Function

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Function

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01

Assignments :)

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02

Function

Function

Definition

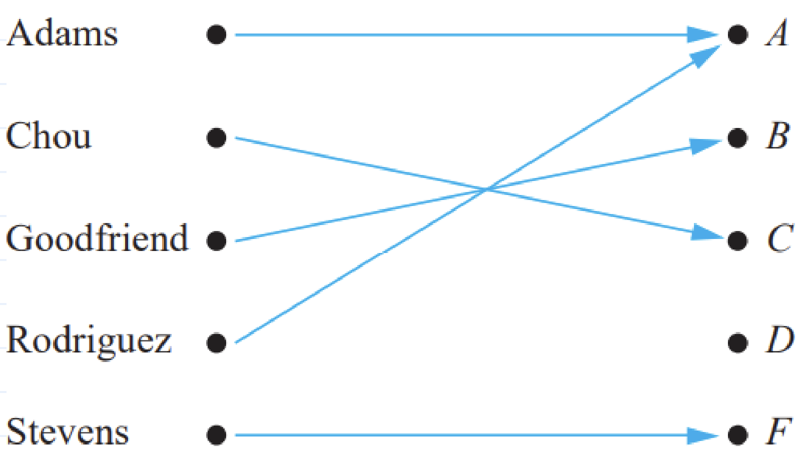
A function f from set A to set B is an assignment of exactly one element of B to each element of A. Written as **f (a) = b**. f is **f : A → B.**

Example

Set of grade is {A, B, C, D, F}.

Grades are A for Adams, C for Chou, B for Goodfriend, A for Rodriguez, and F for Stevens.

