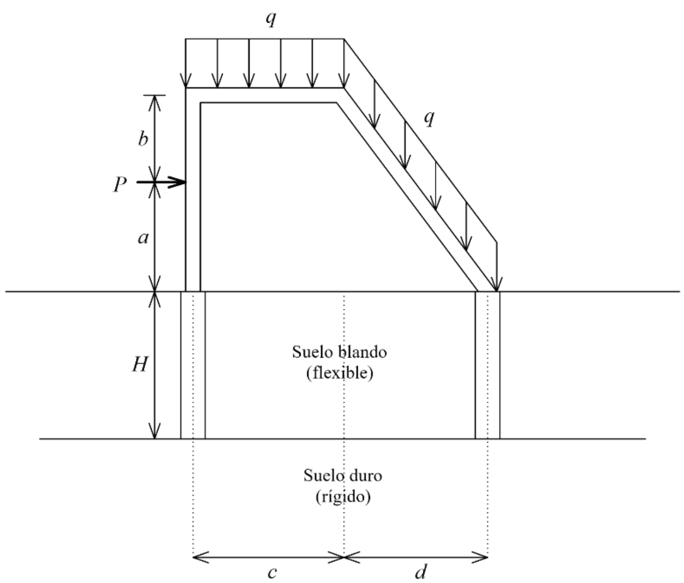
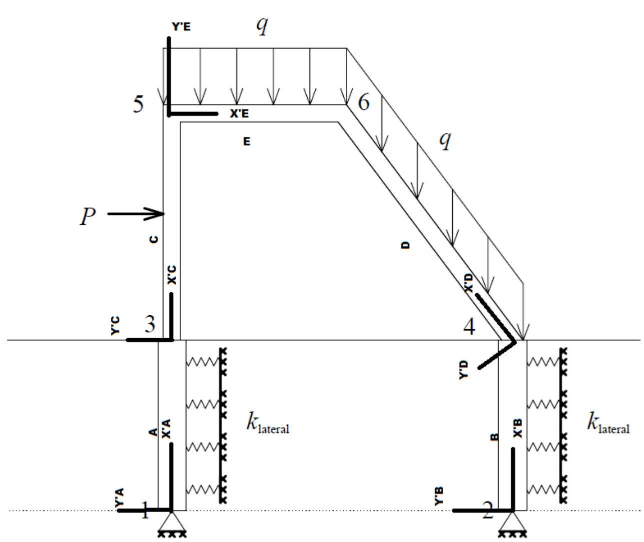
El pórtico plano de concreto ($E = 2x10^7$ kN/m^2) cuyas propiedades son a =2.05 m, b =1.8 m, c =4 m, d =2.7 m, H =5 m, q =60 kN/m, P =70 kN y sus elementos tienen una sección transversal de 30 cm de base por 35 cm de altura, está soportado por un par de pilas como se muestra en la siguiente figura:



Para analizar esta estructura, el ingeniero geotecnista propone hacer un modelo estático del conjunto estructura + cimentación + suelo, empleando las siguientes propiedades (ver figura a continuación):

1. Pilas circulares con diámetro de 80 cm.
2. Despreciar la fricción entre fuste de las pilas y el suelo blando.
3. Modelar la rigidez lateral del uso empleando $k_{Lateral}$=5000 kN/m^2 e incluir el efecto de la fuerza axial desacoplado.
4. Emplear el efecto de la punta de las pilas por medio de apoyo simple.

Discretización



Elemento A

Coordenadas Locales

$$\begin{Bmatrix} F'X1^A \\ F'Y1^A \\ M'1^A \\ F'X3^A \\ F'Y3^A \\ M'3^A \end{Bmatrix} = \begin{bmatrix} 2010619 & 0 & 0 & -2010619 & 0 & 0 \\ 0 & 47820 & 102984 & 0 & -35453 & 92710 \\ 0 & 102984 & 327574 & 0 & -92710 & 156460 \\ -2010619 & 0 & 0 & 2010619 & 0 & 0 \\ 0 & -35453 & -92710 & 0 & 47820 & -102984 \\ 0 & 92710 & 156460 & 0 & -102984 & 327574 \end{bmatrix} * \begin{Bmatrix} u1'^A \\ v1'^A \\ \theta1'^A \\ u3'^A \\ v3'^A \\ \theta3'^A \end{Bmatrix}$$

$$[TA] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad [TA]^T = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Coordenadas Globales

$$\begin{Bmatrix} FX1^A \\ FY1^A \\ M1^A \\ FX3^A \\ FY3^A \\ M3^A \end{Bmatrix} = \begin{bmatrix} 47820 & 0 & -102984 & -35453 & 0 & 92710 \\ 0 & 2010619 & 0 & 0 & -2010619 & 0 \\ 102984 & 0 & 327574 & 92710 & 0 & 156460 \\ -35452 & 0 & 92710 & 47820 & 0 & 102984 \\ 0 & -2010619 & 0 & 0 & 2010619 & 0 \\ -92710 & 0 & 156460 & 102984 & 0 & 327574 \end{bmatrix} * \begin{Bmatrix} u1^A \\ v1^A \\ \theta1^A \\ u3^A \\ v3^A \\ \theta3^A \end{Bmatrix}$$

