

# PHIL 220: Introduction to Logic

## Week 13 Discussion (11/21/2025)

**Exercises 1** Which lines of the following proofs are wrong, and why?

(1). **2 is wrong: it's not allowed to apply EA twice within one step.**

1.  $Ax Ay Rxy$  : Assumption

2.  $Rab$  : EA1

(3). **3 is wrong:  $a$  is in an undischarged assumption so using IA to  $a$  is not allowed.**

1.  $Ax Rxa$  : Assumption

2.  $Rba$  : EA1

3.  $Ay Rby$  : IA2

(2). **2 is wrong: it's not allowed to apply IE twice within one step.**

1.  $Rac$  : Assumption

2.  $Ex Ey Rxy$  : IE1

(4). **3 is wrong:  $a$  is in  $Ex Rxa$  so using EE to  $a$  is not allowed.**

1.  $Pa \rightarrow Ex Rxa$  : Assumption

2.  $Ex Px$  : assumption

3.  $Ex Rxa$  : EE1, 2

(5). **5 is wrong because  $b$  is in an undischarged assumption. Using EE to  $b$  is not allowed**

1.  $Ex Rxb$  : assumption

2.  $Rbb$  : assumption

3.  $Ex Rxx$  : IE2

4.  $Rbb \rightarrow Ex Rxx$  : I  $\rightarrow$  2-3

5.  $Ex Rxx$  : EE1, 4

**Exercises 2** Prove the following arguments:

(1).  $\forall x(Px \rightarrow Rxa), \forall x \neg Rbx \vdash \neg Pb$

$\forall x(Px \rightarrow Rxa), \forall x \neg Rbx \vdash \neg Pb$		✓
1. $Ax(Px \rightarrow Rxa)$ : assumption		+
2. $Ax \neg Rbx$ : assumption		+
3. $Pb \rightarrow Rba$ : EA 1		+
4. $Pb$ : assumption		+
5. $Rba$ : E $\rightarrow$ 3, 4		+
6. $\neg Rba$ : EA2		+
7. $!?$ : E $\sim$ 5, 6		+
8. $\neg Pb$ : I $\sim$ 4-7		+

(2).  $(\forall x Px \wedge \forall x Qx) \vdash \forall x(Px \wedge Qx)$

$(\forall x Px \wedge \forall x Qx) \vdash \forall x (Px \wedge Qx)$	✓
1. $Ax \ Px \wedge Ax \ Qx$ :assumption	+
2. $Ax \ Px$ :E $\wedge$ 1	+
3. $Pa$ :EA2	+
4. $Ax \ Qx$ :E $\wedge$ 1	+
5. $Qa$ :EA4	+
6. $Pa \wedge Qa$ :I $\wedge$ 3,5	+
7. $Ax \ (Px \wedge Qx)$ :IA6	+

(3).  $\forall x(Px \rightarrow Rxb), \exists yPy \vdash \exists zRzb$

$\forall x(Px \rightarrow Rxb), \exists yPy \vdash \exists zRzb$	✓
1. $Ax(Px \rightarrow Rxb)$ :assumption	+
2. $Ey \ Py$ :assumption	+
3. $Pa \rightarrow Rab$ :EA1	+
4. $Pa$ :assumption	+
5. $Rab$ :E $\rightarrow$ 3,4	+
6. $EzRzb$ :IE 5	+
7. $Pa \rightarrow EzRzb$ :I $\rightarrow$ 4-6	+
8. $EzRzb$ :EE2,7	+

(4).  $\forall x(\exists y(Qy \wedge Rxy) \rightarrow Px), \exists x(Sx \wedge \exists y(Qy \wedge Rxy)) \vdash \exists x(Sx \wedge Px)$

$\forall x(\exists y(Qy \wedge Rxy) \rightarrow Px), \exists x(Sx \wedge \exists y(Qy \wedge Rxy)) \vdash \exists x(Sx \wedge Px)$	✓
1. $Ax(\exists y(Qy \wedge Rxy) \rightarrow Px)$ :assumption	+
2. $Ex(Sx \wedge \exists y(Qy \wedge Rxy))$ :assumption	+
3. $Sa \wedge \exists y(Qy \wedge Rxy)$ :assumption	+
4. $\exists y(Qy \wedge Ray)$ :E $\wedge$ 3	+
5. $\exists y(Qy \wedge Ray) \rightarrow Pa$ :EA1	+
6. $Pa$ :E $\rightarrow$ 4,5	+
7. $Sa$ :E $\wedge$ 3	+
8. $Sa \wedge Pa$ :I $\wedge$ 6,7	+
9. $Ex(Sx \wedge Px)$ :IE8	+
10. $(Sa \wedge \exists y(Qy \wedge Ray)) \rightarrow Ex(Sx \wedge Px)$ :I $\rightarrow$ 3-9	+
11. $Ex(Sx \wedge Px)$ :EE2,10	+