This assignment of grades is illustrated in the right side



Function

Terminology

Given f is a function from A to B

f *maps* A to B

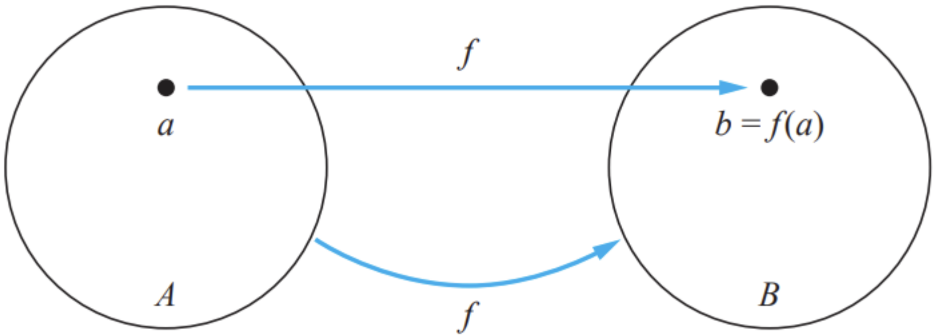
A is the *domain* of f and

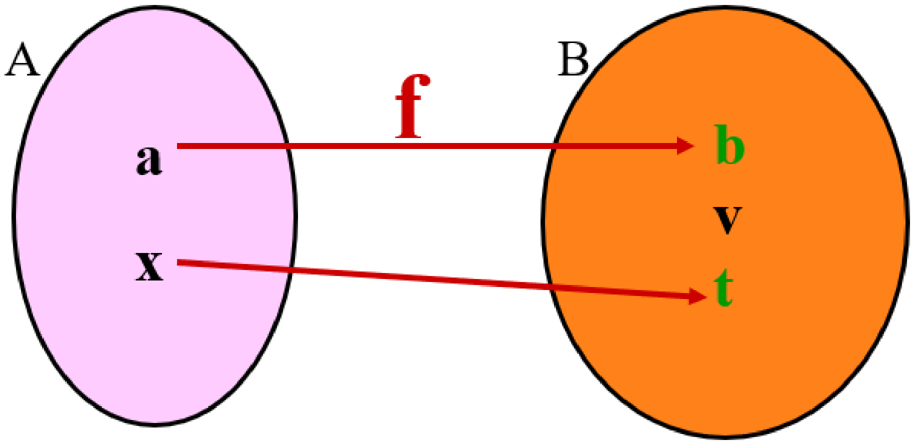
B is the *codomain* of f

Given f(a) = b

b is the *image* of a and a is a *preimage* of b

range or *image*, of f is the set of all images of elements of A





Function

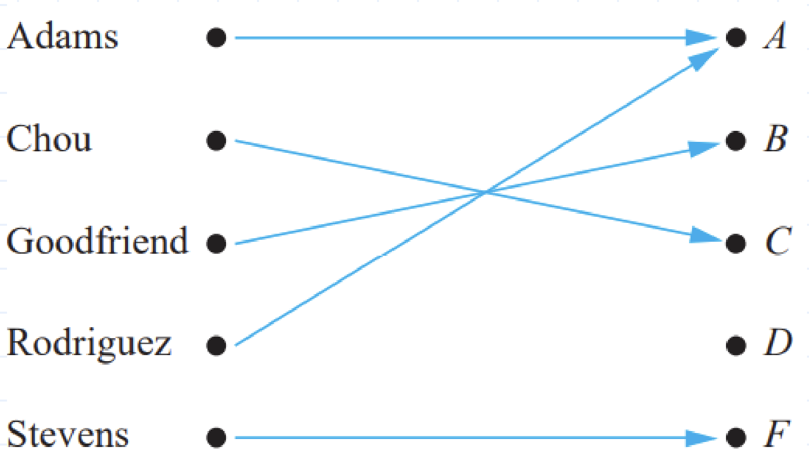
Example

domain is the set

{Adams, Chou, Goodfriend, Rodriguez, Stevens}

codomain is the set {A, B, C, D, F}

range is the set {A, B, C, F}



Example

Let f : Z → Z assign the square of an integer to this integer.

Then, f (x) = x^2, where the domain of f is the set of all integers,

The codomain of f is the set of all integers,

The range of f is the set of all integers that are perfect squares, {0, 1, 4, 9, ...}.

Function

Example

On programming

Java statement

```
int floor(float real){…}
```

C++ function statement

```
int floor (float x){…}
```

domain of the floor function is the set of real numbers

codomain is the set of integers

Function

Definition

Let f1 and f2 be functions from A to R.

Then f1 + f2 and f1f2 are also functions from A to R defined for all x ∈ A by

(f1 + f2)(x) = f1(x) + f2(x)

(f1f2)(x) = f1(x)f2(x)

Example

Let f1 and f2 be functions from R to R such that f1(x) = x^2 and f2(x) = x – x^2.

What are the functions f1 + f2 and f1f2?

(f1 + f2)(x) = f1(x) + f2(x) = x2 + (x – x2) = x

(f1f2)(x) = x^2(x – x^2) = x^3 – x^4

Function

Definition

Let f be a function from A to B and let S be a subset of A.

The image of S under the function f is the subset of B that consists of the images of the elements of S. We denote the image of S by f (S), so

$\mathbf{f(S) = \{t \mid \exists s \in S\ (t = f(s))\}}$ or $\mathbf{\{f(s) \mid s \in S\}}$

Example

Let A = {a, b, c, d, e} and B = {1, 2, 3, 4}

with f (a) = 2, f (b) = 1, f (c) = 4, f (d) = 1, and f (e) = 1.

The image of the subset S = {b, c, d} is the set f (S) = {1, 4}.

Function

One to One Function

A function f is said to be *one-to-one*, or an *injunction*,

if and only if f (a) = f (b) implies that a = b for all a and b in the domain of f.