Elemento B

Coordenadas Locales

$$\begin{Bmatrix} F'X2^B \\ F'Y2^B \\ M'2^B \\ F'X4^B \\ F'Y4^B \\ M'4^B \end{Bmatrix} = \begin{bmatrix} 2010619 & 0 & 0 & -2010619 & 0 & 0 \\ 0 & 47820 & 102984 & 0 & -35453 & 92710 \\ 0 & 102984 & 327574 & 0 & -92710 & 156460 \\ -2010619 & 0 & 0 & 2010619 & 0 & 0 \\ 0 & -35453 & -92710 & 0 & 47820 & -102984 \\ 0 & 92710 & 156460 & 0 & -102984 & 327574 \end{bmatrix} * \begin{Bmatrix} u2'^B \\ v2'^B \\ \theta2'^B \\ u4'^B \\ v4'^B \\ \theta4'^B \end{Bmatrix}$$

$$[TB] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad [TB]^T = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Coordenadas Globales

$$\begin{Bmatrix} FX2^B \\ FY2^B \\ M2^B \\ FX4^B \\ FY4^B \\ M4^B \end{Bmatrix} = \begin{bmatrix} 47820 & 0 & -102984 & -35453 & 0 & 92710 \\ 0 & 2010619 & 0 & 0 & -2010619 & 0 \\ 102984 & 0 & 327574 & 92710 & 0 & 156460 \\ -35452 & 0 & 92710 & 47820 & 0 & 102984 \\ 0 & -2010619 & 0 & 0 & 2010619 & 0 \\ -92710 & 0 & 156460 & 102984 & 0 & 327574 \end{bmatrix} * \begin{Bmatrix} u2^B \\ v2^B \\ \theta2^B \\ u4^B \\ v4^B \\ \theta4^B \end{Bmatrix}$$

Elemento C

Cálculo de empotramientos para una viga doblemente empotrada sometida a una fuerza puntual vertical

$$-P\varphi 2(\varepsilon) = \frac{Pb^2(3a+b)}{(a+b)^3} = 31.595\ KN$$

$$-P\varphi 3(\varepsilon) = \frac{Pab^2}{(a+b)^2} = 31.367\ \ KN * m$$

$$-P\varphi 5(\varepsilon) = \frac{Pb^2(a+3b)}{(a+b)^3} = 38.404\ \ KN$$

$$-P\varphi 6(\varepsilon) = \frac{-Pa^2b}{(a+b)^2} = -35.723\ \ KN * m$$

Coordenadas Locales

$$\begin{Bmatrix} F'X3^C \\ F'Y3^C \\ M'3^C \\ F'X5^C \\ F'Y5^C \\ M'5^C \end{Bmatrix} = \begin{bmatrix} 545454 & 0 & 0 & -545454 & 0 & 0 \\ 0 & 4507 & 8677 & 0 & -4507 & 8677 \\ 0 & 8677 & 22272 & 0 & -8677 & 11136 \\ -545454 & 0 & 0 & 545454 & 0 & 0 \\ 0 & -4507 & -8677 & 0 & 4507 & -8677 \\ 0 & 8677 & 11136 & 0 & -8677 & 22272 \end{bmatrix} * \begin{Bmatrix} u3'^C \\ v3'^C \\ \theta 3'^C \\ u5'^C \\ v5'^C \\ \theta 5'^C \end{Bmatrix} + \begin{Bmatrix} 0 \\ 31.595 \\ 31.367 \\ 0 \\ 38.404 \\ -35.723 \end{Bmatrix}$$

$$[TC] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad [TC]^T = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Coordenadas Globales

$$\boldsymbol{Para\ las\ fuerzas\ de\ empotramiento\ globales} = [\boldsymbol{Tc}]^{\boldsymbol{T}} * [\boldsymbol{FEmlocal}]$$

$$\begin{Bmatrix} FX3^C \\ FY3^C \\ M3^C \\ FX5^C \\ FY5^C \\ M5^C \end{Bmatrix} = \begin{bmatrix} 4507 & 0 & -8677 & -4507 & 0 & -8677 \\ 0 & 545454 & 0 & 0 & -545454 & 0 \\ -8677 & 0 & 22272 & 8677 & 0 & 11136 \\ -4507 & 0 & 8677 & 4507 & 0 & 8677 \\ 0 & -545454 & 0 & 0 & 545454 & 0 \\ -8677 & 0 & 11136 & 8677 & 0 & 22272 \end{bmatrix} * \begin{Bmatrix} u3^C \\ v3^C \\ \theta 3^C \\ u5^C \\ v5^C \\ \theta 5^C \end{Bmatrix} + \begin{Bmatrix} -31.595 \\ 0 \\ 31.367 \\ -38.404 \\ 0 \\ -35.723 \end{Bmatrix}$$

Elemento D

Cálculo de fuerzas empotradas

$$\frac{-qx*L}{2} = 66.317\ KN$$
$$\frac{-qy*L}{2} = -46.508\ KN$$
$$qx = -28.20576\ \frac{KN}{m} \qquad \frac{-qy*L^2}{12} = -36.45\ KN*m$$
$$qy = 19.78066\ \frac{KN}{m} \qquad \frac{-qx*L}{2} = 66.317\ KN$$
$$\frac{-qy*L}{2} = -46.508\ KN$$
$$\frac{qy*L^2}{12} = 36.45\ KN*m$$

