PHIL 220: Introduction to Logic

Week 10 Discussion (10/31/2025)

Notes: Assignment 5 is due by midnight next Monday (11/03).

Today: Do more exercises on the syntax of predicate logic and translation.

Exercise 1: Atomic formulas and Formulas

Which of the expressions below are atomic formulas, or formulas, or neither?

1. (Fa)
2. $F(x)$
3. $\neg(Bxyz)$
4. $Qaxcy$
5. $\forall x(Gx \rightarrow Hx)$
6. $(\forall xGx \rightarrow Hx)$
7. $FFab$
8. $\neg Gxy$
9. $\exists yGx$
10. $\exists x \forall yHyz$
11. $\forall x \neg(Fx \wedge Gb)$
12. $\exists z(Hz \wedge \neg Gz)$

Exercise 2: Free and Bound Variables

For each formula below

- Identify any **free** variable occurrences (state the variable and where it is).
- Identify any **bound** variable occurrences.
- State whether the formula is an **open formula** or a **sentence**.

1. $Qabcdaxc$
2. $Px \wedge Qay$
3. $\forall x(Px \wedge Qay)$
4. $\exists zLza$
5. $\forall yPy \wedge Qy$
6. $\forall x \exists y(Rxy \rightarrow Pz)$
7. $\forall x \forall yRzxy \wedge \exists zFz$
8. $\neg \forall x(Px \vee \exists yGxy)$
9. $\forall xQx \rightarrow \exists y(Gxy \vee Hy)$
10. $\forall x \exists yRabc$

Exercise 3: Translation

Translation key:

Domain: people

$P\_$: _ is a philosopher.

$Q\_$: _ is a logician.

$R\_\ \_$: _ admires _.

a : Plato

b: Aristotle

1. Not every philosopher is a logician.
2. Aristotle is a logician, but not every philosopher is.
3. Some philosophers are logicians and some philosophers are not logicians.
4. Not every philosopher admires some logician.
5. If Plato is a logician, then he is a philosopher only if some logicians are philosophers.
6. Plato is a philosopher, and Aristotle is a philosopher too, but some logicians admire neither of them.
7. No logicians are philosophers, unless Plato is a logician.
8. Plato is not a logician, unless every philosopher is a logician.