A function is said to be *injective* if it is one-to-one

Quantifier expression

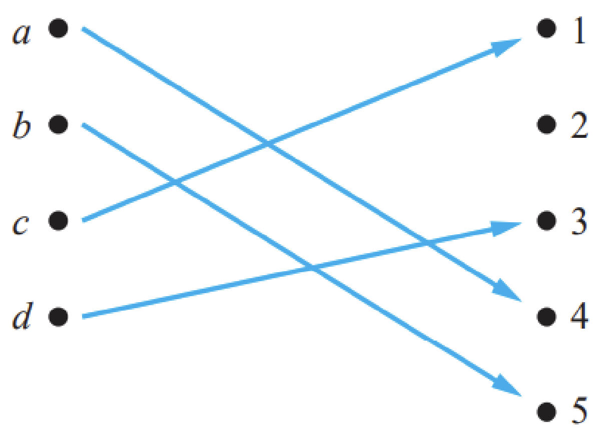
$\forall a \forall b(f(a) = f(b) \rightarrow a = b)$

$\forall a \forall b(a \neq b \rightarrow f(a) \neq f(b))$

Function

Example

Determine whether the function f from {a, b, c, d} to {1, 2, 3, 4, 5} with f (a) = 4, f (b) = 5, f (c) = 1, and f (d) = 3 is one-to-one



Answer: Yes, f is one-to-one because f takes on different values at the four elements of its domain.

Example

Determine whether the function f (x) = x^2 from the set of integers to the set of integers is one-to-one.

Answer: No, f is not one-to-one because for instance, f (1) = f (-1) = 1, but 1 ≠ -1.

Function

Onto Function

A function f from A to B is called *onto*, or a *surjection*,

if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.

A function f is called *surjective* if it is onto

Quantifier expression

$\forall y \exists x (f(x) = y)$

domain for x is the domain of the function

domain for y is the codomain of the function.

Function

Example

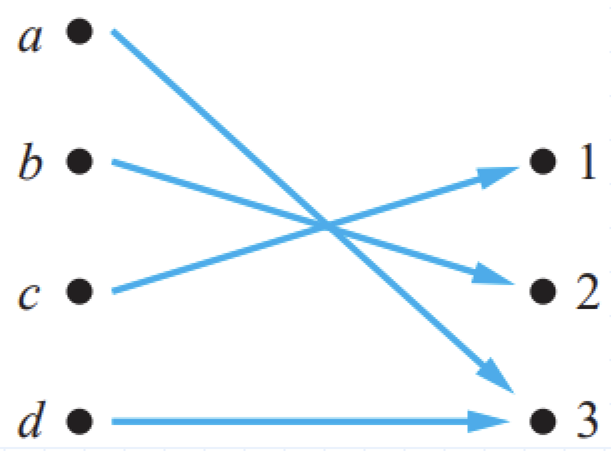
Let f be the function from {a, b, c, d} to {1, 2, 3}

defined by f (a) = 3, f (b) = 2, f (c) = 1, and f (d) = 3.

Is f an onto function?

Answer: Yes, all three elements of the codomain

are images of elements in the domain



Example

Is the function f (x) = x^2 from the set of integers to the set of integers onto?

Answer: No, because there is no integer x with x^2 = −1, for instance

Function

One-to-one Correspondence Function

The function f is a one-to-one correspondence, or a *bijection*,

if it is both one-to-one and onto. We also say that such a function is *bijective*.

Example

Let f be the function from {a, b, c, d} to {1, 2, 3, 4}

with f (a) = 4, f (b) = 2, f (c) = 1, and f (d) = 3. Is f a bijection?

Answer: Yes, .