Coordenadas Locales

$$\begin{Bmatrix} F'X4^D \\ F'Y4^C \\ M'4^D \\ F'X6^D \\ F'Y6^D \\ M'6^D \end{Bmatrix} = \begin{bmatrix} 446581 & 0 & 0 & -446581 & 0 & 0 \\ 0 & 2437 & 5816 & 0 & -2473 & 5816 \\ 0 & 5816 & 18235 & 0 & -5816 & 9117 \\ -446581 & 0 & 0 & 446581 & 0 & 0 \\ 0 & -2473 & -5816 & 0 & 2473 & -5816 \\ 0 & 5816 & 9117 & 0 & -5816 & 18235 \end{bmatrix} * \begin{Bmatrix} u4'^D \\ v4'^D \\ \theta 4'^D \\ u6'^D \\ v6'^D \\ \theta 6'^D \end{Bmatrix} + \begin{Bmatrix} 66.317 \\ -46.508 \\ -36.45 \\ 66.317 \\ -46.508 \\ 36.45 \end{Bmatrix}$$

$$[TD] = \begin{bmatrix} -0.5741 & 0.8187 & 0 & 0 & 0 & 0 \\ -0.8187 & -0.5741 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5741 & 0.8187 & 0 \\ 0 & 0 & 0 & -0.8187 & -0.5741 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[TD]^T = \begin{bmatrix} -0.5741 & -0.8187 & 0 & 0 & 0 & 0 \\ 0.8187 & -0.5741 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5741 & -0.8187 & 0 \\ 0 & 0 & 0 & 0.8187 & -0.5741 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{Para\ las\ fuerzas\ de\ empotramiento\ globales} = [\boldsymbol{TD}]^{\boldsymbol{T}} * [\boldsymbol{FEmlocal}]$$

Coordenadas Globales

$$\begin{Bmatrix} FX4^D \\ FY4^C \\ M4^D \\ FX6^D \\ FY6^D \\ M6^D \end{Bmatrix} = \begin{bmatrix} 148886 & -208773 & -4762 & -148886 & 208773 & -4762 \\ -208773 & 300168 & -3339 & 208773 & -300168 & -3339 \\ -4762 & -3339 & 18235 & 4762 & 3339 & 9117 \\ -148886 & 208773 & 4762 & 148886 & -208773 & 4762 \\ 208773 & -300168 & 3339 & -208773 & 300168 & 3339 \\ -4762 & -3339 & 9117 & 4762 & 3339 & 18235 \end{bmatrix} * \begin{Bmatrix} u4^D \\ v4^D \\ \theta 4^D \\ u6^D \\ v6^D \\ \theta 6^D \end{Bmatrix} + \begin{Bmatrix} 0 \\ 81 \\ -36.45 \\ 0 \\ 81 \\ 36.45 \end{Bmatrix}$$

Elemento E

Fuerzas de empotramiento

$$\frac{qL}{2} = \quad 120\ KN$$
$$\frac{qL^2}{12} = \ 80\ KN * m$$
$$\frac{qL}{2} = \quad 120\ \ KN$$
$$-\frac{qL^2}{12} = \ -80\ KN * m$$

Coordenadas Locales

$$\begin{Bmatrix} F'X5^E \\ F'Y5^E \\ M'5^E \\ F'X6^E \\ F'Y6^E \\ M'6^E \end{Bmatrix} = \begin{bmatrix} 525000 & 0 & 0 & -525000 & 0 & 0 \\ 0 & 4019 & 8039 & 0 & -4019 & 8039 \\ 0 & 8039 & 21437 & 0 & -8039 & 10718 \\ -525000 & 0 & 0 & 525000 & 0 & 0 \\ 0 & -4019 & -8039 & 0 & 4019 & -8039 \\ 0 & 8039 & 10718 & 0 & -8039 & 21437 \end{bmatrix} * \begin{Bmatrix} u5'^E \\ v5'^E \\ \theta5'^E \\ u6'^E \\ v6'^E \\ \theta6'^E \end{Bmatrix} + \begin{Bmatrix} 0 \\ 120 \\ 80 \\ 0 \\ 120 \\ -80 \end{Bmatrix}$$

$$[TE] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[TE]^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Como el sistema local y global es el mismo.

Coordenadas Globales

$$\begin{Bmatrix} FX5^E \\ FY5^E \\ M5^E \\ FX6^E \\ FY6^E \\ M6^E \end{Bmatrix} = \begin{bmatrix} 525000 & 0 & 0 & -525000 & 0 & 0 \\ 0 & 4019 & 8039 & 0 & -4019 & 8039 \\ 0 & 8039 & 21437 & 0 & -8039 & 10718 \\ -525000 & 0 & 0 & 525000 & 0 & 0 \\ 0 & -4019 & -8039 & 0 & 4019 & -8039 \\ 0 & 8039 & 10718 & 0 & -8039 & 21437 \end{bmatrix} * \begin{Bmatrix} u5^E \\ v5^E \\ \theta5^E \\ u6^E \\ v6^E \\ \theta6^E \end{Bmatrix} + \begin{Bmatrix} 0 \\ 120 \\ 80 \\ 0 \\ 120 \\ -80 \end{Bmatrix}$$

