Semi-Lagrangian method on Octree

Project Proposal

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1. Introduction

The octree data structure[?][?] with adaptive mesh refinement proves to be a flexible and allows accurate and efficient tracking of flow features. On the other hand, semi-Lagrangian method[?][?] enables the choice of a larger time steps in the simulation of fluids which would be crucial for an adaptive mesh approach. Popinet[?] and Gibou[?] have previously worked on solving Euler equation on octree data structure. In this project, we try to implement a parallelized semi-Lagrangian method on octree data structure for a simple transport equation and Euler equation if time permits.

2. Problem

Solve transport equation or incompressible Euler equation on *Octree* Data structure with cell centered discretization and semi-Lagrangian scheme.

2.1. Non adaptive mesh

1. Regular Cartesian grid

   A three time level semi-Lagrangian method will be adopted. See section ...

2. Time-invariant octree

   We will adopt the implementation of 2:1 balance octree.

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2.2. Adaptive mesh

1. Refine criterion

   An attractive option is to use Richardson extrapolation. Simple criterion based on the norm of the local vorticity vector can also be considered, e.g. $\frac{h||\nabla\times U||}{\max||U||} > \tau$. [?]

2. Balance octree after refining

   Refinement of a single cell may cause dismatch of level with neighboring cells. Thus a rebalancing of the octree is required. [?][?]

3. The level of refinement

   After each refinement and re-balancing we will need to do one more cycle of calculation. This will be expensive if a few cells need multilevel refinement.

   Possible solution:

   1. In Gerris they would do at most 1-level refinement each time-step.
   2. Maximum refinement can be taken as parameter in the program.

3. Semi-Lagrangian

Use time level temporal discretization and update each grid point using

$$\frac{F(x_m, t_n + \Delta t) - F(x_m - 2\alpha_m, t_n - \Delta t)}{2\Delta t} = 0 \tag{1}$$

$F(x_m - 2\alpha_m, t_n - \Delta t)$ second order interpolation Find α_m by

$$\alpha_m = \Delta t\, U(x_m - \alpha_m, t_n) \tag{2}$$

where $U(x, t)$ is the velocity field.

3.1. Iteration

Use fix point iteration as following,

$$\alpha_m^{k+1} = \Delta t\, U(x_m - \alpha_m^k, t_n) \tag{3}$$

The condition for above iteration to converge is

$$\Delta t \max_x |\nabla_x U| \leq 1 \tag{4}$$

3.2. Computing Concerns

1. Interpolation

   When neighboring cells are of different level, we will need to do interpolation to calculate gradient flux on the interface of cells in Euler equation. Here we plan to adopt the strategy introduced in Gerris.

2. Boundary condition

   For Euler equation, we shall use no-flow condition.

4. Complexity for work-depth model

4.1. Semi-Lagrangian

Suppose the tolerance of each iteration is tol, then steps for convergence is $\Omega(|\log(tol)|)$. Since each iteration is independent, the time complexity is $\Omega(1/\Delta t|\log(tol)|)$, work complexity is $\Omega(N^d/\Delta t|\log(tol)|)$.

4.2. Octree

The complexity here depends on specific implementations. [**?**],[**?**].

And the octree related operations required:

1. Time invariant:

   (a) Generation of octree.

   (b) Fetch data from neighboring cells.

2. Extra operations for adaptive:

   (a) (Check refinement/coarsening criterion)

   (b) Refinement/coarsening of octree

5. Milestone

☑ 2-D and 3-D regular grid semi-Lagrangian

☐ Parallelized semi-Lagrangian on regular grid – (1 week)

☐ Semi-Lagrangian on time-invariant Octree – (1 week)

☐ Semi-Lagrangian on adaptive Octree – (2 week)

6. Possible issues

☑ Octree Package interface

☐ Overhead of the adaptive octree method

7. Bitbucket Repository

The project is hosted on https://bitbucket.org/nlmd/cse-proj.

References

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