Numerical Approach to Cauchy Elliptic Problem

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1 Brief Intro

The aimed Cauchy problem stated as

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\psi\frac{\mathrm{d}u}{\mathrm{d}x}\right)+\frac{\mathrm{d}}{\mathrm{d}y}\left(\phi\frac{\mathrm{d}u}{\mathrm{d}y}\right)=0 \tag{1}$$

with Cauchy condition as

$$u=h \quad \text{on} \quad \Gamma_1 \tag{2}$$
$$\nabla u\cdot\mathbf{n}=g \quad \text{on} \quad \Gamma_1 \tag{3}$$

where Γ_1 is part of the boundary of the domain Ω.

1.1 Uniqueness & Stability

It is well known the problem is ill-posed, the solution is extreme sensitive to small perturbation on data. The famous example is

EXAMPLE 1.1.

$$\triangle u=0 \tag{4}$$

with Cauchy data as

$$u(x,0) = 0 \tag{5}$$
$$u_y(x,0) = A_n\sin nx \tag{6}$$

for all $(x,y)\in\mathbb{R}\times\mathbb{R}^+$. *Since the solution is given as*

$$u_n(x,y)=\frac{A_n}{n}\sin nx\sinh ny\to\infty \tag{7}$$

as $n\to\infty$.