PHIL 220: Introduction to Logic

Week 10 Discussion (10/31/2025)

Exercise 1: Atomic formulas and Formulas

Which of the expressions below are atomic formulas, or formulas, or neither?

1. (Fa)
2. $F(x)$
3. $\neg(Bxyz)$
4. $Qaxcy$
5. $\forall x(Gx \rightarrow Hx)$
6. $(\forall xGx \rightarrow Hx)$
7. $FFab$
8. $\neg Gxy$
9. $\exists yGx$
10. $\exists x\forall yHyz$
11. $\forall x\neg(Fx \wedge Gb)$
12. $\exists z(Hz \wedge \neg Gz)$

Solutions:

1. (Fa) — **Neither**
2. $F(x)$ — **Neither**
3. $\neg(Bxyz)$ — **Neither**
4. $Qaxcy$ — **Both**
5. $\forall x(Gx \rightarrow Hx)$ — **Formula**
6. $(\forall xGx \rightarrow Hx)$ — **Formula**
7. $FFab$ — **Neither**
8. $\neg Gxy$ — **Formula**
9. $\exists yGx$ — **Formula**
10. $\exists x\forall yHyz$ — **Formula**
11. $\forall x\neg(Fx \wedge Gb)$ — **Formula**
12. $\exists z(Hz \wedge \neg Gz)$ — **Formula**

Exercise 2: Free and Bound Variables

For each formula below

- Identify any **free** variable occurrences (state the variable and where it is).
- Identify any **bound** variable occurrences.
- State whether the formula is an **open formula** or a **sentence**.

1. $Qabcdaxc$
2. $Px \land Qay$
3. $\forall x(Px \land Qay)$
4. $\exists zLza$
5. $\forall yPy \land Qy$
6. $\forall x\exists y(Rxy \to Pz)$
7. $\forall x\forall yRzxy \land \exists zFz$
8. $\neg\forall x(Px \lor \exists yGxy)$
9. $\forall xQx \to \exists y(Gxy \lor Hy)$
10. $\forall x\exists yRabc$

Solutions:

1. $Qabcdaxc$ — **Free**: x. **Bound**: none. **Open formula**.
2. $Px \land Qay$ — **Free**: x (in Px), y (in Qay). **Bound**: none. **Open formula**.
3. $\forall x(Px \land Qay)$ — **Free**: y (in Qay). **Bound**: x (in Px). **Open formula**.
4. $\exists zLza$ — **Free**: none. **Bound**: z (in Lza). **Sentence**.
5. $\forall yPy \land Qy$ — **Free**: y (in Qy). **Bound**: y (in Py). **Open formula**.
6. $\forall x\exists y(Rxy \to Pz)$ — **Free**: z (in Pz). **Bound**: x, y (in Rxy). **Open formula**.
7. $\forall x\forall yRzxy \land \exists zFz$ — **Free**: y (in $Rzxy$). **Bound**: x, y (in $Rzxy$), z (in Fz). **Open formula**.
8. $\neg\forall x(Px \lor \exists yGxy)$ — **Free**: none. **Bound**: x (in Px and Gxy), y (in Gxy). **Sentence**.
9. $\forall xQx \to \exists y(Gxy \lor Hy)$ — **Free**: x (in Gxy). **Bound**: x (in Qx), y (in Gxy and Hy). **Open formula**.
10. $\forall x\exists yRabc$ — **Free**: none. **Bound**: x, y (vacuously). **Sentence**.

Exercise 3: Translation

Translation key:

Domain: people

$P\_$: _ is a philosopher.

$Q\_$: _ is a logician.

$R\_\ \_$: _ admires _.

a : Plato

b: Aristotle

1. Not every philosopher is a logician.

   Solution: $\neg\forall x(Px \rightarrow Qx)$ or equivalently $\exists x(Px \wedge \neg Qx)$

2. Aristotle is a logician, but not every philosopher is.

   Solution: $Qb \wedge \neg\forall x(Px \rightarrow Qx)$ or $Qb \wedge \exists x(Px \wedge \neg Qx)$

3. Some philosophers are logicians and some philosophers are not logicians.

   Solution: $\exists x(Px \wedge Qx) \wedge \exists y(Py \wedge \neg Qy)$

4. Not every philosopher admires some logician.

   Solution: $\neg\forall x(Px \rightarrow \exists y(Qy \wedge Rxy))$ or $\exists x(Px \wedge \neg\exists y(Qy \wedge Rxy))$

5. If Plato is a logician, then he is a philosopher only if some logicians are philosophers.

   Solution: $Qa \rightarrow (Pa \rightarrow \exists x(Qx \wedge Px))$

6. Plato is a philosopher, and Aristotle is a philosopher too, but some logicians admire neither of them.

   Solution: $Pa \wedge Pb \wedge \exists x(Qx \wedge \neg Rxa \wedge \neg Rxb)$

7. No logicians are philosophers, unless Plato is a logician.

   Solution: $\neg Qa \rightarrow \neg\exists x(Qx \wedge Px)$ or equivalently $\neg Qa \rightarrow \forall x(Qx \rightarrow \neg Px)$

8. Plato is not a logician, unless every philosopher is a logician.

   Solution: $\neg\forall x(Px \rightarrow Qx) \rightarrow \neg Qa$ or equivalently $Qa \vee \forall x(Px \rightarrow Qx)$