PHIL 220: Introduction to Logic

Week 6 Discussion (10/03/2025)

Reminders:

Assignment 3 is due by midnight next Monday (10/06/2025).

Don't forget to post your questions on the weekly question sheet.

Goals for today:

Get more familiar with natural deduction proofs involving conjunctions and conditionals.

Truth tables vs. Natural deduction

A truth table is a *semantic* method. It checks the *meaning* of the statements by testing every single possible truth value assignment.

A proof is a *syntactic* method. It focuses on the *form* or *structure* of the argument, using rules of inference to derive a conclusion.

Two rules for conjunctions:

Conjunction Elimination (E∧) 1:

m	$p \wedge q$	
n	p	E∧ m

Conjunction Elimination (E∧) 2:

m	$p \wedge q$	
n	q	E∧ m

Conjunction Introduction (I∧):

l	p	
m	q	
n	$p \wedge q$	I∧ l, m

Two rules for conditionals:

Conditional Elimination (E→) 1:

l	$p \rightarrow q$	
m	p	
n	q	E→ l, m

Conjunction Introduction (I→):

l	p	
m	q	
n	$p \rightarrow q$	I→ l, m

Identify the missing rules in the following proofs:

Exercise 2

1:

1	r	Assumption	
2	$p \land q$	Assumption	
3	q	? 2	
4	$q \land r$	? 1,3	

2:

1	$p \land q$	Assumption
2	$p \to (q \to r)$	Assumption
3	p	? 1
4	$q \to r$	? 2,3
5	q	? 1
6	r	? 4,5

3:

1	$p \land q$	Assumption	
2	$r \land s$	Assumption	
3	p	? 1	
4	s	? 2	
5	$p \land s$	? 3,4	
6	r	? 2	
7	$(p \land s) \land r$	? 5,6	

4:

1	$p \to (q \land r)$	Assumption
2	p	Assumption
3	$q \land r$	? 1,2
4	r	? 3
5	q	? 3
6	$r \land q$	? 4,5
7	$p \to (r \land q)$	? 2-6

Prove the following arguments:

Exercise 3

(a). $(p \land q) \land (r \land s) \vdash s$

(b). $(p \land q) \land (r \land s) \vdash q \land s$

(c). $\vdash p \to (q \to p)$

(d). $p \to (q \to r) \vdash q \to (p \to r)$