Simple Linear Regression

Introduction

Regression analysis is commonly used for modeling the relationship between a single dependent variable Y and one or more predictors. When we have one predictor, we call this "simple" linear regression:

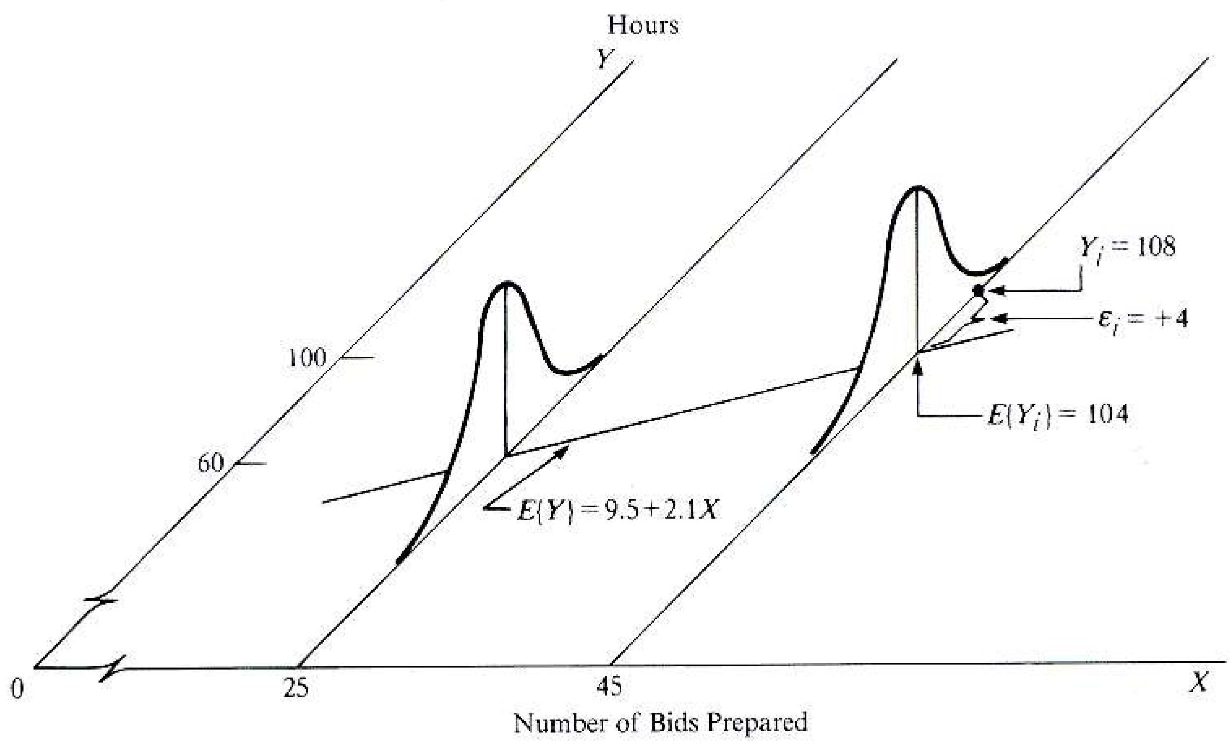
$$E[Y] = \beta_0 + \beta_1 X$$

That is, the ***expected value*** of Y is a straight-line function of X. The betas are selected by choosing the line that minimizing the squared distance between each Y value and the line of best fit. The betas are chose such that they minimize this expression:

$$\sum_i (y_i - (\beta_0 + \beta_1 X))^2$$

An instructive graphic I found on the Internet

FIGURE 1.6 Illustration of Simple Linear Regression Model (1.1).



