University of Illinois Fall 2024

ECE 364 Programming Methods for Machine Learning

Homework 2

Finished by Jianchong Chen

Due on Thursday, September 26, 2024, 11:59pm on Gradescope

1. **[15 points]** PyTorch Linear Algebra

   (a) (3 points) Compute the addition of two matrices by completing the missing line of code below and state the result of the addition.

```
import torch
a  = torch.tensor([[0, 2, 4], [1, 3, 5]])
b  = torch.tensor([[7, 9, 1], [0, 8, 4]])
c = a+b #completed
print(c)
```

   Your answer:I have modified it above c=tensor([[7, 11, 5], [1, 11, 9]])

   (b) (3 points) Calculate the matrix multiplication of the following two matrices, if possible. a=$\begin{bmatrix}0 & 2 & 4\\1 & 3 & 5\end{bmatrix}$; b=$\begin{bmatrix}7 & 9 & 1\\0 & 8 & 4\end{bmatrix}$. If it is not possible, please explain your answer.

   Your answer: It is not possible. Because the number of column of the first matrix is not the same as the row of second matrix.

   (c) (3 points) Using the same matrices from part (b), calculate the matrix multiplication of $a^\top b$, if possible. If it is not possible, please explain your answer.

   Your answer: it is possible, $\begin{bmatrix}0 & 8 & 4\\14 & 42 & 14\\28 & 76 & 24\end{bmatrix}$

(d) (3 points) Complete the code below to compute the transpose of matrix `b` in Pytorch. Please also state what $b^\top$ is.

```
import torch
b  = torch.tensor([[7, 9, 1], [0, 8, 4]])
b_transpose = b.t() #completed
```

Your answer:I have modified it above $b^\top = \begin{bmatrix} 7 & 0 \\ 9 & 8 \\ 1 & 4 \end{bmatrix}$

(e) (3 points) Compute the matrix multiplication of the following two matrices by completing the code below and state the result of the multiplication.

```
import torch
a  = torch.tensor([[0, 2, 4], [1, 3, 5]])
b  = torch.tensor([[0, 7], [8, 9], [10, 11]])
c =  torch.matmul(a,b) #completed
print("c=", c)
```

Your answer:I have modified it above c = tensor([[56, 62], [74, 89]])

2. **[20 points]** PyTorch Solve Linear Equation

(a) (4 points) Given $\begin{cases} x + y = 5 \\ 2x + 3y = 8 \end{cases}$, write down the above equations in matrix form

Your answer: $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

(b) (6 points) The solution of the above equations can be found via matrix inversion. Find the necessary matrix inverse and solve for x and y

Your answer: let's assume that $A=\begin{bmatrix}1 & 1\\2 & 3\end{bmatrix}$, $x=\begin{bmatrix}x\\y\end{bmatrix}$, $B=\begin{bmatrix}5\\8\end{bmatrix}$ we get that $Ax = B$, then $x = A^{-1}B$,then we get $A^{-1} = \begin{bmatrix}3 & -1\\-2 & 1\end{bmatrix}$The final result is $x = 7, y = -2$

(c) (4 points) Given $\begin{cases} 2x + 4y + 3z = 5 \\ 9x + 6y + 8z = 7 \\ 11x + 13y + 10z = 12 \end{cases}$ Compute $A^{-1} = \begin{bmatrix} 2 & 4 & 3 \\ 9 & 6 & 8 \\ 11 & 13 & 10 \end{bmatrix}^{-1}$ using Pytorch by completing the below code. Please complete the code and state A^{-1}.

```
import torch
a=torch.tensor([[2,4,3],[9,6,8],[11,13,10]],dtype=torch.float32)#
    completed
print(a)
a_inversed=torch.inverse(a) #completed
print(a_inversed)
```

Your answer: $A^{-1} = \begin{bmatrix} -0.7719 & -0.0175 & 0.2456 \\ -0.0351 & -0.2281 & 0.1930 \\ 0.8947 & 0.3158 & -0.4211 \end{bmatrix}$

(d) (6 points) Write a Pytorch code to solve the equations $\left\{ \begin{array}{c} 2x + 4y + 3z = 5 \\ 9x + 6y + 8z = 7 \\ 11x + 13y + 10z = 12 \end{array} \right\}$.

Please complete the code below and state the solution values for x, y, and z.

```
import torch
a=torch.tensor([[2,4,3],[9,6,8],[11,13,10]],dtype=torch.float32)#
    completed
b=torch.tensor([[5],[7],[12]],dtype=torch.float32) #completed
print(a)
print(b)
X=torch.matmul(torch.inverse(a),b) #completed
print(X)
```

Your answer: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1.0351 \\ 0.5439 \\ 1.6316 \end{bmatrix}$

3. **[15 points]** Gradients and orthonormal basis

(a) (6 points) Consider $f(x) = x^\top x$ where $x \in \mathbb{R}^n$. Determine $\frac{\partial f}{\partial x}$ and show your work.