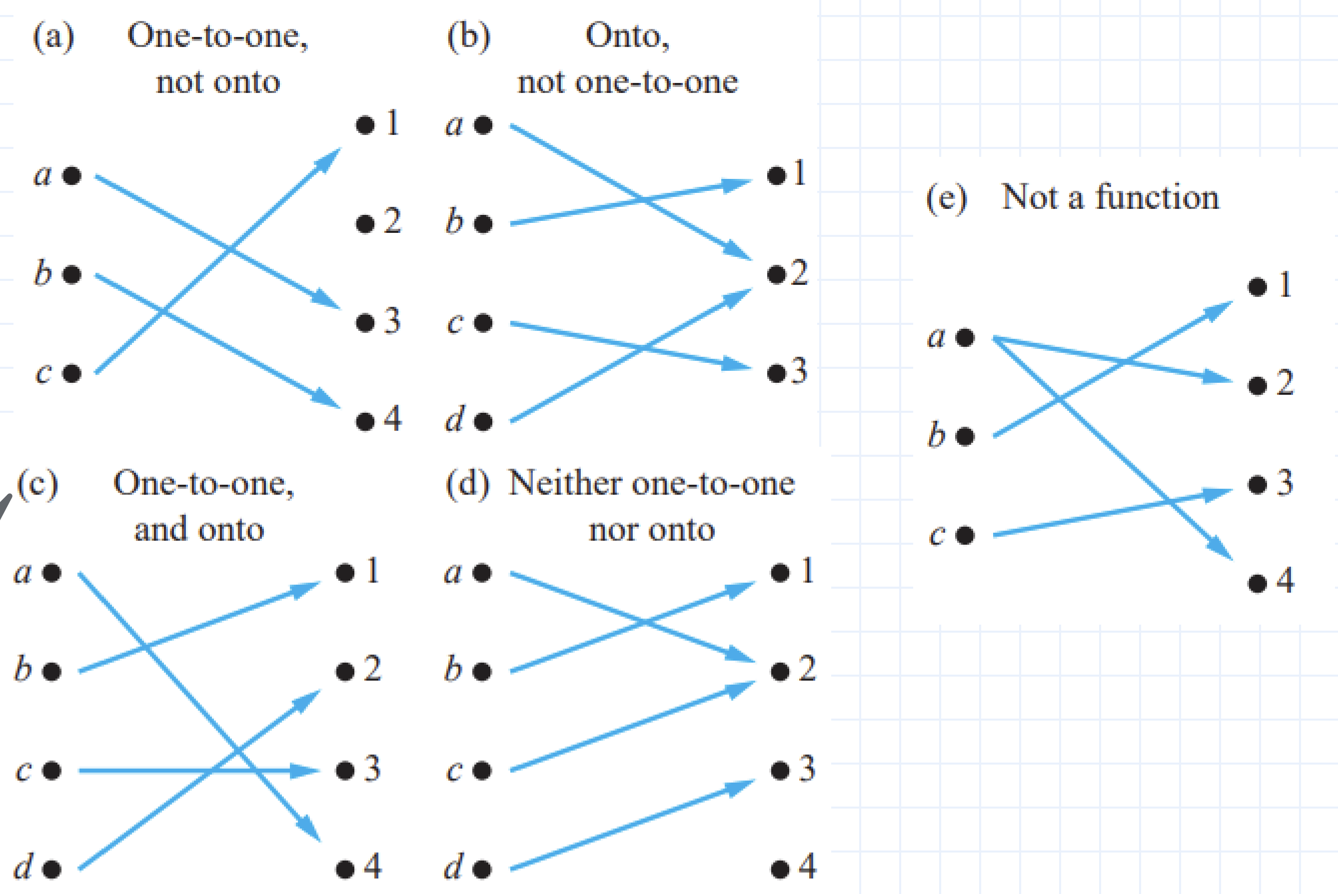
It is one-to-one because

no two values in the domain are assigned the same function value.

It is onto because

all four elements of the codomain are images of elements in the domain

Function



bijec
tive

Function

Other definitions

A function f whose domain and codomain are subsets of the set of real numbers is called *increasing* if $f(x) \leq f(y)$, and *strictly increasing* if $f(x) < f(y)$, whenever $x < y$ and x and y are in the domain of f

A function f is called *decreasing* if $f(x) \geq f(y)$, and *strictly decreasing* if $f(x) > f(y)$, whenever $x < y$ and x and y are in the domain of f

Let A be a set. The *identity function* on A is the function $\iota_A : A \to A$, where

$\iota_A(x) = x$ for all $x \in A$

function ι_A is the function that assigns each element to itself

Function

Proving

Suppose that $f : A \to B$.