Cálculo de los desplazamientos

$$\left\{\begin{matrix} M1 \\ M2 \\ FX3 \\ FY3 \\ M3 \\ FX4 \\ FY4 \\ M4 \\ FX5 \\ FY5 \\ M5 \\ FX6 \\ FY6 \\ M6 \end{matrix}\right\} = \left\{\begin{matrix} M1 \\ M2 \\ FX3^{A} + FX3^{C} \\ FY3^{A} + FY3^{C} \\ M3^{A} + M3^{C} \\ FX4^{B} + FX4^{D} \\ FY4^{B} + FY4^{D} \\ M4^{B} + M4^{D} \\ FX5^{C} + FX5^{E} \\ FY5^{C} + FY5^{E} \\ M5^{C} + M5^{E} \\ FX6^{D} + FX6^{E} \\ FY6^{D} + FY6^{E} \\ M6^{E} + M6^{E} \end{matrix}\right\} = [KDesplazamientos] * \begin{bmatrix} \theta 1 \\ \theta 2 \\ u3 \\ v3 \\ \theta 3 \\ u4 \\ v4 \\ \theta 4 \\ u5 \\ v5 \\ \theta 5 \\ u6 \\ v6 \\ \theta 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -31.595 \\ 0 \\ 31.367 \\ 0 \\ 81 \\ -36.45 \\ -38.404 \\ 120 \\ 44.276 \\ 0 \\ 201 \\ -43.550 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

[KDesplazamientos]

$$\begin{bmatrix} 327574 & 0 & 92710 & 0 & 156460 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 327574 & 0 & 0 & 0 & 92710 & 0 & 156460 & 0 & 0 & 0 & 0 & 0 & 0 \\ 92710 & 0 & 52327 & 0 & 94306 & 0 & 0 & 0 & -4507 & 0 & -8677 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2556073 & 0 & 0 & 0 & 0 & 0 & -545454 & 0 & 0 & 0 & 0 \\ 156460 & 0 & 94306 & 0 & 349846 & 0 & 0 & 0 & 8677 & 0 & 11136 & 0 & 0 & 0 \\ 0 & 92710 & 0 & 0 & 0 & 196706 & -208773 & 98221 & 0 & 0 & 0 & -148886 & 208773 & -4762 \\ 0 & 0 & 0 & 0 & 0 & -208773 & 2310788 & -3339 & 0 & 0 & 0 & 208773 & -300168 & -3339 \\ 0 & 156460 & 0 & 0 & 0 & 98221 & -3339 & 345809 & 0 & 0 & 0 & 4762 & 3339 & 9117 \\ 0 & 0 & -4507 & 0 & 8677 & 0 & 0 & 0 & 529507 & 0 & 8677 & -525000 & 0 & 0 \\ 0 & 0 & 0 & -545454 & 0 & 0 & 0 & 0 & 0 & 549474 & 8039 & 0 & -4019 & 8039 \\ 0 & 0 & -8677 & 0 & 11136 & 0 & 0 & 0 & 8677 & 8039 & 43710 & 0 & -8039 & 10718 \\ 0 & 0 & 0 & 0 & 0 & -148886 & 208773 & 4762 & -525000 & 0 & 0 & 673886 & -208773 & 4762 \\ 0 & 0 & 0 & 0 & 0 & 208773 & -300168 & 3339 & 0 & -4019 & -8039 & -208773 & 304188 & -4699 \\ 0 & 0 & 0 & 0 & 0 & -4762 & -3339 & 9117 & 0 & 8039 & 10718 & 4762 & -4699 & 39672 \end{bmatrix}$$

Resolviendo el sistema anterior da como resultado los desplazamientos en cada nodo en sistema global

$$\{Desplazamientos\} = \begin{bmatrix} 2.46880761x10^{-4} \\ -1.2875228x10^{-3} \\ -1.73254419x10^{-3} \\ -7.80851790x10^{-5} \\ 5.09726792x10^{-4} \\ 6.11781565x10^{-3} \\ -1.21853218x10^{-4} \\ -9.29451559x10^{-4} \\ -6.60291670x10^{-3} \\ -3.65917720x10^{-4} \\ -2.48127415x10^{-3} \\ -6.75047454x10^{-3} \\ -9.63525729x10^{-3} \\ 2.44907734x10^{-3} \end{bmatrix}$$

Cálculo de las reacciones desconocidas

$$\begin{Bmatrix} FX1 \\ FY1 \\ FX2 \\ FY2 \end{Bmatrix} = \begin{bmatrix} -102984 & 0 & -35452 & 0 & -92709 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2010619 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -102984 & 0 & 0 & 0 & -35452 & 0 & -92709 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2010619 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 2.46880x10^{-4} \\ -1.28752x10^{-3} \\ -1.73254x10^{-3} \\ -7.80851x10^{-5} \\ 5.09726x10^{-4} \\ 6.11781x10^{-3} \\ -1.21853x10^{-4} \\ -9.29451x10^{-4} \\ -6.60291x10^{-3} \\ -3.65917x10^{-4} \\ -2.48127x10^{-3} \\ -6.75047x10^{-3} \\ -9.63525x10^{-3} \\ 2.44907x10^{-3} \end{bmatrix}$$

$$\begin{Bmatrix} FX1 \\ FY1 \\ FX2 \\ FY2 \end{Bmatrix} = \begin{bmatrix} -11.2581723\ KN \\ 156.9995679\ KN \\ 1.87090088\ KN \\ 245.0004321\ KN \end{bmatrix}$$

