

# StockTracker Implementation Report

## Time Complexity Requirements

The project specifies the following operations with their required time complexities:

- `insert_new_stock(x, p)`: Insert a stock with ID  $x$  and price  $p$ ,  $O(\log n)$ .
- `update_price(x, p)`: Update the price of stock  $x$  to  $p$ ,  $O(\log n)$ .
- `increase_volume(x, vinc)`: Increase the trading volume of stock  $x$  by  $vinc$ ,  $O(\log n)$ .
- `lookup_by_id(x)`: Return the price and volume of stock  $x$ ,  $O(1)$ .
- `price_range(p1, p2)`: Return IDs of stocks with prices in  $[p1, p2]$ ,  $O(\log n + k)$ , where  $k$  is the number of stocks in the range.
- `max_vol()`: Return the maximum trading volume and its stock ID,  $O(1)$  for retrieval,  $O(\log n)$  for heap cleanup.

Here,  $n$  is the number of stocks stored.

## Data Structures and Algorithms

To meet these time guarantees, the StockTracker class employs a combination of data structures: a hash table, a Binary Search Tree (BST), a defaultdict for price-to-ID mapping, and a max heap with UUIDs for volume tracking. Below, each operation is analyzed with its supporting data structure and algorithm.

### 1. Hash Table (`self.stocks`)

- **Purpose**: Stores stock information (price and volume) for each stock ID.
- **Implementation**: A Python dictionary (`self.stocks`), where keys are stock IDs ( $x$ ) and values are lists `[price, volume]`.
- **Operations**:
  - **Lookup**: Checking if a stock exists ( $x$  in `self.stocks`) or retrieving its data (`self.stocks[x]`) is  $O(1)$  on average due to the hash table's efficient key-value mapping.

- **Insert/Update:** Adding or updating a stock (`self.stocks[x] = [p, 0]`) is  $O(1)$ .
- **Usage:** Critical for `lookup_by_id`, `insert_new_stock`, `update_price`, and `increase_volume` to achieve fast access and updates.

## 2. Binary Search Tree (`self.price_sorted`)

- **Purpose:** Maintains stock IDs sorted by price for efficient range queries.
- **Implementation:** A custom `BinarySearchTree` class with `BSTNode` objects, each storing a price and id. The BST supports:
  - **Insert:** Adds a node with price and id,  $O(\log n)$  in a balanced BST (average case).
  - **Delete:** Removes a node with price and id,  $O(\log n)$ .
  - **Range Query:** Retrieves IDs with prices in  $[p1, p2]$  using in-order traversal,  $O(\log n + k)$ .
- **Algorithm Details:**
  - **Insert:** Traverses the tree based on price (and id for ties), inserting a new node at the appropriate leaf.
  - **Delete:** Finds the node, handles three cases (no children, one child, two children), and uses the minimum node in the right subtree as a successor for two-child cases.
  - **Range Query:** Performs an in-order traversal, pruning branches outside  $[p1, p2]$ , collecting IDs within the range.
- **Usage:** Supports `insert_new_stock`, `update_price` (delete and insert), and `price_range`.

## 3. Defaultdict (`self.price_to_ids`)

- **Purpose:** Maps prices to sets of stock IDs to handle multiple stocks with the same price efficiently.
- **Implementation:** A `defaultdict(set)` where keys are prices and values are sets of stock IDs.
- **Operations:**
  - **Add/Remove:** Adding an ID (`self.price_to_ids[p].add(x)`) or removing an ID (`self.price_to_ids[p].discard(x)`) is  $O(1)$ .
  - **Cleanup:** Deleting an empty set (`del self.price_to_ids[old_price]`) is  $O(1)$ .
- **Usage:** Used in `insert_new_stock` and `update_price` to track IDs per price, ensuring

efficient BST updates.

#### 4. Max Heap (`self.volume_heap`) with UUIDs (`self.id_to_uuid`)

- **Purpose:** Tracks the stock with the maximum trading volume.
- **Implementation:** A Python list (`self.volume_heap`) used as a max heap via `heapq`, storing tuples `(-volume, x, uuid)`. `self.id_to_uuid` maps stock IDs to their latest UUID to invalidate outdated heap entries.
- **Operations:**
  - **Push:** Adding a new volume entry (`heapq.heappush`) is  $O(\log n)$ .
  - **Peek/Cleanup:** Checking the top entry and removing invalid entries (due to volume updates) is  $O(1)$  for peeking and  $O(\log n)$  for popping invalid entries.
- **Algorithm Details:**
  - Each volume update generates a new UUID, pushing a new `(-volume, x, uuid)` tuple.
  - `max_vol` checks the heap's top entry, popping entries with outdated UUIDs or deleted stocks until a valid entry is found.
- **Usage:** Supports `increase_volume` (push) and `max_vol` (peek/cleanup).

