

13.3 Arc Length and Curvature

Some Formulas

Curvature

$$\kappa = \left| \frac{\mathbf{T}}{ds} \right| = \frac{|\mathbf{T}'|}{|\mathbf{r}'|} = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3} = \frac{|f''(x)|}{\left[1 + (f'(x))^2\right]^{3/2}}$$

The Normal and Binormal Vectors

$$\mathbf{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|}$$

$$\mathbf{N} = \frac{\mathbf{T}'}{|\mathbf{T}'|}$$

$$\mathbf{B} = \mathbf{T} \times \mathbf{N}$$

Torsion

$$\tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N} = -\frac{\mathbf{B}' \cdot \mathbf{N}}{|\mathbf{r}'|} = \frac{[\mathbf{r}' \times \mathbf{r}''] \cdot \mathbf{r}'''}{|\mathbf{r}' \times \mathbf{r}''|^2}$$