Math 241 Calculus III (Prof. Honold) **Midterm Exam 1** 2022/10/19

Name: ____________________ Student ID : ____________________ Group B

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. The volume of the parallelepiped spanned by the vectors $(1,-1,2)$, $(-b,1,-1)$, $(-1,2,2)$ is equal to 1 for

   ☐ $b=-2$ ☐ $b=-1$ ☐ $b=0$ ■ $b=1$ ☐ $b=2$

2. The tangent to $f(t)=(t,t^2,t^4)$ in the point $(1,1,1)$ meets the plane $x+y-z=3$ in

   ☐ no point. ☐ $(-1,3,-5)$ ☐ $(2,3,5)$ ■ $(-1,-3,-7)$ ☐ $(2,3,-7)$

3. The unit normal vector $\mathbf{N}(1)$ of the curve $f(t)=(t,t^2/2,t^3/3)$ is a positive multiple of

   ☐ $(0,1,-1)$ ☐ $(0,-1,1)$ ☐ $(0,0,1)$ ■ $(-1,0,1)$ ☐ $(1,0,-1)$

4. If $f\colon[0,2\pi]\to\mathbb{R}^3$ satisfies $f(0)=(0,0,0)$, $f'(0)=(0,1,-1)$ and $f''(t)=(1,\cos t,\sin t)$, the point $f(2\pi)$ is equal to

   ☐ $(2\pi^2,0,2\pi)$ ☐ $(\pi^2,0,2\pi)$ ■ $(2\pi^2,2\pi,0)$ ☐ $(0,2\pi,2\pi)$ ☐ $(\pi^2,2\pi,0)$

5. The 2-contour (level-2 set) of $f(x,y)=\dfrac{1}{x^2+y^2-1}$ is

   ☐ empty ☐ a point ☐ a line ■ a circle ☐ a sphere

6. The paths of the curves $f(t)=(t,t^2,t^3)$ and $g_a(t)=\big(1+2t,(1-a)t,t\big)$ intersect for

   ■ $a=2$ ☐ no $a\in\mathbb{R}$ ☐ $a=1$ ☐ all $a\in\mathbb{R}$ ☐ $a=0$

7. The length of the arc of $\gamma(t)=(t^3-1,6t,3t^2-3)$ between $(0,6,0)$ and $(-2,-6,0)$ is

   ☐ 6 ☐ 8 ☐ 10 ☐ 12 ■ 14

8. For a C^2-curve $\mathbf{r}\colon I\to\mathbb{R}^3\setminus\{\mathbf{0}\}$ with nonzero curvature and $t\in I$, the derivative $\dfrac{\mathrm{d}}{\mathrm{d}t}\dfrac{\mathbf{r}(t)}{|\mathbf{r}(t)|}$ is perpendicular to

   ■ $\mathbf{r}(t)$ ☐ $\mathbf{r}'(t)$ ☐ $\mathbf{r}''(t)$ ☐ $\mathbf{N}(t)$ ☐ $\mathbf{B}(t)$

9. For $\mathbf{A}=\begin{pmatrix}\sqrt{3}/2 & 1/2\\ 1/2 & -\sqrt{3}/2\end{pmatrix}$ the smallest positive integer k such that $\mathbf{A}^k=\mathbf{I}_2$ (the 2×2 identity matrix) is

   ■ 2 ☐ 3 ☐ 6 ☐ 12 ☐ 24

10. The distance between the lines $\mathbb{R}(1,-1,1)$ and $(2,1,-3)+\mathbb{R}(1,1,-1)$ is

    ☐ $1/2$ ☐ $1/\sqrt{2}$ ☐ 1 ■ $\sqrt{2}$ ☐ 2

Time allowed: 45 min CLOSED BOOK ***Good luck!***