To show that f is injective

Show that if $f(x) = f(y)$ for arbitrary $x, y \in A$, then $x = y$.

To show that f is not injective

Find particular elements $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$.

To show that f is surjective

Consider an arbitrary element $y \in B$ and find an element $x \in A$ such that $f(x) = y$.

To show that f is not surjective

Find a particular $y \in B$ such that $f(x) = y$ for all $x \in A$

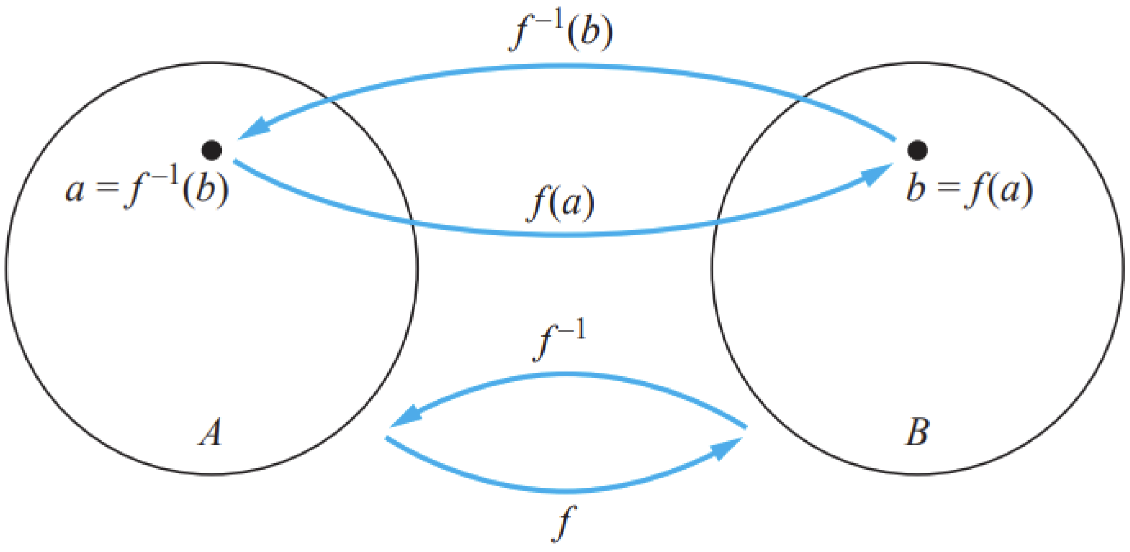
Function

Inverse Function

Let f be a one-to-one correspondence from the set A to the set B.

The *inverse function* of f is the function that assigns to an element b belonging to B the unique element a in A such that f (a) = b.

The inverse function of f is denoted by $\boldsymbol{f^{-1}}$. Hence, $f^{-1}(b) = a$ when f (a) = b.



Function

Example

Let f be the function from {a, b, c} to {1, 2, 3} such that f (a) = 2, f (b) = 3, and f (c) = 1.

Is f invertible, and if it is, what is its inverse?

Answer: The function f is invertible because it is a one-to-one correspondence. The inverse function f^{-1} reverses the correspondence given by f , so $f^{-1}(1) = c$, $f^{-1}(2) = a$, and $f^{-1}(3) = b$.

Example

Let f be the function from R to R with f (x) = x^2. Is f invertible?

Answer: Because f (−2) = f (2) = 4, f is not one-to-one. If an inverse function were defined, it would have to assign two elements to 4. Hence, f is not invertible.