Cálculo de las funciones

Elemento A

- Desplazamiento axial

$$U_{A(xA)} = -1.56170358090579 \cdot 10^{-5} * \mathrm{xA}$$

- Desplazamiento vertical

$$\begin{aligned} V_{A(XA)} = & -3.28736848615449 \cdot 10^{-19} * \sin(0.236122592374754\mathrm{xA}) * \sinh(0.236122592374754\mathrm{xA}) \\ & +0.00105444266200369 * \sin(0.236122592374754\mathrm{xA}) * \cosh(0.236122592374754\mathrm{xA}) \\ & -8.88086983100818 \cdot 10^{-6} \cos(0.236122592374754\mathrm{xA}) * \sinh(0.236122592374754\mathrm{xA}) \end{aligned}$$

Elemento B

- Desplazamiento axial

$$U_{B(xB)} = -2.43706436427808 \cdot 10^{-5} * xB$$

- Desplazamiento vertical

$$\begin{aligned} V_{B(XB)} = & -1.31494739446179 \cdot 10^{-18} * \sin(0.236122592374754\mathrm{xB}) * \sinh(0.236122592374754\mathrm{xB}) \\ & -0.00281473886860202 * \sin(0.236122592374754\mathrm{xB}) * \cosh(0.236122592374754\mathrm{xB}) \\ & -0.00263803408191698 * \cos(0.236122592374754\mathrm{xB}) \sinh(0.236122592374754\mathrm{xB}) \end{aligned}$$

Elemento C

- Desplazamiento axial

$$U_{C(xC)} = -7.47616989997487 \cdot 10^{-5} \mathrm{xC} - 7.80851790452896 \cdot 10^{-5}$$

Desplazamiento vertical

- Campo de desplazamiento empotrado

$$V_f(xc) = \begin{cases} \{0.000245641987478206 * xC^3 - 0.000731595994781789 * xC^2\} & 0 \leq xc \leq 2.05 \\ \{0.000245641987478206 xC^3 - 0.000731595994781789 * xC^2 - 0.00468850340136054 * (0.487804878048781 * \mathrm{xC} - 1)^3\} & 2.05 \leq x \leq 3.85 \end{cases}$$

- Campo de desplazamiento homogéneo

$$\begin{aligned} V_h(\mathrm{xc}) = & -0.000170690749936751 * xC^3 - 0.000644486790993086 xC^2 \\ & * (0.25974025974026 * \mathrm{xC} - 1) + 0.000985739080884738 * xC^2 \\ & + 0.000509726791951021 * \mathrm{xC} * (1 - 0.25974025974026\mathrm{xC})^2 + 0.0017325441943997 \end{aligned}$$

- Campo de desplazamiento de la parte inferior

$$\begin{aligned} \boldsymbol{V_{C(XC)Inferior}} = & \, 7.49512375414548 * 10^{-5} * xC^3 - 0.000644486790993086 * xC^2 \\ & * (0.25974025974026 * \mathrm{xC} - 1) + 0.00025414308610294 * xC^2 \\ & + 0.000509726791951021 * \mathrm{xC} * (1 - 0.25974025974026\mathrm{xC})^2 \\ & + 0.0017325441943997 \end{aligned}$$

- Campo de desplazamiento de la parte superior

$$\boldsymbol{V_c(xc)Superior} = 7.49512375414548 * 10^{-5} * xC^3 - 0.000644486790993086 * xC^2(0.25974025974026 * \mathrm{xC} - 1) + 0.000254143086102949 * xC^2 + 0.000509726791951021 * \mathrm{xC} * (1 - 0.25974025974026\mathrm{xC})^2 - 0.00468850340136054 * (0.487804878048781 * \mathrm{xC} - 1)^3 + 0.0017325441943997$$

Elemento D

- Desplazamiento axial

$$uD(x) = uD_{h(xD)} + uD_{f(xD)}$$

$$uD_{h(xD)} = -8.51202781179226 * 10^{-5} * \mathrm{xD} - 0.00361246648492818$$

$$uD_{f(xD)} = 6.71565856416054 * 10^{-6} * xD^2 - 3.1579665875019 * 10^{-5}\mathrm{xD}$$

$$uD(x) = 6.71565856416054 * 10^{-6} * xD^2 - 0.000116699943992942 * \mathrm{xD} - 0.00361246648492818$$

- Desplazamiento vertical

$$VD_h(xD) = -0.000307709013191146 * \mathrm{xD}^3 + 0.000520815111082221 * \mathrm{xD}^2 * (0.212657682660061 * \mathrm{xD} - 1) + 0.00217045306811013 * \mathrm{xD}^2 - 0.000929451558848926 * \mathrm{xD} * (1 - 0.212657682660061 * \mathrm{xD})^2 - 0.00493888675765332$$

$$VD_f(xD) = 3.84463888115208 * 10^{-5} * xD^4 - 0.000361580059846494 * xD^3 + 0.000850145772594753 * xD^2$$

- Campo de desplazamiento vertical

$$vD(x) = 3.84463888115208 * 10^{-5} * \mathrm{xD}^4 - 0.00066928907303764 * xD^3 + 0.000520815111082221 * \mathrm{xD}^2 * (0.212657682660061 * \mathrm{xD} - 1) + 0.00302059884070488 * \mathrm{xD}^2 - 0.000929451558848926 * \mathrm{xD} * (1 - 0.212657682660061\mathrm{xD})^2 - 0.00493888675765332$$

