Puffin: New System of Equations

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Compared to the original POP Puffin paper, now we use the peak magnetic fields are used to scale the system of eqns (as opposed to the *rms* values used previously). Instead of p_2, the energy exchange is modelled directly through the scaled variable $\Gamma_j = \gamma_j/\gamma_r$, so that:

$$\Big[\frac{1}{2}\Big(\frac{\partial^2}{\partial\bar{x}^2}+\frac{\partial^2}{\partial\bar{y}^2}\Big)-\frac{\partial^2}{\partial\bar{z}\partial\bar{z}_2}\Big]A_\perp = -\frac{1}{\bar{n}_p}\frac{\partial}{\partial\bar{z}_2}\sum_{j=1}^{N}\frac{\bar{p}_{\perp j}}{\Gamma_j}(1+\eta p_{2j})\delta^3(\bar{x}_j,\bar{y}_j,\bar{z}_{2j}) \tag{1}$$

$$\frac{d\bar{p}_{\perp j}}{d\bar{z}} = \frac{1}{2\rho}\Big[ib_\perp - \frac{\eta p_{2j}}{\kappa^2}A_\perp\Big] - i\kappa\frac{\bar{p}_{\perp j}}{\Gamma_j}(1+\eta p_{2j})b_z \tag{2}$$

$$\frac{d\Gamma_j}{d\bar{z}} = -\rho\frac{(1+\eta p_{2j})}{\Gamma_j}(\bar{p}_{\perp j}A^*_{\perp j} + c.c.) \tag{3}$$

$$\frac{d\bar{z}_{2j}}{d\bar{z}} = p_{2j} \tag{4}$$

$$\frac{d\bar{x}_j}{d\bar{z}} = \frac{2\rho\kappa}{\sqrt{\eta}\Gamma_j}(1+\eta p_{2j})\Re(\bar{p}_{\perp j}) \tag{5}$$

$$\frac{d\bar{y}_j}{d\bar{z}} = -\frac{2\rho\kappa}{\sqrt{\eta}\Gamma_j}(1+\eta p_{2j})\Im(\bar{p}_{\perp j}). \tag{6}$$

where $\kappa = \dfrac{a_w}{2\rho\gamma_r}$, which is actually the scaled natural focussing wavenumber $\kappa = \sqrt{2}\bar{k}_\beta$.

p_{2j} may be worked out analytically from

$$p_{2j} = \frac{1}{\eta}\Big[\Big(1 - \frac{(1+a_w^2|\bar{p}_{\perp j}|^2)}{\gamma_r^2\Gamma_j^2}\Big)^{-1/2} - 1\Big] \tag{7}$$

at each step. It is NOT output from Puffin.

Outputs from Puffin are the scaled radiation field $A_\perp$, and electron phase space coords $\bar{x}, \bar{y}, \bar{z}_2, \bar{p}_\perp, \Gamma$, and scaled distance through the undulator $\bar{z}$. The normalized electron weights are in the file NormChiDataFile.dat (normailsed to the peak *spatial* (3D) density).

Scaled parameters are:-

$$\begin{aligned}
\bar{z}_{2j} &= \frac{ct_j - z}{l_c}, & \bar{z} &= \frac{z}{l_g}, \\
\bar{p}_\perp &= \frac{p_\perp}{mca_u}, & A_\perp &= \frac{ea_u l_g}{2mc^2\gamma_0^2\rho} E_\perp, \\
(\bar{x}, \bar{y}) &= \frac{(x, y)}{\sqrt{l_g l_c}}, & l_g &= \frac{\lambda_w}{4\pi\rho}, \\
l_c &= \frac{\lambda_r}{4\pi\rho}, & \Gamma_j &= \frac{\gamma_j}{\gamma_0}, \\
\rho &= \frac{1}{\gamma_0}\left(\frac{a_u\omega_p}{4ck_u}\right)^{2/3}, & a_u &= \frac{eB_0}{mck_u}, \\
\kappa &= \frac{a_u}{2\rho\gamma_0}, & b_\perp &= b_x - ib_y,
\end{aligned}$$

B_0 is the peak magnetic field in the wiggler. $\omega_p = \sqrt{e^2 n_p/\epsilon_0 m}$ is the (non-relativistic) plasma frequency, and n_p is the peak spatial number density of the electron beam ($N_e/\delta_x\delta_y\delta_z$). $E_\perp = E_x - iE_y$ are the x and y radiation electric field vectors. γ_0 is the reference energy (Lorentz factor), which is usually taken as the mean beam energy.

The scaled reference velocity,

$$\eta = \frac{1-\beta_{zr}}{\beta_{zr}} = \frac{\lambda_r}{\lambda_u} = \frac{l_c}{l_g}, \tag{8}$$

where β_{zr} is some reference velocity scaled to c, which is sensible (but not, strictly speaking, necessary) to take as the mean longitudinal electron velocity in the wiggler, so that

$$\beta_{zr} = \sqrt{\left(1 - \frac{1}{\gamma_r^2}\left(1 + \bar{a}_u^2\right)\right)^{-1/2} - 1}, \tag{9}$$

where $\bar{a}_u$ is the rms undulator parameter. This defines the velocity at which the electrons travel in the scaled $\bar{z}_2$ frame. More generally, η describes an *ideal* resonance condition - electrons resonant with wavelength λ_r will travel with velocity $p_2 = 1$ through the $\bar{z}_2$ frame.