Your answer: $f(x) = x^\top x = \sum x_i^2$, $\frac{\partial f(x_i)}{\partial x_i} = 2 * x_i$ so we get that $\frac{\partial f}{\partial x} = 2 * x$

(b) (6 points) Consider $f(x) = x^\top \mathbf{A} x$ where $x \in \mathbb{R}^n$ and $\mathbf{A} \in \mathbb{R}^{n \times n}$. Determine $\frac{\partial f}{\partial x}$ and show your work.

Your answer: $\frac{\partial f}{\partial x} = \frac{\partial x^T A x}{\partial x} = \frac{\partial x^T A}{\partial x} + \frac{\partial A x}{\partial x} = (A^T + A) * x$

(c) (3 points) Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

Determine an orthonormal basis for the span($\mathbf{A}$).

Your answer:

$\mathrm{v}_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$ $\mathbf{v}_2 = \begin{bmatrix}0\\2\\-1\end{bmatrix}$ $\mathbf{v}_3 = \begin{bmatrix}0\\-1\\2\end{bmatrix}$

Set:

$\mathrm{u}_1 = \mathbf{v}_1$.

Normalize $\mathbf{u}_1$ to get $\mathbf{e}_1$:

$\|\mathbf{u}_1\| = \sqrt{1^2 + 0^2 + 0^2} = 1$.

$\mathrm{e}_1 = \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} = \begin{bmatrix}1\\0\\0\end{bmatrix}$.

Compute the projection of $\mathbf{v}_2$ onto $\mathbf{u}_1$:

$\operatorname{proj}_{\mathbf{u}_1} \mathbf{v}_2 = \frac{\mathbf{v}_2 \cdot \mathbf{u}_1}{\mathbf{u}_1 \cdot \mathbf{u}_1} \mathbf{u}_1$.

$\mathrm{u}_2 = \mathbf{v}_2 - \operatorname{proj}_{\mathbf{u}_1} \mathbf{v}_2 = \begin{bmatrix}0\\2\\-1\end{bmatrix}$.

Normalize $\mathbf{u}_2$ to get $\mathbf{e}_2$:

$\|\mathbf{u}_2\| = \sqrt{0^2 + 2^2 + (-1)^2} = \sqrt{5}$.

$\mathrm{e}_2 = \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|} = \frac{1}{\sqrt{5}} \begin{bmatrix}0\\2\\-1\end{bmatrix} = \begin{bmatrix}0\\ \frac{2}{\sqrt{5}}\\ \frac{-1}{\sqrt{5}}\end{bmatrix}$.

$\operatorname{proj}_{\mathbf{u}_1} \mathbf{v}_3 = \frac{\mathbf{v}_3 \cdot \mathbf{u}_1}{\mathbf{u}_1 \cdot \mathbf{u}_1} \mathbf{u}_1$.

$\operatorname{proj}_{\mathbf{u}_1} \mathbf{v}_3 = 0 \cdot \mathbf{u}_1 = \begin{bmatrix}0\\0\\0\end{bmatrix}$.

Compute the projection onto $\mathbf{u}_2$:

$\operatorname{proj}_{\mathbf{u}_2} \mathbf{v}_3 = \frac{\mathbf{v}_3 \cdot \mathbf{u}_2}{\mathbf{u}_2 \cdot \mathbf{u}_2} \mathbf{u}_2$.

$\mathrm{u}_3 = \mathbf{v}_3 - \operatorname{proj}_{\mathbf{u}_1} \mathbf{v}_3 - \operatorname{proj}_{\mathbf{u}_2} \mathbf{v}_3 = \begin{bmatrix}0\\-1\\2\end{bmatrix} - \begin{bmatrix}0\\0\\0\end{bmatrix} - \begin{bmatrix}0\\ \frac{-8}{5}\\ \frac{4}{5}\end{bmatrix}$.

Normalize $\mathbf{u}_3$ to get $\mathbf{e}_3$: Compute the norm:

$\|\mathbf{u}_3\| = \sqrt{0^2 + \left(\frac{3}{5}\right)^2 + \left(\frac{6}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{36}{25}} = \sqrt{\frac{45}{25}} = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$.

Normalize $\mathbf{u}_3$:

$\mathrm{e}_3 = \frac{\mathbf{u}_3}{\|\mathbf{u}_3\|} = \frac{1}{\frac{3}{\sqrt{5}}} \begin{bmatrix}0\\ \frac{3}{5}\\ \frac{6}{5}\end{bmatrix} = \frac{\sqrt{5}}{3} \begin{bmatrix}0\\ \frac{3}{5}\\ \frac{6}{5}\end{bmatrix} = \begin{bmatrix}0\\ \frac{\sqrt{5}}{5}\\ \frac{2\sqrt{5}}{5}\end{bmatrix}$.