$G(\_) : \{1\}$

3. Invalidity

(a). $\forall x \exists y \exists z(Pz \wedge Rxzy), Rabc \vdash (Racb \wedge \neg Pb)$

Solution: Invalid.

Domain: $\{0, 1, 2\}$

$P(\_) : \{0, 1, 2\}$

$R(\_, \_, \_) : \{[0, 1, 2], [0, 2, 1]\}$

$a : 0, b : 1, c : 2$

(b). $\exists x \exists y Rxy, \forall x \forall y (Rxy \to Ryx), \forall x \forall y \forall z ((Rxy \land Ryz) \to Rxz) \vdash \forall x \forall y Rxy$

Solution: Invalid.

Domain: $\{0, 1, 2\}$

$R(\_, \_) : \{[0, 0], [1, 1], [2, 2]\}$

(c). $\neg \forall x (Px \to Qx), \forall x (Qx \to Px) \vdash \exists x (\neg Px \land \neg Qx)$

Solution: Invalid.

Domain: $\{0, 1\}$

$P(\_) : \{0, 1\}$

$Q(\_) : \{0\}$