Elemento E

- Desplazamiento axial

$$UE(x) = -3.68894599238872 * 10^{-5} * xE - 0.00660291670320438$$

- Desplazamiento vertical

$$VE_h(xE) = 0.000289666861466532 * xE^3 + 0.000612269334180085 * xE^2 * \left(\frac{\mathrm{xE}}{4} - 1\right) - 0.00173800116879919 * xE^2 - 0.00248127414532338 * \mathrm{xE} * \left(1 - \frac{xE}{4}\right)^2 - 0.000365917720194322$$

$$VE_f(xE) = -0.000116618075801749 * \mathrm{xE}^4 + 0.000932944606413994 * \mathrm{xE}^3 - 0.00186588921282799 * \mathrm{xE}^2$$

- Campo de desplazamiento vertical

$$vE(x) = -0.000116618075801749 * \mathrm{xE}^4 + 0.00122261146788053 * \mathrm{xE}^3 + 0.000612269334180085 * xE^2 * \left(\frac{\mathrm{xE}}{4} - 1\right) - 0.00360389038162718 * xE^2 - 0.00248127414532338 * xE * \left(1 - \frac{xE}{4}\right)^2 - 0.000365917720194322$$

Fuerza que el suelo le ejerce a la pila

$$S1 = \int_0^H (K * vA) dxA = 18.7260381431672 \;\; KN$$

$$S2 = \int_0^H (K * vB) dxB = -79.3387667237668 \, KN$$

Equilibrio de la estructura

Sumatoria de fuerzas en X

$$\sum Fx = Fx1 + Fx2 + S1 + S2 + P = -11.2581723 \, KN \;+\; 1.87090088 \;\; KN \;+ 18.7260381431672 \;\; KN - 79.3387667237668 \, KN + 70 = 0$$

Sumatoria de fuerzas en Y

$$\sum FY = FY1 + FY2 - q * (LE + d) = \; 156.9995679 \;\; KN + 245.0004321 \;\; KN - 402 = 0$$

Sumatoria de momentos

$$\sum SM1 = FY2 * (LE + d) - P * (H + a) - \left(\frac{qLE^2}{2}\right) - q * d * \left(LE + \frac{d}{2}\right) - \int_0^H (K * vA * xA) dxA - \int_0^H (K * vB * xB) dxB \quad = 1641.5028 - 493.5 - 480 - 866.699 - 64.5150 + 263.212 = 0$$

Cálculo de las funciones de momento, fuerza axial y fuerza cortante

<u>Elemento A</u>

- Fuerza Axial

$$PA(xA) = A * E\frac{duA}{dxA}(xA) = -156.999567$$

- Fuerza de momento

$$MA(xA) = E * I\frac{d^2vA}{xA^2}$$
$$=0.398217579174191 * \sin(0.236122592374754 * \mathrm{xA}) * \cosh(0.236122592374754 * \mathrm{xA}) +47.2811348698073 * \cos(0.236122592374754 * \mathrm{xA}) * \sinh(0.236122592374754 * \mathrm{xA}) -1.47405371919674 * 10^{-14} * \cos(0.236122592374754 * \mathrm{xA}) * \cosh(0.236122592374754 * \mathrm{xA})$$

- Fuerza cortante

$$VA(xA) = -E * I = \frac{d^3vA}{xA^3}$$
$$= 11.0701159687555 * \sin(0.236122592374754 * \mathrm{xA}) * \sinh(0.236122592374754 * \mathrm{xA}) - 3.48057385476382 * 10^{-15} * \sin(0.236122592374754 * \mathrm{xA}) * \cosh(0.236122592374754 * \mathrm{xA}) + 3.48057385476382 * 10^{-15} * \cos(0.236122592374754 * \mathrm{xA}) * \sinh(0.236122592374754 * \mathrm{xA}) - 11.2581723030031 * \cos(0.236122592374754 * \mathrm{xA}) * \cosh(0.236122592374754 * \mathrm{xA})$$

Elemento B

- Fuerza Axial

$$\mathrm{P}B(xB) = A * E\frac{duB}{dxB}(xB) =- 245.0004321$$

- Fuerza de momento

$$MB(xB) = E * I\frac{d^2vB}{xB}$$
$$=118.289262861624 * \sin(0.236122592374754 * \mathrm{xB}) * \cosh(0.236122592374754 * \mathrm{xB}) -126.212693079746 * \cos(0.236122592374754 * \mathrm{xB}) * \sinh(0.236122592374754 * \mathrm{xB}) -5.89621487678696 * 10^{-14} * \cos(0.236122592374754 * \mathrm{xB}) * \cosh(0.236122592374754 * \mathrm{xB})$$

- Fuerza cortante

$$VB(xB) = -E * I = \frac{d^3vB}{xB^3}$$
$$= -57.732435677574 * \sin(0.236122592374754 * \mathrm{xB}) * \sinh(0.236122592374754 * \mathrm{xB}) - 1.39222954190553 * 10^{-14} * \sin(0.236122592374754 * \mathrm{xB}) * \cosh(0.236122592374754 * \mathrm{xB}) + 1.39222954190553 * 10^{-14} * \cos(0.236122592374754 * \mathrm{xB}) * \sinh(0.236122592374754 * \mathrm{xB}) + 1.87090088360326 * \cos(0.236122592374754 * \mathrm{xB}) * \cosh(0.236122592374754 * \mathrm{xB})$$