Function

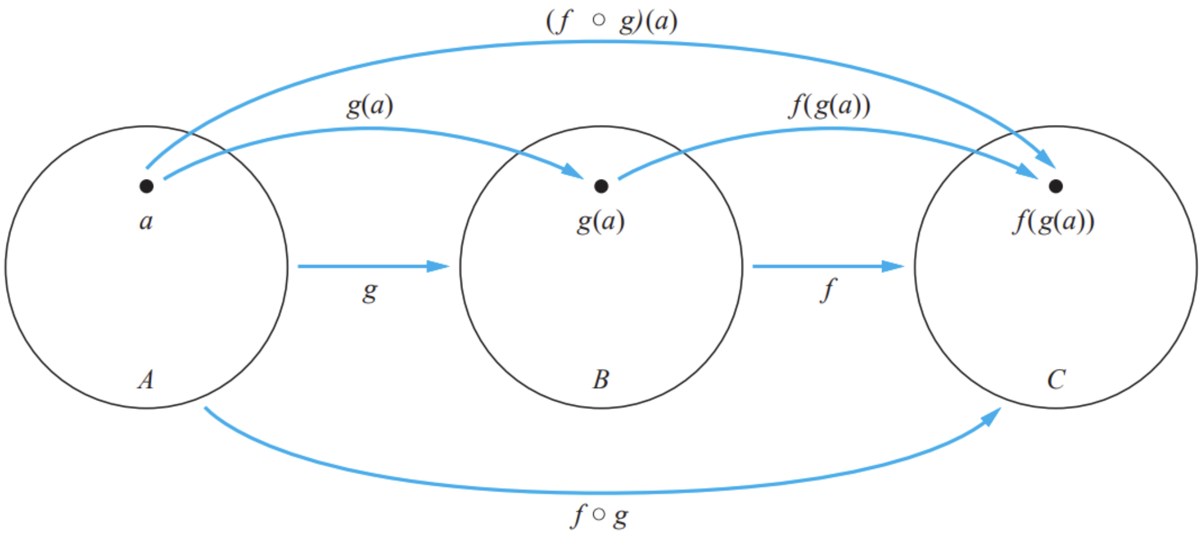
Composition Function

Let g be a function from the set A to the set B

and let f be a function from the set B to the set C.

The composition of the functions f and g, denoted for all a ∈ A by f ∘ g,

is defined by **(f ∘ g)(a) = f (g(a)).**



Function

Example

Let g be the function from the set $\{a, b, c\}$ to itself such that

$g(a) = b$, $g(b) = c$, and $g(c) = a$.

Let f be the function from the set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that

$f(a) = 3$, $f(b) = 2$, and $f(c) = 1$.

What is the composition of f and g, and what is the composition of g and f ?

Answer: The composition $f \circ g$ is defined by

$(f \circ g)(a) = f(g(a)) = f(b) = 2$,

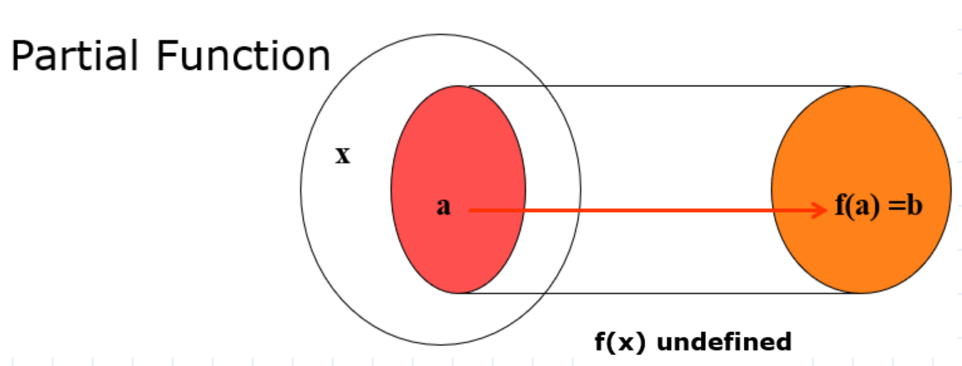
$(f \circ g)(b) = f(g(b)) = f(c) = 1$, and

