SALVADOR MARTI-GARCIA

TRACK FITTING, VERTEX FITTING AND DETECTOR ALIGNMENT

EXERCISES BOOK

This "Track Fitting, Vertex Fitting and Detector Alignment: Exercises Book" is intended for Physics Students following a course on Experimental Techniques in Nuclear and Particle Physics.

The book contains several exercises, which are explained with step by step solutions and also with supporting graphics whenever needed.

The aim of the exercises is also to help the student understand some basic features of the track fitting, vertex fitting and detector alignment.

Of course, students may get support from algebra solving software and applications, like Wolfram *Mathematica*®, but nor very complex, neither tedious calculations are needed to work out the solutions.

Salvador Martí-García
12th January, 2016

Excercise 1

Consider a charged particle moving across an axial uniform magnetic field $\mathbf{B} = (0, 0, B)$. Prove that it describes a circular trajectory and write its equations as a function of time (t).

Excercise 2

Consider a particle with charge q moving inside an axial magnetic field of intensity B and describing a circular trajectory with curvature radius of ρ. It is known that its momentum can be expressed as:

$$p_{\mathrm{T}} = \kappa \, qB\rho$$

where κ is a proportionality constant depending on the used units.

Find the value of κ when using the following units: p_{T} in GeV, q in electron charge units, B in Tesla and ρ in m (those are commonly used in High Energy Physics).

Excercise 3

A particle physics experiment uses a track reconstruction system with a cylindrical shape, with 1 m radius. This system is immersed in a uniform axial magnetic field of 2 Tesla pointing along the cylinder axis. During the experiment, several charged particles with charge $\pm e$ (being e the electron charge unit) emerge from the center of the system. Compute:

a) What is the minimum transverse momentum of a particle with respect to the magnetic field axis in order for that particle leaving the system?

b) Consider a particle with a $p_T = 100$ GeV with respect the magnetic field axis. What is the sagitta of the trajectory? In other words: how much the trajectory of this particle deviates from a straight line?

c) What should be the spatial resolution on the sagitta if the system goal is to measure the $p_T = 100$ GeV with a 1% precision.

Hint: work just on the transverse (XY) plane. The magnetic field serves to bend the trajectory of the particles in this plane.

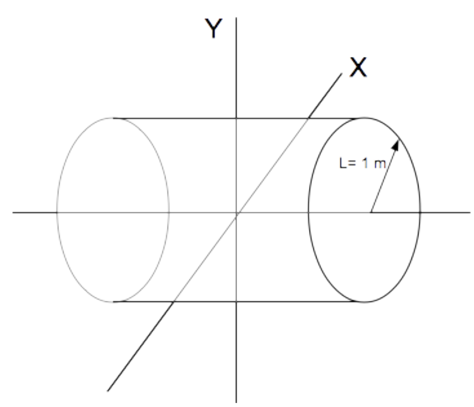


Figure 1: A cylindrical tracking system, with uniform magnetic field of 2 Tesla pointing along Z axis.

Excercise 4

The rapidity (y) and pseudorapidity (η) of a particle are defined as follows:

$$y = \frac{1}{2} \ln \left(\frac{E + P_z}{E - Pz} \right)$$

$$\eta = - \ln \left(\tan \frac{\theta}{2} \right)$$

Show that for massless particles $y = \eta$.

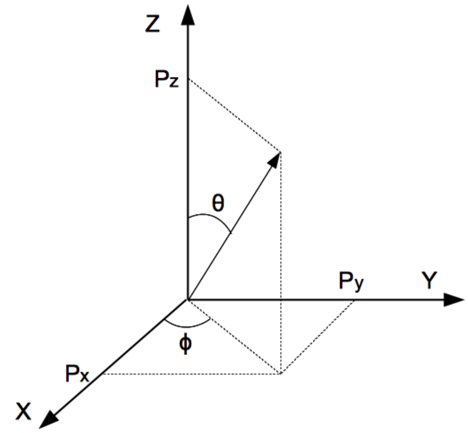


Figure 2: The momentum vector and its components

Exercise 5

In a High Energy Physics experiment, physicist are interested in reconstructing the invariant mass of a neutral resonance that decays into two charged particles (for example: $Z \to \mu^+\mu^-$). Unfortunately, the track reconstruction introduces a small but systematic bias in the pseudorapidity of the reconstructed particle (η). Check if the reconstructed invariant mass is altered by the bias in η.

Hint: One can neglect the muon mass (105.7 MeV) as the momentum acquired from the Z decay is generally of the order of 40-50 GeV. Therefore it is convenient to express the four momentum as:

$$p = P_T(\cosh \eta, \ \cos \phi, \ \sin \phi, \ \sinh \eta)$$

Besides, one has to bear in mind that in the experiment physicists have access only to the reconstructed quantities. Then, try to express the true invariant mass of the resonance as a function of the reconstructed four-momentum.

Excercise 6

Consider a 2D space. In this space one counts with a 1D sensor. This sensor has a pitch along its sensing coordinate of 50 μm.

Given a sensor of this kind, located at 50 mm of the origin of the global coordinate system, along the 45° direction, and rotated $-30°$ with respect to the global system, compute:

a) The local to global transformation.

b) The covariance matrix of the sensor measurements as given in the global frame.

Hint: as the sensor is 1D, it can only measure points along its X axis.

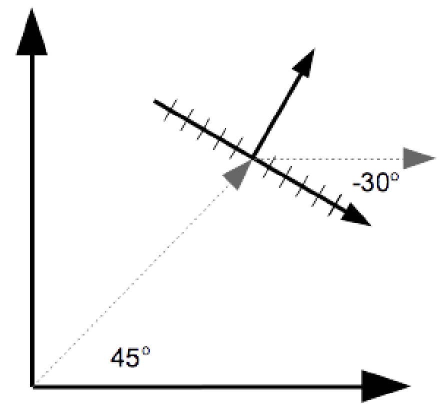


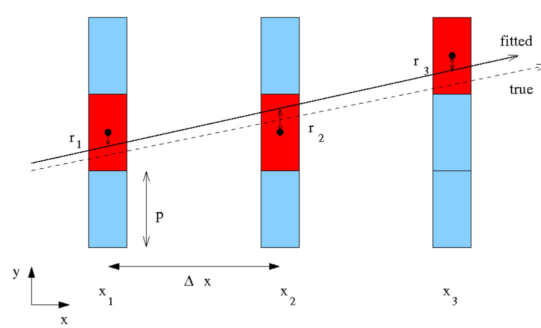
Figure 3: The local frame of the sensor and the global reference frame.

Excercise 7

Let's consider a tracking system built using 3 equidistant identical planes. The incident particles fly along a straight path. Given a set of three measurements (one in each plane), compute the track parameters with a χ^2 minimization.

Hint: the track can be paramterized as: $y = y_0 + x \tan \phi$, so the track parameters set is given by:

$$\mathbf{t} = (y_0, \tan \phi)$$



Excercise 8

Consider a silicon planar sensor (pixel or microstrips type) that has to be aligned. Computed the derivative of the track-hit residuals with respect to the within plane rotation.

Hint: the local reference frame can be set in such a way that the silicon sensor plane matches the XY plane, whilst the Z axis is pointing out of the sensor plane. With this convention: the within plane rotation is then the rotation around the local Z axis.