Source: http://www.unc.edu/~nielsen/soci709/m1/m1005.gif

When we have more than one predictor, we call it **multiple linear regression**:

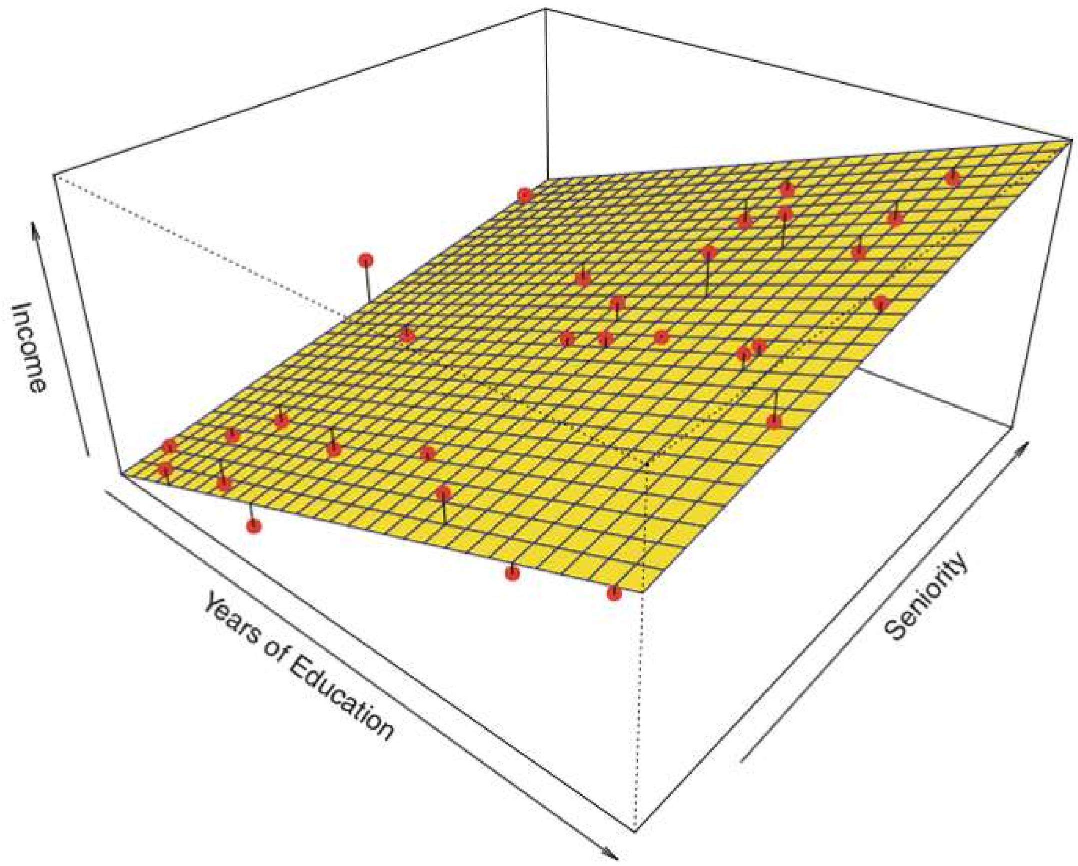
$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_2 X_3 + \ldots + \beta_k X_k$$

The **fitted** values (i.e., the **predicted** values) are defined as those values of Y that are generated if we plug our X values into our fitted model.

The **residuals** are the fitted values minus the actual observed values of Y.

Here is an example of a linear regression with two predictors and one outcome:

Instead of the "line of best fit," there is a "**plane of best fit**."



Source: James et al. *Introduction to Statistical Learning* (Springer 2013)

There are four assumptions associated with a linear regression model:

1. **Linearity**: The relationship between X and the mean of Y is linear.
2. **Homoscedasticity**: The variance of residual is the same for any value of X.
3. **Independence**: Observations are independent of each other.
4. **Normality**: For any fixed value of X, Y is normally distributed.

We will review how to assess these assumptions later in the module.

Let's start with simple regression. In R, models are typically fitted by calling a model-fitting function, in our case **lm()**, with a **"formula" object** describing the model and a **"data.frame" object** containing the variables used in the formula. A typical call may look like

```
> myfunction <- lm(formula, data, …)
```

and it will return a fitted model object, here stored as **myfunction**. This fitted model can then be subsequently printed, summarized, or visualized; moreover, the fitted values and residuals can be extracted, and we can make predictions on new data (values of X) computed using functions such as **summary()**, **residuals()**,**predict()**, etc. Next, we will look at how to fit a simple linear regression.