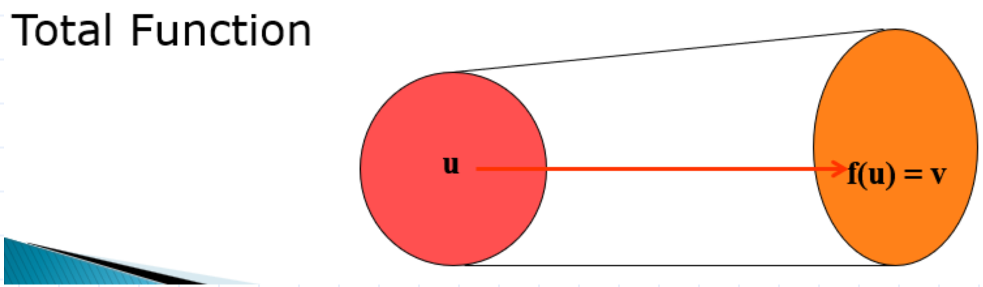
$(f \circ g)(c) = f(g(c)) = f(a) = 3$.

Function

Partial and Total Function

- *Partial function* f from a set A to a set B is an assignment to each element a in a subset of A, called the domain of definition of f , of a unique element b in B.
- We say that f is *undefined* for elements in A that are not in the domain of definition of f.
- When the domain of definition of f equals A, we say that f is a *total function*.





Function

Example

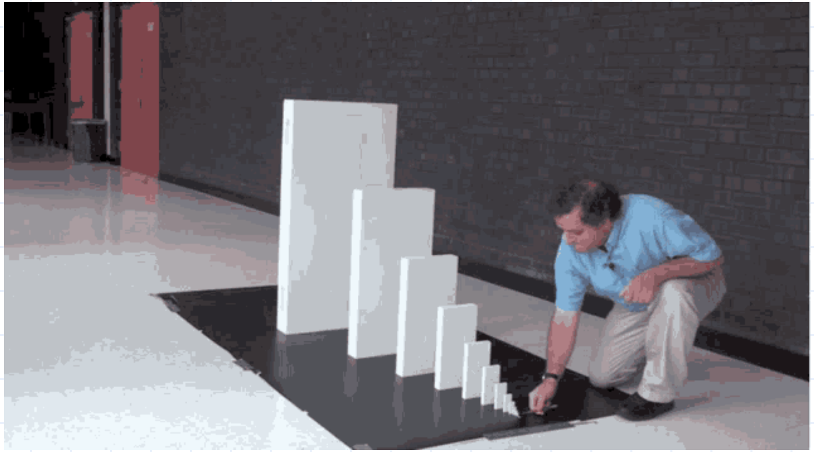
Function $f : Z \to R$ where $f(n) = \sqrt{n}$ is a partial function from Z to R

where the domain of definition is the set of nonnegative integers.

Note that f is undefined for negative integers.

e.g.

For $n = -1$, $f(n)$ is undefined



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03

Sequences & Summation

Sequences & Summation

Sequence (Indonesian: Baris)

A discrete structure used to represent an ordered list

$\{\boldsymbol{a_n}\}$ denotes sequence

$\boldsymbol{a_n}$ denote the image of the integer n. $\boldsymbol{a_n}$ is called *term* of the sequence.

Example

Given sequence $\{a_n\}$ where $a_n = {}^1/_n$

The list of the terms of this sequence, beginning with a_1, namely, $a_1, a_2, a_3, a_4, \ldots$

Starts with 1, ${}^1/_2$, ${}^1/_3$, ${}^1/_4$, …

Sequences & Summation

Geometric Progression

Sequence in a form of a, ar, ar^2, ..., ar^n, ...

where the initial term a and the common ratio r are real numbers.

