Semi-Lagrangian method on Octree

Project Proposal

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1. Introduction

2. Problem

Solve transport equation or incompressible Euler equation on *Octree* Data structure with cell centered discretization and semi-Lagrangian scheme.

2.1. Non adaptive mesh

1. Regular grid
2. 2:1 balance octree
3. Finite volume method

2.2. Adaptive mesh

1. Refine criterion
2. The level of refinement
3. Balance octree after refining
4. Recalculation

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Re-partition after refinement and then do calculation? Otherwise could require a multi-level interpolation. Seems very complex. In Gerris they would do at most 1-level refinement each time-step.

Maximum refinement can be taken as parameter in the program.

3. Semi-Lagrangian

Use time level temporal discretization and update each grid point using

$$\frac{F(x_m, t_n + \Delta t) - F(x_m - 2\alpha_m, t_n - \Delta t)}{2\Delta t} = 0 \tag{1}$$

$F(x_m - 2\alpha_m, t_n - \Delta t)$ second order interpolation Find α_m by

$$\alpha_m = \Delta t\, U(x_m - \alpha_m, t_n) \tag{2}$$

where $U(x, t)$ is the velocity field.

3.1. Iteration

Use fix point iteration as following,

$$\alpha_m^{k+1} = \Delta t\, U(x_m - \alpha_m^k, t_n) \tag{3}$$

The condition for above iteration to converge is

$$\Delta t \max_x |\nabla_x U| \le 1 \tag{4}$$

3.2. Computing Concerns

1. Interpolation

2. Boundary condition

4. Complexity for work-depth model

5. Milestone

☑ 2-D and 3-D regular grid semi-Lagrangian

☐ Parallelized semi-Lagrangian on regular grid – (1 week)

☐ Semi-Lagrangian on time-invariant Octree – (1 week)

☐ Semi-Lagrangian on adaptive Octree – (2 week)

6. Possible issues

☑ Octree Package interface

☐ Overhead of the adaptive octree method

7. Bitbucket Repository

The project is hosted on https://bitbucket.org/nlmd/cse-proj.

References

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