Math 241 Calculus III (Prof. Honold) **Midterm Exam 3** 2022/12/14

Name: ____________________ Student No.: ____________________ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. The vector field $G(x,y) = (x^2 + cy^2, xy)$, $(x,y) \in \mathbb{R}^2$ is a gradient field if

   ☐ $c = 0$ ■ $c = 1/2$ ☐ $c = 1$ ☐ $c = -1$ ☐ $c = -1/2$

2. The line integral of $y\,\mathrm{d}x + 2x\,\mathrm{d}y$ along the half circle $\gamma(t) = (\cos t, \sin t)$, $t \in [0, \pi]$ equals

   ☐ $-\pi$ ☐ $-\pi/2$ ☐ 0 ■ $\pi/2$ ☐ π

3. Let $D \subseteq \mathbb{R}^2$ be the region bounded from below by the lines $y = \pm x$ and from above by the unit circle. The integral $\int_D x + y\,\mathrm{d}^2(x,y)$ is equal to

   ☐ $-\frac{1}{3}\sqrt{2}$ ☐ $-\frac{1}{6}\sqrt{2}$ ☐ 0 ☐ $\frac{1}{6}\sqrt{2}$ ■ $\frac{1}{3}\sqrt{2}$

4. Let R be the (solid) rectangle in $\mathbb{R}^2$ with vertices $(0,0)$, $(1,0)$, $(0,2)$, $(1,2)$. The integral $\displaystyle\int_R xy(x+y)\,\mathrm{d}^2(x,y)$ equals

   ☐ 0 ☐ $2/3$ ☐ $4/3$ ■ 2 ☐ $8/3$

5. The volume of $\{(x,y,z) \in \mathbb{R}^3; 0 \leq x \leq 4 - y^2 - z^2\}$ is

   ☐ π ☐ 2π ☐ 4π ■ 8π ☐ 16π

6. For $F(x) = \displaystyle\int_0^2 \frac{\sin(xt^2)}{t^3+1}\,\mathrm{d}t$ the derivative $F'(0)$ is equal to

   ☐ $\frac{2}{3}\ln(2)$ ■ $\frac{2}{3}\ln(3)$ ☐ $\frac{1}{3}\ln(2)$ ☐ $\frac{3}{2}\ln(2)$ ☐ $\frac{1}{2}\ln(3)$

7. The function $f(x,y) = \cos x + \sin y$ has in $(0,0)$

   ☐ a local minimum ■ no extremum ☐ a global extremum
   ☐ a saddle point ☐ a local maximum

8. The tangent plane to the surface $x^2 - yz = 3$ in $(1,2,-1)$ contains the point $(0,0,c)$ for

   ■ $c = -3$ ☐ $c = 11$ ☐ $c = 5$ ☐ $c = -7$ ☐ $c = 2$

9. The function $z = g(x,y)$ implicitly defined by the equation $xy^3 + yz^3 + zx^3 = -1$ and $g(1,1) = -1$ has $g_x(1,1)$ equal to

   ☐ -1 ☐ $-1/2$ ☐ 0 ■ $1/2$ ☐ 1

10. The 1-dimensional surface integral (integral with respect to arc length) of $f(x,y) = x^2$ over the cicle $x^2 + y^2 = 4$ is

    ☐ π ☐ 2π ☐ 4π ■ 8π ☐ 16π

Time allowed: 45 min CLOSED BOOK ***Good luck!***