14.3 Partial Derivatives

Definition

If f is a function of two or more variables, its **partial derivatives** are the functions f_x and f_y defined by

$$f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$$

$$f_y(x,y) = \lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h}$$

Notations for Partial Derivatives

If $z = f(x,y)$, we write

$$f_x(x,y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x,y) = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f$$

$$f_y(x,y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} f(x,y) = \frac{\partial z}{\partial y} = f_2 = D_2 f = D_y f$$

Rule for Finding Partial Derivatives of $z = f(x,y)$

1. To find f_x, regard y as a constant and differentiate $f(x,y)$ with respect to x.
2. To find f_y, regard x as a constant and differentiate $f(x,y)$ with respect to y.

Functions of Three or More Variables

$$f_x(x,y,z) = \lim_{h\to 0} \frac{f(x+h,y,z)-f(x,y,z)}{h}$$

$$\frac{\partial u}{\partial x_i} = \lim_{h\to 0} \frac{f(x_1,\ldots,x_{i-1},x_i,\ldots,x_n)-f(x_1,\ldots,x_{i-1},x_i-h,\ldots,x_n)}{h}$$

Higher Derivatives

$$(f_x)_x = f_{xx} = f_{11} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 z}{\partial x^2}$$

$$(f_x)_y = f_{xy} = f_{12} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 z}{\partial y \partial x}$$

$$(f_y)_x = f_{yx} = f_{21} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 z}{\partial x \partial y}$$

$$(f_y)_y = f_{yy} = f_{22} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 z}{\partial y^2}$$

Clairaut's Theorem:

Suppose f is defined on a Disk D \hat{t} contains the point (a,b). If the functions f_{xy} and f_{yx} are both continuous on D, then

$$f_{xy}(a,b) = f_{yx}(a,b)$$