Elemento C

- Fuerza axial

$$PC(xC) = A * E\frac{duC}{dxC}(xC) = -156.999567$$

- Fuerza de momento

$$MC - INF(xC) = E * I\frac{d^2vCInf}{xC^2} = 27.1757501597652 - 7.46786584016407 * \mathrm{xC}$$

$$MC - SUP(xC) = E * I\frac{d^2vCSup}{xC^2} = 170.675750159765 - 77.4678658401641 * \mathrm{xC}$$

- Fuerza cortante

$$VC - INF(xC) = -E * I = \frac{d^3vCInferior}{xC^3} = 7.46786584016407$$

$$VC - INF(xC) = -E * I = \frac{d^3vCSuperior}{xC^3} = 77.4678658401641$$

Elemento D

- Fuerza Axial

$$PD(xD) = A * E\frac{duD}{dxD}(xD) = 28.2057659694743 * \mathrm{xD} - 245.069882385177$$

- Fuerza de momento

$$MD(xD) = E * I\frac{d^2vD}{xD^2} = 9.89033352176371 * xD^2 - 77.247878385042 * \mathrm{xD} + 124.127144913774$$

- Fuerza cortante

$$VD(xD) = -E * I = \frac{d^3vD}{xD^3} = 77.247878385042 - 19.7806670435274 * \mathrm{xD}$$

Elemento E

- Fuerza Axial

$$PE(xE) = A * E\frac{duE}{dxE}(xE) = -77.4678658401631$$

- Fuerza de momento

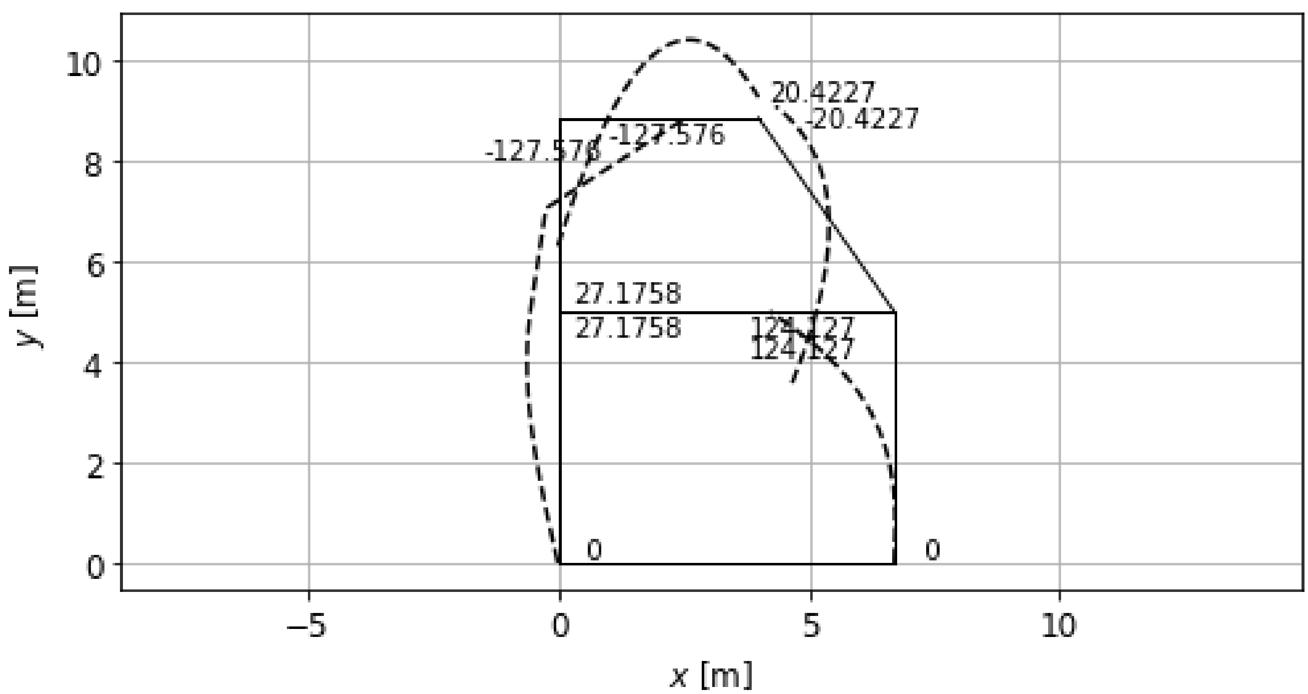
$$ME(xE) = E * I\frac{d^2vE}{xE^2} = -30.0 * \mathrm{xE}^2 + 156.999567899472 * \mathrm{xE} - 127.575533324866$$

