PHIL 220: Introduction to Logic

Week 12 Discussion (11/14/2025)

Translation

> (#3) France borders only European countries.
>
> (#8) Anyone louder than every soprano is a tenor.
>
> (#10) The Rose Bowl is neither close to USC nor close to any stadium at which USC plays.

Models

> **Invalidity**
>
> **(#28)** $\exists x \exists y Rxy$, $\forall x \forall y (Rxy \rightarrow Ryx)$, $\forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz) \vdash \forall x \forall y Rxy$
>
> Domain: $0, 1$; $R : [0, 0]$
>
> **(#22)** $\neg \forall x (Px \rightarrow Qx)$, $\forall x (Qx \rightarrow Px) \vdash \exists x (\neg Px \wedge \neg Qx)$
>
> Domain: 1
>
> P: 1
>
> Q: (empty)
>
> **(#25)** $\forall x \exists y \exists z (Pz \wedge Rxzy)$, $Rabc \vdash (Racb \wedge \neg Pb)$
>
> Domain: $0, 1, 2$
>
> P: $1, 2$
>
> R: $[0, 1, 2], [1, 1, 1], [2, 2, 2]$
>
> a: 0; b: 1; c: 2
>
> **(#27)** $\forall x \neg Rxx$, $\forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz) \vdash \forall x \forall y \neg Rxy$
>
> Domain: $0, 1$; R: $[0, 1]$

Extra Problems

1. $\forall x \exists y Rxy, \forall x \forall y (Rxy \rightarrow Ryx) \vdash \forall x \forall y (Rxy \vee Ryx)$

Domain: $0, 1; R: [0, 0], [1, 1]$

2. $\exists x (Px \wedge Qx), \forall x \forall y ((Px \wedge Qy) \rightarrow Rxy) \vdash \forall x Rxx$

Domain: $0, 1$

$P: 0; Q: 0; R: [0, 0]$

3. $\exists x (Px \wedge \forall y Rxy), \neg \forall x \forall y Rxy \vdash \neg \forall x Px$

Domain: $0, 1$

$P: 0, 1; R: [1, 1], [1, 0]$

4. $\forall x \forall y (Rxy \rightarrow \forall z (Szy \rightarrow Szx)), \forall x \forall y (Sxy \rightarrow \forall z (Rzy \rightarrow Rzx)), Rab \wedge Sbc \vdash \neg \exists x (Rxb \wedge Sxa)$

Domain: $0, 1, 2;$

$R: [0, 1], [2, 1]; S: [1, 2], [2, 0]$

$a: 0; b: 1; c: 2$

5. $\exists x (Qx \wedge \exists y (Py \wedge Rxy)), Pa \vdash \neg Qa \rightarrow \exists x (Qx \wedge Rxa)$

Domain: $0, 1;$

$P: 0, 1; Q: 1; R: [1, 1]$

$a: 0; b: 1$