PHIL 220: Introduction to Logic

Week 11 Discussion (11/07/2025)

Today: Do more exercises on translations and models of predicate logic.

Exercise 1: Translation

Domain: people

P**_ : _ is a philosopher.**

Q**_ : _ is a logician.**

R**_ _ : _ admires _.**

S**_ _ : _ is greater than _.**

a **: Plato**

b**: Aristotle**

c**: Hume**

1. Plato admires only philosophers.
2. No philosopher admires a logician.
3. Not every logician admires Plato.
4. Some logicians admire every philosopher.
5. No logician admires every philosopher.
6. Anyone who admires every philosopher is a logician.
7. Hume neither admires Plato nor admires anyone greater than Aristotle.
8. No philosopher greater than Hume admires Aristotle.

Exercise 2: Models

1. Consistency

(a). $Raa, (Raa \rightarrow Qa), (\neg Pa \wedge Qb), Rbc, (Rcb \rightarrow (Pc \wedge Qb)), \neg Rac$

(b). $Qa \rightarrow (Raa \rightarrow Pa), \neg Pa, \forall x Rxa$

(c). $\forall x Px \rightarrow \forall x Qx, \neg \forall x (Px \rightarrow Qx)$

2. Non-equivalence

(a). $\exists x \neg Qx, \forall x Qx$

(b). $(\forall x Px \vee \forall x Qx), \forall x (Px \vee Qx)$

(c). $\forall x (Px \rightarrow Gx), \exists x Px \rightarrow \exists x Gx$

3. Invalidity

(a). $\forall x \exists y \exists z (Pz \wedge Rxzy), Rabc \vdash (Racb \wedge \neg Pb)$

(b). $\exists x \exists y Rxy, \forall x \forall y (Rxy \rightarrow Ryx), \forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz) \vdash \forall x \forall y Rxy$

(c). $\neg \forall x (Px \rightarrow Qx), \forall x (Qx \rightarrow Px) \vdash \exists x (\neg Px \wedge \neg Qx)$