- Fuerza cortante

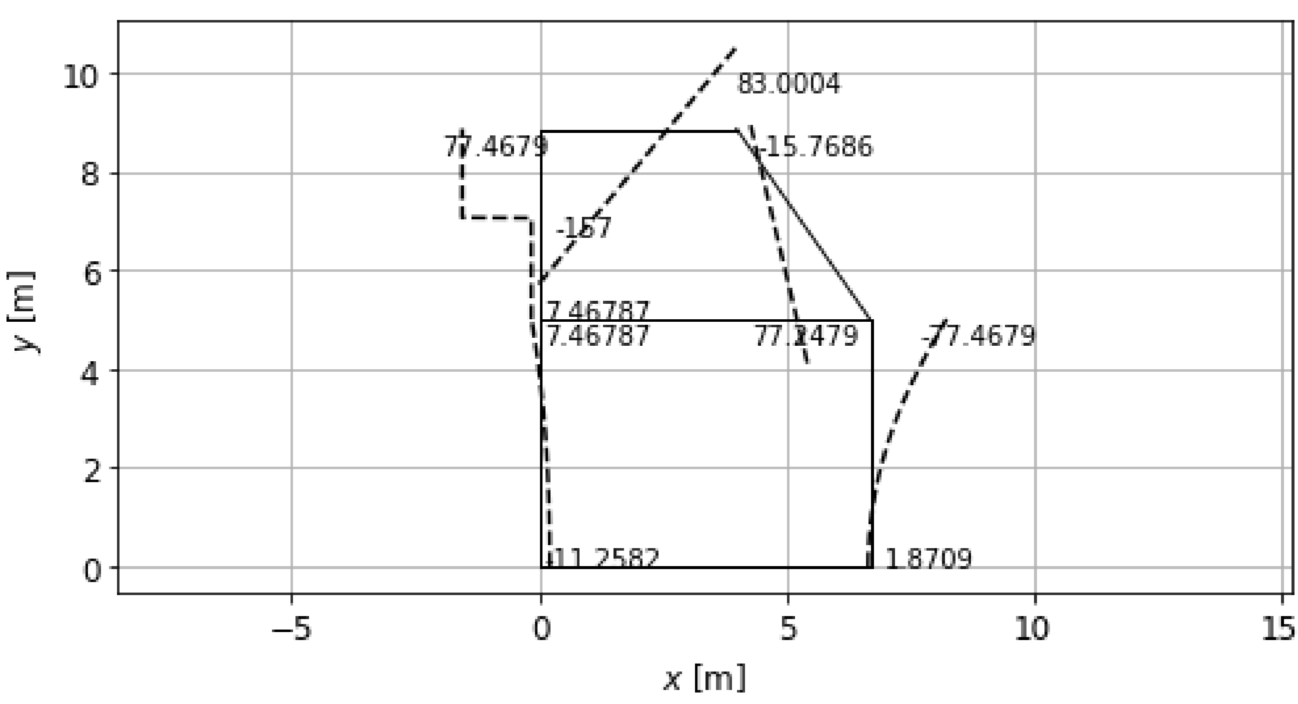
$$VE(xE) = -E * I = \frac{d^3vE}{xE^3} = 60.0 * \mathrm{xE} - 156.999567899472$$

GRAFICAS DE LAS FUNCIONES

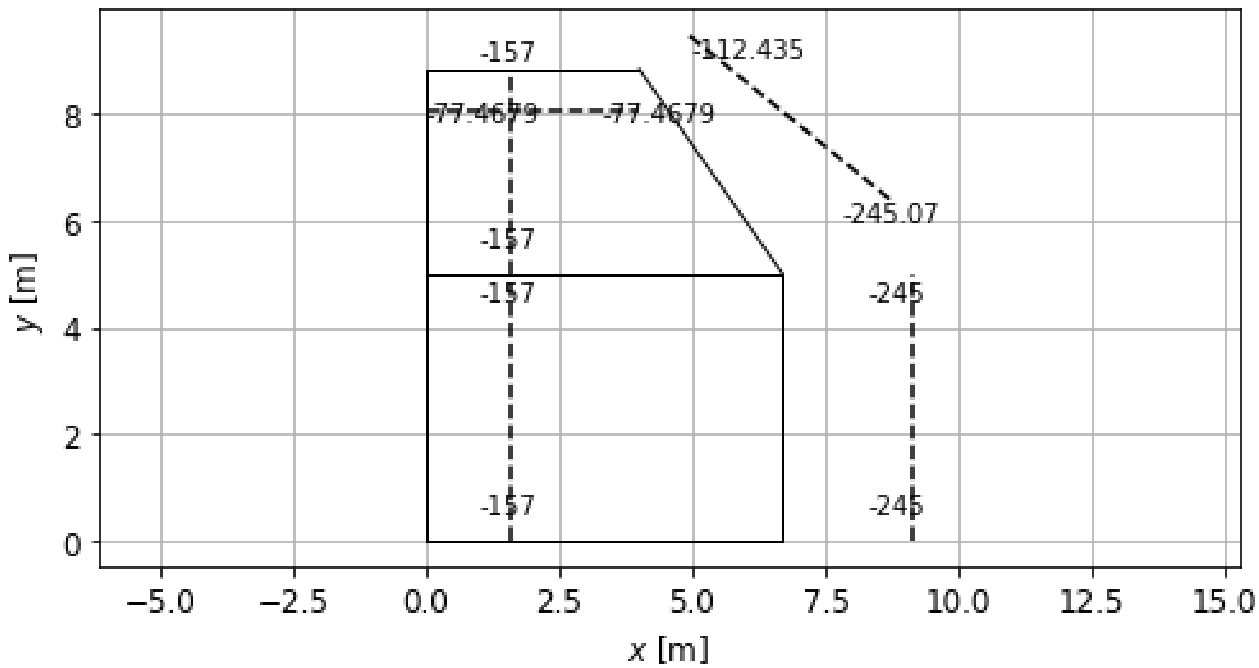
GRAFICA DE MOMENTO [KN*m]



GRAFICA DE LA CORTANTE [KN]



GRAFICA DE LA FUERZA AXIAL [KN]



GRAFICA DE LA DEFORMADA AMPLIADA 100 VECES