Example

Given sequence $\{b_n\}$ where $b_n = (-1)^2$

The list of the terms of this sequence, beginning with b_1, namely, b_1, b_2, b_3, b_4, ...

Starts with 1, −1, 1, −1, ...

Sequences & Summation

Arithmetic Progression

Sequence in a form of a, a + d, a + 2d, …, a + nd, …

where the initial term a and the common difference d are real numbers

Example

Given sequence $\{s_n\}$ where $s_n = -1 + 4n$

The list of the terms of this sequence, beginning with s_1, namely, s_1, s_2, s_3, s_4, …

Starts with −1, 3, 7, 11, …

Sequences & Summation

Recurrence Relation

Sequence $\{a_n\}$ is an equation that expresses an in terms of one or more of the previous terms of the sequence, namely, a_0, a_1, …, a_{n-1}

for all integers n with n ≥ n_0, where n_0 is a nonnegative integer

Example

Fibonacci sequence

$f_n = f_{n-1} + f_{n-2}$

$f_2 = f_1 + f_0 = 1 + 0 = 1$

$f_3 = f_2 + f_1 = 1 + 1 = 2$

$f_4 = f_3 + f_2 = 2 + 1 = 3$

Sequences & Summation

Solving Recurrence Relation

forward substitution successive terms starting with initial condition and ending with a_n

Example

$a_n = a_{n-1} + 3$ for $n \geq 2$ with $a_1 = 2$

$$\begin{aligned}
a_2 &= 2 + 3 \\
a_3 &= (2 + 3) + 3 = 2 + 3 \cdot 2 \\
a_4 &= (2 + 2 \cdot 3) + 3 = 2 + 3 \cdot 3 \\
&\vdots \\
a_n &= a_{n-1} + 3 = (2 + 3 \cdot (n - 2)) + 3 = 2 + 3 \cdot (n - 1)
\end{aligned}$$

Sequences & Summation

Solving Recurrence Relation

backward substitution began with a_n and iterated to express it in terms of falling terms of the sequence until we found it in terms of a_1

Example

$a_n = a_{n-1} + 3$ for $n \geq 2$ with $a_1 = 2$

$$
\begin{aligned}
a_n &= a_{n-1} + 3 \\
&= (a_{n-2} + 3) + 3 = a_{n-2} + 3 \cdot 2 \\
&= (a_{n-3} + 3) + 3 \cdot 2 = a_{n-3} + 3 \cdot 3 \\
&\;\;\vdots \\
&= a_2 + 3(n-2) = (a_1 + 3) + 3 \cdot (n-2) = 2 + 3 \cdot (n-1)
\end{aligned}
$$

Sequences & Summation

Summation (Indonesian: Deret)

Addition of the terms of a sequence, <u>*summation notation*</u>

$$\boldsymbol{a_m, a_{m+1}, \ldots, a_n}$$

can be written in following notation:

upper limit
batas atas

$$\sum_{j=m}^{n} a_j$$

index notation

lower limit
batas bawah

Sequences & Summation

Example

Use summation notation to express the sum of the first 100 terms of the sequence {aj }, where a_j = 1/j for j = 1, 2, 3,....

Answer

$$\sum_{j=1}^{100} \frac{1}{j}$$

Example

What is the value of $\sum_{j=1}^{5} j^2$

Answer

$$\sum_{j=1}^{5} j^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$$

Sequences & Summation

Double Summation

$$\sum_{i=m}^{n}\sum_{j=p}^{q} j = \sum_{i=m}^{n}\left[p + (p+1) + (p+2) + \ldots + q\right]$$

Example

What is the value of

$$\sum_{i=1}^{4}\sum_{j=1}^{3} ij$$

Answer

$$\sum_{i=1}^{4}\sum_{j=1}^{3} ij = \sum_{i=1}^{4}\left(i + 2i + 3i\right)$$

$$= \sum_{i=1}^{4} 6i$$

Thanks

Do you have any questions?

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