

PHIL 220: Introduction to Logic

Week 13 Discussion (11/21/2025)

Exercises 1 Which lines of the following proofs are wrong, and why?

(1). 2 is wrong: it's not allowed to apply EA twice within one step.

1. $AxAyRxy : Assumption$

2. $Rab : EA1$

(3). 3 is wrong: a is in an undischarged assumption so using IA to a is not allowed.

1. $AxRxa : Assumption$

2. $Rba : EA1$

3. $AyRby : IA2$

(2). 2 is wrong: it's not allowed to apply IE twice within one step.

1. $Rac : Assumption$

2. $ExEyRxy : IE1$

(4). 3 is wrong: a is in $ExRxa$ so using EE to a is not allowed.

1. $Pa \rightarrow ExRxa : Assumption$

2. $ExPx : assumption$

3. $ExRxa : EE1, 2$

(5). 5 is wrong because b is in an undischarged assumption. Using EE to b is not allowed

1. $Ex Rxb : assumption$
2. $\neg Rbb : assumption$
3. $ExRxx : IE2$
4. $Rbb \rightarrow ExRxx : I \rightarrow 2-3$
5. $ExRxx : EE1, 4$

Exercises 2 Prove the following arguments:

(1). $\forall x(Px \rightarrow Rxa), \forall x \neg Rbx \vdash \neg Pb$

$\forall x(Px \rightarrow Rxa), \forall x \neg Rbx$	✓
1. $Ax(Px \rightarrow Rxa) : assumption$	+
2. $Ax \neg Rbx : assumption$	+
3. $Pb \rightarrow Rba : EA 1$	+
4. $\neg Pb : assumption$	+
5. $\neg Rba : EA2$	+
6. $\neg \neg Rba : E \neg \neg 5$	+
7. $! ? : E \neg \neg 5, 6$	+
8. $\neg Pb : I \neg \neg 4-7$	+

(2). $(\forall x Px \wedge \forall x Qx) \vdash \forall x(Px \wedge Qx)$

$(\forall x Px \wedge \forall x Qx) \vdash \forall x(Px \wedge Qx)$	✓
1. $\forall x Px \wedge \forall x Qx$:assumption	+
2. $\forall x Px$:E \wedge 1	+
3. $\exists x Qx$:EA2	+
4. $\forall x Qx$:E \wedge 1	+
5. $\exists x Qx$:EA4	+
6. $\forall x Px \wedge \forall x Qx$:I \wedge 3,5	+
7. $\forall x(Px \wedge Qx)$:IA6	+

(3). $\forall x(Px \rightarrow Rx b), \exists y Py \vdash \exists z Rz b$

$\forall x(Px \rightarrow Rx b), \exists y Py \vdash \exists z Rz b$	✓
1. $\forall x(Px \rightarrow Rx b)$:assumption	+
2. $\exists y Py$:assumption	+
3. $\exists x Rx b$:EA1	+
4. $\exists x Rx b$:assumption	+
5. $\exists x Rx b$:E \rightarrow 3,4	+
6. $\exists x Rx b$:IE 5	+
7. $\exists x Rx b$:I \rightarrow 4-6	+
8. $\exists x Rx b$:EE2,7	+

(4). $\forall x(\exists y(Qy \wedge Rxy) \rightarrow Px), \exists x(Sx \wedge \exists y(Qy \wedge Rxy)) \vdash \exists x(Sx \wedge Px)$

$\forall x(\exists y(Qy \wedge Rxy) \rightarrow Px), \exists x(Sx \wedge \exists y(Qy \wedge Rxy))$	✓
$\vdash \exists x(Sx \wedge Px)$	
1. $\forall x(\exists y(Qy \wedge Rxy) \rightarrow Px)$:assumption	+
2. $\exists y(Qy \wedge Rxy) \rightarrow Px$:assumption	+
3. $\exists y(Qy \wedge Rxy)$:assumption	+
4. $\exists y(Qy \wedge Rxy)$:E \wedge 3	+
5. $\exists y(Qy \wedge Rxy) \rightarrow Pa$:EA1	+
6. $\exists y(Qy \wedge Rxy)$:E \rightarrow 4,5	+
7. $\exists y(Qy \wedge Rxy)$:E \wedge 3	+
8. $\exists y(Qy \wedge Rxy)$:I \wedge 6,7	+
9. $\exists y(Qy \wedge Rxy)$:IE8	+
10. $(\exists y(Qy \wedge Rxy)) \rightarrow \exists x(Sx \wedge Px)$:I \rightarrow 3-9	+
11. $\exists x(Sx \wedge Px)$:EE2,10	+