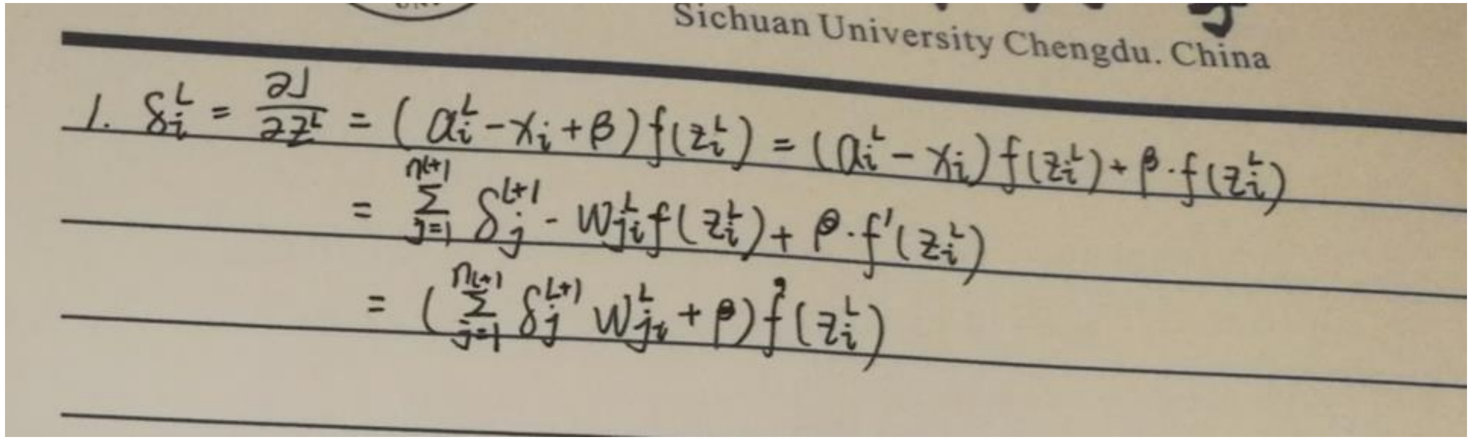
1. Given the cost function

$$J = \frac{1}{2}\sum_{i=1}^{m}\left(a_i^L - x_i\right)^2 + \beta \cdot \sum_{i=1}^{n_l} a_i^l$$

Prove that

$$\delta_i^l = \dot{f}\left(z_i^l\right) \cdot \left(\sum_{j=1}^{n_{l+1}} w_{ji}^l \delta_j^{l+1} + \beta\right)$$

Answer with digital formulation or take a picture of your handcraft manuscript.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1. $\delta_i^L = \frac{\partial J}{\partial z_i^L} = (a_i^L - x_i + \beta) f(z_i^L) = (a_i^L - x_i) f(z_i^L) + \beta \cdot f(z_i^L)$

$= \sum_{j=1}^{n_{L+1}} \delta_j^{L+1} \cdot w_{ji}^L f(z_i^L) + \beta \cdot f'(z_i^L)$

$= \left(\sum_{j=1}^{n_{L+1}} \delta_j^{L+1} w_{ji}^L + \beta\right) \dot{f}(z_i^L)$

2. Given the optimization problem

$$\begin{cases} \max \sum_{i=1}^{n} w_{ij}^{(1)} x_j \\ s.t. \sum_{i=1}^{n} x_i^2 \leq 1 \end{cases}$$

Prove that

$$x_j = \frac{w_{ij}^{(1)}}{\sqrt{\sum_{j=1}^{n} \left(w_{ij}^{(1)}\right)^2}}, (j = 1, \cdots, n)$$

Answer with digital formulation or take a picture of your handcraft manuscript.

2. 令 $a = \sqrt{\sum_{j=1}^{n} W_{ij}^{(1)} x_j}$，则求 $\max \sum_{j=1}^{n} x_j W_{ij}^{(1)}$ 等价 $\frac{\max \sum_{j=1}^{n} x_j W_{ij}^{(1)}}{a}$

$$\frac{1}{a} \sum_{j=1}^{n} x_j W_{ij}^{(1)} \leq \frac{1}{2a} \sum_{j=1}^{n} x_j^2 + W_{ij}^{(1)2}$$

$\Rightarrow \sum_{j=1}^{n} x_j \frac{W_{ij}}{a}$ ① 要取①式最大，则 $x_j = \frac{W_{ij}}{a}$，即 $x = \frac{W_{ij}}{\sum_{j=1}^{n} x_j W_{ij}^{(1)}}$