## Implementation in Source Code

The `stock_tracker.py` file implements the above data structures and algorithms as follows:

### 1. `insert_new_stock(self, x, p) - O(\log n)`

- **Code:**

```
def insert_new_stock(self, x, p):  
    if x in self.stocks:  
        return  
    self.stocks[x] = [p, 0]  
    self.price_sorted.insert(p, x)  
    self.price_to_ids[p].add(x)  
    unique_id = str(uuid.uuid4())  
    self.id_to_uuid[x] = unique_id  
    heapq.heappush(self.volume_heap, (0, x, unique_id))
```

- **Steps:**
  - Checks if  $x$  exists in `self.stocks` ( $O(1)$ ).
  - Adds `[p, 0]` to `self.stocks` ( $O(1)$ ).
  - Inserts `(p, x)` into the BST (`self.price_sorted.insert`,  $O(\log n)$ ).
  - Adds  $x$  to `self.price_to_ids[p]` ( $O(1)$ ).
  - Generates a UUID and pushes `(0, x, uuid)` to the max heap ( $O(\log n)$ ).
- **Time Guarantee:** Dominated by BST insert and heap push, both  $O(\log n)$ .

## 2. `update_price(self, x, p) - O(\log n)`

- **Code:**

```
def update_price(self, x, p):
    if x not in self.stocks:
        return
    old_price, volume = self.stocks[x]
    self.stocks[x][0] = p
    self.price_to_ids[old_price].discard(x)
    if not self.price_to_ids[old_price]:
        del self.price_to_ids[old_price]
    self.price_sorted.delete(old_price, x)
    self.price_sorted.insert(p, x)
    self.price_to_ids[p].add(x)
```
- **Steps:**
  - Checks if  $x$  exists ( $O(1)$ ).
  - Updates price in `self.stocks` ( $O(1)$ ).
  - Removes  $x$  from `self.price_to_ids[old_price]` and cleans up if empty ( $O(1)$ ).
  - Deletes `(old_price, x)` from BST ( $O(\log n)$ ).
  - Inserts `(p, x)` into BST ( $O(\log n)$ ).
  - Adds  $x$  to `self.price_to_ids[p]` ( $O(1)$ ).
- **Time Guarantee:** Dominated by BST delete and insert,  $O(\log n)$ .

## 3. `increase_volume(self, x, vinc) - O(\log n)`

- **Code:**

```
def increase_volume(self, x, vinc):
    if x not in self.stocks:
```

```

        return
    self.stocks[x][1] += vinc
    new_volume = self.stocks[x][1]
    unique_id = str(uuid.uuid4())
    self.id_to_uuid[x] = unique_id
    heapq.heappush(self.volume_heap, (-new_volume, x, unique_id))

```

- **Steps:**
  - Checks if x exists ( $O(1)$ ).
  - Increments volume in self.stocks ( $O(1)$ ).
  - Generates a new UUID and pushes (-new\_volume, x, uuid) to the max heap ( $O(\log n)$ ).
- **Time Guarantee:** Dominated by heap push,  $O(\log n)$ .

#### 4. lookup\_by\_id(self, x) - $O(1)$

- **Code:**

```
def lookup_by_id(self, x):
    if x not in self.stocks:
        return None
    return self.stocks[x]
```
- **Steps:**
  - Checks if x exists in self.stocks ( $O(1)$ ).
  - Returns self.stocks[x] or None ( $O(1)$ ).
- **Time Guarantee:** Hash table operations are  $O(1)$  on average.

#### 5. price\_range(self, p1, p2) - $O(\log n + k)$

- **Code:**

```
def price_range(self, p1, p2):
    return self.price_sorted.range_query(p1, p2)
```
- **Steps:**
  - Calls BinarySearchTree.range\_query, which:
    - Traverses the BST to the first node where price  $\geq p1$  ( $O(\log n)$ ).
    - Performs in-order traversal, collecting IDs where  $p1 \leq \text{price} \leq p2$

( $O(k)$ ).

- Prunes branches where price > p2 (reduces unnecessary traversal).
- **Time Guarantee:**  $O(\log n)$  to reach the range,  $O(k)$  to collect k IDs.

## 6. max\_vol(self) - $O(1)$ retrieval, $O(\log n)$ cleanup

- **Code:**

```
def max_vol(self):
    while self.volume_heap:
        neg_vol, x, uuid = self.volume_heap[0]
        if x not in self.stocks or self.id_to_uuid.get(x) != uuid:
            heapq.heappop(self.volume_heap)
            continue
        return -neg_vol, x
    return 0, None
```
- **Steps:**
  - Peeks at the heap's top entry ( $O(1)$ ).
  - If the entry is invalid (stock deleted or UUID outdated), pops it ( $O(\log n)$ ) and repeats.
  - Returns the valid  $(-neg\_vol, x)$  or  $(0, None)$  if empty.
- **Time Guarantee:**  $O(1)$  for valid top entry,  $O(\log n)$  per pop for cleanup (amortized low due to infrequent updates).

## Implementation Notes

- **BST Design:** The BST uses price as the primary key and id to break ties, ensuring unique nodes. The implementation avoids balancing (e.g., AVL or Red-Black Tree) for simplicity, assuming random insertions approximate balance ( $O(\log n)$  average case).
- **Heap Cleanup:** The max heap uses lazy deletion (checking UUIDs in max\_vol), reducing the need for immediate updates in increase\_volume.
- **Hash Table Efficiency:** Python's dictionary ensures  $O(1)$  for lookup\_by\_id and other hash-based operations, critical for frequent lookups in testing.